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Methodology for determining the parameters of drilling mode for directional straight sections of well using screw downhole motors

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Article presents results of study on possibility of increasing the efficiency of drilling directional straight sections of wells using screw downhole motors (SDM) with a combined method of drilling with rotation of drilling string (DS). Goal is to ensure steady-state operation of SDM with simultaneous rotation of DS by reducing the amplitude of oscillations with adjusting the parameters of drilling mode on the basis of mathematical modeling for SDM – DS system.

Results of experimental study on determination of extrema distribution of lateral and axial oscillations of SDM frame depending on geometrical parameters of gerotor mechanism and modes ensuring stable operation are presented.

Approaches to development of a mathematical model and methodology are conceptually outlined that allow determining the range of self-oscillations for SDM – DS system and boundaries of rotational and translational wave perturbations for a heterogeneous rod with an installed SDM at drilling directional straight sections of well. This mathematical model of SDM – DS system's dynamics makes it possible to predict optimal parameters of directional drilling mode that ensure stable operation of borehole assembly.

Key words: well drilling; screw downhole motor; vibration; drilling string

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Introduction. During drilling of extended directional and horizontal sections of wells using volumetric principle engines, part of axial load on the bit is not transmitted due to frictional force arising between walls of well and drilling tool [2].

To ensure required load on the bit a combined drilling method is used in production process. Distinctive feature of this method is in the joint operation of drilling string (DS) and screw downhole motor (SDM) [9]. In the process of their joint work, torsional, lateral and axial oscillations can occur depending on type of SDM, its energy characteristics and DS, which acts as an elastic unbalanced rod [4, 6, 10].

It should be noted that SDM that is located in lower part of DS has its own beating of the frame, nature of occurrence of which is associated with work of its power section, represented by a planetary reductor. Moreover, frequency, amplitude and direction of the frame beats depend on design of gerotor mechanism, hydraulic component of drilling mud flow, as well as load on the bit [3].

To determine parameters of well drilling mode by a combined method, it is necessary to develop a technique that allows providing forecast and control of stable operation of borehole assembly (BHA), based on mathematical modeling of elastic properties of DS stress-strain state and characteristics of SDM [11, 12].

Methodology and results of the research. Stability of SDM operation is characterized by working mode of power section, in which there is no intensive decrease in rotor rotation frequency with increasing torque on motor shaft [1].

It is known that axis of rotor rotates around its own axis, and also makes a transferring movement around axis of stator, directed counterclockwise. Moreover, frequency of transferring (planetary) rotation of rotor's axis relative to stator's axis is higher than rotor rotation frequency around its own axis.



Angular rotation velocity of rotor's axis relative to stator's axis, which determines beat frequency of the frame,

$$\omega_n = -z_z \; \omega_r, \tag{1}$$

where z_z – number of rotor teeth; ω_r – angular rotation velocity of rotor around its own axis.

Motor's frame beats depend on inertial F_{in} and hydraulic F_h forces acting on rotor,

$$F_{in} = m \, z_2 \, \omega^2 e, \tag{2}$$

$$F_h = M_{ind}/ez_1,\tag{3}$$

where M_{ind} – indicator moment; e – eccentricity; z_1 and z_2 – number of stator and rotor teeth; m – rotor mass; ω – angular velocity.

During engine start, a skew moment arises, causing instability of rotor rolling along stator teeth and leading to additional beating of SDM frame.

Skew moment is

$$M_s = \frac{P_p D t^2}{4\pi} \,. \tag{4}$$

where D – stator diameter at tooth cavities; P_p – pressure difference; t – rotor pitch.

Experimental study of motor's frame beats is performed at the test bench. Bench is equipped with an automatic control system that provides real-time output of SDM main energy characteristics to panel of a personal computer. To study beats of SDM, oscillation sensors are installed on frame.

Study results for vibration acceleration and amplitude of motor's frame oscillations under different operating conditions are shown in Fig.1.

Based on experimental study, shaft rotation frequency is determined, which ensures minimal lateral oscillations and optimal axial beats of motor.

Modeling of tool operation is carried out on an advanced mathematical model of E.K.Yunin and V.K.Khegai [8].

At well drilling, it is required to determine the combination of load on the bit along depth P and rotor rotation frequency n_0 so that drilling time t of specified interval is minimal under condition of optimal energy costs [7].

DS can be represented as a composite rod, including a section of length L_1 with outer and inner diameters D_{L_1} , d_{L_1} , interval of heavy drilling pipes (HDP) with length L_2 and outer and inner diameters D_{L_2} , d_{L_2} and interval, represented by SDM frame and navigation system with length L_3 and diameters D_{L_3} , d_{L_3} . Current well depth $H = L_1 + L_2 + L_3$ in process of drilling a certain interval increases due to deepening of bottomhole. At this, let us assume L_2 , $L_3 = \text{const}$ and due to increasing of $L_1 + \Delta L$, H also rises.







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Fig.2. Computational scheme for study of rotational and translational oscillations of SDM - DS system operation

Let us consider that sections are made of various materials. Therefore, first, second and third section corresponds to propagation velocity of rotational oscillations, respectively λ_{L_1} , λ_{L_2} , λ_{L_3} , propagation velocity of translational oscillations, respectively $\chi_{L_1}, \chi_{L_2}, \chi_{L_3}$. Computational scheme for analyzing DS behavior during rotational and translational motion is shown in Fig. 2.

Differential equation of rotational and translational motion of composite heterogeneous rod with boundary and initial conditions [5]:

$$\begin{cases} \frac{\partial^{2} \varphi_{1}}{\partial t} + f_{\tau_{1}} \left(\frac{\partial \varphi_{1}}{\partial t} \right) = \lambda_{L_{1}}^{2} \frac{\partial^{2} \varphi_{1}}{\partial s_{1}^{2}}, s_{1} \in [0, L_{1}]; \\ \frac{\partial^{2} \varphi_{2}}{\partial t} + f_{\tau_{2}} \left(\frac{\partial \varphi_{2}}{\partial t} \right) = \lambda_{L_{2}}^{2} \frac{\partial^{2} \varphi_{2}}{\partial s_{2}^{2}}, s_{2} \in [0, L_{2}]; \\ \frac{\partial^{2} \varphi_{3}}{\partial t} + f_{\tau_{3}} \left(\frac{\partial \varphi_{3}}{\partial t} \right) = \lambda_{L_{3}}^{2} \frac{\partial^{2} \varphi_{3}}{\partial s_{3}^{2}}, s_{3} \in [0, L_{3}]; \end{cases} \qquad \begin{cases} \frac{\partial^{2} u_{1}}{\partial t} + f_{\tau_{1}} \left(\frac{\partial u_{1}}{\partial t} \right) = \chi_{L_{1}}^{2} \frac{\partial^{2} u_{2}}{\partial s_{1}^{2}}, s_{1} \in [0, L_{1}]; \\ \frac{\partial^{2} u_{2}}{\partial t} + f_{\tau_{2}} \left(\frac{\partial u_{2}}{\partial t} \right) = \chi_{L_{2}}^{2} \frac{\partial^{2} u_{2}}{\partial s_{2}^{2}}, s_{2} \in [0, L_{2}]; \end{cases} \qquad \begin{cases} \frac{\partial^{2} u_{1}}{\partial t} + f_{\tau_{3}} \left(\frac{\partial u_{2}}{\partial t} \right) = \chi_{L_{2}}^{2} \frac{\partial^{2} u_{2}}{\partial s_{2}^{2}}, s_{2} \in [0, L_{2}]; \\ \frac{\partial^{2} u_{3}}{\partial t} + f_{\tau_{3}} \left(\frac{\partial u_{3}}{\partial t} \right) = \chi_{L_{3}}^{2} \frac{\partial^{2} u_{3}}{\partial s_{3}^{2}}, s_{3} \in [0, L_{3}]. \end{cases}$$

Boundary conditions for rotational and translational motion:

$$s_{1} = 0; \ \varphi = n_{0}t, \ M = G_{1}J_{1} \frac{\partial \varphi_{1}}{\partial s_{1}}, \qquad s_{1} = 0; \ u_{1} = h, \ N = E_{1}F_{1} \frac{\partial u_{1}}{\partial s_{1}}, \\$$

$$s_{1} = L_{1}; \ s_{2} = 0; \ G_{1}J_{1} \frac{\partial \varphi_{1}}{\partial s_{1}} = G_{2}J_{2} \frac{\partial \varphi_{2}}{\partial s_{2}}, \qquad s_{1} = L_{1}; \ s_{2} = 0; \ E_{1}F_{1} \frac{\partial u_{1}}{\partial s_{1}} = E_{2}F_{2} \frac{\partial u_{2}}{\partial s_{2}}, \\$$

$$s_{1} = L_{1}; \ s_{2} = 0; \ \varphi_{1} = \varphi_{2}, \qquad s_{1} = L_{1}; \ s_{2} = 0; \ u_{1} = u_{2},$$

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$$s_{2} = L_{2}; s_{3} = 0; G_{2}J_{2} \frac{\partial \varphi_{2}}{\partial s_{2}} = G_{3}J_{3} \frac{\partial \varphi_{3}}{\partial s_{3}}, \qquad s_{2} = L_{2}; s_{3} = 0; E_{2}F_{2}\frac{\partial u_{2}}{\partial s_{2}} = E_{3}F_{3}\frac{\partial u_{3}}{\partial s_{3}},$$

$$s_{2} = L_{3}; s_{3} = 0; \varphi_{2} = \varphi_{3}, \qquad s_{2} = L_{3}; s_{3} = 0; u_{2} = u_{3},$$

$$s_{3} = L_{3}; G_{3}J_{3} \frac{\partial \varphi_{3}}{\partial s_{3}} = -M_{\rm H}(P, n_{\rm H}). \qquad s_{3} = L_{3}; E_{3}F_{3}\frac{\partial u_{3}}{\partial s_{3}} = P(n_{\rm H}).$$

Initial conditions for rotational motion at t = 0:

$$\begin{split} \varphi_{1}(s_{1}, t = 0) &= \frac{f_{\tau_{1}}(n_{0})}{2\lambda_{L_{1}}^{2}} s_{1}^{2} - \left\{ \frac{f_{\tau_{1}}(n_{0})L_{1}}{\lambda_{L_{1}}^{2}} + \theta \left[\frac{f_{\tau_{2}}(n_{0})L_{2}}{\lambda_{L_{2}}^{2}} + \varepsilon \left(\frac{f_{\tau_{2}}(n_{0})L_{3}}{\lambda_{L_{3}}^{2}} + \frac{M_{H}(P, n_{H})}{G_{3}J_{3}} \right) \right] \right\} s_{1}; \\ \varphi_{2}(s_{2}, t = 0) &= f_{1}(L_{1}) + \frac{f_{\tau_{2}}}{2\lambda_{L_{2}}^{2}} s_{2}^{2} - \left[\frac{f_{\tau_{2}}(n_{0})L_{2}}{\lambda_{L_{2}}^{2}} + \varepsilon \left(\frac{f_{\tau_{3}}(n_{0})L_{3}}{\lambda_{L_{3}}^{2}} + \frac{M_{H}(P, n_{H})}{G_{3}J_{3}} \right) \right] s_{2}; \\ \varphi_{3}(s_{3}, t = 0) &= f_{1}(L_{1}) + f_{2}(L_{2}) + \frac{f_{\tau_{2}}(n_{0})}{2\lambda_{L_{2}}^{2}} s_{3}^{2} - \left(\frac{f_{\tau_{3}}(n_{0})L_{3}}{\lambda_{L_{3}}^{2}} + \frac{M_{H}(P, n_{H})}{G_{3}J_{3}} \right) s_{3}; \\ s_{1} &\in [0, L_{1}], \ s_{2} &\in [0, L_{2}], \ s_{3} &\in [0, L_{3}]; \\ \partial r_{0} &= \partial r_{0} &= \partial r_{0} \end{split}$$

$$\frac{\partial \varphi_1}{\partial t} = n_0, \quad \frac{\partial \varphi_2}{\partial t} = n_0, \quad \frac{\partial \varphi_3}{\partial t} = n_0.$$

Initial conditions for translational motion at t = 0:

$$\begin{split} u_{1}(s_{1}, t = 0) &= \frac{f_{\tau_{1}}(n_{0})}{2\chi_{L_{1}}^{2}} s_{1}^{2} - \left\{ \frac{f_{\tau_{1}}(n_{0})L_{1}}{\chi_{L_{1}}^{2}} + \theta \left[\frac{f_{\tau_{2}}(n_{0})L_{2}}{\chi_{L_{2}}^{2}} + \varepsilon \left(\frac{f_{\tau_{2}}(n_{0})L_{3}}{\chi_{L_{3}}^{2}} + \frac{P(n_{H})}{E_{3}F_{3}} \right) \right] \right\} s_{1}; \\ u_{2}(s_{2}, t = 0) &= f_{1}(L_{1}) + \frac{f_{\tau_{2}}}{2\chi_{L_{2}}^{2}} s_{2}^{2} - \left[\frac{f_{\tau_{2}}(n_{0})L_{2}}{\chi_{L_{2}}^{2}} + \varepsilon \left(\frac{f_{\tau_{3}}(n_{0})L_{3}}{\chi_{L_{3}}^{2}} + \frac{P(n_{H})}{E_{3}F_{3}} \right) \right] s_{2}; \\ u_{3}(s_{3}, t = 0) &= f_{1}(L_{1}) + f_{2}(L_{2}) + \frac{f_{\tau_{2}}(n_{0})}{2\chi_{L_{2}}^{2}} s_{3}^{2} - \left(\frac{f_{\tau_{3}}(n_{0})L_{3}}{\chi_{L_{3}}^{2}} + \frac{P(n_{H})}{E_{3}F_{3}} \right) s_{3}; \\ s_{1} &\in [0, L_{1}], \ s_{2} &\in [0, L_{2}], \ s_{3} &\in [0, L_{3}]; \end{split}$$

$$\frac{\partial u_1}{\partial t} = \chi_{L_1} u_1, \quad \frac{\partial u_2}{\partial t} = \chi_{L_2} u_2, \quad \frac{\partial u_3}{\partial t} = \chi_{L_3} u_3,$$

where $\varphi_1(s_1, t)$, $\varphi_2(s_2, t)$, $\varphi_3(s_3, t)$ – rotation angles of current cross-sections of string in corresponding sections; s_1 , s_2 , s_3 – current position of cross-section; $u_1(s_1, t)$, $u_2(s_2, t)$, $u_3(s_3, t)$ – translational movements of current cross-sections of string in corresponding sections; s_1 , s_2 , s_3 – current position of cross-section; h – value of translational movement with transfer, lateral to truncation of round;

 $f_{\tau_1}\left(\frac{\partial \varphi_1}{\partial t}\right), f_{\tau_2}\left(\frac{\partial \varphi_2}{\partial t}\right), f_{\tau_3}\left(\frac{\partial \varphi_3}{\partial t}\right)$ - dissipative members characterizing the resistance to rotation of



drilling string in corresponding sections; $f_{\tau_1}\left(\frac{\partial u_1}{\partial t}\right)$, $f_{\tau_2}\left(\frac{\partial u_2}{\partial t}\right)$, $f_{\tau_3}\left(\frac{\partial u_3}{\partial t}\right)$ – dissipative members characterizing the resistance of drilling string translational movement; n_0 – rotation velocity for upper end of string; G_1 , G_2 , G_3 – shear modules for materials in corresponding sections; E_1 , E_2 , E_3 – elastic moduli of materials in corresponding sections under tension or compression; J_1 , J_2 , J_3 – polar moments of inertia in cross-section of string in corresponding sections; F_1 , F_2 , F_3 – cross-sectional area of string in corresponding sections; $M_{\rm H}(P, n_{\rm H})$ – moment of resistance to rotation of composite rod lower section from the side of rock; P – axial load on end of composite rod lower section;

 $P(n_{\rm H})$ – axial load on end of composite rod lower section; $n_{\rm H} = \frac{\partial \varphi_3}{\partial t}\Big|_{s_3 = L_3}$ – rotation frequency for

end of composite rod lower section, $\theta = \frac{G_1 J_1}{G_2 J_2}$ – coefficient of moment-force ratio of the first and

second sections during rotation; $\varepsilon = \frac{G_2 J_2}{G_3 J_3}$ – coefficient of moment-force ratio of the second and

third sections during rotation; $\theta = \frac{E_1 F_1}{E_2 F_2}$ – coefficient of moment-force ratio of the first and second

sections during translational movement; $\varepsilon = \frac{E_2 F_2}{E_3 F_3}$ – coefficient of moment-force ratio of the second

and third sections during translational movement.

This problem is most clearly solved for case in which the values of dissipative members of system are equal to zero. Following formulas are used:

$$\begin{cases} H = \frac{\lambda_{L_{1}}}{\mu_{1}} \ln \frac{\lambda_{L_{2}} \left[\frac{\lambda_{L_{2}}}{\mu_{2}} \ln \frac{\lambda_{L_{2}} \left[\frac{\lambda_{L_{3}}}{\mu_{3}} \ln \frac{\lambda_{L_{3}} \Delta M_{H} + G_{3} J_{3} n_{H}}{\lambda_{L_{3}} \Delta M_{H} - G_{3} J_{3} n_{H}} \right] + G_{2} J_{2} n_{0}}{\lambda_{L_{2}} \left[\frac{\lambda_{L_{3}}}{\mu_{3}} \ln \frac{\lambda_{L_{3}} \Delta M_{H} + G_{3} J_{3} n_{H}}{\lambda_{L_{3}} \Delta M_{H} - G_{3} J_{3} n_{H}} \right] - G_{2} J_{2} n_{0}} \right] + G_{I} J_{1} n_{0}}{\lambda_{L_{1}} \left[\frac{\lambda_{L_{2}}}{\mu_{2}} \ln \frac{\lambda_{L_{2}} \left[\frac{\lambda_{L_{3}}}{\mu_{3}} \ln \frac{\lambda_{L_{3}} \Delta M_{H} + G_{3} J_{3} n_{H}}{\lambda_{L_{3}} \Delta M_{H} - G_{3} J_{3} n_{H}} \right] - G_{2} J_{2} n_{0}} \right] - G_{I} J_{1} n_{0}}{\lambda_{L_{2}} \left[\frac{\lambda_{L_{3}}}{\mu_{3}} \ln \frac{\lambda_{L_{3}} \Delta M_{H} + G_{3} J_{3} n_{H}}{\lambda_{L_{3}} \Delta M_{H} - G_{3} J_{3} n_{H}} \right] - G_{2} J_{2} n_{0}} \right] - G_{I} J_{1} n_{0}} \\ H \leq \frac{\lambda_{L_{1}}}{\mu_{1}} \ln \frac{\lambda_{L_{1}} \left[\frac{\lambda_{L_{2}}}{\mu_{2}} \ln \frac{\lambda_{L_{2}} \left[\frac{\lambda_{L_{3}}}{\mu_{3}} \ln \frac{\lambda_{L_{3}} \Delta M_{H} + G_{3} J_{3} n_{H}}{\lambda_{L_{3}} \Delta M_{H} - G_{3} J_{3} n_{H}} \right] - G_{2} J_{2}} \right] + G_{I} J_{1} n_{0}}{\lambda_{L_{2}} \left[\frac{\lambda_{L_{3}}}{\mu_{3}} \ln \frac{\lambda_{L_{3}} \Delta M_{H} + G_{3} J_{3} n_{H}}{\lambda_{L_{3}} \Delta M_{H} - G_{3} J_{3} n_{H}} \right] - G_{2} J_{2}} \right] + G_{I} J_{1} n_{0}} \\ K_{L_{1}} \left[\frac{\lambda_{L_{1}}} {\lambda_{L_{2}}} \left[\frac{\lambda_{L_{3}}}{\mu_{3}} \ln \frac{\lambda_{L_{3}} \Delta M_{H} + G_{3} J_{3} n_{H}}{\lambda_{L_{3}} \Delta M_{H} - G_{3} J_{3} n_{H}}} \right] - G_{2} J_{2}} \right] - G_{I} J_{1} n_{0}} \\ K_{L_{1}} \left[\frac{\lambda_{L_{2}}} {\mu_{2}} \ln \frac{\lambda_{L_{2}} \left[\frac{\lambda_{L_{3}}}{\mu_{3}} \ln \frac{\lambda_{L_{3}} \Delta M_{H} + G_{3} J_{3} n_{H}}{\lambda_{L_{3}} \Delta M_{H} - G_{3} J_{3} n_{H}}} \right] - G_{2} J_{2}} \right] - G_{I} J_{1} n_{0}} \\ K_{L_{1}} \left[\frac{\lambda_{L_{2}}} {\mu_{2}} \ln \frac{\lambda_{L_{2}} \left[\frac{\lambda_{L_{3}}}{\mu_{3}} \ln \frac{\lambda_{L_{3}} \Delta M_{H} + G_{3} J_{3} n_{H}}}{\lambda_{L_{3}} \Delta M_{H} - G_{3} J_{3} n_{H}}} \right] - G_{2} J_{2}} - G_{I} J_{1} n_{0} \right]$$



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$$P_{\mu} = \frac{G_{3}J_{3}}{\lambda_{L_{3}}} \binom{n_{0}^{2}}{1 + ke^{\frac{\mu_{3}L_{3}}{2\lambda_{L_{3}}}}}, \\P_{\mu} = \frac{1 + ke^{\frac{\mu_{3}L_{3}}{\lambda_{L_{3}}}} P\left(ch\left(\frac{\mu_{1}L_{1}}{2\lambda_{L_{1}}} + \frac{\mu_{2}L_{2}}{2\lambda_{L_{2}}} + \frac{\mu_{3}L_{3}}{2\lambda_{L_{3}}}\right) + kch\left(\frac{\mu_{1}L_{1}}{2\lambda_{L_{1}}} - \frac{\mu_{2}L_{2}}{2\lambda_{L_{2}}} - \frac{\mu_{3}L_{3}}{2\lambda_{L_{3}}}\right)\right)}{1 - ke^{\frac{\mu_{3}L_{3}}{\lambda_{L_{3}}}} n_{0}\left(sh\left(\frac{\mu_{1}L_{1}}{2\lambda_{L_{1}}} + \frac{\mu_{2}L_{2}}{2\lambda_{L_{2}}} + \frac{\mu_{3}L_{3}}{2\lambda_{L_{3}}}\right) - ksh\left(\frac{\mu_{1}L_{1}}{2\lambda_{L_{1}}} - \frac{\mu_{2}L_{2}}{2\lambda_{L_{2}}} - \frac{\mu_{3}L_{3}}{2\lambda_{L_{3}}}\right)\right)}, \\P_{\mu} = \frac{1 + ke^{\frac{\mu_{3}L_{3}}{\lambda_{L_{3}}}} P\left(sh\left(\frac{\mu_{1}L_{1}}{2\lambda_{L_{1}}} + \frac{\mu_{2}L_{2}}{2\lambda_{L_{2}}} + \frac{\mu_{3}L_{3}}{2\lambda_{L_{3}}}\right) - ksh\left(\frac{\mu_{1}L_{1}}{2\lambda_{L_{1}}} - \frac{\mu_{2}L_{2}}{2\lambda_{L_{2}}} - \frac{\mu_{3}L_{3}}{2\lambda_{L_{3}}}\right)\right)}{1 - ke^{\frac{\mu_{3}L_{3}}{\lambda_{L_{3}}}} n_{0}\left(ch\left(\frac{\mu_{1}L_{1}}{2\lambda_{L_{1}}} + \frac{\mu_{2}L_{2}}{2\lambda_{L_{2}}} + \frac{\mu_{3}L_{3}}{2\lambda_{L_{3}}}\right) - kch\left(\frac{\mu_{1}L_{1}}{2\lambda_{L_{1}}} - \frac{\mu_{2}L_{2}}{2\lambda_{L_{2}}} - \frac{\mu_{3}L_{3}}{2\lambda_{L_{3}}}\right)\right)}, \\n_{0}^{*} = \frac{1 - k^{2}}{sh^{2}\left(\frac{\mu_{1}L_{1}}{2\lambda_{L_{1}}} + \frac{\mu_{2}L_{2}}{2\lambda_{L_{2}}} + \frac{\mu_{3}L_{3}}{2\lambda_{L_{3}}}\right) + kch^{2}\left(\frac{\mu_{1}L_{1}}{2\lambda_{L_{1}}} - \frac{\mu_{2}L_{2}}{2\lambda_{L_{2}}} - \frac{\mu_{3}L_{3}}{2\lambda_{L_{3}}}\right)},$$

where $L_1 - DS$ length; $L_2 - HDP$ length; $L_3 - length$ of SDM frame and navigation system; $k - coefficient of wave reflection, rotational oscillations at the interface of heterogeneous sections of a composite rod, <math>k = \frac{k_1 - k_2}{k_1 + k_2}$, $k_1 = \frac{\lambda_{L_2}G_1J_1 - \lambda_{L_1}, G_2J_2}{\lambda_{L_2}G_1J_1 + \lambda_{L_1}G_2J_2}$, $k_2 = \frac{\lambda_{L_3}G_2J_2 - \lambda_{L_2}G_3J_3}{\lambda_{L_3}G_2J_2 + \lambda_{L_2}G_3J_3}$; μ_1 , μ_2 , $\mu_3 - dissipation coefficients in corresponding sections, <math>\lambda_{L_1}$, λ_{L_2} , $\lambda_{L_3} - propagation velocity of rotational oscillations in corresponding sections; <math>P_B$ and $P_H - axial loads on bottom end of SDM frame corresponding to upper and lower boundaries of self-oscillations; <math>n_0^*$ - rotor rotation frequency at $P_B = P_H$.

Task for case, in which the values of dissipative members of system are equal to zero, and propagation depth of translational oscillations of drilling tool, represented as a composite rod of three heterogeneous sections, is solved by system (6). At the same time G_1 , G_2 , G_3 are replaced by E_1 , E_2 , E_3 and J_1 , J_2 , J_3 by F_1 , F_2 , F_3 , and also λ_{L_1} , λ_{L_2} , λ_{L_3} – propagation velocity of rotational oscillations by propagation velocity of translational oscillations by χ_{L_1} , χ_{L_2} , χ_{L_3} in corresponding sections. Obtained equations determine conditions for occurrence possibility of translational self-oscillations of DS, represented as a composite rod in process of translating to deepen bottomhole of well [13]. Axial loads on lower end of SDM frame, corresponding to upper and lower boundaries of self-oscillations during translational movement of $P_{\rm B}$ and $P_{\rm H}$, are determined by the equation (7). At the same time propagation velocity of rotational oscillations λ_{L_1} , λ_{L_2} , λ_{L_3} , is replaced by χ_{L_1} , χ_{L_2} , χ_{L_3} , elastic modulus G_1 , G_2 , G_3 and polar moments of inertia in cross-section J_1 , J_2 , J_3 are replaced by E_1 , E_2 , E_3 and F_1 , F_2 , F_3 respectively.

As a result of calculations based on developed mathematical model (5), range of selfoscillations' onset during rotation and translational movement of SDM – DS system was revealed.

Input parameters for calculating rotational and translational movements:

$$L_1 = 1800 \text{ m}; L_2 = 190 \text{ m}; L_3 = 10 \text{ m}; J_1 = 5.841 \cdot 10^{-6} \text{ m}^4; J_2 = 1.941 \cdot 10^{-6} \text{ m}^4; J_3 = 4.928 \cdot 10^{-6} \text{ m}^4;$$

 $k = 0.106; G_1 = G_2 = G_3 = 8 \cdot 10^{10} \text{ Pa}; \lambda_{L_1}, \lambda_{L_2}, \lambda_{L_3} = 3200 \text{ m/s}; n_0 = [0; 7] \text{ rad/s};$



 $\mu_1 = 0.1; \ \mu_2 = 0.2; \ \mu_3 = 0.3;$ $L_1 = 1800 \text{ m}; \ L_2 = 190 \text{ m}; \ L_3 = 10 \text{ m}; \ F_1 = 1.018 \cdot 10^{-3} \text{ m}^2; \ F_2 = 1.81 \cdot 10^{-3} \text{ m}^2; \ F_3 = 8.042 \cdot 10^{-4} \text{ m}^2;$ $k = 0.106; \ E_1 = E_2 = E_3 = 2 \cdot 10^{10} \text{ Pa}; \ \chi_{L_1}, \chi_{L_2}, \chi_{L_3} = 5320 \text{ m/s}; \ n_0 = [0; 7] \text{ rad/s};$ $\mu_1 = 0.1; \ \mu_2 = 0.2; \ \mu_3 = 0.3.$

Results of mathematical modeling are presented in fig.3. Comparison of obtained study results for SDM frame oscillations in bench conditions with calculated values of boundaries of DS self-oscillations allows determining the range of stable operation for SDM – DS system. Values located under the line indicated by lower boundary of $P_{\rm B}$ self-oscillations mean absence of vibration – uniform translational and rotational movement of tool, between upper $P_{\rm H}$ and lower $P_{\rm B}$ boundaries – a temporary stop (jamming), above the upper $P_{\rm H}$ – braking (no rotation).

Developed methodology of determining required parameters for drilling mode of inclined sections in a well, ensuring stable operation of BHA, is as follows.

SDM is started and pressure drop is determined during its operation in idle mode. Then, required load on the bit (according to work plan and geological and technical schedule) is created and pressure drop is fixed taking into account loading of gerotor mechanism. On the basis of SDM test bench diagram, optimal range of shaft rotation frequency with corresponding pressure drop is graphically determined. At the same time, maximum allowable decrease in rotation frequency of SDM shaft is noted, which corresponds to optimum amplitudes of frame lateral oscillations.

According to developed mathematical model, boundaries of DS self-oscillations' onset are calculated. After constructing the graphical dependencies, required frequency and load on the bit are determined, at which DS is in permissible range of stable operation. Noting modes of DS stable operation, correlation is made with load on the bit, at which SDM will also be in mode of optimal energy characteristics. If rotation frequency of SDM shaft (according to test bench diagram), determined by pressure drop, has decreased by more than 70 %, load on the bit is reduced. Based on graphical dependences (Fig.3) for boundaries range of self-oscillations' onset at given rotation frequencies of DS and load on the bit, rotation frequency of top drive is adjusted to ensure stable operation of the system while maintaining mechanical drilling speed [14, 15].

Conclusion. On the basis of experimental study, amplitudes and frequencies of oscillations of SDM frame were determined along entire length of power section of working bodies and spindle part for various operating modes of motor. It has been established that to reduce amplitude of motor's lateral oscillations and ensure its stable operation, range of shaft rotation frequencies must be maintained within 70 % of SDM rotation frequency in idle mode.







Mathematical model of SDM - DS system has been developed, which allows predicting the range of DS self-oscillations and boundaries of rotational and translational wave disturbances for case of string modeling as a heterogeneous rod at drilling directionally straight sections of well.

Methodology has been developed for determining required parameters of drilling mode for directional straight sections of well that ensure stable operation of BHA, based on conditions of maintaining stable operation of system, taking into account maximum permissible rotational frequency of SDM and boundaries of DS self-oscillations' onset.

The developed methodology and technical recommendations aimed at ensuring stable operation of SDM with simultaneous rotation of drilling string at drilling directional wells are used in the branch of "LUKOIL-Inzhiniring" LLC – "KogalymNIPIneft".

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