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Meson-Baryon Effective Chiral Lagrangian at $\mathcal{O}(q^3)$ Revisited

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Abstract: After our work [1] was published, Frink and Meißner [2] pointed out that the $\mathcal{O}(q^3)$ three-flavour meson-baryon chiral Lagrangian presented there was not minimal. Here, we critically review their paper and revise ours. Remarkably, we find that the effective meson-baryon chiral Lagrangian at $\mathcal{O}(q^3)$ contains 76 monomials, i.e. eight less than in [1] and two less than in [2].

Keywords: Chiral Lagrangians, NLO Calculations, QCD.

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Recently, we presented in [1] the first complete and by then minimal SU(3) chiral invariant relativistic meson-baryon Lagrangian at $\mathcal{O}(q^3)$ with the presence of external sources (scalar, pseudoscalar, vector and axial ones). This was a clear improvement over the standard reference at the moment [3], since the Lagrangian presented there was not complete nor reduced in minimal form. A detailed comparison between the Lagrangians of [1] and [3] is given in ref.[1]. Later on, Frink and Meißner [2] pointed out that one can further reduce the number of monomials present in the $\mathcal{O}(q^3)$ Lagrangian of [1] by six, passing from 84 in [1] to 78 in [2]. Here, we discuss the findings of [2] and show that the SU(3) meson-baryon chiral Lagrangian of [2] can be further reduced by two further monomials, i.e. to 76 monomials instead of the 78 ones presented in [2]. We refer to [1] for the presentation of the building blocks and techniques employed in the construction of the monomials, where it is discussed in detail.

Some Cayley-Hamilton relations involving monomials with five flavour matrices were missed in [1], as correctly noticed in [2]. The technicalities of this point were explained in detail in the Appendix A of [2]. Along these lines, we find three Cayley-Hamilton relations between the monomials O_{12} to O_{25} of [1] that were not taken into account there. If these Cayley-Hamilton relations are used to eliminate only monomials involving the product of two flavour traces, then one monomial between O_{20}, O_{22} and O_{24} and two more ones between O_{21}, O_{23} and O_{25} can be eliminated. We choose to eliminate O_{22}, O_{23} and O_{25}. Thus, we agree with [2] that Cayley-Hamilton relations can be used to further eliminate three monomials from O_{12} to O_{25} in [1]. However, it is not possible to simultaneously eliminate the monomials O_{20}, O_{21} and O_{22} from the basis in [1], as wrongly claimed in [2].

We find other two Cayley-Hamilton relations between the monomials O_{31} to O_{37} in [1] not considered there. They allow to eliminate two monomials between O_{35}, O_{36} and O_{37}, as already remarked in [2]. We choose to eliminate O_{35} and O_{36}.

Another Cayley-Hamilton relation, not used in [1], is found between the monomials O_{38} to O_{43} in [1] that was not used there. This is not commented either in [2]. In this way one can eliminate another monomial that we chose to be O_{43} of [1].

In [1] we used a Cayley-Hamilton relation to eliminate the one flavour trace monomial,

$$\widehat{O}_{33} = i \left(\langle \bar{B} \{u^\nu, u^\rho\} \sigma^{\lambda\tau} D_\rho B u^\mu \rangle - \langle \bar{B} \overleftarrow{D}_\rho \{u^\nu, u^\rho\} \sigma^{\lambda\tau} B u^\mu \rangle \right) \varepsilon_{\mu\nu\lambda\tau} \ , \tag{1}$$

while all the other monomials eliminated using the Cayley-Hamilton theorem contained more that one flavour trace. Here, we prefer, because of large N_c counting, to eliminate the two trace monomial O_{42} in [1] and put back $\widehat{O}_{33}$ in our new basis for the $\mathcal{O}(q^3)$ Lagrangian.

Apart from the missed Cayley-Hamilton relations in [1], Frink and Meißner [2] also realized that only the symmetric combination of O_9 and O_{10} in [1] is independent. Hence, only one of these two monomials should be considered and we keep O_9. Since we found difficulties in understanding the argumentation given in [2], we reproduce here our way of deriving such relationship between O_9 and O_{10}. We proceed as follows. Taking into account that

$$D_\nu u_\rho - D_\rho u_\nu = f^-_{\rho\nu} \ , \tag{2}$$

see eq.(2.10) of [1], the difference between $O_9 = i\langle \bar{B} u^\mu \sigma_{\mu\nu} D_\rho B\, h^{\nu\rho}\rangle - i\langle \bar{B}\overleftarrow{D}_\rho u^\mu \sigma_{\mu\nu} B h^{\nu\rho}\rangle$, and $i\langle \bar{B} u^\mu \sigma_{\mu\nu} D_\rho B\, D^\nu u^\rho\rangle - i\langle \bar{B}\overleftarrow{D}_\rho u^\mu \sigma_{\mu\nu} B D^\nu u^\rho\rangle$, is accounted for by the monomial O_{82} of ref.[1], or by our present $\widehat{O}_{74}$ of Table 1. Then, neglecting a global divergence,

$$\begin{aligned} O_9 \to & -i\langle D^\nu \bar{B} u^\mu \sigma_{\mu\nu} D_\rho B\, u^\rho\rangle - i\langle \bar{B} D^\nu u^\mu \sigma_{\mu\nu} D_\rho B\, u^\rho\rangle - i\langle \bar{B} u^\mu \sigma_{\mu\nu} D^\nu D_\rho B\, u^\rho\rangle \\ & + i\langle D^\nu D_\rho \bar{B} u^\mu \sigma_{\mu\nu} B\, u^\rho\rangle + i\langle D_\rho \bar{B} D^\nu u^\mu \sigma_{\mu\nu} B\, u^\rho\rangle + i\langle D_\rho \bar{B} u^\mu \sigma_{\mu\nu} D^\nu B\, u^\rho\rangle \;, \end{aligned} \tag{3}$$

where other monomials already accounted for are not written and this is why we use the right pointing arrow. The second terms on each of the lines of eq.(3) can be written again in terms of monomials with $f^-_{\mu\nu}$ because of eq.(2), since $D^\nu u^\mu$ is contracted with the antisymmetric tensor $\sigma_{\nu\mu}$. The resulting structures are taken into account by the monomial $\widehat{O}_{75}$ in Table 1. In this way we are left with

$$\begin{aligned} O_9 \to & -i\langle D^\nu \bar{B} u^\mu \sigma_{\mu\nu} D^\rho B\, u_\rho\rangle - i\langle \bar{B} u^\mu \sigma_{\mu\nu} D^\nu D^\rho B\, u_\rho\rangle \\ & + i\langle D^\rho \bar{B} u^\mu \sigma_{\mu\nu} D^\nu B\, u_\rho\rangle + i\langle D^\nu D^\rho \bar{B} u^\mu \sigma_{\mu\nu} B\, u_\rho\rangle \;. \end{aligned} \tag{4}$$

Employing the relation $-i\sigma_{\mu\nu} = g_{\mu\nu} - \gamma_\nu\gamma_\mu$ in the first and fourth monomials above and $+i\sigma_{\mu\nu} = g_{\mu\nu} - \gamma_\mu\gamma_\nu$ in the second and third ones, one can write

$$O_9 \to -4\langle \bar{B} u^\nu D_\nu D_\rho B\, u^\rho\rangle - 2\langle \bar{B} u^\nu D_\rho B\, D_\nu u^\rho\rangle - 2\langle \bar{B} D_\rho u^\nu D_\nu B\, u^\rho\rangle \;, \tag{5}$$

where the equation of motion of baryons has been used to remove those terms involving $\gamma^\nu D_\nu B$ and $D_\nu \bar{B}\gamma^\nu$, see eq.(4.2) of [2]. One can proceed analogously for O_{10} and then exactly the same combination of monomials as in (5) is found. Hence, only the symmetric combination of O_9 and O_{10} is independent, while the difference can be written in terms of other monomials already taken into account.

Frink and Meißner also noticed that the index ordering in the monomials O_{31}, O_{33} and O_{34} in [1] do not match the conditions imposed by charge conjugation. We want to point out that the difference between the index ordering in [1] and that which is exactly invariant under charge conjugation is $\mathcal{O}(q^4)$. However, we prefer –see our comments in [1]– monomials in the Lagrangian which are exactly charge conjugation invariant, because charge conjugation is a symmetry of strong interactions. Then, we now take the ordering in the indices such that these monomials are exactly charge conjugation invariant.

As pointed out in [2] the relative sign between the flavour traces in O_{41} should be plus instead of the minus in [1]. Once this is corrected O_{41} becomes of $\mathcal{O}(q^4)$. Then, the comment at the end of Section 5 of [1], though correct, is not relevant.

Summarizing the discussion above, we can further eliminate from the $\mathcal{O}(q^3)$ three-flavour meson-baryon Lagrangian in [1] the following monomials: O_{10}, O_{22}, O_{23}, O_{25}, O_{35}, O_{36}, O_{41} and O_{43}. In addition, we exchange O_{42} by $\widehat{O}_{33}$. We therefore end with 76 independent monomials in the SU(3) meson-baryon chiral Lagrangian at $\mathcal{O}(q^3)$, eight less than in [1] and two less than in [2]. We give the full list of the monomials present in the minimal SU(3) meson-baryon chiral invariant Lagrangian in Table 1.

$$\mathcal{L}^{(3)}_{MB} = \sum_{i=1}^{76} h_i\, \widehat{O}_i \,. \tag{6}$$

i	$\widehat{O}_i$	Contributes to vertex
1	$i\left(\langle \bar{B}\gamma_\mu D_{\nu\rho}B[u^\mu,h^{\nu\rho}]\rangle + \langle \bar{B}\overleftarrow{D}_{\nu\rho}\gamma_\mu B[u^\mu,h^{\nu\rho}]\rangle\right)$	$M_1B_1 \to M_2B_2$
2	$i\left(\langle \bar{B}[u^\mu,h^{\nu\rho}]\gamma_\mu D_{\nu\rho}B\rangle + \langle \bar{B}\overleftarrow{D}_{\nu\rho}[u^\mu,h^{\nu\rho}]\gamma_\mu B\rangle\right)$	$M_1B_1 \to M_2B_2$
3	$i\left(\langle \bar{B}u^\mu\rangle\langle h^{\nu\rho}\gamma_\mu D_{\nu\rho}B\rangle - \langle \bar{B}\overleftarrow{D}_{\nu\rho}h^{\nu\rho}\rangle\langle u^\mu\gamma_\mu B\rangle\right)$	$M_1B_1 \to M_2B_2$
4	$i\langle \bar{B}[u_\mu,h^{\mu\nu}]\gamma_\nu B\rangle$	$M_1B_1 \to M_2B_2$
5	$i\langle \bar{B}\gamma_\nu B[u_\mu,h^{\mu\nu}]\rangle$	$M_1B_1 \to M_2B_2$
6	$i\left(\langle \bar{B}u_\mu\rangle\langle h^{\mu\nu}\gamma_\nu B\rangle - \langle \bar{B}h^{\mu\nu}\rangle\langle u_\mu\gamma_\nu B\rangle\right)$	$M_1B_1 \to M_2B_2$
7	$i\langle \bar{B}\sigma_{\mu\nu}D_\rho B\{u^\mu,h^{\nu\rho}\}\rangle - i\langle \bar{B}\overleftarrow{D}_\rho\sigma_{\mu\nu}B\{u^\mu,h^{\nu\rho}\}\rangle$	$M_1B_1 \to M_2B_2$
8	$i\langle \bar{B}\{u^\mu,h^{\nu\rho}\}\sigma_{\mu\nu}D_\rho B\rangle - i\langle \bar{B}\overleftarrow{D}_\rho\{u^\mu,h^{\nu\rho}\}\sigma_{\mu\nu}B\rangle$	$M_1B_1 \to M_2B_2$
9	$i\langle \bar{B}u^\mu\sigma_{\mu\nu}D_\rho Bh^{\nu\rho}\rangle - i\langle \bar{B}\overleftarrow{D}_\rho u^\mu\sigma_{\mu\nu}Bh^{\nu\rho}\rangle$	$M_1B_1 \to M_2B_2$
10	$i\left(\langle \bar{B}\sigma_{\mu\nu}D_\rho B\rangle - \langle \bar{B}\overleftarrow{D}_\rho\sigma_{\mu\nu}B\rangle\right)\langle u^\mu h^{\nu\rho}\rangle$	$M_1B_1 \to M_2B_2$
11	$\langle \bar{B}\gamma_5\gamma_\nu B\{u_\mu u^\mu,u^\nu\}\rangle$	$M_1B_1 \to M_2M_3B_2$
12	$\langle \bar{B}\gamma_5\gamma_\nu Bu_\mu u^\nu u^\mu\rangle$	$M_1B_1 \to M_2M_3B_2$
13	$\langle \bar{B}u_\mu\gamma_5\gamma_\nu B\{u^\mu,u^\nu\}\rangle$	$M_1B_1 \to M_2M_3B_2$
14	$\langle \bar{B}u_\mu u^\mu\gamma_5\gamma_\nu Bu^\nu\rangle$	$M_1B_1 \to M_2M_3B_2$
15	$\langle \bar{B}\{u_\mu u^\mu,u^\nu\}\gamma_5\gamma_\nu B\rangle$	$M_1B_1 \to M_2M_3B_2$
16	$\langle \bar{B}\{u^\mu,u^\nu\}\gamma_5\gamma_\nu Bu_\mu\rangle$	$M_1B_1 \to M_2M_3B_2$
17	$\langle \bar{B}u_\mu u^\nu u^\mu\gamma_5\gamma_\nu B\rangle$	$M_1B_1 \to M_2M_3B_2$
18	$\langle \bar{B}u^\nu\gamma_5\gamma_\nu Bu_\mu u^\mu\rangle$	$M_1B_1 \to M_2M_3B_2$
19	$\langle \bar{B}\{u^\nu,\gamma_5\gamma_\nu B\}\rangle\langle u_\mu u^\mu\rangle$	$M_1B_1 \to M_2M_3B_2$
20	$\langle \bar{B}[u^\nu,\gamma_5\gamma_\nu B]\rangle\langle u_\mu u^\mu\rangle$	$M_1B_1 \to M_2M_3B_2$
21	$\langle \bar{B}\gamma_5\gamma_\nu B\rangle\langle u_\mu u^\mu u^\nu\rangle$	$M_1B_1 \to M_2M_3B_2$
22	$i\langle \bar{B}\gamma^\tau B\{[u^\mu,u^\nu],u^\rho\}\rangle\varepsilon_{\mu\nu\rho\tau}$	$M_1B_1 \to M_2M_3B_2$
23	$i\langle \bar{B}\{[u^\mu,u^\nu],u^\rho\}\gamma^\tau B\rangle\varepsilon_{\mu\nu\rho\tau}$	$M_1B_1 \to M_2M_3B_2$
24	$i\langle \bar{B}[u^\mu,u^\nu]\gamma^\tau Bu^\rho\rangle\varepsilon_{\mu\nu\rho\tau}$	$M_1B_1 \to M_2M_3B_2$
25	$i\langle \bar{B}u^\rho\gamma^\tau B[u^\mu,u^\nu]\rangle\varepsilon_{\mu\nu\rho\tau}$	$M_1B_1 \to M_2M_3B_2$
26	$i\langle \bar{B}\gamma^\tau B\rangle\langle [u^\mu,u^\nu]u^\rho\rangle\varepsilon_{\mu\nu\rho\tau}$	$M_1B_1 \to M_2M_3B_2$

Table 1:

i	$\widehat{O}_i$	Contributes to vertex
27	$\langle\bar{B}\gamma_5\gamma_\mu D_{\nu\rho}Bu^\mu u^\nu u^\rho\rangle + \langle\bar{B}\overleftarrow{D}_{\nu\rho}\gamma_5\gamma_\mu Bu^\rho u^\nu u^\mu\rangle$	$M_1B_1 \to M_2M_3B_2$
28	$\langle\bar{B}u^\mu\gamma_5\gamma_\mu D_{\nu\rho}Bu^\nu u^\rho\rangle + \langle\bar{B}\overleftarrow{D}_{\nu\rho}u^\mu\gamma_5\gamma_\mu Bu^\rho u^\nu\rangle$	$M_1B_1 \to M_2M_3B_2$
29	$\langle\bar{B}u^\mu u^\nu\gamma_5\gamma_\mu D_{\nu\rho}Bu^\rho\rangle + \langle\bar{B}\overleftarrow{D}_{\nu\rho}u^\nu u^\mu\gamma_5\gamma_\mu Bu^\rho\rangle$	$M_1B_1 \to M_2M_3B_2$
30	$\langle\bar{B}u^\mu u^\nu u^\rho\gamma_5\gamma_\mu D_{\nu\rho}B\rangle + \langle\bar{B}\overleftarrow{D}_{\nu\rho}u^\rho u^\nu u^\mu\gamma_5\gamma_\mu B\rangle$	$M_1B_1 \to M_2M_3B_2$
31	$\left(\langle\bar{B}\gamma_5\gamma_\mu D_{\nu\rho}B\rangle + \langle\bar{B}\overleftarrow{D}_{\nu\rho}\gamma_5\gamma_\mu B\rangle\right)\langle u^\mu u^\nu u^\rho\rangle$	$M_1B_1 \to M_2M_3B_2$
32	$i\left(\langle\bar{B}u^\mu\sigma^{\lambda\tau}D_\rho B\{u^\nu,u^\rho\}\rangle - \langle\bar{B}\overleftarrow{D}_\rho u^\mu\sigma^{\lambda\tau}B\{u^\nu,u^\rho\}\rangle\right)\varepsilon_{\mu\nu\lambda\tau}$	$M_1B_1 \to M_2M_3B_2$
33	$i\left(\langle\bar{B}\{u^\nu,u^\rho\}\sigma^{\lambda\tau}D_\rho Bu^\mu\rangle - \langle\bar{B}\overleftarrow{D}_\rho\{u^\nu,u^\rho\}\sigma^{\lambda\tau}Bu^\mu\rangle\right)\varepsilon_{\mu\nu\lambda\tau}$	$M_1B_1 \to M_2M_3B_2$
34	$i\left(\langle\bar{B}\{u^\mu,\sigma^{\lambda\tau}D_\rho B\}\rangle - \langle\bar{B}\overleftarrow{D}_\rho\{u^\mu,\sigma^{\lambda\tau}B\}\rangle\right)\langle u^\nu u^\rho\rangle\varepsilon_{\mu\nu\lambda\tau}$	$M_1B_1 \to M_2M_3B_2$
35	$i\left(\langle\bar{B}[u^\mu,\sigma^{\lambda\tau}D_\rho B]\rangle - \langle\bar{B}\overleftarrow{D}_\rho[u^\mu,\sigma^{\lambda\tau}B]\rangle\right)\langle u^\nu u^\rho\rangle\varepsilon_{\mu\nu\lambda\tau}$	$M_1B_1 \to M_2M_3B_2$
36	$\langle\bar{B}u^\mu\gamma_5\gamma_\mu B\chi_+\rangle$	$B_1 \to M_1B_2$
37	$\langle\bar{B}\chi_+\gamma_5\gamma_\mu Bu^\mu\rangle$	$B_1 \to M_1B_2$
38	$\langle\bar{B}u^\mu\gamma_5\gamma_\mu B\rangle\langle\chi_+\rangle$	$B_1 \to M_1B_2$
39	$\langle\bar{B}\gamma_5\gamma_\mu Bu^\mu\rangle\langle\chi_+\rangle$	$B_1 \to M_1B_2$
40	$\langle\bar{B}\gamma_5\gamma_\mu B\rangle\langle u^\mu\chi_+\rangle$	$B_1 \to M_1B_2$
41	$\langle\bar{B}\gamma_5\gamma_\mu B\{u^\mu,\chi_+\}\rangle$	$B_1 \to M_1B_2$
42	$\langle\bar{B}\{u^\mu,\chi_+\}\gamma_5\gamma_\mu B\rangle$	$B_1 \to M_1B_2$
43	$\langle\bar{B}\{\chi_-,\gamma_5 B\}\rangle$	$B_1 \to M_1B_2$
44	$\langle\bar{B}[\chi_-,\gamma_5 B]\rangle$	$B_1 \to M_1B_2$
45	$\langle\bar{B}\gamma_5 B\rangle\langle\chi_-\rangle$	$B_1 \to M_1B_2$
46	$\langle\bar{B}\gamma_\mu B[\chi_-,u^\mu]\rangle$	$B_1M_1 \to M_2B_2$
47	$\langle\bar{B}[\chi_-,u^\mu]\gamma_\mu B\rangle$	$B_1M_1 \to M_2B_2$
48	$\langle\bar{B}u^\mu\rangle\langle\chi_-\gamma_\mu B\rangle - \langle\bar{B}\chi_-\rangle\langle u^\mu\gamma_\mu B\rangle$	$B_1M_1 \to M_2B_2$
49	$\langle\bar{B}\{D_\mu f_+^{\mu\nu},\gamma_\nu B\}\rangle$	$B_1 \to \gamma B_2$
50	$\langle\bar{B}[D_\mu f_+^{\mu\nu},\gamma_\nu B]\rangle$	$B_1 \to \gamma B_2$
51	$i\langle\bar{B}\gamma_5\gamma_\nu B[u_\mu,f_+^{\mu\nu}]\rangle$	$\gamma B_1 \to M_2B_2$
52	$i\langle\bar{B}[u_\mu,f_+^{\mu\nu}]\gamma_5\gamma_\nu B\rangle$	$\gamma B_1 \to M_2B_2$

Table 1:

i	$\widehat{O}_i$	Contributes to vertex
53	$i\left(\langle\bar{B}u_\mu\rangle\langle f_+^{\mu\nu}\gamma_5\gamma_\nu B\rangle - \langle\bar{B}f_+^{\mu\nu}\rangle\langle u_\mu\gamma_5\gamma_\nu B\rangle\right)$	$\gamma B_1 \to M_2 B_2$
54	$\langle\bar{B}\gamma^\tau B\{u^\mu, f_+^{\nu\rho}\}\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \to M_2 B_2$
55	$\langle\bar{B}\{u^\mu, f_+^{\nu\rho}\}\gamma^\tau B\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \to M_2 B_2$
56	$\langle\bar{B}u^\mu\gamma^\tau Bf_+^{\nu\rho}\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \to M_2 B_2$
57	$\langle\bar{B}f_+^{\nu\rho}\gamma^\tau Bu^\mu\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \to M_2 B_2$
58	$\langle\bar{B}\gamma^\tau B\rangle\langle u^\mu f_+^{\nu\rho}\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \to M_2 B_2$
59	$\left(\langle\bar{B}[u^\mu, f_+^{\nu\rho}]\sigma^{\lambda\tau}D_\mu B\rangle - \langle\bar{B}\overleftarrow{D}_\mu[u^\mu, f_+^{\nu\rho}]\sigma^{\lambda\tau}B\rangle\right)\varepsilon_{\nu\rho\lambda\tau}$	$\gamma B_1 \to M_2 B_2$
60	$\left(\langle\bar{B}\sigma^{\lambda\tau}D_\mu B[u^\mu, f_+^{\nu\rho}]\rangle - \langle\bar{B}\overleftarrow{D}_\mu\sigma^{\lambda\tau}B[u^\mu, f_+^{\nu\rho}]\rangle\right)\varepsilon_{\nu\rho\lambda\tau}$	$\gamma B_1 \to M_2 B_2$
61	$\left(\langle\bar{B}u^\mu\rangle\langle f_+^{\nu\rho}\sigma^{\lambda\tau}D_\mu B\rangle + \langle\bar{B}\overleftarrow{D}_\mu f_+^{\nu\rho}\rangle\langle u^\mu\sigma^{\lambda\tau}B\rangle\right)\varepsilon_{\nu\rho\lambda\tau}$	$\gamma B_1 \to M_2 B_2$
62	$\langle\bar{B}\{D_\mu f_-^{\mu\nu}, \gamma_5\gamma_\nu B\}\rangle$	$\gamma B_1 \to M_2 B_2$
63	$\langle\bar{B}[D_\mu f_-^{\mu\nu}, \gamma_5\gamma_\nu B]\rangle$	$\gamma B_1 \to M_2 B_2$
64	$\langle\bar{B}\gamma_5\gamma^\tau B\{u^\mu, f_-^{\nu\rho}\}\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \to M_2 M_3 B_2$
65	$\langle\bar{B}\{u^\mu, f_-^{\nu\rho}\}\gamma_5\gamma^\tau B\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \to M_2 M_3 B_2$
66	$\langle\bar{B}f_-^{\nu\rho}\gamma_5\gamma^\tau Bu^\mu\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \to M_2 M_3 B_2$
67	$\langle\bar{B}u^\mu\gamma_5\gamma^\tau Bf_-^{\nu\rho}\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \to M_2 M_3 B_2$
68	$\langle\bar{B}\gamma_5\gamma^\tau B\rangle\langle u^\mu f_-^{\nu\rho}\rangle\varepsilon_{\mu\nu\rho\tau}$	$\gamma B_1 \to M_2 M_3 B_2$
69	$i\langle\bar{B}[u_\mu, f_-^{\mu\nu}]\gamma_\nu B\rangle$	$\gamma B_1 \to M_2 M_3 B_2$
70	$i\langle\bar{B}\gamma_\nu B[u_\mu, f_-^{\mu\nu}]\rangle$	$\gamma B_1 \to M_2 M_3 B_2$
71	$i\left(\langle\bar{B}u_\mu\rangle\langle f_-^{\mu\nu}\gamma_\nu B\rangle - \langle\bar{B}f_-^{\mu\nu}\rangle\langle u_\mu\gamma_\nu B\rangle\right)$	$\gamma B_1 \to M_2 M_3 B_2$
72	$i\left(\langle\bar{B}\sigma_{\nu\rho}D_\mu B\{u^\mu, f_-^{\nu\rho}\}\rangle - \langle\bar{B}\overleftarrow{D}_\mu\sigma_{\nu\rho}B\{u^\mu, f_-^{\nu\rho}\}\rangle\right)$	$\gamma B_1 \to M_2 M_3 B_2$
73	$i\left(\langle\bar{B}\{u^\mu, f_-^{\nu\rho}\}\sigma_{\nu\rho}D_\mu B\rangle - \langle\bar{B}\overleftarrow{D}_\mu\{u^\mu, f_-^{\nu\rho}\}\sigma_{\nu\rho}B\rangle\right)$	$\gamma B_1 \to M_2 M_3 B_2$
74	$i\left(\langle\bar{B}u^\mu\sigma_{\nu\rho}D_\mu Bf_-^{\nu\rho}\rangle - \langle\bar{B}\overleftarrow{D}_\mu u^\mu\sigma_{\nu\rho}Bf_-^{\nu\rho}\rangle\right)$	$\gamma B_1 \to M_2 M_3 B_2$
75	$i\left(\langle\bar{B}f_-^{\nu\rho}\sigma_{\nu\rho}D_\mu Bu^\mu\rangle - \langle\bar{B}\overleftarrow{D}_\mu f_-^{\nu\rho}\sigma_{\nu\rho}Bu^\mu\rangle\right)$	$\gamma B_1 \to M_2 M_3 B_2$
76	$i\left(\langle\bar{B}\sigma_{\nu\rho}D_\mu B\rangle - \langle\bar{B}\overleftarrow{D}_\mu\sigma_{\nu\rho}B\rangle\right)\langle u^\mu f_-^{\nu\rho}\rangle$	$\gamma B_1 \to M_2 M_3 B_2$

Table 1: Minimal set of linearly independent monomials of the $SU(3)$ chiral meson-baryon Lagrangian of $\mathcal{O}(q^3)$. On the third column we give the vertex with the minimal number of mesons and photons to which each term contributes.

In the previous list, the symbol $D_{\nu\rho} = D_\nu D_\rho + D_\rho D_\nu$. For the other symbols we refer to [1]. In addition, a covariant derivative acts only on one hadronic matrix field, the one immediately to the right or left (in the latter case there is a left pointing arrow over D). E.g., $D_\rho B u_\nu$ must be understood such that the covariant derivative acts only on B. We also want to remark that our way of presenting the monomials of the $\mathcal{O}(q^3)$ meson-baryon chiral Lagrangian here and in [1] is much more compact and easy to manipulate than the one employed in [2]. We also prefer not to introduce dimensionful parameters to change artificially the dimension of the coefficients h_i.

Regarding the Lagrangian presented in [2], monomials $O_{32}^{(3)}$ and $O_{33}^{(3)}$ are not linearly independent from the rest of monomials in the $\mathcal{O}(q^3)$ meson-baryon chiral Lagrangian and can be removed. We first note that $\sigma^{\mu\nu}[u_\mu,[u_\nu,u_\rho]]$, the combination used in [2], is proportional to $\sigma^{\mu\nu}[u_\rho,[u_\mu,u_\nu]]$. This can be seen by explicitely expanding the commutators and taking into account that $\sigma^{\mu\nu}$ is antisymmetric in the indices μ and ν. In this way, the two monomials $O_{32}^{(3)}$ and $O_{33}^{(3)}$ of [2] are accounted for by the structures $\langle \bar{B}\gamma_5\sigma^{\rho\eta}[[u_\rho,u_\eta],u_\sigma]D^\sigma B\rangle$ and $\langle \bar{B}\gamma_5\sigma^{\rho\eta}D^\sigma B\,[[u_\rho,u_\eta],u_\sigma]\rangle$, plus the corresponding charge conjugated terms. We consider in detail the first of these monomials and employ the relation,

$$[[u_\rho,u_\eta],X] = 4[D_\rho,D_\eta]X + 2i[f_{\rho\eta}^+,X] \;, \tag{7}$$

see eq.(2.9) of [1]. Hence,

$$\langle \bar{B}\gamma_5\sigma^{\rho\eta}[[u_\rho,u_\eta],u_\sigma]D^\sigma B\rangle = 4\langle \bar{B}\gamma_5\sigma^{\rho\eta}[D_\rho,D_\eta]u_\sigma\, D^\sigma B\rangle + 2i\langle \bar{B}\gamma_5\sigma^{\rho\eta}[f_{\rho\eta}^+,u_\sigma]D^\sigma B\rangle \;. \tag{8}$$

The last term in the previous equation is accounted for by the monomials $O_{61}^{(3)}$ and $O_{62}^{(3)}$ of ref.[2] and corresponds to O_{67} of [1], once the identity $\sigma_{\alpha\beta}\varepsilon^{\alpha\beta\rho\eta} = 2i\gamma_5\sigma^{\rho\eta}$ is employed. Then, we do not consider this term any further and concentrate on the first one on the right hand side of the equality.

$$\begin{aligned}
&\langle \bar{B}\gamma_5\sigma^{\rho\eta}[D_\rho,D_\eta]u_\sigma D^\sigma B\rangle = 2\langle \bar{B}\gamma_5\sigma^{\rho\eta}D_\rho D_\eta u_\sigma\, D^\sigma B\rangle \\
&= -2\langle D_\rho\bar{B}\gamma_5\sigma^{\rho\eta}D_\eta u_\sigma D^\sigma B\rangle - 2\langle \bar{B}\gamma_5\sigma^{\rho\eta}D_\eta u_\sigma\, D_\rho D^\sigma B\rangle \;,
\end{aligned} \tag{9}$$

where a total divergence has been dropped out in the last equality. Now, since $\sigma^{\rho\eta} = i\gamma^\rho\gamma^\eta - ig^{\rho\eta}$, we have for the first term on the second line,

$$i\langle D_\rho\bar{B}\gamma_5\gamma^\rho\gamma^\eta D_\eta u_\sigma D^\sigma B\rangle - i\langle D_\rho\bar{B}\gamma_5 g^{\rho\eta}D_\eta u_\sigma\, D^\sigma B\rangle \;. \tag{10}$$

The first term in the previous equation can be removed by the baryon equations of motion. Using now that $\sigma^{\rho\eta} = -i\gamma^\eta\gamma^\rho + ig^{\rho\eta}$ to the last term in eq.(9), one has,

$$i\langle \bar{B}\gamma_5 g^{\rho\eta}D_\eta u_\sigma\, D_\rho D^\sigma B\rangle - i\langle \bar{B}\gamma_5\gamma^\eta\gamma^\rho D_\eta u_\sigma\, D_\rho D^\sigma B\rangle \;. \tag{11}$$

Employing again the baryon equations of motion the last term can be disregarded. When summing eqs.(10) and (11), as corresponds to the last line of eq.(9), the terms proportional to $g^{\rho\eta}$ cancel each other. As a result the monomial on the left hand side of eq.(8) can be removed as stated before. One can proceed in a similar way to remove the monomial

$\langle \bar{B}\gamma_5\sigma^{\rho\eta}D^\sigma B\,[[u_\rho,u_\eta],u_\sigma]\rangle$ as well. This discussion shows that one can further reduce the number of monomials by two in [2] passing from 78 to 76, in agreement with the number of monomials we found above.

In addition, we notice that the monomial $O_{40}^{(3)}$ of [2] is not exactly charge conjugation invariant since those terms involving two covariant derivatives acting on the mesonic fields u_α are missed. These contributions, though are of $\mathcal{O}(q^4)$, are needed to guarantee exact charge conjugation invariance.

Here, we have discussed the findings of [2] in relation to [1] and showed that one can reduce in eight the number of monomials in the $SU(3)$ meson-baryon chiral Lagrangian of $\mathcal{O}(q^3)$ presented in [1] and in two when comparing with the Lagrangian in [2]. Thus, we end up with 76 monomials, instead of the 84 in [1] and of the 78 presented in [2].

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