An evaluation of the alpha-cluster formation factor in (n, α) reactions

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Abstract. In this work we suggest some methods based on the statistical and knock-on models, for evaluation of the α -clustering factor or α -clustering probability in (n, α) reactions induced by slow and fast neutrons. The main purpose of this study is to compare the values of the α -clustering factors obtained by the compound and direct mechanisms for the same nuclear reactions. Also, our results are compared with values estimated by other authors.

1 Introduction

Alpha clustering in nuclei is important to understand mechanisms of α -decay, α -particle scattering, α -particle transfer and emission reactions, and nuclear structure [1–3]. The α -clustering effect has been investigated for a long time using different methods based on various theoretical approaches. Most of these studies were focused on the α -decay [4–11] and molecule like α -particle structure of light nuclei [12–14]. Several papers were dedicated to the determination of the α -particle formation factor in the (n, α) reaction [15–19]. However, the results of these studies are not consistent and up to now a common explanation of the α -clustering in a nucleus and unified method to obtain the α -clustering factor (or formation probability) are not available. Recently, we have determined the α -clustering factor for fast neutron ($E_n = 2 - 20 \text{ MeV}$) induced (n, α) reactions, by analogy with the spectroscopic factor [20], using the ratio of experimental cross sections to theoretical ones calculated by the statistical model [21].

In this work, we suggest some methods to derive α -clustering factors from the analysis of experimental data for slow ($E_n \leq 30 \text{ keV}$) and fast ($E_n = 4-6 \text{ MeV}$) neutron induced (n, α) reactions using the statistical model and knock-on mechanism. The obtained α -clustering factors are compared with our previous results and those determined by other authors.

2 Theoretical formulae

2.1 Slow neutron-induced (n, α) reactions

The statistical model will be used to derive the α -clustering factor for the slow neutron induced (n, α)

reaction. Then, taking into account the α -clustering in the compound nucleus, Weisskopf's formula [22] of an average α -width of a level for given spin, J, and angular momentum, l, can be written in the following form:

$$\langle \Gamma_{\alpha}(J,l) \rangle = \frac{D(J)}{2\pi} T_{\alpha}(l) \phi_{\alpha},$$
 (1)

where D(J) is the average level spacing for given J; $T_{\alpha}(l)$ is the transmission factor of an α -particle through the potential barrier of the daughter nucleus; ϕ_{α} is the α -clustering factor. From (1), the α -clustering factor is given by

$$\phi_{\alpha} = 2\pi \frac{\langle \Gamma_{\alpha}(J, l) \rangle}{D(J) T_{\alpha}(l)}.$$
 (2)

The formula (2) is utilized to estimate the α -clustering factor for the (n,α) reaction induced by resonance neutrons. Experimental data of the average α -widths [18] and the average level spacing for s-resonances [23] were used in the calculation. The transmission factors, $T_{\alpha}(l)$, were obtained using Rasmussen's formula [24] for zero angular momentum, l=0, to simplify the calculations.

In the case of intermediate neutrons, using the statistical model, the average (n, α) cross section can be expressed as [25]

$$\langle \sigma(n,\alpha) \rangle = 2\pi^2 \left(\frac{\lambda_n}{2\pi}\right)^2 \sum_{l} \sum_{J} \frac{g(J)}{D(J)} \frac{\langle \Gamma_n(J,l) \rangle \langle \Gamma_\alpha(J,l) \rangle}{\langle \Gamma(J,l) \rangle} F_l,$$
(3)

where λ_n is the wave length of the incident neutron; $\langle \Gamma_n(J,l) \rangle$, $\langle \Gamma_\alpha(J,l) \rangle$ and $\langle \Gamma(J,l) \rangle$ are the average neutron, alpha and total level widths, respectively; g(J) is the spin factor; F_l is the level width fluctuation factor comprised within the range of 0.6-1.0. For the intermediate neutrons can be assumed $\Gamma_n \gg \Gamma_\gamma \gg \Gamma_\alpha$. If we neglect the angular momentum and spin dependences of the total (n,α) cross

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	Resonance	Intermediate	Statical model,		Knock-on mechanism,	
Target	neutrons	neutrons	by formula (6)		by formula (8)	
nuclei	$(E_n \le 5 \text{ keV})$ by formula (2)	$(E_n = 24 - 30 \text{keV})$ by formula (4)	$E_n = 4 \text{MeV}$	$E_n = 6$ MeV	$E_n = 4 \text{MeV}$	$E_n = 6 \text{MeV}$
⁵⁴ Fe	=	=	0.02	0.02	3.0E-04	-
⁵⁸ Ni	-	-	0.0024	-	0.004	0.037
⁶³ Cu	-	-	0.013	0.046	1.2E-04	0.003
⁶⁴ Zn	0.30	-	0.0022	-	0.024	0.037
⁶⁷ Zn	0.21	-	-	-	0.003	0.004
⁹⁵ Mo	0.53	0.53	-	-	2.8E-04	0.001
¹²³ Te	0.81	0.52	-	-	-	-
¹⁴³ Nd	0.37	0.20	-	-	4.8E-05	1.5 <i>E</i> -04
¹⁴⁷ Sm	0.24	0.25	-	-	-	1.4E-04
¹⁴⁹ Sm	0.52	=	-	-	-	4.9E-05

Table 1. The α -clustering factor, ϕ_{α} , for slow and fast neutron induced (n, α) reactions

Empty places in the Table 1 mean that experimental data for given isotopes are not available.

section averaged over the wide neutron energy range and assume $F_l \approx 1$, from (1) and (3) is obtained the following simple formula for the α -clustering factor:

$$\phi_{\alpha} \approx \frac{\langle \sigma(n,\alpha) \rangle}{\pi(\frac{\lambda_n}{2\pi})^2 T_{\alpha}(l)}.$$
 (4)

The equation (4) is used to estimate the α -clustering factor for the 24-30 keV neutron-induced (n, α) reactions. Experimental values of the (n, α) cross sections were taken from Ref. [18].

2.2 The fast neutron-induced (n, α) reaction

For fast neutron induced (n, α) reactions two methods based on the statistical model and knock-on mechanism were used. In the framework of the statistical model the proton-clustering factor for (n,p) reactions, by analogy with (4), can be written as

$$\phi_p \approx \frac{\langle \sigma(n, p) \rangle}{\pi (\frac{\lambda_n}{2\pi})^2 T_p}.$$
 (5)

If we assume $\phi_p = 1$, the α -clustering factor for the (n, α) reaction induced by quasimonoenergetic fast neutrons can be obtained from (4) and (5) as following

$$\phi_{\alpha} \approx \frac{\sigma(n,\alpha)T_p}{\sigma(n,p)T_{\alpha}}.$$
 (6)

The α -clustering factor in (6) is defined as the probability of an interaction of the incident neutron with an α -cluster relative to that with a proton. (6) is used to estimate the α -clustering factor for the (n,α) reaction induced by 4-6 MeV neutrons where experimental (n,α) and (n,p) cross sections for the same isotopes are simultaneously available.

In the framework of the knock-on mechanism, by analogy of Bohr's postulate of the compound mechanism, we assume that the (n, α) cross section for fast neutrons can be expressed as two stages process:

$$\sigma(n,\alpha) = \phi_{\alpha} \cdot \sigma_n^{tot}(^4He). \tag{7}$$

Here, the (n,α) cross section is defined as the multiplication of the α -cluster formation probability on the target nucleus, ϕ_{α} , and total neutron cross section for the 4 He, $\sigma_{n}^{tot}({}^{4}He)$. The α -clustering factor can be from (7) obtained as following:

$$\phi_{\alpha} = \frac{\sigma(n,\alpha)}{\sigma_n(^4He)}.$$
 (8)

For evaluation of the α -clustering factor by formula (8) the experimental data of the (n, α) cross sections and the total neutron cross sections for the ⁴He were taken from the EXFOR [26] and other references.

3 Results and Discussion

Results of our evaluations for the α -clustering factors in (n, α) reactions induced by slow (resonance and intermediate) and fast $(E_n=4-6 \text{ MeV})$ neutrons are given in Table 1.

Table 1 shows that the statistical model gives almost the same α -clustering factors for each isotope for the resonance and intermediate neutrons. Furthermore, the α -clustering factors for slow neutrons obtained in this work are in a satisfactorily agreement with the results of Refs. [15–18]. In the case of fast neutrons the α -clustering factors are lower than those for slow neutrons. α -clustering factors obtained by the statistical model formula (6) for $E_n = 4 - 6$ MeV neutrons vary in the range of 0.0022 to 0.046 and are on average a little lower than our previous results of 0.02-0.33, which were obtained, by analogy with the spectroscopic factor, from the ratio of experimental (n, α) cross sections to the theoretical ones for $E_n = 2 - 20 \text{ MeV}$ [21]. It can be seen, also, from Table 1 that the α -clustering factors, $\phi_{\alpha} \approx 3.7 \cdot 10^{-2} - 4.9 \cdot 10^{-5}$, obtained by the knock-on mechanism formula (8) are on average appreciably lower than the values calculated by the statistical model. At the same time, these results are close to the Kadmensky and Furman's cluster model conclusions for α -decay: $7 \cdot 10^{-4}$ and $3 \cdot 10^{-5}$ for favoured and semifavoured α -transitions, respectively.

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