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Designs**

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# Cap-and-trade Properties under Different Scheme Designs\*

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## Abstract

This paper examines the key design mechanisms of existing and proposed cap-and-trade markets. First, it is shown that the hybrid systems under investigation (safety-valve with offsets, price floor using a subsidy, price collar, allowance reserve, and options offered by the regulator) can be decomposed into a combination of an ordinary cap-and-trade scheme with European- or American-style call and put options. Then, we quantify and discuss the advantages and disadvantages of the proposed hybrid schemes by investigating whether pre-set objectives (enforcement of permit price bounds and reduction of potential costs for relevant companies) can be accomplished while maintaining the original environmental targets.

**Keywords:** Allowance reserve, Price ceiling, Price collar, Price floor, Safety valve.

**JEL Classifications:** H23, Q28, Q54, Q58.

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# 1 Introduction

Behind the global interest in marketable permits for air pollution is the recognition that any meaningful climate change policy has to put a price on carbon dioxide emissions.<sup>1</sup> As explained in Baumol and Oates (1988), pricing greenhouse gas emissions is a fundamental lesson from environmental economics and the theory of externalities. Environmental economists consider that the absence of a price charge for scarce environmental resources such as clean air leads to air pollution. They prescribe, therefore, the introduction of surrogate prices in the form of unit taxes or marketable emission permits in order to induce people to economize in the use of these resources.<sup>2</sup>

Probably because markets for permits are easier to implement politically, carbon markets are currently quite popular among policy makers around the world. In 2005, the European Emissions Trading Scheme (EU ETS) was launched. It is the world's largest carbon market to date, covering more than 40% of the carbon dioxide emitted in Europe. European member states agreed in December 2008 to extend this scheme until 2020 and open it up to new sectors, most notably aviation. In January 2009, the official launch of the Regional Greenhouse Gas Initiative (RGGI) signed by 10 north-eastern US States, ushered in the carbon market era in North America. A plan to introduce a US-wide cap-and-trade scheme has recently been proposed by the new U.S. administration. Canada demonstrated its interest in linking up with the US scheme, abandoning its own plans for developing an efficiency-based system. In the Pacific area, Australia's Carbon Pollution Reduction Scheme (CPRS) and New Zealand's Emissions Trading Scheme (NZ ETS) are in different stages of development. Finally, Japan is timidly considering different options for the development of a market for emission permits. Most of these schemes, however, differ in the way they address common objectives. For instance, several existing and proposed regional trading schemes are surrounded by concerns about the range of acceptable prices for emission permits. Policy regulators, therefore, have suggested the introduction of specific mechanisms to keep the permit price from rising too high or falling too low. The aim of this paper is to investigate currently proposed hybrid schemes in an equilibrium framework for the market of permits. We assess the ability of these schemes to achieve pre-set objectives (enforcement of permit price bounds and reduction of potential costs for relevant

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<sup>1</sup>Stern (2007) provides a comprehensive discussion on the economics of climate change.

<sup>2</sup>Under textbook assumptions, marketable emission permits are essentially equivalent to taxes. However, in real world situations with market power, imperfect information, and transaction costs, there are important differences between permits and taxes. We refer to Taschini (2009) for an introductory review of factors that impinge on the effectiveness of marketable permits schemes and to references therein for in-depth proofs of specific differences.

companies) and, at the same time, maintain their original environmental targets.

Few stochastic models describing the equilibrium price dynamics of emission permits are currently available in the literature. Carmona et al. (2009) show in a general setting that the price of emission permits equals the discounted penalty multiplied by the probability of the event of excess supply (i.e. the aggregated cumulative emissions exceed total amount of permits). The models of Seifert et al. (2008), Chesney and Taschini (2008) and Gröll and Kiesel (2009) specify the process for the cumulative emissions in the framework of Carmona et al. (2009). In the first paper the emission rate of the representative agent follows an arithmetic Brownian motion, while in the other papers firms' emission rate follows a geometric Brownian motion. This implies the total amount of pollution is described by the integral over an arithmetic and a geometric Brownian motion, respectively. The approaches of Chesney and Taschini (2008) and Gröll and Kiesel (2009) differ in the way such an integral is approximated. In particular, Carmona et al. (2009) analyze the effect of windfall profits, Chesney and Taschini (2008) investigate the effect of asymmetric information on the permit price and Gröll and Kiesel (2009) provide a sound theoretical discussion about the permit price slump in 2006 in the EU ETS. These models realistically depict the dynamic price formation of emission permits, accounting for the most important features of the cap-and-trade scheme implemented under the EU ETS. Introducing a stylized version of the equilibrium price formula of emission permits of Carmona et al. (2009) and then extending it, we first investigate the most relevant existing and proposed schemes' design mechanisms by evaluating their apparent objectives. Second, after Keeler's (1991) results highlighting that the success of pollution control strategies reliant on market permits deeply depends on the enforcement structure, we systematically compare the expected enforcement costs of each hybrid scheme to the enforcement costs of an ordinary scheme. Finally, we assess the impact of each scheme on the original environmental targets.

The present paper is organized as follows. In Section 2 we introduce a stylized version of the stochastic equilibrium price of Carmona et al. (2009). Then, we derive the price properties of emission permits under five different alternatives of scheme designs (safety-valve with offsets, price floor using a subsidy, price collar, permit reserve, and options offered by the regulator). We determine how each hybrid system affects the equilibrium permit price and whether pre-set objectives (enforcement of permit price bounds and reduction of potential costs for relevant companies) are realistically enforced. Also, we assess the potential impacts on the original environmental targets of the scheme. Finally, we quantify

the financial burden of each hybrid system and compare it to a standard cap-and-trade system. Section 3 concludes and summarizes in three tables the rationale behind each hybrid system, its advantages and disadvantages, and the description of its mechanisms.

## 2 Current and Proposed Scheme Design Mechanisms

In this section we concentrate on the most relevant scheme alternatives proposed by policy regulators to keep the permit price from rising or falling to an inordinate degree. Among suggested mechanisms, relaxing the amount of usable offsets (so-called safety-valve), introducing a subsidy to ensure a minimum price level, setting a price ceiling and price floor (so-called price collar), and creating a permit reserve to be deployed when permit prices are too high, are the most popular hybrid systems. A hybrid system is generally considered as a tailored combination of price (tax) and quantity (permit) instruments. The idea of creating a hybrid system by combining these two policy tools was first introduced by the seminal papers of Weitzman (1974) and Roberts and Spence (1976).<sup>3</sup> In any cap-and-trade scheme, there will be always a penalty for non-compliance. If payment of the penalty is an alternative to compliance, as in the framework of 2a, the penalty is effectively a price ceiling in a hybrid scheme as discussed by Jacoby and Ellerman (2004). In contrast, if payment of the penalty does not amount to compliance and the company is still obliged to comply as soon as possible, then the scheme is not directly equivalent to a conventional hybrid scheme. In the following subsections we consistently compare cap-and-trade schemes supplied with a specific mechanism (hereafter hybrid systems) to the cap-and-trade system described in Section 2a (hereafter ordinary system). Our analysis is performed in the one-period framework of Carmona et al. (2009) where banking and borrowing are not allowed. Banking and borrowing options have been proposed by environmental economists with the aim of enforcing the credibility of cap-and-trade schemes and allowing a greater flexibility over time. Past literature on the analysis of how banking and borrowing mechanisms affect the price formation of emission permits is extensive. We refer to Rubin (1996) and Schennach (2000) for an analysis of the consequences of banking and borrowing on the inter-temporal trading of emission permits. In this paper we do not explicitly account for those features that would simply complicate formulae and distract the reader. However, the statements derived for the hybrid and for the ordinary systems also hold in a setting where banking is allowed.

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<sup>3</sup>We refer to Hepburn (2006) for a recent overview on the possible combination of price and quantity instruments.

By distinguishing the mechanisms under investigation with respect to the use of external offsets for controlling the permit price in the market, we classify the hybrid systems under study into two main groups. The first group encompasses those cap-and-trade schemes that recognize offsets as functional for compliance purposes. In particular, we study a mechanism where the amount of offsets is a function of the permit price observed in the market. The higher the permit price, the larger the amount of offsets that can be employed for compliance purposes. Conversely, the second group of scheme mechanisms relies on the ability of each policy regulator to purchase or sell an (un)limited amount of emission permits. The remainder of the hybrid systems under study belongs to this group. Neglecting possible interdependence with any offset market for the ease of exposure, we investigate these systems from Section 2c to Section 2f.

## 2a Ordinary cap-and-trade scheme

Allowing for stochastic production and abatement costs, revenues from selling produced goods and emission quantities, Carmona et al. (2009) derived the theoretical futures price of permits in the EU ETS framework, where the total pollution net of abatement reductions (the so-called aggregated cumulative emissions) in  $[0, t]$  is measured by the stochastic process  $q_{[0,t]}$ . Let us define  $P$  as the penalty that has to be paid for each emission unit that is not covered by a permit at the compliance date  $T$ . Also,  $N$  is the total amount of permits allocated by the policy regulator to relevant companies, i.e. the cap. Both  $P$  and  $N$  are known values. We can then express a stylized version of the Carmona et al. (2009) equilibrium price formula at time  $t$  in terms of the demand ( $q_{[0,t]}$ ) and supply ( $N$ ) of permits, and the enforcement level ( $P$ ) in monetary units:

$$F(t, T) = P \cdot \mathbb{P}(q_{[0,T]} > N | \mathcal{F}_t), \quad (1)$$

where, after abatement reductions,  $\mathbb{P}(q_{[0,T]} > N | \mathcal{F}_t)$  measures the probability of the total amount of emissions exceeding the initial amount of permits. In other words, it is the probability of the event of a shortage of permits. In what follows, we refer to the permit price in the ordinary system by  $F(t, T)$ , as given in Equation (1). The specific variables needed to describe each hybrid system will be introduced separately in every subsection.

The model of Carmona et al. (2009) does not explicitly consider the interdependence be-

tween the markets of the emission permits and the emission offsets. In order to incorporate such an interdependence we first define the maximum amount of offsets functional for compliance purposes as  $\lambda \cdot N$ . The stylized permit price becomes:

$$F(t, T) = P \cdot \mathbb{P} \left( q_{[0, T]} - \min\{c_{[0, T]}, \lambda N\} > N | \mathcal{F}_t \right), \quad (2)$$

where  $c_{[0, T]}$  is the stochastic process that denotes the total amount of offsets purchased for compliance purposes. We now derive the theoretical price bounds (lower and upper) for emission permits, in the presence of restrictions on the use of offsets.

**Theorem 1 (Bounds for emission permit price)**

Let  $\lambda \in [0, \infty)$ . Let  $c_{[0, T]}$  be a continuous random variable on  $[0, C] \subseteq [0, \infty)$ . Then

(a)  $F(t, T) \in [F_l(t, T), F_u(t, T)]$  where

$$F_l(t, T) = P \cdot \mathbb{P} \left( q_{[0, T]} > (1 + \lambda)N | \mathcal{F}_t \right), \quad (3)$$

$$F_u(t, T) = P \cdot \mathbb{P} \left( q_{[0, T]} > N | \mathcal{F}_t \right). \quad (4)$$

(b)  $F(t, T)$  is a non-increasing function in  $\lambda$  for  $\lambda \in [0, \frac{C}{N})$  and constant in  $\lambda$  for  $\lambda \geq \frac{C}{N}$ .

*Proof :*

(a) The lower and upper bound are derived by using

$$\min\{c_{[0, T]}, \lambda N\} \leq \lambda N,$$

$$\min\{c_{[0, T]}, \lambda N\} \geq 0.$$

(b) Let  $c_{[0, T]}$  be a random variable on  $[0, C)$ . If  $\lambda \geq \frac{C}{N}$ , then

$$\min\{c_{[0, T]}, \lambda N\} = c_{[0, T]}.$$

Thus for  $\lambda \geq \frac{C}{N}$  the permit price is equal to:  $P \cdot \mathbb{P} \left( q_{[0, T]} - c_{[0, T]} > N | \mathcal{F}_t \right)$ . Let  $0 < \lambda < \frac{C}{N}$ . Then we have that  $\min\{c_{[0, T]}, \lambda N\} \leq \min\{c_{[0, T]}, \frac{C}{N}\}$  almost surely which completes the proof.  $\diamond$

**2b Safety-valve with offsets**

A popular mechanism which aims to keep the price of emission permits from rising too high is the so-called safety-valve. This mechanism works by relaxing the limitations on



the amount of offsets that can be used for compliance purposes. This mechanism is implemented in the Regional Greenhouse Gas Initiative (RGGI) in the United States. The RGGI is the first mandatory, market-based scheme in the United States to reduce greenhouse gas emissions. As mentioned in Section 1, under the RGGI ten Northeastern and Mid-Atlantic states agreed to cap and reduce their CO<sub>2</sub> emissions from the power sector by 10% by 2018. The RGGI allows power companies to buy offsets to meet their compliance.<sup>4</sup> However, the use of these offsets is constrained to 3.3 percent of a power plant's total compliance obligation. The safety-valve expands this limit to 5 percent and 10 percent if given CO<sub>2</sub> permit price thresholds are reached in the market. Let us now study this scheme considering the situation where such a mechanism might be extended to an international or global market. Using the price properties shown in Section 2a, we first derive the theoretical pattern of the price of permits in the presence of this type of safety-valve. Then, we discuss its effectiveness and quantify the corresponding expected enforcement costs for regulated companies.

Assuming that the price of the offsets is solely determined by the level of emission of relevant companies, and using the approach presented in Section 2a, the permit price is given by:

$$\bar{F}(t, T) = P \cdot \mathbb{P} \left( q_{[0, T]} - \min \{ \lambda(t)N, c_{[0, T]} \} > N | \mathcal{F}_t \right), \quad (5)$$

where  $P$  is the penalty level and  $N$  denotes the number of allowances handed out by the regulator. Let the stochastic variable  $c_{[0, T]}$  represent the total number of offsets that regulated companies purchase in the presence of unrestricted offset-use. Let  $\lambda(t)$  be an increasing step function, taking the values  $0 < \lambda_0 < \lambda_1 < \lambda_2 < \dots < \lambda_n$ . As in the RGGI scheme, at each instant  $t$  the regulator allows utilities to use  $\lambda(t) \cdot N$  offsets for compliance. Let  $\{\bar{F}_1, \dots, \bar{F}_n\}$  be the increasing ordered constants corresponding to permit price thresholds set by the policy regulator at the beginning of the scheme. In this framework,  $t_i = \inf \{ t, \bar{F}(t, T) = \bar{F}_i \}$ , where  $i = 1, \dots, n$  defines the instant when the permit price  $\bar{F}(t, T)$  hits the price threshold  $\bar{F}_i$ . Especially, we have that  $\lambda(t_i) = \lambda_i$ . This means that the amount of offsets that can be used for compliance at time  $t$  depends on the permit price  $\bar{F}(t, T)$  observed on the market. Such a system implies that, as soon as the permit

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<sup>4</sup>A RGGI offset permit represents a project-based greenhouse gas emission reduction outside the capped electric power generation sector. The RGGI participating states limit the award of offset permits only to five project categories. Furthermore, all offset projects must be located within one of the RGGI participating states.

price reaches a pre-specified price barrier  $\bar{F}_i$ ,  $\lambda(\cdot)$  jumps from  $\lambda_{i-1}$  to  $\lambda_i$ .<sup>5</sup> This additional amount of offsets functionals for compliance results in an immediate increase in the supply base of the permits and, possibly, causes a permit price drop. Looking at the price level around each instant  $t_i$ , we can observe that at time  $t < t_i$  the permit price is:

$$\bar{F}(t, T) = P \cdot \mathbb{P} \left( q_{[0, T]} - \min \{ \lambda_{i-1} N, c_{[0, T]} \} > N | \mathcal{F}_t \right).$$

By definition, at time  $t = t_i$  the permit price is equal to:

$$\bar{F}(t, T) = \bar{F}_i.$$

At time  $t > t_i$ , after the safety-valve has been used and the amount of offset that can be used has been increased, the permit price equals:

$$\bar{F}(t, T) = P \cdot \mathbb{P} \left( q_{[0, T]} - \min \{ \lambda_i N, c_{[0, T]} \} > N | \mathcal{F}_t \right).$$

Similarly to the proof in Theorem 1, it can be shown that  $\bar{F}(t, T)$  is a non-increasing function in  $\lambda(t)$ . However, the response of the permit price to the increase in  $\lambda(\cdot)$  heavily depends on the random variable  $q_{[0, T]}$ . A larger amount of usable offsets, therefore, does not necessarily lead to a permit price decrease. As such, the effectiveness of this safety-valve in terms of capping a permit price increase is rather limited.

We now quantify the financial burden imposed on regulated companies by this hybrid system and compare it with an ordinary system. Let us assume that the amount  $\lambda$  of offsets functional for compliance purposes in the ordinary cap-and-trade system corresponds to  $\lambda \equiv \lambda_0 > 0$ , and  $\lambda_0 < \lambda_1 < \lambda_2 < \dots < \lambda_n$ . As we have already shown (cf. Theorem 1) that the emission permit price is a non-increasing function in  $\lambda$ , it is trivial to show that prices of emission permits in a hybrid system with safety-valve are lower than in an ordinary cap-and-trade system. This statement clearly implies lower expected enforcement costs for regulated companies. In other words, the overall financial burden for relevant companies in a situation of high permit prices is lowered by the presence of a mechanism that literally functions as a relief-valve for the cap-and-trade scheme.

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<sup>5</sup>It is interesting to observe that the EU ETS implements a specific case of this mechanism. There the function  $\lambda(t)$  is constant, i.e.  $\lambda(t) \equiv \lambda^{EU}$ , whereas in the RGGI it is an increasing step function where the step values are  $\lambda_0 = 0.033$ ,  $\lambda_1 = 0.05$ ,  $\lambda_2 = 0.1$ .

The advantage of such a safety-valve is that it reduces expected enforcement costs for regulated companies without imposing an extra cost on the policy regulator. Furthermore, reducing the restrictions on the use of offsets is relatively easy to implement. However, as mentioned above, this mechanism does not guarantee an effective price ceiling under all circumstances. Also, and more remarkably, its success depends highly on the ability of the policy regulator to set correct price thresholds  $\bar{F}_i$ . This requires good skills in modeling and forecasting the supply ( $c_{[0,T]}$ ) and demand ( $q_{[0,T]}$ ) of permits. Finally, the fact that the amount of offsets useful for compliance purposes is a function of the (stochastic) price of emission permits, is a disadvantage for offsets project developers because it increases the overall project uncertainty.

## 2c Price Floor using a Subsidy

A severe permit price drop, followed by a price hovering above zero for more than five months during the first phase of the EU ETS, persuaded several policy makers that new cap-and-trade schemes would need additional safety-valve features. Apart from the usual presence of banking and borrowing options, therefore, policy makers have been discussing the introduction of additional mechanisms to reinforce economic incentives at the basis of market-based instruments. In particular, policy makers have been concerned about permit prices that are either too low or too high. The most obvious provision to limit such price variations is to set a price floor or ceiling. This type of mechanism will be investigated in the next section. Instead of a direct intervention on the permit price path, some economists envisage the possibility of eliminating the unfortunate consequences of extremely low permit prices by a proper combination of price (subsidy) and quantity (permit) instruments. Roberts and Spence (1976), for instance, propose to remunerate virtuous companies, i.e. companies able to massively reduce their pollution emission below their permits allocation, by means of a subsidy.

Similar to situations involving an ordinary system, a company with a permit shortage at compliance date faces a penalty  $P$ . On the contrary, when a company ends up with an excess of permits, it receives a subsidy  $S$  per unit of permit. Let  $0 < S \leq P$  and let  $N$  be the initial amount of permits allocated to relevant companies. We first prove that the permit price is indeed bounded by  $S$  from below. We show that the introduction of a subsidy in fact creates a price floor equal to the subsidy. In particular, the futures permit

price denoted by  $\tilde{F}(t, T)$  in this hybrid system stays in the interval  $[S, P]$ :

$$\begin{aligned}
\tilde{F}(t, T) &= P \cdot \mathbb{P}(q_{[0, T]} > N \mid \mathcal{F}_t) + S \cdot \mathbb{P}(q_{[0, T]} \leq N \mid \mathcal{F}_t) \\
&= P \cdot \mathbb{P}(q_{[0, T]} > N \mid \mathcal{F}_t) + S \cdot (1 - \mathbb{P}(q_{[0, T]} > N \mid \mathcal{F}_t)) \\
&= S + (P - S) \cdot \mathbb{P}(q_{[0, T]} > N \mid \mathcal{F}_t) = S + \frac{P - S}{P} \cdot P \cdot \mathbb{P}(q_{[0, T]} > N \mid \mathcal{F}_t) \\
&= S + \frac{P - S}{P} \cdot F(t, T) = F(t, T) + S \left(1 - \frac{F(t, T)}{P}\right),
\end{aligned}$$

where  $F(t, T) = P \cdot \mathbb{P}(q_{[0, T]} > N \mid \mathcal{F}_t)$  is the futures permit price in an ordinary system. The subsidy  $S$ , ensured by the policy regulator at the end of the compliance period, plays effectively the role of a price-floor. More interestingly, we can disentangle this hybrid scheme, emerging with an ordinary system and a European-style put option with strike price  $S$ .<sup>6</sup>

We now quantify the financial impact on regulated companies of this hybrid system as opposed to the standard one. Let us define  $f_q$  as the probability density function of the cumulative emissions  $q_{[0, T]}$  in the entire regulated period. The expected enforcement costs for relevant companies in an ordinary system are described by:

$$EEC = P \int_N^\infty (x - N) f_q(x) dx \geq 0. \quad (6)$$

Similarly, the expected enforcement costs for regulated companies in this hybrid system are:

$$EEC^{PF} = P \int_N^\infty (x - N) f_q(x) dx - S \int_0^N (N - x) f_q(x) dx. \quad (7)$$

Because  $S \leq P$ , a lower bound for  $EEC^{PF}$  is given by  $P(\mathbb{E}[q_{[0, T]}] - N)$ . Indeed,

$$EEC^{PF} \geq P \int_N^\infty (x - N) f_q(x) dx - P \int_0^N (N - x) f_q(x) dx = P(\mathbb{E}[q_{[0, T]}] - N).$$

Considering Equations (6) and (7), the total expected enforcement costs for regulated companies under this hybrid system are lower than under an ordinary system. In particular,

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<sup>6</sup>These calculations are an alternative derivation for pricing European call and put options written on emission permits with maturity corresponding to the end of the compliance period. We refer to Chesney and Taschini (2008) for the derivation of a closed-form pricing formula for European-style options on emission permits.

the difference between these costs is:

$$EEC - EEC^{PF} = S \int_0^N (N - x) f_q(x) dx \geq 0.$$

A price floor ensured by the presence of a subsidy is relatively easy to implement and has the further advantage of lowering the expected enforcement costs for regulated companies. Furthermore, the presence of the subsidy could induce a higher stimulus in technology and abatement investments, favoring the achievement of emission reduction targets. However, the implementation of such a hybrid system might result in a significant financial burden for the environmental policy regulator. The magnitude of this burden is hardly quantifiable a priori.

## 2d Price collar

The experience of the first phase of the EU ETS, and the threat of an extremely volatile price of emission permits, have persuaded several policy makers about the need for a more stable market, ideally enforceable by a strict price control mechanism. Policy makers, therefore, have been discussing the possible introduction a fixed price-range (the so-called price collar) within which the permit price can fluctuate.<sup>7</sup> This mechanism has long been discussed and was recently endorsed by some economists in their recommendations for a US cap-and-trade program. The rationale behind the introduction of a price collar is that the presence of a minimum (floor) and a maximum (ceiling) price of permits would lower the volatility of the price, potentially providing a higher level of price predictability. According to policy makers, such a hybrid scheme can reduce the price risk faced by innovating firms, possibly promoting higher investments in abatement technologies.

We now investigate the implications of a price collar on the trading strategies of relevant companies and on the pattern of the permit price. We use the framework of Carmona et al. (2009) for illustration. However, the results can also be extended to more complex settings. Let  $P$  be again the penalty fee;  $p^{max}$  the price ceiling, i.e. the price at which the policy regulator sells an unlimited amount of permits; and  $p^{min}$  the price floor, i.e. the price at which the policy regulator buys an unlimited amount of permits.<sup>8</sup> Such a hybrid system

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<sup>7</sup>It should be noted that a price collar can be implemented also by means of a proper combination of price (tax) and quantity (permit) instruments - see Roberts and Spence (1976).

<sup>8</sup>When the price collar is set symmetrically around a certain permit price level  $\bar{p}$ , where  $\bar{p} = \frac{1}{2}(p^{min} + p^{max})$ , we have the so-called symmetric price collar.

can be broken down into a combination of an ordinary cap-and-trade system and a sum of free-of-charge American-style call and put options. In fact, when the permit price moves above a pre-specified  $p^{max}$  level, regulated companies can (have the right to) purchase at  $p^{max}$  as many permits as they need. This optionality can be quantified by summing up the values of all exercised American call options with strike price  $p^{max}$ . Similarly, when the permit price moves below a pre-specified  $p^{min}$  level, regulated companies can (have the right to) sell at  $p^{min}$  their extra permits. This optionality can be quantified by summing up the values of all exercised American put options with strike price  $p^{min}$ . However, since the amount of options on offer is unlimited, it is difficult for a policy regulator to foresee the quantity of permits needed to inject into or withdraw from the market when the permit price is respectively above  $p^{max}$  or below  $p^{min}$ . Breaking this hybrid system down into an ordinary system plus free-of-charge American options maturing at compliance time, we argue that it is complex to quantify the amount of exercised American options a priori. Let  $N_{t-}$  and  $N_t$  be, respectively, the amount of outstanding permits before ( $t-$ ) and after ( $t$ ) the intervention of the policy regulator on the market for permits. Let  $\alpha_t = N_t - N_{t-}$  denote the amount of permits added ( $\alpha_t > 0$ ) or subtracted ( $\alpha_t < 0$ ) to the market at time  $t$ . At each instant of time  $t = 0, \dots, T$  we can identify three possible situations:

1. If the permit price is between the price collar,  $F(t, T) \in (p^{min}, p^{max})$ , then  $\alpha_t = 0$  and there is no market intervention by the regulator on the amount of outstanding permits, i.e.  $N_t = N_{t-}$ .
2. If the permit price exceeds  $p^{max}$ , the policy regulator is then ready to supply an unlimited amount of additional permits. This means that regulated companies that buy permits at the price ceiling are in fact exercising American call options with a strike price  $p^{max}$ . Therefore, relying on standard arbitrage arguments, the theoretical amount of permits  $\alpha_t > 0$  (corresponding to the exercised amount of American call options) that drives the market price of permits back to  $p^{max}$  is:

$$P \cdot \mathbb{P}(q_{[0, T]} > N_{t-} + \alpha_t | \mathcal{F}_t) = p^{max}$$

The rationale behind this equality is based on a standard supply-demand mechanism: a larger supply of permits increases the downside pressure on the permit market price. However, as described below, the extra amount  $\alpha_t$  has further potential side effects.

3. If the permit price drops below  $p^{min}$ , the policy regulator is then ready to buy an unlimited amount of permits at the price floor. This means that regulated compa-

nies that sell permits at the price floor are exercising American put options with strike price  $p^{min}$ . Similarly to the previous case, and relying on the same arbitrage arguments, the theoretical amount of permits  $\alpha_t < 0$  (corresponding to the exercised amount of American put options) that drives the market price of permits up to  $p^{min}$  is:

$$P \cdot \mathbb{P}(q_{[0,T]} > N_{t-} + \alpha_t | \mathcal{F}_t) = p^{min}$$

The supply-demand mechanism is exactly the same, but works in the opposite direction.

In an ordinary emission trading system, Equation (1) shows the manifest relationship between the permit price and cumulative emissions. As such, a desirable feature of the price of emission permits is that they convey most of the relevant information concerning expectations of the market about the cumulative emissions of relevant companies. This is the basic rationale behind market-based instruments: the market sets the price for scarce resources. Based on this concept, Gröll and Kiesel (2009) justify the permit price slump in 2006 in the EU ETS market that followed the publication of the verified emission data by the European Commission.<sup>9</sup> Intuitively, the (unknown) amount  $\alpha_t$  has a clear impact on such a price formation mechanism. Blending in with expectations on cumulative emissions, the extra stochastic factor  $\alpha_t$ , enhances uncertainty on the supply side and, consequently, on the permit price level. When  $F(t, T) > p^{max}$ , the amount of additional permits  $\alpha_t$  that would drive the permit price back below the price ceiling is unknown prior to the compliance time. In practice regulated companies will never exercise their American call options before maturity.<sup>10</sup> The rationale behind such a strategy is based on the fact that companies do not physically need the permits to produce and, more importantly, they have to achieve compliance only at time  $T$ . As in the case of an American call option written on a financial underlying that pays no dividends, it is never optimal to exercise American call before maturity. Similarly, when  $F(t, T) < p^{min}$ , an advisable trading strategy for regulated companies is to wait until the permit price is sufficiently small, and then exercise their American put options. In such a way their put options will be more valuable or, in the financial terminology, deep in the money. So, in contrast to American call options, it makes sense to exercise American put options before maturity. Yet, the amount  $\alpha_t$  is hardly foreseeable a priori as it depends on the option-exercising strategies of regulated

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<sup>9</sup>The sudden expectation of a permit market severely in excess of permits caused an immediate price adjustment and, backed by the banking limitations in phase I, accelerated the price decrease.

<sup>10</sup>Because regulated companies never exercise their American call options prior to maturity, the penalty level is effectively reduced from  $P$  to  $p^{max}$ .

companies.

Therefore, a possible side effect of  $\alpha_t$  is intimately related to a larger uncertainty level about the net amount of permits available on the market. When  $F(t, T) > p^{max}$ , relevant companies with extra permits would be better off by selling their permits as soon as possible, before the regulator intervenes on the market. This action, in addition to the extra permits offered by the regulator, might result in an excessive over-supply and, consequently, in a permit price collapse. Otherwise, these companies might prefer to hold on to their permits, and wait for market price developments. This action might result in a severe decrease of permit trading volumes, possibly leading to a deadlocked market. A similar situation might occur when  $F(t, T) < p^{min}$ . Relevant companies would be better off holding on to their permits while the permit price stays below the price floor. Because the price floor corresponds to American put options, as time goes by the situation cannot get worse. If the price does not recover above  $p^{min}$ , American put options will be exercised for a (guaranteed minimum) strike price equal to  $p^{min}$ . Either way,  $F(t, T) > p^{max}$  or  $F(t, T) < p^{min}$ , the permit price will no longer reflect the real expectations of the market about the cumulative emissions of relevant companies.

The expected enforcement costs for regulated companies in a hybrid system with a price collar are lower than in an ordinary system. Intuitively, this system corresponds to an ordinary scheme with American call and put options with strike price  $p^{max}$  and  $p^{min}$ , respectively. Unfortunately, the difference in the expected enforcement costs is hardly quantifiable a priori because the regulator offers an unlimited number of additional permits at the price ceiling. However, offering an unlimited number of American call options does not result in a financial burden for the regulator. When an American call option is exercised, the regulator creates the corresponding permit (loosening its original environmental targets) and sells it for  $p^{max}$ . Conversely, the regulator faces a financial burden by offering American put options for free. When an American put option is exercised, the regulator buys back permits (leaving unaffected its original environmental targets) at a price  $p^{min}$ . As with expected enforcement costs, the determination of an priori financial burden for the regulator is then quite complicated. As the policy regulator can at most buy back the total amount of initial permits, the lower bound of the cost of this hybrid scheme can be trivially quantified as  $N \cdot p^{min}$ .

The price collar is a hybrid system whose objectives (setting a minimum and a maximum permit price) are always achieved. Therefore, the expected enforcement costs for regulated



companies compared to an ordinary system are lower. However, this scheme has three major disadvantages. First, after the regulator market intervention, the permit price no longer reflects the real market expectations on the cumulative emissions of relevant companies. Second, the policy regulator might face severe expenses that are unquantifiable a priori or, conversely, its original environmental targets might be significantly loosened. This last consequence might be difficult to justify to public stakeholders.

## 2e Allowance Reserve

Another common mechanism proposed by economists to manage the economic (and unpopular) consequences of excessively high permit prices is to set a permit (or allowance) reserve.<sup>11</sup> This hybrid scheme has again been proposed by Murray et al. (2009).<sup>12</sup> The allowance reserve is very similar to the mechanism of the price collar. The main difference is that the maximum amount of permits available in the market equals  $N^{max}$ . In other words, the regulator sets the allowance reserve  $\eta$  equal to  $N^{max} - N$ , where  $N^{max} > N$ . Similar to the price collar, the allowance reserve can be broken down into an ordinary cap-and-trade system and a limited sum of free-of-charge American-style call options. In practice, when the permit price moves above a pre-specified  $p^{max}$  level, regulated companies can (have the right to) purchase permits at  $p^{max}$  up to a limited amount  $\eta$ . This optionality can be quantified as the value of  $\eta$  American call options with strike price  $p^{max}$ .

Unlike the price collar, the finite nature of the reserve  $\eta$  cannot guarantee the price ceiling once the reserve has been completely exploited. As opposed to the previous hybrid system, the limitation in the available extra amount of permits allows us to quantify expected enforcement costs. In particular, the difference between the expected enforcement costs of an ordinary system and the hybrid system with allowance reserve equals:

$$EEC - EEC^{AR} = (N^{max} - N) P^c \geq 0, \quad (8)$$

where  $P^c$  is the price of an American call option with strike price  $p^{max}$ . We can quantify an upper bound for the difference of the expected enforcement costs relying on the fact that  $P^c \leq P - p^{max}$ :

$$EEC - EEC^{AR} = \eta \cdot P^c \leq \eta \cdot (P - p^{max}).$$

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<sup>11</sup>Here we consider situations, where the permit reserve is solely employed to control excessively high permit prices.

<sup>12</sup>For a comprehensive discussion of the merits of the allowance reserve, we refer to Murray et al. (2009).

The smaller the price ceiling, the lower the expected enforcement costs of this hybrid system. Conversely, and unsurprisingly,  $EEC^{AR} = EEC$  when  $p^{max}$  tends to the penalty level  $P$ .<sup>13</sup>

The major disadvantage of the allowance reserve is its inability to guarantee the price ceiling once the reserve has been completely exploited. Similar to the price collar, the intervention of the regulator on the market affects the expectations of market participants regarding the cumulative emissions of relevant companies. Finally, in order to implement this scheme and partially lower the expected enforcement costs for regulated companies, the policy regulator faces new costs. Unlike the price collar, these costs are bounded. Yet, price control is possible at the expense of original environmental targets.

## 2f Plain-vanilla Options offered by the Regulator

The final mechanism under investigation concerns the offering of European- and American-style options at the inception of the compliance period for a certain price. This hybrid scheme has been proposed by Unold and Requate (2001), although they do not specify the type of options under discussion. This mechanism is closely related to the previous mechanisms (the price floor with a subsidy, the price collar and the allowance reserve). Accordingly, all these mechanisms belong to the group of hybrid systems that rely on the faculty of the policy regulator to create or withdraw permits. As described in Section 2c, a price floor which has been enforced using a subsidy is equivalent to an ordinary cap-and-trade system coupled with European put options. The price collar and the allowance reserve described in Sections 2d and 2e can be broken down into an ordinary system coupled with an unlimited or limited amount of American-style options. By offering standard American and/or European options at the beginning of the compliance period, a policy regulator can replicate the results enforced by a subsidy, or a price collar, or an allowance reserve. More importantly, this mechanism avoids the undesirable manipulation of expectations about the net amount of emission permits which is caused by the other hybrid systems. Clearly, as in any standard financial market, an extremely large amount of outstanding options, perhaps concentrated in the hands of few companies, might result again in undesired market price manipulation. Such an event, however unlikely, can be prevented by the policy regulator employing necessary corrective actions, such as screening options buyers.

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<sup>13</sup>This corresponds to the case discussed by Jacoby and Ellerman (2004).

Under the assumption that the regulator offers the options at a fair market price, the expenses borne by the regulator to implement this scheme are zero, as concluded by Unold and Requate (2001).<sup>14</sup> Furthermore, the permit price bounds are guaranteed for regulated companies that require this protection and are willing to pay for this optionality. Yet, the price of emission permits reflects the real expectations of market participants about the cumulative emissions of relevant companies.

### 3 Conclusions

Using a stylized equilibrium permit price, we analyze five different cap-and-trade schemes characterized by specific price mechanisms. These hybrid systems are implemented by the policy regulators in order to prevent the permit price from rising too high or falling too low. By distinguishing those mechanisms that employ offsets (so-called safety-valve) from those that rely on the ability of the policy regulator to control permit quantity (price floor with a subsidy, price collar, allowance reserve, standard options) we quantify their impact on the equilibrium permit price. Price bounds of emission permits can be always guaranteed in a hybrid system with a subsidy or in a system with price collar. A system where the regulator sells options to regulated companies guarantees price bounds for those companies that are willing to pay for such a protection. The two other systems under study (safety-valve with offset and allowance reserve) cannot guarantee that the permit price will be capped under all possible circumstances.

An interesting result of this paper is that all the hybrid systems that we have investigated, with the exception of the safety-valve, can be translated into an ordinary cap-and-trade scheme combined with European-style put options (price floor with a subsidy); with an unlimited amount of American-style call and put options (price collar); with a limited amount of American-style call and put options (allowance reserve); with a limited amount of European- and American-style call and put options (standard options offered by the regulator).

Employing such a breakdown, we assess the price collar and the allowance reserve to be unable to guarantee a reduction in the volatility of the market price of emission permits.

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<sup>14</sup>More precisely, the policy regulator bears the typical risks related to writing option contracts. As a consequence, Unold and Requate (2001) raise the delicate question of whether the state or a private institution should offer these options.

On the contrary, they might enhance the permit price volatility. The intuition behind such a result is that, after the intervention of the policy regulator on the permit market, the price of emission permits will not reflect the real expectation of market participants regarding the net cumulative emissions of relevant companies. More precisely, the unknown quantity of permits released into or withdrawn from the market alters this information.

Moreover, because the success of cap-and-trade schemes depends significantly on the enforcement structure, we systematically compare the expected enforcement costs of each hybrid scheme to the enforcement costs of an ordinary scheme. We show that all proposed hybrid systems reduce the expected economic burden of the cap-and-trade for relevant companies. In implementing these schemes the regulator faces substantial costs (price collar), limited costs (price floor, allowance reserve), or no-costs at all (safety-valve with offsets). At the same time, the original environmental targets are severely loosened (price collar), or lowered (allowance reserve and safety-valve). The hybrid scheme with standard options maintains the environmental targets under control but does not impose extra costs on the policy regulator.

A cap-and-trade system where plain-vanilla options are available (written by either the policy regulator or by private institutions) can replicate the intentional results of the hybrid systems under investigation, while at the same time avoiding their undesirable effects. We recommend, therefore, implementing an ordinary cap-and-trade system such as that described in Section 2a where private institutions write options on permits. The challenge in the coming years will be the creation of properly designed option contracts on emission permits backed by sufficiently liquid option markets. Further studies on the ability of financial options to offer a real hedge against the risk of compliance is in our agenda for future research.

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Scheme	Price bound	Prices can exceed bounds	Link with offsets market	Description of the mechanism
<b>Existing cap-and-trade scheme</b>				
Offset safety-valve	Upper	Yes	Yes	Flexible limit on the use of offsets
<b>Proposed safety-valve mechanisms for cap-and-trade schemes</b>				
Subsidy price floor	Lower	No	No	Subsidy
Price collar	Upper & Lower	No	No	Regulator sells unlimited amount of permits at the price ceiling and buys unlimited amount of permits at the price floor
Allowance reserve	Upper & Lower	Yes	No	Regulator sells limited amount of permits at the price ceiling and buys limited amount permits at the price floor
Regulator offers options	Upper & Lower	No (for owner of options)	No	Regulator sells options at a market price

Table 1: Overview on the main results of the mechanisms under investigation in this paper and description of how they work in practice.

Mechanism	Advantages	Disadvantages
Offset safety valve	<ul style="list-style-type: none"> <li>(a) Relatively simple to implement</li> <li>(b) Lower expected enforcement costs for regulated companies than in an ordinary cap-and-trade system</li> <li>(c) Regulator faces no financial burden</li> </ul>	<ul style="list-style-type: none"> <li>(a) Price ceiling is not guaranteed under all circumstances</li> <li>(b) Creates uncertainty on the projects for active emission reduction</li> <li>(c) Weakens the pressure for actions within the system, i.e. environmental targets are not ensured</li> </ul>
Subsidy	<ul style="list-style-type: none"> <li>(a) Relatively simple to implement</li> <li>(b) Reduces investment uncertainty under all circumstance</li> <li>(c) Stimulates reduction efforts in the system</li> </ul>	<ul style="list-style-type: none"> <li>(a) Regulator might face a significant financial burden whose size is hardly quantifiable a priori</li> </ul>
Price collar	<ul style="list-style-type: none"> <li>(a) Price collar is guaranteed under all circumstances</li> <li>(b) Lower expected enforcement costs for regulated companies than in an ordinary cap-and-trade system</li> </ul>	<ul style="list-style-type: none"> <li>(a) Permit prices do not reflect real expectations on the level of cumulative emissions after market intervention. The permit price volatility is not necessarily reduced</li> <li>(b) Regulator might face a significant financial burden when the price floor is reached</li> <li>(c) Regulator cannot plan the size of the financial burden and when the cash outflows will occur</li> <li>(d) Environmental targets are loosened when the price ceiling is reached.</li> </ul>
Allowance reserve	<ul style="list-style-type: none"> <li>(a) Compared to price collar, environmental target is only weakened up to a certain level</li> </ul>	<ul style="list-style-type: none"> <li>(a) Price bounds cannot be guaranteed under all circumstances</li> <li>(b) Drawbacks of price collar (see above)</li> </ul>
Regulator offers options	<ul style="list-style-type: none"> <li>(a) Regulator faces no financial burden</li> <li>(b) Price bounds are guaranteed for those companies willing to pay for these options</li> <li>(c) Environmental targets are not affected</li> </ul>	<ul style="list-style-type: none"> <li>(a) Policy regulator bears the price risk of the options written</li> </ul>

Table 2: Advantages and disadvantages of the different schemes under investigation.



Mechanism		Corresponds to a combination of an ordinary cap-and-trade system and
Subsidy	-	Free of charge European-style put option with strike price equal to the price floor offered for free
Price collar	-	Free of charge American-style call option with strike price equal to the price ceiling (unlimited amount) Free of charge American-style put option with strike price equal to the price floor (unlimited amount)
Allowance reserve	-	Free of charge American-style call option with strike price equal to the price ceiling (limited amount) Free of charge American-style put option with strike price equal to the price floor (limited amount)
Options	-	European-style or American-style put and call options offered at a certain price

Table 3: Disentangling the schemes under investigation into an ordinary cap-and-trade system and standard financial type of options.