CP violation in K decays: experimental aspects

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Abstract
CP violation was originally discovered in neutral K mesons. Over the last few years, it has also been seen in B mesons, and most of the research in the field is currently concentrating on the B system. However, there are some parameters which could be best measured in kaons. In order to see to which extent our present understanding of CP violation within the framework of the CKM matrix is correct, one has to check for possible differences between the K system and the B system. After an historical overview, I discuss a few of the most important recent results, and give an outlook on experiments that are being prepared.

1 The discovery of CP violation

Symmetries are a salient feature of our world, but so is the breaking of approximate symmetries. Still, for a long time physicists believed that at the level of elementary particles, a high level of symmetry should prevail. In particular, it was expected that all fundamental interactions should be symmetric under the discrete transformations of spatial inversion (parity transformation P), substitution of antiparticles for particles (charge conjugation C), and time inversion (T). However, in 1956 Lee and Yang concluded from experimental data that the weak interactions might not be invariant under spatial inversion, in other words that parity might be violated. This was then explicitly shown in an experiment by Wu in 1957 [1].

For a few years, physicists were inclined to believe that although parity was broken, this symmetry violation was exactly compensated by the charge symmetry violation, and that the symmetry under a combined charge and parity transformation (CP) was exactly conserved. An obvious example was the helicity of the neutrino, which was always observed to be negative ('left-handed neutrino'). Parity transformation would transform it into a right-handed neutrino, which has not been observed in nature. In other words, it appears that parity is maximally violated. However, by performing charge conjugation in addition, one arrives at the right-handed anti-neutrino, which does exist in nature.

However, only a few years after the discovery of parity violation, it turned out that this so-called 'CP symmetry' was also violated, although to a much smaller extent than parity itself. In an experiment carried out at the Brookhaven Alternating Gradient Synchrotron (AGS), Christenson, Cronin, Fitch and Turlay found out that the longer-lived of the two neutral kaons, the K_L^0, which frequently decays into three pions and should therefore be assigned odd parity, in rare cases decayed into a parity-even two-pion state [2]. For some physicists, this was hard to believe and a number of possible explanations were looked at [3] before it was accepted that CP had to be broken at the per mil level.

While at first CP violation was regarded by some physicists as a sort of unwelcome guest and an unnecessary complication of nature, it later turned out that it is in fact vital for our very existence! According to the Big Bang model of the origin of the universe, particles and antiparticles were at first produced in equal numbers. We know that at present, however, the universe contains almost no antimatter. How could matter survive and not be annihilated right away by antimatter, in which case the universe would now be a fairly dull place made up largely of photons, without much of a structure and without physicists wondering about it? In 1967 Andrei Sakharov found three necessary conditions for creating such a 'baryon asymmetry' [4]:

– baryon number violation
– absence of thermal equilibrium
– CP violation.

While this in itself is certainly a very good reason to study CP violation, it must be said that the effects that have been found in the K and now also in the B system are far too weak to explain the size of the baryon asymmetry in the universe, and physicists are looking for new, stronger sources of CP violation.

After the experimental discovery of CP violation, various theories were developed, one of the first being the so-called 'superweak' theory developed by Lincoln Wolfenstein [5]. This theory introduced a fifth fundamental interaction (the 'superweak interaction') on top of the four known interactions: gravity, electromagnetism, strong interaction, and weak interaction. It is interesting to note that this theory was published less than two months after the publication of the experimental discovery but it took 35 years to decide by experiment if it really gave a satisfactory description of CP violation in nature. The paper concluded with the following remark:

"The most interesting point of the model discussed here lies in the possibility that the experiment ... may measure an interaction as much as 10^7 or 10^8 times weaker than the standard weak interactions. If this is the case it may prove extremely difficult to observe CP violation (or T violation) in independent ways."

Almost nine years went by before Makoto Kobayashi and Toshihide Maskawa discovered that CP violation could be described in an organic, natural way by a theory with at least three quark generations [6]. In a model with three generations, there is one physical complex phase (i.e., a phase which cannot be made to disappear, or 'rotate away', by phase conventions), and it is this very phase which gives rise to CP violation.

Soon this model was preferred by most theorists on 'aesthetic' grounds but it was not so easy to decide between these theories by experiment. In the 'superweak' theory CP violation is caused exclusively by state mixing, where the K_L^0 meson consists mostly of the CP-odd state K_2 but has a tiny admixture $\tilde{\epsilon}$ of the CP-even state K_1 while the K_S^0 meson corresponds to the CP-even state K_1 with only a small admixture of the K_2 state:

$$|K_L> \ \approx \ |K_2> + \tilde{\epsilon}|K_1> \tag{1}$$

$$|K_S> \ \approx \ |K_1> + \tilde{\epsilon}|K_2> \tag{2}$$

where

$$|K_1> \ = \ \frac{1}{\sqrt{2}}(|K^0> + |\bar{K}^0>) \tag{3}$$

and

$$|K_2> \ = \ \frac{1}{\sqrt{2}}(|K^0> - |\bar{K}^0>)\ . \tag{4}$$

If CP violation is, however, caused by the phase of the three-generation quark mixing matrix (which is now known as the Cabibbo–Kobayashi–Maskawa or CKM matrix) there should also exist a 'direct' violation of CP in the decay amplitude itself. From experiments it soon became clear that this effect had to be even much more suppressed than the CP violation due to state mixing.

2 The quest for direct CP violation

2.1 Direct CP violation in two-pion decays

If CP violation were exclusively due to state mixing as in Eq. (1), the amount of CP violation in any decay would only be determined by the mixing parameter $\tilde{\epsilon}$ and would therefore be the same for all decay channels. So, in the case of two-pion decays, the ratio of the CP-violating and the CP-conserving decay rates would be the same for the charged decay channel ($K^0 \to \pi^+\pi^-$) and for the corresponding neutral channel ($K^0 \to \pi^0\pi^0$). In other words, we should find

$$\frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^0\pi^0)} = \frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)} \ . \tag{5}$$

In case of 'direct' CP violation in the decay amplitudes, however, this would not have to be so. The strength of direct CP violation is usually parametrized by a parameter ϵ' which can be obtained from the 'double ratio' R defined by the following equation:

$$R = \frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^0\pi^0)} / \frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)} = 1 - 6 \times \mathrm{Re}(\epsilon'/\epsilon) \ . \tag{6}$$

Here, ϵ is a measure of mixing-induced CP violation and related to the mixing parameter $\tilde{\epsilon}$ introduced in Eqs. (1) and (2) by

$$\epsilon = \tilde{\epsilon} + i\frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} \tag{7}$$

where A_0 is the isospin = 0 amplitude of the $K^0 \to \pi\pi$ decay.

Experimental results soon showed that the 'double ratio' R was very close to unity, and ϵ' had to be much smaller than ϵ. So, R had to be measured with very high precision, and this could only be obtained in a *relative* measurement of the four decay rates entering Eq. (6), which would allow one to reduce the systematic error. Over many years, a series of competing experiments tried to reach the precision needed for establishing a non-zero value of Re(ϵ'/ϵ). At the beginning of the 1990s, the NA31 experiment at CERN had found a more than three-sigma deviation of Re(ϵ'/ϵ) from zero (Re(ϵ'/ϵ) $= (23.0 \pm 6.5) \times 10^{-4}$, Ref. [7]) while the E731 experiment at Fermilab had measured a value compatible both with zero and with NA31 (Re(ϵ'/ϵ) $= (7.4 \pm 5.9) \times 10^{-4}$, Ref. [8]).

In the hope of finally finding a sign of direct CP violation, both laboratories built still more refined experiments: NA48 at CERN and KTeV at Fermilab. In order to minimize systematic errors due to acceptance or changes in the detector over time, both experiments aimed at measuring the four decay rates simultaneously in the same apparatus.

A fundamental problem in this measurement is the fact that K_L^0 and K_S^0 cannot be produced separately. At an accelerator, kaons are produced in strong interaction processes, and the eigenstates of neutral kaons from the point of view of strong interactions (the strangeness eigenstates) are not K_L^0 and K_S^0 but their linear combinations K^0 and $\bar{K}^0$. So, equal amounts of K_L^0 and K_S^0 are created. The large difference in lifetime (the K_L^0 lives 580 times longer than the K_S^0) allows, however, to obtain strongly enhanced samples of K_L^0 or K_S^0 decays.

Figure 1 shows the setup of the NA48 experiment [9]. At a first target, 450-GeV protons produce neutral kaons along with other particles. Charged particles are deflected by magnets while neutral particles continue along the beamline over 120 m. Most K_S^0 particles decay here while the K_L^0 mesons pass a final collimator and enter the fiducial volume of the experiment, which is observed by the detector. A proton beam of relatively low intensity continues along the same axis as the neutral kaon beam. Shortly before the collimator, the protons are deflected onto a second target where again neutral kaons (and other particles) are produced, which enter the detector's fiducial volume through a collimator close to the K_L^0 collimator. Within the fiducial volume of the detector, most of the K_S^0 mesons but only a tiny fraction of the K_L^0 mesons from this second beam decay. By detecting the individual protons which go to the

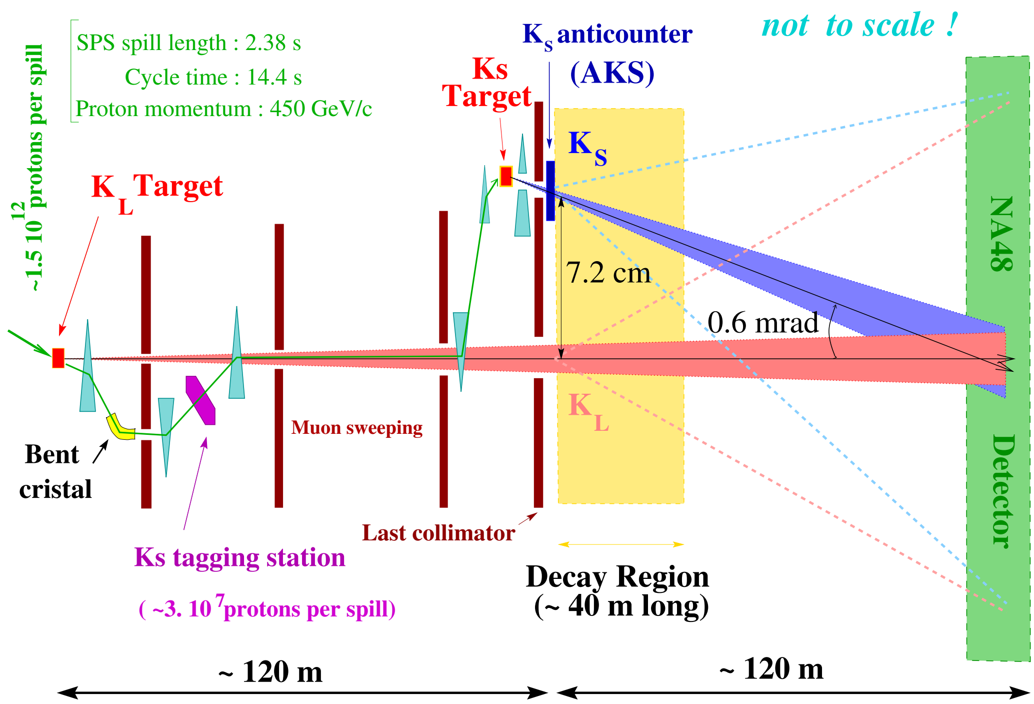

Fig. 1: Setup of the NA48 experiment

second target (by 'tagging' the decays), it can be decided if a particular decay stems from a K_L^0 or from a K_S^0 meson.

By identifying decays into two neutral or two charged pions, the decay rates of all four channels in Eq. (6) can be measured simultaneously, and the double ratio and thus the parameter Re ϵ'/ϵ can be computed. The KTeV experiment [10] used a similar setup. Instead of producing kaons at a second target, however, one of two K_L^0 beams hit a regenerator where its K_S^0 content was strongly enhanced.

These two experiments measured a value of Re ϵ'/ϵ about seven sigmas away from zero and thus established the existence of direct CP violation beyond any doubt [10, 9]. So, the CKM matrix and the Standard Model seem to explain the CP violation we observe in the kaon system, and the superweak model is excluded. However, on account of quantum chromodynamics effects it is very hard to calculate a theoretical value for ϵ' and thus to check how well the theory really describes experimental data. Before a major theoretical breakthrough is achieved (which might come from lattice QCD) it does not make much sense to improve the current experimental measurements of ϵ' (see Fig. 2).

2.2 Direct CP violation in three-pion decays of charged kaons

In order to really understand direct CP violation, it is important to also find it in other channels than in the two-pion decay of neutral kaons. One possibility could be the decays of charged kaons into three pions ($K^\pm \to \pi^\pm\pi^+\pi^-$ and $K^\pm \to \pi^\pm\pi^0\pi^0$). Differently from the neutral kaon system, K^+ and K^- cannot mix because of their different charge, and any difference in the decays for positive and negative kaons would be a sign of direct CP violation. The amount of CP violation in this channel as predicted by the Standard Model is very small and hardly measurable at present. However, certain theories have predicted a significant enhancement of the effect, which could be within the reach of present experiments (see Fig. 3).

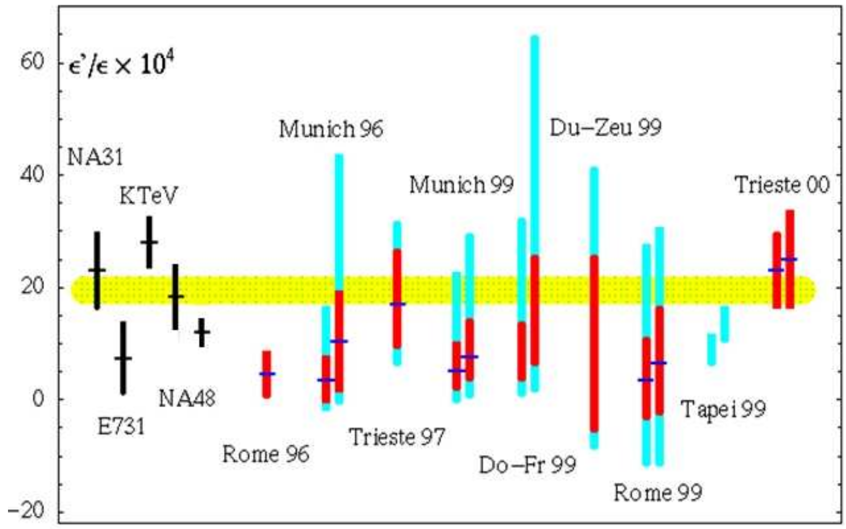

Fig. 2: Measurements of Re ϵ'/ϵ by the four most recent experiments (left) and calculations by various theory groups. The experimental errors are now far smaller than even the most optimistic theoretical errors. Over the last few years, theoretical calculations of Re ϵ'/ϵ have not achieved much progress, and at the moment it would be rather useless to carry out more refined experimental measurements of this quantity.

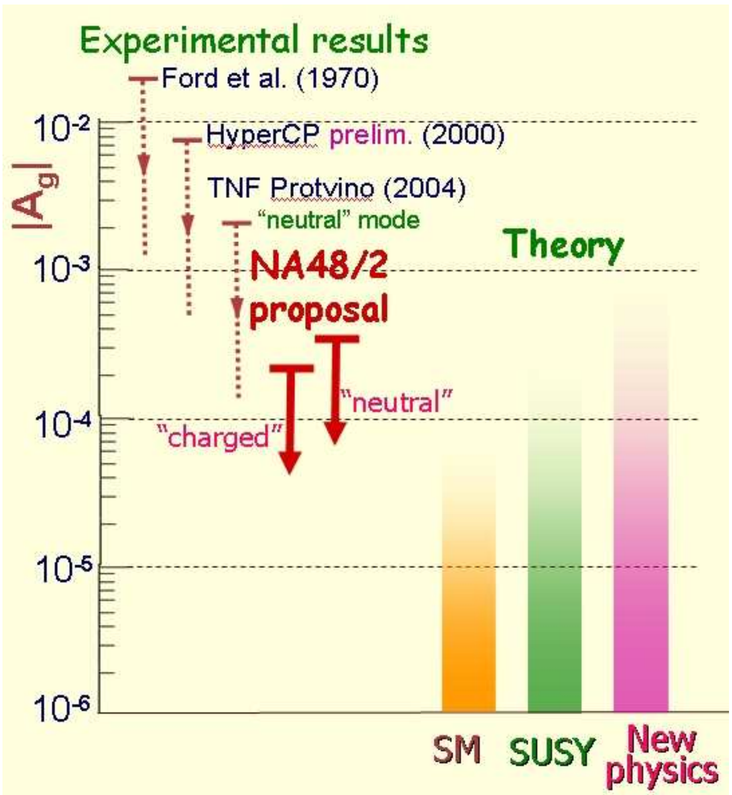

Fig. 3: Experimental limits and theoretical expectations for the size of CP violation visible in the charge asymmetry in the decay of charged kaons to three pions

Measuring a possible difference in the branching ratios of these decays does not look very promising. In all models such differences are predicted to be very small. Moreover, such a measurement would require an accurate knowledge of the kaon flux for both charge signs, which is very hard to achieve from the experimental point of view. However, somewhat larger differences are predicted for the distribution of the decay in phase space (the shape of the Dalitz plot), and these distributions can be measured independently of the kaon flux.

It is usual to parametrize the phase space of the kaon decay products in terms of the Dalitz-plot parameters u and v defined as

$$u = \frac{s_3 - s_0}{m_\pi^2} \qquad\qquad v = \frac{s_2 - s_1}{m_\pi^2} \tag{8}$$

where $s_i = (p_K - p_i)^2$, p_K is the four-momentum of the decaying kaon and p_i are the four-momenta of the pions; p_3 corresponds to the 'odd' pion, i.e., the one that differs in charge from the other two, and $s_0 = (s_1 + s_2 + s_3)/3$. The matrix element can then be expanded as

$$|M|^2 \approx 1 + gu + hu^2 + kv^2 + ... \tag{9}$$

with the linear g term being the dominant one (see Fig. 4).

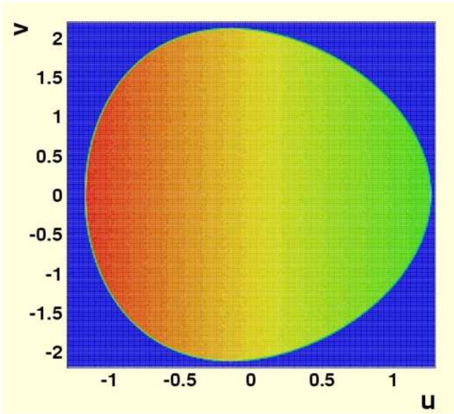

Fig. 4: The Dalitz plot for the decay $K^\pm \to \pi^\pm\pi^+\pi^-$ describes the phase space distribution of the decay products in terms of the kinematic variables u and v defined in Eq. (8)

This allows to define an asymmetry parameter

$$A_g = \frac{g_+ - g_-}{g_+ + g_-} \tag{10}$$

where g_+ and g_- are the values of g measured for positive and negative kaons.

Such measurements have been made by several experiments, the latest and by far the most accurate one being the NA48/2 experiment [11] at CERN. Owing to the smallness of the expected effect a large amount of data must be recorded, and great care has to be taken to minimize systematic errors. As in the case of the measurement in neutral kaons described above, simultaneous K^+ and K^- beams have been used to avoid systematic effects from variations in the detector and the magnetic fields over time. The fields in the beam and spectrometer magnets have been reversed at regular intervals to achieve cancellation of the effects of detector inefficiencies.

So far, part of the data has been analysed and no signal has been seen. The measured value of the asymmetry parameter from the data analysed so far is $A_g = (1.7 \pm 2.9) \times 10^{-4}$ (see Ref. [11]). This is in keeping with the Standard Model but excludes some theories that suggested a possible strong enhancement of the effect.

3 The decay $K_S \to \pi^0\pi^0\pi^0$ and CPT symmetry

Originally, CP violation was discovered in the decay of K_L^0 mesons into two pions. It is harder to study the analogous CP-violating decay of K_S^0 mesons into three pions for two reasons.

Firstly, the K_S^0 meson's decay constant is much larger (the lifetime is much shorter) than for the K_L^0 meson, so that comparable partial decay widths translate into much smaller branching ratios. In other words: while in K_L^0 the CP-violating decay (into two pions) is favoured by phase space over the CP-conserving decay (into three pions), in K_S^0 the decay into three pions is disfavoured both by being CP-violating *and* by the smaller phase space.

Secondly, it is impossible to produce K_S^0 without producing the same amount of K_L^0 mesons at the same time (while a rather pure K_L^0 sample can be obtained by waiting for the K_S^0 component to decay, as in the NA48 experiment described above). In fixed-target experiments, $K_S^0 \to \pi\pi\pi$ decays have been studied by investigating the interference of these decays with the corresponding K_L^0 decays (see Fig. 5). It is, however, also possible to carry out a direct search for such decays by 'tagging' the decaying neutral kaon: if, for example, a ϕ meson decays into two neutral kaons, and one of them is a K_L^0, the other one must be a K_S^0 because the two kaons form an entangled quantum mechanical system. So, a kaon can be identified by measuring the decay of its partner. This method is being used by the KLOE experiment at the DAΦNE e^+e^- collider in Frascati, Italy (see Fig. 6 and Ref. [12]).

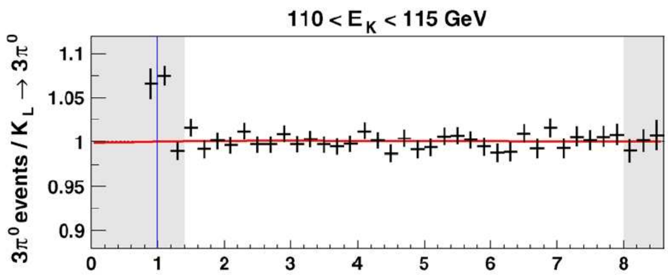

Fig. 5: Ratio of K_S^0/K_L^0 interference data over purely exponentially decaying K_L^0 data (from a target further upstream) as a function of kaon lifetime (in units of the K_S^0 lifetime; points with error bars). The almost constant line is the fit result for the interference signal.

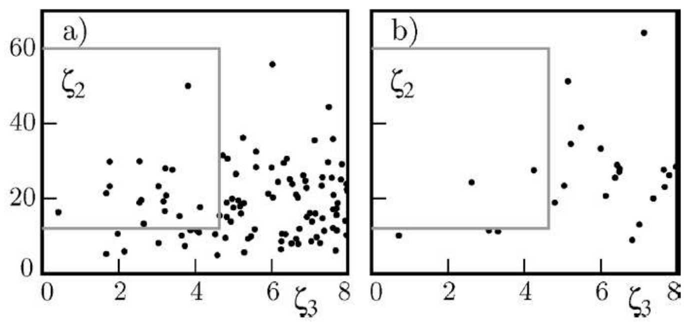

Fig. 6: Events in signal box from the KLOE experiment, where K_S^0 decays are tagged by measuring the K_L^0 which is produced together with the K_S^0. Left: Monte Carlo for 900 pb^{-1}, right: data, 450 pb^{-1}.

The decay $K_S^0 \to \pi^+\pi^-\pi^0$ may be CP violating or CP conserving, depending on the isospin (and thus the angular momentum) in the final state. The CP conserving component of this decay, which is suppressed by the higher angular momentum, has been measured (see, for example, Ref. [13]) and limits for the CP-violating component have been established [14].

For $K_S^0 \rightarrow \pi^0\pi^0\pi^0$, not all isospin states are possible that are accessible to $K_S^0 \rightarrow \pi^+\pi^-\pi^0$, and therefore this decay is always CP violating. It is of particular interest because of the so-called Bell–Steinberger relation which links possible CPT violation in the $K^0\bar{K}^0$ mixing matrix to CP violating amplitudes in K_L^0 and K_S^0 decays via unitarity (conservation of probability, see Ref. [15]). When parametrizing a possible violation of CPT by a parameter δ, this relation states that

$$(1 + i \tan \phi_{SW})[\text{Re}(\epsilon) - i\,\text{Im}(\delta)] = \sum A(K_L \rightarrow f)^* A(K_S \rightarrow f)/\Gamma_S\,. \qquad (11)$$

Here ϕ_{SW} is the so-called 'superweak phase': $\tan \phi_{SW} = \frac{2\Delta m}{\Delta\Gamma}$; Δm and $\Delta\Gamma$ are the differences in mass and decay rate between K_L^0 and K_S^0. For some time, the uncertainty in the right-hand side of this equation was dominated by the uncertainty in the $K_S^0 \rightarrow \pi^0\pi^0\pi^0$ branching ratio.

Of course, there are very good theoretical reasons to believe in CPT symmetry. It is an almost inescapable consequence of Lorentz-invariant quantum field theories. There are, however, ways to theoretically envisage CPT violation, e.g., in superstring theories, which have a fundamentally non-local structure (cf. Ref. [16]). So, the experimental verification of CPT symmetry is not an academic exercise, but an important task!

Recent measurements of the CP-violating parameter

$$\eta_{000} \equiv \frac{A(K_S^0 \rightarrow \pi^0\pi^0\pi^0)}{A(K_L^0 \rightarrow \pi^0\pi^0\pi^0)} \qquad (12)$$

(see Figs. 5, 6, 7, and Refs. [17], [12]) have not allowed one to see the decay $K_S^0 \rightarrow \pi^0\pi^0\pi^0$. In fact, the best limit on its branching ratio ($BR(K_S^0 \rightarrow \pi^0\pi^0\pi^0) < 1.2 \times 10^{-7}$, from Ref. [12]) is still almost two orders of magnitude away from the Standard Model prediction of $BR(K_S^0 \rightarrow \pi^0\pi^0\pi^0) \sim 1.9 \times 10^{-9}$. However, these recent experimental results have significantly reduced the *error* on the branching ratio and thus improved the constraint on CPT violation from the Bell–Steinberger relation. At present, the uncertainty in the right-hand side of Eq. (11) is dominated by the uncertainty on the decay into $\pi^+\pi^-$— the very first decay in which CP violation was seen!

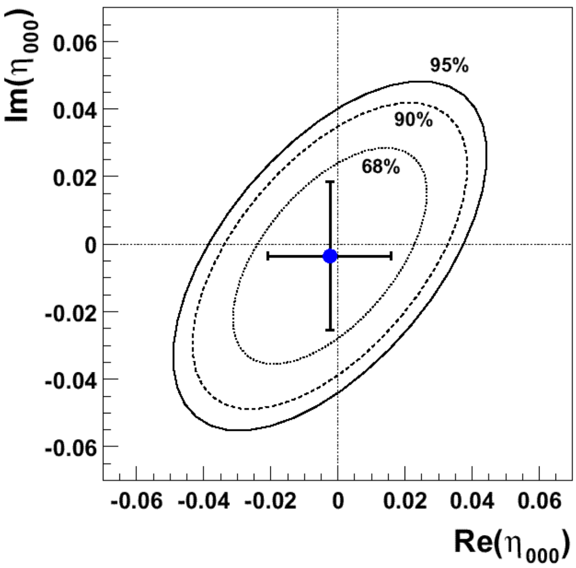

Fig. 7: The experimental result for the parameter η_{000} of the decay $K_S^0 \rightarrow 3\pi^0$ measured by the NA48 experiment

4 Time's arrow: the violation of T symmetry

From daily life, we are used to the fact that time always appears to move in the same direction. Travels into the past seem to be restricted to the realm of science fiction. If they were possible, this would completely upset our notion of causality, our understanding of how the world works. So, although for a long time philosophers have thought that space and time have something in common (cf. Ref. [18]), and this vague feeling has developed into the physical concept of spacetime in special relativity, we are convinced that there is a fundamental difference between space and time: we can move around freely in space, but time always progresses in the same direction, and there is nothing we can do about it.

So it might seem ridiculous to even think about something as symmetry in time. When we watch a film and see fragments of china scattered over the floor and coffee spilt over a carpet, and suddenly the fragments and the drops of coffee fly upwards and assemble into a nice cup with good hot coffee inside while the carpet below turns clean, we will be convinced that the film was taken in the reverse direction. But why is this so? After all, the fragments of the cup and the drops of coffee could move in any direction in space. It is the difference in the initial and final states which creates this asymmetry. For just one initial state of the cup being whole and the coffee being in it, there are billions of states for each fragment, for each drop of coffee being in a different place. One 'macroscopic' state is thus presented by an enormous multitude of different 'microscopic' states. If we accept that each microscopic state is equally likely, it becomes clear why the inverse transition between macroscopic states is never observed. So, the explanation for the obvious arrow of time we experience in everyday life lies in thermodynamics and the increase of entropy and has nothing to do with possible asymmetries in the interactions themselves.

If we watch a game of billiards with just three balls and look at the positions of the balls between the shots, it will not be obvious at all in which direction time is going, although the laws of mechanics should be the same as in the previous example. In fact, what we saw in the first example was not a time asymmetry in the interaction itself, but in the initial and final states. All the configurations of three balls on a billiard table are about equally likely, so that we cannot make out the direction in time in this example.

In fact, if we look at the interactions themselves, Newton's laws are symmetric in time. According to all observations, most of the basic interactions in nature—the strong, electromagnetic, and gravitational interactions—are all time symmetric. What about the weak interactions? For a long time it has been believed that the product of the three transformations of parity (P), particle–antiparticle exchange (C) and time inversion (T), in short 'CPT', is conserved under all interactions. This is true for any kind of interaction in a relativistic field theory (see Ref. [19]). Although recently it has been envisaged that CPT invariance might still be violated on a very small scale, it is an experimental fact that it is conserved to a very good approximation. So, CP violation should entail T violation, and one may say that in this indirect way, T asymmetry in weak interactions was discovered back in 1964 when CP violation was first observed.

Still, if weak interactions are really not symmetric under T, it is desirable to observe this in a more direct way. There are processes between particles whose inverse processes can also be observed. However, it is not straightforward to demonstrate T violation in this way. One major problem is due to finite-state interactions which may exist between the particles that are produced in the process. So, if in the decay of a particle more than one hadron is produced, they will interact strongly while they are sufficiently close to each other, thus influencing the rate of the decay. Likewise, charged particles produced in a decay or an interaction will continue to interact electromagnetically even at a distance, again influencing the rate for the process in question. So, different rates may be observed when looking at a process and its inverse, but this difference is not necessarily due to the basic interaction itself [20].

Again, neutral kaons have allowed us to carry out the first unequivocal measurement of T violation. $K^0 \leftrightarrow \bar{K}^0$ oscillations may serve to compare a process with its inverse, by comparing the number of neutral kaons that are created as K^0 and decay as $\bar{K}^0$ with the opposite process. For this, one needs to know the flavour state of the neutral kaon (if it is a K^0 or a $\bar{K}^0$) both at production and at the time ofy decay. As stated above, the production of kaons at an accelerator is mediated by strong interactions, which conserve strangeness. At the CPLEAR experiment at CERN (see Ref. [21]) antiprotons impinged on a hydrogen target and the processes $p\bar{p} \to K^+\pi^-\bar{K}^0$ and $p\bar{p} \to K^-\pi^+K^0$ were selected. As K^+ and K^0 each contain a strange quark while K^- and $\bar{K}^0$ each contain an anti-strange quark, and a strange quark can only be produced together with an anti-strange quark, one can determine the flavour content of the neutral kaon by measuring the charge of the charged kaon.

The selected decay channels were the semileptonic channels $K^0 \to \pi^-e^+\nu$ and $\bar{K}^0 \to \pi^+e^-\bar{\nu}$ (so-called 'Ke3 decays'). Here, the charge of the pion allows one to determine the flavour state of the neutral kaon by means of the so-called 'ΔS = ΔQ rule', which is an expression of the experimental fact that no flavour-changing neutral currents are observed at tree level. This fact has been explained by the so-called 'GIM mechanism', which led to the prediction of the charm quark (see Ref. [22]). When a neutral kaon decays semileptonically, the s quark turns into a u quark (strangeness and charge change by +1), or the $\bar{s}$ quark into a $\bar{u}$ quark (strangeness and charge change by –1). The other quark (the $\bar{d}$ or the d quark) flies on as a 'spectator' (see Fig. 8). Owing to the absence of flavour-changing neutral currents it never happens, however, that the s quark transforms into a d quark (or the $\bar{s}$ quark into a $\bar{d}$ quark).

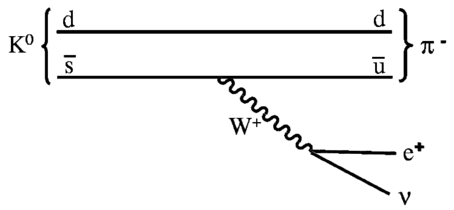

Fig. 8: Semileptonic decay of neutral kaons

So, the flavour of the neutral kaon can be determined both at production and at decay, and the difference in the rates of $K^0 \to \bar{K}^0$ and $\bar{K}^0 \to K^0$ can be measured:

$$A = \frac{R(\bar{K}^0 \to K^0) - R(K^0 \to \bar{K}^0)}{R(\bar{K}^0 \to K^0) + R(K^0 \to \bar{K}^0)} \,. \tag{13}$$

The measurement by the CPLEAR experiment at CERN yielded a value of $A = (6.6 \pm 1.3_{\text{stat}} \pm 1.0_{\text{syst}}) \times 10^{-3}$ (Ref. [21]), and T violation was thus established by a direct measurement, without making use of any assumptions on CPT symmetry.

It is not obvious that the analysis does not rely on implicit assumptions. An in-depth investigation [23] into the theoretical framework has shown that the measurement does not rely on the assumption of general CPT symmetry. It does, however, have to assume that semileptonic kaon decays are CPT symmetric, or that the Bell–Steinberger relation in its conventional form is valid [16]. This relation is a consequence of unitarity if we assume that all relevant kaon decay channels are known. As the experimental error on the branching ratios is of the order of a per cent, it would in principle be possible (although this may seem unlikely) that there exist hitherto unobserved decays with a branching fraction of 10^{-3}. Although this may appear somewhat far-fetched, this possibility is not necessarily more exotic than the possibility of CPT violation.

There are decays which show T-odd correlations between variables, which might be interpreted as a sign of T violation. One of them is the decay $K_L \to \pi^+\pi^-e^+e^-$, where a strong T-odd correlation has been measured [24]. However, the interpretation of this effect as T violation relies on the assumption of CPT conservation [16] although this is not as obvious as when simply deducing T violation from CP violation. Another experiment, E246 at KEK (Japan), is looking for T violation in the decays $K^+ \to \pi^0\mu^+\nu$ ('$K\mu3$') and $K^+ \to \mu^+\nu\gamma$ ('$K\mu2\gamma$'; see, for example, Ref. [25]). The expected Standard Model branching ratio is small and the signal has to be seen against the background from electromagnetic final-state interactions. However, non-Standard-Model mechanisms of CP violation could possibly lead to a strong enhancement of the effect. So far, no signal has been seen.

5 Rare kaon decays: hard to measure but easier to calculate

As stated above in the discussion of the ϵ'/ϵ measurement, it is not always straightforward to derive a theoretical Standard Model prediction of decay rates. While the basic weak-interaction processes are thought to be under control, it is well known that strong interactions between the decay products give rise to large corrections which are very hard to calculate (cf. Ref. [26]). The Standard Model with three generations of quarks whose coupling is described by the Cabibbo–Kobayashi–Maskawa matrix, yields a plausible description of the phenomena of CP violation we have discovered. However, in most cases technical difficulties in the calculations do not allow us to make accurate predictions, so that possible limitations of the Standard Model that would require modifications in the theory may escape us. This is, however, exactly what physicists are looking for. There are good reasons to believe that the Standard Model cannot be ‘the whole truth’ and that there must be some sort of ‘New Physics’. (With regard to CP violation, the observed baryon asymmetry in the universe discussed in Section 1 is one of these reasons.)

There are, however, a few rare kaon decays that can be calculated with much better accuracy. These decays feature only one strongly interacting particle in the final state, so that QCD corrections play a much smaller role. For the very same reason, and because of their small branching ratios, they are problematic from the experimental point of view. Their accurate measurement will be the main target of kaon physics over the coming years.

5.1 $K^0 \rightarrow \pi^0 l^+ l^-$

An accurate experimental determination of the directly CP-violating component of the decay $K_L^0 \rightarrow \pi^0 e^+ e^-$ (or $K_L^0 \rightarrow \pi^0 \mu^+ \mu^-$) would yield a value for the height of the so-called ‘unitarity triangle’ (designated by η), which is a measure of the overall strength of CP violation (see Fig. 9). One complication consists in the fact that these decays also have a CP-conserving part and an indirectly CP-violating component due to state mixing. The CP-conserving component is predicted by theory with good accuracy by making use of experimental data on the decay $K_L^0 \rightarrow \pi^0 \gamma\gamma$. For the electronic mode ($K_L^0 \rightarrow \pi^0 e^+ e^-$) it is negligible. The indirectly CP-violating contribution can be obtained by measuring the same decay for K_S^0 mesons.

The measurement of $K_L^0 \rightarrow \pi^0 e^+ e^-$ itself is complicated by the large background from $K_L^0 \rightarrow \gamma\gamma e^+ e^-$, whose branching ratio is $(5.95 \pm 0.33) \times 10^{-7}$ (Ref. [27]), while for $K_L^0 \rightarrow \pi^0 e^+ e^-$ the Standard Model predicts a branching ratio of only $10^{-12} - 10^{-11}$ (in some SUSY scenarios it could be up to 10^{-10}). As the π^0 decays almost instantaneously into two γ's, both decays show the same particles in the final state. Of course, one expects the invariant mass of the two γ's in the signal channel to be close to the known mass of the π^0, but due to the much higher rate of the background channel there may be some events in it where this also happens by accident (see Fig. 10). Simulation studies predict somewhat different distributions in a few kinematic variables for signal and background events but this is of limited use in a very rare decay where one might find only a handful of events. The best measurement so far found a number of events consistent with background expectations and allowed one to set an upper threshold on the branching ratio: $BR(K_L^0 \rightarrow \pi^0 e^+ e^-) < 2.8 \times 10^{-10}$ (see Ref. [28] and Fig. 11).

What has been measured are the (not so strongly suppressed, CP conserving) decay rates for the corresponding K_S^0 decays. The branching ratios are $BR(K_S^0 \rightarrow \pi^0 e^+ e^-) = (5.8^{+2.9}_{-2.4}) \times 10^{-9}$ (Ref. [29]) and $BR(K_S^0 \rightarrow \pi^0 \mu^+ \mu^-) = (2.9^{+1.5}_{-1.2}) \times 10^{-9}$ (Ref. [30]). As one sees from the number of identified signal events (seven events for the $K_S^0 \rightarrow \pi^0 e^+ e^-$ channel, see Fig. 12) this was not an easy measurement either, although the branching ratio is at least one order of magnitude higher than that of the corresponding K_L^0 decay.

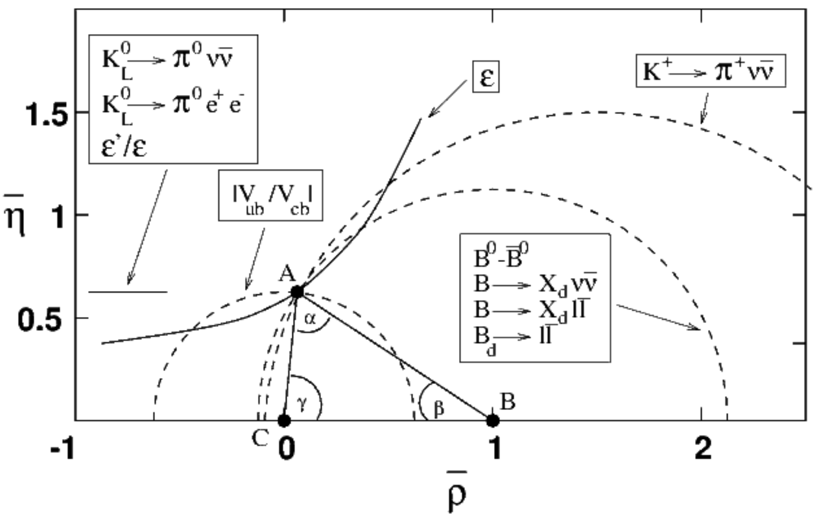

Fig. 9: The unitarity triangle and the various experimental ways to measure its parameters. If one assumes the CKM matrix to be unitary and multiplies it with its Hermitian conjugate, the off-diagonal elements must be zero. In the matrix multiplication, this means that certain sums of three products of CKM matrix elements add up to zero. When graphically representing these products as vectors in the complex plane, this yields a triangle, the so-called 'unitarity triangle'. By choosing the appropriate phase and normalization, two of its end points can be made to lie at (0,0) and (1,0) and the experimentalist's task is to determine the position of the third end point, the triangle's tip. Of course, if the CKM theory is not completely correct, the triangle may not close, and various measurements—in particular those derived from K physics and those derived from B physics—may yield contradictory results. Therefore it is very important to 'overconstrain' the unitarity triangle.

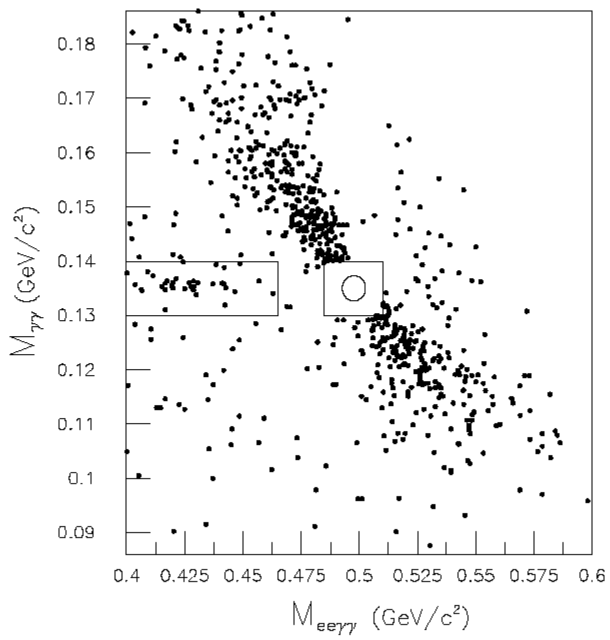

Fig. 10: The invariant mass of the photon pair (vertical axis) against the invariant $ee\gamma\gamma$ mass (horizontal axis). For K^0_L decays where only two electrons and two photons are produced, the invariant $ee\gamma\gamma$ mass should be close to the K^0_L mass (0.498 GeV/c^2). If the two photons have been produced in the decay of a neutral pion, their invariant mass should be close to the π^0 mass (0.135 GeV/c^2). The part of the plot where a signal from $K^0_L \rightarrow \pi^0 e^+ e^-$ should be expected has been masked out by the circular 'signal region' and the square 'control region'. From looking at the rest of the plot, some background from other events is expected for the signal region.

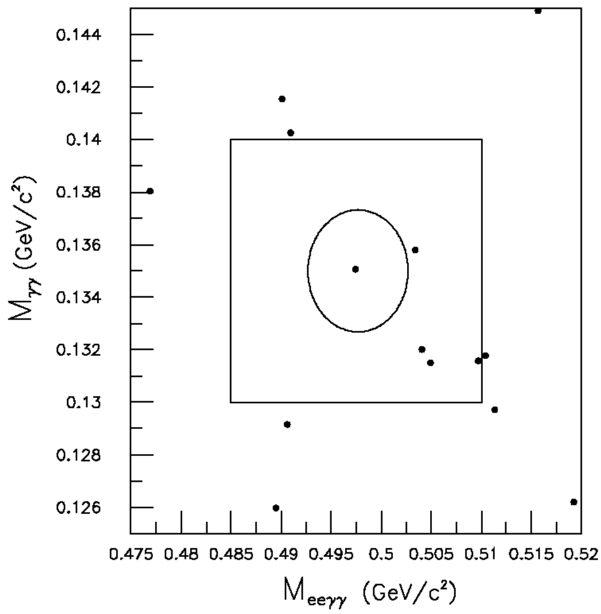

Fig. 11: The circular 'signal region' and the square 'control region' have been unmasked. The event in the signal region is compatible with the background expected from looking at the rest of the plot. So, it cannot be claimed that this should be a signal event.

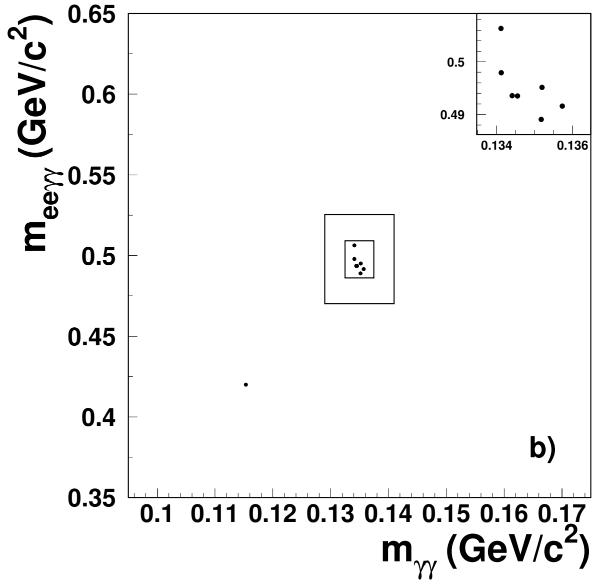

Fig. 12: The signal box for the $K_S^0 \to \pi^0 e^+ e^-$ decay (enlarged in the top right-hand corner). As in Fig. 10, for K_S^0 decays where only two electrons and two photons are produced, the invariant $ee\gamma\gamma$ mass should be close to the K_S^0 mass (0.498 GeV/c^2). If the two photons have been produced in the decay of a neutral pion, their invariant mass should be close to the π^0 mass (0.135 GeV/c^2). For this decay, it has been possible to choose cuts that suppress the background very efficiently, and the nearest background event is very far away from the signal box. The cuts were chosen while the 'signal box' in the centre and the 'control box' that surrounds it were masked. Then the control box was 'opened' to check if for some reason there was an accumulation of background close to the signal box. Only then was the signal box itself opened, thus giving confidence that no bias was introduced by a specific choice of cuts based on the experimentalists' expectations (or 'hopes') concerning the signal value.

Unfortunately, even when the branching ratio of $K_L^0 \to \pi^0 e^+ e^-$ is measured, these numbers by themselves will not be sufficient to determine the relative contribution of indirect and direct CP violation in $K_L^0 \to \pi^0 l^+ l^-$ because of the interference between these two decay amplitudes. Using chiral perturbation theory, the K_S^0 decay's branching ratio can be written as

$$BR(K_S^0 \to \pi^0 e^+ e^-) = 5.2 \times 10^{-9} a_s^2 \, , \tag{14}$$

while the branching ratio of the CP-violating component of the corresponding K_L^0 decay is written as [31]

$$BR_{\text{CPV}}(K_L^0 \to \pi^0 e^+ e^-) = \{15.3 a_s^2 - 6.8 a_s (\text{Im}\, \lambda_t \times 10^4) + 2.8 (\text{Im}\, \lambda_t \times 10^4)^2\} \times 10^{-12} \, . \tag{15}$$

While the measurement of the K_S^0 decay rates fixes the absolute size of the K_S^0 decay amplitudes, it does not tell us if the interference term is positive or negative (constructive or destructive interference), which will have to be decided by theory.

Owing to the above-mentioned experimental and theoretical difficulties, this decay channel does not appear to be the most promising for the near future.

5.2 $K^+ \to \pi^+ \nu \bar{\nu}$

This rare decay is not CP violating. When considering the unitarity triangle (see Fig. 9), the rate of this decay yields an ellipse around a point on the abscissa, so that the tip of the unitarity triangle should lie on this ellipse. Its measurement would allow one to derive in an independent way the length of the right side of the unitarity triangle, which has already been measured from B–$\bar{B}$ oscillations. The systematics which enter into these two kinds of measurement are different, so that they are complementary to each other. In case a significant difference in the results should be observed, this would be a strong hint towards new physics.

An advantage for the theoretical treatment of this decay is the fact that the hadronic matrix element can be calculated from other, measured processes, such as $K^+ \to \pi^0 e^+ \nu$. From the experimental point of view, this is of course a difficult measurement because two of the three decay products, the neutrinos, cannot be seen in the detector. The task of the experiment is thus to look for K^+ decays producing nothing but a π^+. The detector has to be completely hermetic in order to suppress other, much more frequent decay channels, such as $K^+ \to \pi^+ \pi^0$, which could be mistaken for a signal event if the π^0 were not observed. Excellent particle identification is needed to suppress decays such as $K^+ \to \mu^+ \nu_\mu$.

Experiments at Brookhaven running over many years found a total of three signal events with small background (see Fig. 13 and Ref. [32]). Figure 14 shows the detected events and the most important source of background from $K^+ \to \pi^+ \pi^0$. This is a good illustration of the virtues of the so-called 'blind analysis' technique in case of very rare decays. The 'signal box' is defined from background studies before events inside the box are looked at. Only when all experimental cuts have been defined is the signal box 'opened'. This ensures that expectations do not influence the result by tempting observers to arbitrarily change the values of the cuts.

The number of detected events (three events) has been enough to establish the decay but the accuracy to which the branching ratio has been measured ($BR(K^+ \to \pi^+ \nu \bar{\nu}) = 1.47^{+1.30}_{-0.89}$) is still too low to really verify the predictions of the Standard Model (see Fig. 15).

Because of funding problems, the experiments at Brookhaven have been discontinued. Other experiments using different techniques are in preparation (see, for example, Ref. [33]) and it is hoped that over the coming years a total of about 10^2 signal events might be observed.

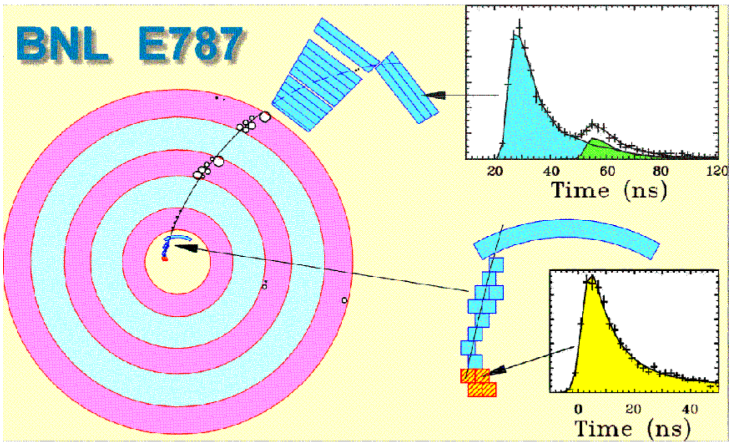

Fig. 13: A $K^+ \to \pi^+\nu\bar{\nu}$ event in the E787 detector at Brookhaven. The kaon is stopped in the target and emits a signal (blow-up and signal shape at bottom right), which shows no extra activity. The only visible particle from the $K^+ \to \pi^+\nu\bar{\nu}$ decay is the π^+, which travels towards the top right in the figure. The top-right graph shows the signal from the travelling pion and a second pulse caused by the $\pi^+ \to \mu^+\nu_\mu$ decay.

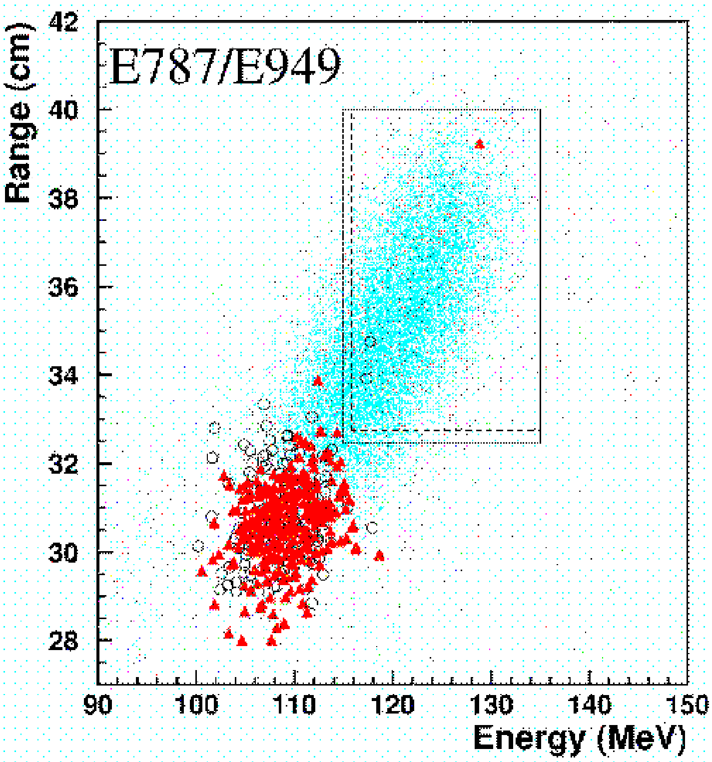

Fig. 14: Kaon range in scintillator versus kaon energy: Monte Carlo generated data for the decay $K^+ \to \pi^+\nu\bar{\nu}$ (blue dots), and signal and $K^+ \to \pi^+\pi^0$ background events measured in the two Brookhaven experiments E787 and E949 (circles and triangles). The signal boxes (frames containing the three signal events, at top right) for the two experiments were slightly different. This graph illustrates the virtue of a 'blind analysis' for such a rare decay, where a small change in the cut parameters (which define the signal box) may significantly influence the result.

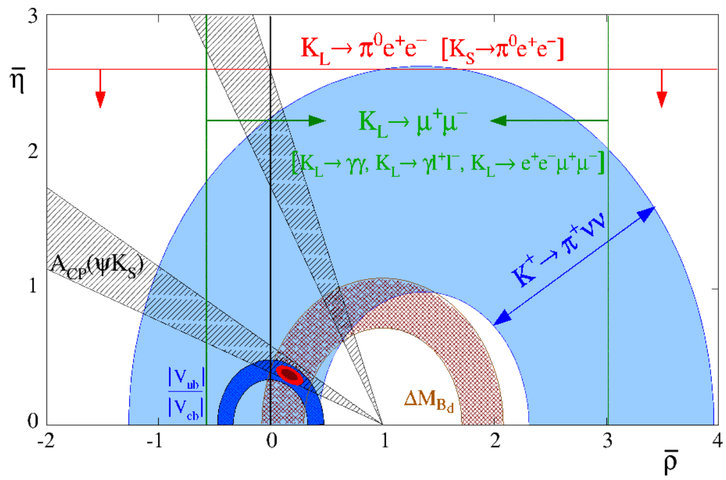

Fig. 15: The branching ratio of the decay $K^+ \to \pi^+ \nu\bar{\nu}$ defines an ellipse in the complex plane, on which the tip of the unitarity triangle should be located. The measurements available so far suffer from a large statistical error and yield a very broad band. By measuring around 100 events, the allowed region could be restricted to a narrow band. Should this band not cover the region of the unitarity triangle's tip as obtained from other measurements, this would be an unequivocal sign of new physics.

5.3 $K^0 \to \pi^0 \nu\bar{\nu}$

This is one of the potentially most instructive decays because it can be calculated with a very small theoretical error, so that any significant deviation between prediction and measurement would be an unequivocal sign of new physics. At the same time, the experiment is extremely challenging, so that this decay, dubbed the 'Holy Grail' of kaon physics, has provoked comments such as "a theorist's dream and an experimentalist's nightmare" where one attempts to measure "nothing goes to nothing plus nothing".

The measurement would directly yield the value of the height of the unitarity triangle, η (cf. Fig. 9). The decay is almost purely directly CP violating, so that its observation would show a second manifestation of direct CP violation in the kaon system. As for the preceding decay, the hadronic matrix element could be obtained from the measured rate of $K^0 \to \pi^+ e^- \nu$, and the total theoretical uncertainty is estimated to be only a few per cent.

Again, care must be taken to fight against the background, which dominates the signal (as expected from Standard Model calculations) by a factor of about 10^{10}. A large number of kaon decays must be observed with a completely hermetic detector, and all possible sources of background must be measured in a convincing way. Experiments were in preparation at BNL and at Fermilab but have been turned down because of funding problems. Hope remains with a new experiment which aims at measuring this decay in the J-PARC facility in Japan [34].

6 Conclusion

The discovery of CP violation in the decays of neutral kaons 41 years ago at first appeared as an unnecessary and unwanted complication of nature. This phenomenon has, however, proved extremely fruitful for our understanding of the world, and has turned out to be a vital ingredient of the universe as we know it. For a long time kaons remained the only particles where CP violation could be observed, but now mainstream research has shifted to the B system, where very promising results have been obtained over the last few years. There are, however, a few outstanding measurements in kaons which pose extreme experimental difficulties but whose results will be indispensable to obtaining a clear overall picture of CP violation.

References

[1] C.S. Wu *et al.*, Phys. Rev. **105** (1957) 1413.

[2] J.H. Christenson, J.W. Cronin, V.L. Fitch and R. Turlay, Phys. Rev. Lett. **13** (1964) 138.

[3] V.L. Fitch, Proc. of Kaon 2001, Frascati Physics Series vol. XXVI, Pisa (2001) 5.

[4] A.D. Sakharov, Violation of CP invariance, C asymmetry, and baryon asymmetry in the universe, JETP Lett. **5** (1966) 24.

[5] L. Wolfenstein, Violation of CP invariance and the possibility of very weak interactions, Phys. Rev. Lett. **13** (1964) 562.

[6] M. Kobayashi and T. Maskawa, CP violation in the renormalizable theory of weak interaction, Prog. Theor. Phys. **49** (1973) 652.

[7] G. Barr *et al.*, Phys. Lett. B **317** (1993) 233.

[8] L.K. Gibbons *et al.*, Phys. Rev. Lett. **70** (1993) 1203.

[9] J.R. Batley *et al.*, A precision measurement of direct CP violation in the decay of neutral kaons into two pions, Phys. Lett. B **544** (2002) 97.

[10] A. Alavi-Harati *et al.*, Measurements of direct CP violation, CPT symmetry, and other parameters in the neutral kaon system, Phys. Rev. D **67** (2003) 012005.

[11] J.R. Batley *et al.*, Search for direct CP violation in the decays $K^{\pm} \to 3\pi^{\pm}$, to be published in Phys. Lett. B (2006); I. Mikulec, Search for direct CP violation in $K^{+} \to \pi^{\pm}\pi^{+}\pi^{-}$ decays by NA48/2, Proc. of the XXXXth Recontres de Moriond, hep-ex/0505081 (2005).

[12] F. Ambrosino *et al.*, A direct search for the CP-violating decay $K_S \to 3\pi^0$ with the KLOE detector at DAΦNE, Phys. Lett. B **619** (2005) 61.

[13] J.R. Batley *et al.*, A measurement of the CP-conserving component of the decay $K^0_S \to \pi^+\pi^-\pi^0$, Phys. Lett. B **630** (2005) 31.

[14] A. Angelopoulos *et al.*, The neutral kaon decays to $\pi^+\pi^-\pi^0$: a detailed analysis of the CPLEAR data, Eur. Phys. J. C **5** (1998) 389.

[15] J.S. Bell and J. Steinberger, Proc. of Oxford International Conference on Elementary Particles, Oxford, 1965 (Chilton, RAL, 1966), p. 195.
`http://cdsweb.cern.ch/search.py?recid=832870&`

[16] I.I. Bigi and A.I. Sanda, On limitations of T invariance in K decays, hep-ph/9904484.

[17] A. Lai *et al.*, Search for CP violation in $K^0 \to 3\pi^0$ decays, Phys. Lett. B **610** (2005) 165.

[18] I. Kant, Critique of Pure Reason (1781).

[19] G. Lüders, Dan. Mat. Fys. Medd. **28** (1954) 5, and Ann. Phys. (N.Y.) **2** (1957) 1.

[20] R.G. Sachs, The Physics of Time Reversal (University of Chicago Press, 1987).

[21] A. Angelopoulos *et al.*, First direct observation of time-reversal non-invariance in the neutral-kaon system, Phys. Lett. B **444** (1998) 43; A. Apostolakis *et al.*, Determination of the T- and CPT-violation parameters in the neutral kaon system using the Bell–Steinberger relation and data from CPLEAR, Phys. Lett. B **456** (1999) 297.

[22] S.L. Glashow, J. Iliopoulos and L. Maiani, Phys. Rev. D **2** (1970) 1285.

[23] L. Alvarez-Gaumé *et al.*, Violation of time-reversal invariance and CPLEAR measurements, Phys. Lett. B **458** (1999) 347.

[24] A. Alavi-Harati *et al.*, Observation of CP violation in $K_L \to \pi^+\pi^-e^+e^-$ decays, Phys. Rev. Lett. **84** (2000) 408; A. Lai *et al.*, Investigation of $K_{L,S} \to \pi^+\pi^-e^+e^-$ decays, Eur. Phys. J. C **30** (2003) 33.

[25] Yu.G. Kudenko, Search for T violation in $K^+ \to \pi^0\mu^+\nu$ and $K^+ \to \mu^+\nu\gamma$ decays, hep-ex/0103007.

[26] I. Scimemi, E. Gámiz and J. Prades, CP violation in kaons: ϵ'_K/ϵ_K vs $K \to 3\pi$, hep-ph 0405204.

[27] Particle Data Group, Review of Particle Physics, Phys. Lett. B **592** (2004) 1.
[28] A. Alavi-Harati *et al.*, Search for the rare decay $K_L \to \pi^0 e^+ e^-$, Phys. Rev. Lett. **93** (2004) 021805.
[29] J.R. Batley *et al.*, Phys. Lett. B **576** (2003) 43.
[30] J.R. Batley *et al.*, Phys. Lett. B **599** (2004) 197.
[31] G. D'Ambrosio, G. Ecker, G. Isidori and J. Portoles, JHEP **08** (1998) 004.
[32] V.V. Anisimovsky *et al.*, Phys. Rev. Lett. **93** (2004) 031801.
[33] Proposal to measure the rare decay $K^+ \to \pi^+ \nu\bar{\nu}$ at the CERN SPS, CERN-SPSC-2005-013, SPSC-P-326, 11.6.2005;
`http://na48.web.cern.ch/NA48/NA48-3/`.
[34] Letter of Intent: Measurement of the $K_L \to \pi^0 \nu\bar{\nu}$ branching ratio,
`http://psux1.kek.jp/~jhf-np/LOIlist/pdf/L05.pdf`.