

Confinement-induced Efimov resonances in Fermi-Fermi mixtures

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A Fermi-Fermi mixture of ⁴⁰K and ⁶Li does not exhibit the Efimov effect in a free space, but the Efimov effect can be induced by confining only ⁴⁰K in one dimension. Here the Efimov's three-body parameter is controlled by the confinement length. We show that the three-body recombination rate in such a system in the dilute limit has a characteristic logarithmic-periodic dependence on the effective scattering length with the scaling factor 22.0 and can be expressed by formulas similar to those for identical bosons in three dimensions. The ultracold mixture of ⁴⁰K and ⁶Li in the one-dimensional–three-dimensional mixed dimensions is thus a promising candidate to observe the Efimov physics in fermions.

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I. INTRODUCTION

Recent realization of an ultracold Fermi-Fermi mixture of ⁴⁰K and ⁶Li with interspecies Feshbach resonances opens up new research directions in cold atomic physics [1–3]. Such examples include the creation of the Bose-Einstein condensate of heteronuclear molecules and the investigation of the effect of mass difference on the superfluidity. More importantly, the two-species mixture offers the possibility of species-selective confinement potentials, which provides novel subjects such as Fermi gases imbalanced in terms of the dimensionality of space [4,5].

It has been shown in Ref. [4] that when ⁴⁰K is confined in one dimension or two dimensions with ⁶Li being in three dimensions, the ⁴⁰K-⁶Li mixture with a resonant interspecies interaction exhibits the Efimov effect characterized by an infinite series of shallow three-body bound states (trimers) composed of two heavy and one light fermions. Their binding energies are given by

$$E_3^{(n)} \rightarrow -e^{-2\pi n/s_0} \frac{\hbar^2 \kappa_*^2}{2m_{\text{KLi}}} \text{ for } n \rightarrow \infty, \quad (1)$$

with $m_{\text{KLi}} = m_{\text{K}} m_{\text{Li}} / (m_{\text{K}} + m_{\text{Li}})$ being the reduced mass and $s_0 = 1.02$ in the one-dimensional–three-dimensional (1D-3D) mixed dimensions and $s_0 = 0.260$ in the two-dimensional–three-dimensional (2D-3D) mixed dimensions [4]. κ_* is the so-called Efimov parameter defined up to multiplicative factors of e^{π/s_0} and will be calculated in this Rapid Communication. The emergence of such Efimov trimers in the ⁴⁰K-⁶Li mixture is remarkable because they are absent in a free space but induced by confining ⁴⁰K in lower dimensions. An alternative way to realize the Efimov effect using the ⁴⁰K-⁶Li mixture would be to apply an optical lattice to ⁴⁰K to increase its effective mass by more than a factor of 2 [6].

In this Rapid Communication, we will show that the Efimov effect in the ⁴⁰K-⁶Li mixture when ⁴⁰K is confined in 1D, is experimentally observable through the three-body recombination rate which has a characteristic logarithmic-periodic behavior as a function of the effective scattering length. In particular, the three-body recombination rate is found to exhibit resonant peaks that can be explained by *confinement-induced Efimov resonances*. We note that the

three-body recombination rate has been successfully employed to obtain evidences for the Efimov trimers in a Bose gas of ¹³³Cs [7,8], a Bose-Bose mixture of ⁸⁷Rb and ⁴¹K [9], and a three-component Fermi gas of ⁶Li [10,11].

II. EFIMOV PARAMETER IN THE BORN-OPPENHEIMER APPROXIMATION

Before developing an exact analysis of the three-body recombination rate, it is worthwhile to demonstrate how the Efimov effect is realized by confining ⁴⁰K in 1D. For generality, we shall consider a three-body problem of two *A* atoms and one *B* atom with the resonant interspecies interaction in which a two-dimensional harmonic potential is applied to only *A* atoms. When the *A* atoms are much heavier than the *B* atom $m_A \gg m_B$, one can first solve the Schrödinger equation for the *B* atom with fixed positions of the *A* atoms, which generates the following effective potential between two *A* atoms: $V(\mathbf{r}) = -\hbar^2 c^2 / (2m_B r^2)$, with $c = 0.567$. Then, the relative motion of *A* atoms is governed by the Schrödinger equation (here and below $\hbar = 1$)

$$\left[-\frac{\nabla_{\mathbf{r}}^2}{m_A} + \frac{1}{4} m_A \omega^2 \mathbf{x}^2 + V(\mathbf{r}) \right] \Psi(\mathbf{r}) = (E_3 + \omega) \Psi(\mathbf{r}), \quad (2)$$

where ω is the oscillator frequency and $\mathbf{r} = (z, \mathbf{x})$ with $\mathbf{x} = (x, y)$ is relative coordinates between two *A* atoms. Fermi statistics of *A* atoms implies $\Psi(-\mathbf{r}) = -\Psi(\mathbf{r})$.

In a free space $\omega = 0$, it is known that the mass ratio $m_A/m_B = 6.67$ for the ⁴⁰K-⁶Li mixture is too small to form Efimov trimers [12]. However, the confinement potential term in Eq. (2) makes it possible by effectively reducing the dimensionality of *A* atoms. When $m_A/m_B > 1/(2c^2) = 1.55$, bound-state wave functions with $E_3 < 0$ behave at long distance $|z| \gg l$ as

$$\Psi(\mathbf{r}) \rightarrow e^{-|x|^2/(4l^2)} \frac{|z|^{3/2}}{z} K_{i\nu}(\sqrt{m_A} |E_3| |z|). \quad (3)$$

Here $\nu \equiv \sqrt{\frac{c^2 m_A}{2m_B} - \frac{1}{4}}$ and $l \equiv \frac{1}{\sqrt{m_A \omega}}$ is the confinement length. For shallow bound states $E_3 \rightarrow -0$, the Bessel function in Eq. (3) oscillates as $K_{i\nu}(\sqrt{m_A} |E_3| |z|)$

$\propto \sin\{\nu \ln(\sqrt{m_A}|E_3||z|/2) - \arg[\Gamma(1+i\nu)]\}$, and their binding energies can be determined by matching this asymptotic behavior with the numerical solution of Eq. (2) for $E_3=0$. The oscillating asymptotic behavior implies that there exists an infinite number of bound states with two successive binding energies separated by a factor of $e^{2\pi/\nu}$. In particular, in the case of ^{40}K - ^6Li mixture with $m_A/m_B=6.67$, we find $E_3^{(n)} \rightarrow -14.0e^{-2\pi n/\nu}/(m_A l^2)$ for $n \rightarrow \infty$, from which we obtain $s_0 \approx \nu$ and the Efimov parameter κ_* in Eq. (1) as $\kappa_* \approx 1.91/l$.

One should bear in mind that those numbers may not be accurate because of the Born-Oppenheimer approximation we employed [13]. However, the analysis presented here reveals the remarkable qualitative aspect of the Efimov effect induced by the confinement potential: the Efimov parameter is determined by the confinement length, and therefore it is tunable by an external optical trap to a certain extent. This is in sharp contrast to other systems in a free space where Efimov parameters are determined by short-range physics that is in general difficult to control.

If the confinement length l is much smaller than mean interatomic distances and the thermal de Broglie wavelength of the system at finite densities and temperature, one can consider A atoms to be fixed on the 1D line neglecting their motion in the confinement direction. Consequently, the resulting system becomes a two-species Fermi gas in the 1D-3D mixed dimensions [4]. However, when the Efimov effect is present, the confinement length scale cannot be removed from the problem completely but appears as the Efimov parameter κ_* in the three-body scattering problem as we will see below.

III. EFFECTIVE FIELD THEORY APPROACH

In order to develop a model-independent analysis of the Efimov effect in our system, it is useful to adopt an effective field theory approach, which has been a powerful method to study the Efimov physics in identical bosons [14]. The two-species fermions in the 1D-3D mixed dimensions is universally described by the action [5]

$$S = \int dt \int dz \left[\psi_A^\dagger \left(i\partial_t + \frac{\nabla_z^2}{2m_A} \right) \psi_A + g_0 \psi_A^\dagger \psi_B^\dagger \psi_B \psi_A \right] + \int dt \int dz d\mathbf{x} \left[\psi_B^\dagger \left(i\partial_t + \frac{\nabla_z^2 + \nabla_{\mathbf{x}}^2}{2m_B} \right) \psi_B \right]. \quad (4)$$

Here $\psi_A(t, z)$ and $\psi_B(t, z, \mathbf{x})$ are fermionic fields for A atoms in 1D and B atoms in 3D, respectively. Their bare propagators in the momentum space are given by $i/[p_0 - p_z^2/2m_A + i0^+]$ for A atoms and $i/[p_0 - (p_z^2 + \mathbf{p}^2)/2m_B + i0^+]$ for B atoms with $\mathbf{p} = (p_x, p_y)$. The interspecies short-range interaction takes place only on the 1D line located at $\mathbf{x} = \mathbf{0}$ and thus the interaction term should be read as $g_0 \psi_A^\dagger(t, z) \psi_B^\dagger(t, z, \mathbf{0}) \psi_B(t, z, \mathbf{0}) \psi_A(t, z)$.

The two-body scattering of A and B atoms is depicted in Fig. 1, and its amplitude \mathcal{A}_2 is given by [5]

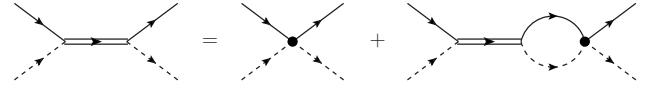


FIG. 1. Two-body scattering of A (solid line) and B (dotted line) atoms. The double line represents the scattering amplitude $i\mathcal{A}_2$.

$$\mathcal{A}_2(p_0, p_z) = \frac{2\pi}{m_B} \frac{1}{\frac{1}{a_{\text{eff}}} - \sqrt{\frac{m_{AB}}{M} p_z^2 - 2m_{AB} p_0 - i0^+}}, \quad (5)$$

where $M = m_A + m_B$ is the total mass. Here the effective scattering length a_{eff} is introduced, which is related to the bare coupling g_0 and the ultraviolet cutoff Λ by $\frac{1}{g_0} - \frac{\sqrt{m_B m_{AB}}}{2\pi} \Lambda = -\frac{m_B}{2\pi a_{\text{eff}}}$. $a_{\text{eff}} \rightarrow -0$ corresponds to the weak attraction and $a_{\text{eff}} \rightarrow +0$ corresponds to the strong attraction between A and B atoms. When $a_{\text{eff}} > 0$, there exists a shallow two-body bound state (dimer) whose binding energy $E_2 = -\frac{1}{2m_{AB} a_{\text{eff}}^2}$ is obtained as a pole of the scattering amplitude in the center-of-mass frame: $\mathcal{A}_2(E_2, 0)^{-1} = 0$. The dependence of a_{eff} on the scattering length a in a free space, which is arbitrarily tunable by means of the interspecies Feshbach resonance, was determined when the A atom is confined in 1D by a harmonic potential [4].

We now proceed to the three-body scattering of two A atoms and one B atom and show the existence of the Efimov trimers. All the relevant diagrams can be summed by solving the integral equation for $\mathcal{T}(E; p_z, q_z)$, which is depicted in Fig. 2. Here E is the total energy in the center-of-mass frame and p_z (q_z) is the momentum of the incoming (outgoing) A atom. \mathcal{T} has a property $\mathcal{T}(E; p_z, q_z) = \mathcal{T}(E; -p_z, -q_z)$ and can be decomposed into even- and odd-parity parts; $\mathcal{T}_{e(o)}(E; p_z, q_z) \equiv [\mathcal{T}(E; p_z, q_z) \pm \mathcal{T}(E; p_z, -q_z)]/2$. The Efimov effect arises in the odd-parity channel \mathcal{T}_o , which satisfies an integral equation

$$\mathcal{T}_o(E; p_z, q_z) = \frac{m_B}{4\pi} K(E + i0^+; p_z, q_z) + \int_0^\Lambda dk_z \frac{\mathcal{T}_o(E; p_z, k_z) K(E + i0^+; k_z, q_z)}{2\pi \sqrt{\frac{m_B + m_{AB}}{M} k_z^2 - 2m_{AB} E - i0^+ - \frac{1}{a_{\text{eff}}}}} \quad (6)$$

with

$$K(E; p_z, q_z) = \ln \left(\frac{p_z^2 + q_z^2 + \frac{2m_A}{M} p_z q_z - 2m_{AB} E}{p_z^2 + q_z^2 - \frac{2m_A}{M} p_z q_z - 2m_{AB} E} \right). \quad (7)$$

When $m_A/m_B > 2.06$, the integration over k_z has to be cut off by $\Lambda \sim l^{-1}$ and the limit $\Lambda \rightarrow \infty$ cannot be taken. Instead, the dependence on the arbitrary cutoff Λ can be eliminated by relating it to the physical parameter κ_* defined in Eq. (1). The spectrum of three-body bound states is obtained by poles

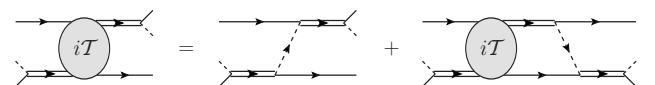


FIG. 2. Three-body scattering of two A and one B atoms.

of $T_0(E)$. When E approaches one of the binding energies $E_3 < -\frac{\theta(a_{\text{eff}})}{2m_{AB}a_{\text{eff}}^2}$, we can write $T_0(E)$ as $T_0(E; p_z, k_z) \rightarrow \mathcal{Z}_0(p_z, q_z)/(E + |E_3|)$. By solving the homogeneous integral equation from Eq. (6) satisfied by $\mathcal{Z}_0(p_z, k_z)$ at the two-body resonance $|a_{\text{eff}}| \rightarrow \infty$, we can obtain an infinite series of binding energies $E_3^{(n)}$ expressed by the form of Eq. (1). The ultraviolet cutoff is found to be related with the Efimov parameter by $\Lambda = 0.460\kappa_*$ for the mass ratio $m_A/m_B = 6.67$ corresponding to the ^{40}K - ^6Li mixture. From now on we shall concentrate on this most important case of $A = ^{40}\text{K}$ and $B = ^6\text{Li}$.

Away from the two-body resonance $|a_{\text{eff}}| < \infty$, there can be a series of resonances associated with the Efimov trimers. On the positive side of the effective scattering length $a_{\text{eff}}^{-1} > 0$, the three-body binding energy $E_3^{(n)}$ for a given n decreases by increasing the value of a_{eff}^{-1} . Eventually $E_3^{(n)}$ merges into the atom-dimer threshold $E_3 = -\frac{1}{2m_{AB}a_{\text{eff}}^2}$ at the critical effective scattering length given by $a_{\text{eff}}\kappa_* = 0.0199e^{n\pi/s_0}$. At such values of a_{eff} , resonant behaviors in the atom-dimer scattering are expected to occur [15].

Similarly, on the negative side of the effective scattering length $a_{\text{eff}}^{-1} < 0$, $E_3^{(n)}$ increases by decreasing the value of a_{eff}^{-1} . Eventually $E_3^{(n)}$ merges into the three-atom threshold $E_3 = 0$ at the critical effective scattering length given by $a_{\text{eff}}\kappa_* = -1.89e^{n\pi/s_0}$. The three-body resonances at such values of a_{eff} shall be referred to as confinement-induced Efimov resonances and bring significant consequences on the three-body recombination rate for $a_{\text{eff}} < 0$.

IV. THREE-BODY RECOMBINATION RATE

The three-body recombination is an inelastic scattering process in which two of three colliding atoms bind to form a diatomic molecule ($A + A + B \rightarrow A + AB$). Assuming the binding energy of the dimer is large enough so that the recoiling atom and dimer escape from the system, the three-body recombination rate can be measured experimentally through the particle loss rate of A atoms: $\dot{n}_A = -2\alpha n_A^2 n_B$. Here $n_{A(B)}$ is the one-(three-)dimensional density of $A(B)$ atoms, and α is the three-body recombination rate constant. The other three-body recombination channel ($A + B + B \rightarrow AB + B$), which does not exhibit the Efimov effect as far as $m_A/m_B > 0.00646$ [4], can also contribute to the particle loss of A atoms. However, it is subleading in the dilute limit $|a_{\text{eff}}| \rightarrow 0$ we consider below and thus negligible.

A convenient way to compute α is to use the optical theorem which relates α to twice the imaginary part of the forward three-body scattering amplitude (Fig. 2). In particular, in the dilute limit where $|a_{\text{eff}}|n_A \ll 1$ and $|a_{\text{eff}}|n_B^{1/3} \ll 1$, the odd-parity channel dominates the three-body scattering, and α can be expressed as $\alpha = 4(2\pi a_{\text{eff}}/m_B)^2 \text{Im } T_0(0; p_z, p_z)|_{p_z \rightarrow 0}$. Because we can find $\text{Im } T_0(0; p_z, q_z)|_{p_z, q_z \rightarrow 0} \propto p_z q_z$ from Eq. (6), it is useful to introduce a dimensionless function $t(q_z)$ by $T_0(0; p_z, q_z)|_{p_z \rightarrow 0} \equiv m_{AB} p_z q_z a_{\text{eff}}^2 t(q_z)/\pi$. Accordingly, the rate constant to the leading order in a_{eff} becomes $\alpha = 16\pi(m_{AB}/m_B)^2 \bar{p}_z^2 a_{\text{eff}}^4 \text{Im } t(0)$. Here \bar{p}_z^2 is the statistical average of the A atom's momentum squared and equals to

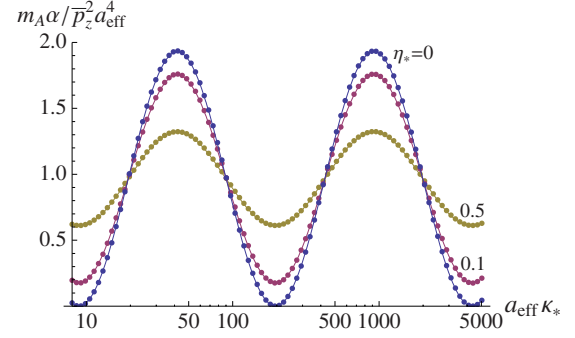


FIG. 3. (Color online) Two periods of $\frac{m_A}{\bar{p}_z^2 a_{\text{eff}}^4} \alpha$ as a function of $a_{\text{eff}}\kappa_* > 0$ at $\eta_* = 0, 0.1$, and 0.5 for $m_A/m_B = 6.67$. Curves behind data points are from the two-parameter fit by Eq. (9).

$(\pi n_A)^2/3$ at zero temperature and $m_A T$ at high temperature. Now $t(q_z)$ satisfies the integral equation

$$t(q_z) = \frac{1}{a_{\text{eff}}^2 q_z^2} + \int_0^\Lambda \frac{dk_z k_z}{2\pi q_z} \frac{t(k_z) K(0; k_z, q_z)}{\sqrt{\frac{m_B + m_{AB}}{M} k_z - \frac{1}{a_{\text{eff}}} - i0^+}}. \quad (8)$$

It is clear that $t(0)$ has a nonzero imaginary part only for $a_{\text{eff}} > 0$ in which the three-body recombination into the shallow dimer is possible. Remarkably the integral equation for $t(q_z)$ is quite similar to that for the s -wave scattering amplitude of three identical bosons in three dimensions [16], and therefore, the solution has the similar property: $\text{Im } t(0)$ is a logarithmic-periodic function of $a_{\text{eff}}\kappa_*$ with a scaling factor $e^{\pi/s_0} = 22.0$. The rate constant α for $a_{\text{eff}} > 0$ is plotted in Fig. 3. We can see that the dimensionless quantity $m_A \alpha / (\bar{p}_z^2 a_{\text{eff}}^4)$ oscillates between zero at $a_{\text{eff}}\kappa_* = 0.404e^{n\pi/s_0}$ and the maximal value 1.93 at $a_{\text{eff}}\kappa_* = 1.89e^{n\pi/s_0}$. Such zeros in α have been explained by the destructive interference effect between two decay pathways in the case of identical bosons [17].

The effect of deeply bound dimers on the three-body recombination rate can be taken into account by analytically continuing the Efimov parameter to a complex value as $\kappa_* \rightarrow e^{i\eta_*/s_0} \kappa_*$ [18]. Here η_* is a real positive parameter to make the Efimov trimers acquire nonzero widths due to decays into the deeply bound dimers. α for $a_{\text{eff}} > 0$ at $\eta_* = 0.1$ and 0.5 are plotted in Fig. 3 as well as at $\eta_* = 0$. We find that our numerical solutions can be excellently fitted by the following formula motivated by that for identical bosons [18]:

$$\frac{m_A}{\bar{p}_z^2 a_{\text{eff}}^4} \alpha = b_+ \frac{\sin^2[s_0 \ln(c_+ a_{\text{eff}} \kappa_*)] + \sinh^2[\eta_*]}{\sinh^2[\pi s_0 + \eta_*] + \cos^2[s_0 \ln(c_+ a_{\text{eff}} \kappa_*)]} + \frac{b_+}{2} \frac{\coth[\pi s_0] \sinh[2\eta_*]}{\sinh^2[\pi s_0 + \eta_*] + \cos^2[s_0 \ln(c_+ a_{\text{eff}} \kappa_*)]}. \quad (9)$$

Here $b_+ = 284$ and $c_+ = 2.48$ are two fitting parameters.

When $\eta_* > 0$, the solution to Eq. (8) can have a nonzero imaginary part even for $a_{\text{eff}} < 0$ because the three-body recombination into the deeply bound dimers becomes possible. The rate constant α for $a_{\text{eff}} < 0$ at $\eta_* = 0.01, 0.1$, and 0.5 are plotted in Fig. 4. Again we find that our numerical solutions can be excellently fitted by the following formula [18]:

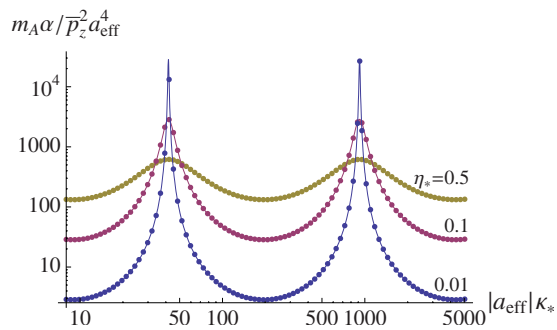


FIG. 4. (Color online) Two periods of $\frac{m_A}{p_z^2 a_{\text{eff}}^4} \alpha$ as a function of $a_{\text{eff}} \kappa_* < 0$ at $\eta_* = 0.01, 0.1,$ and 0.5 for $m_A/m_B = 6.67$. Curves behind data points are from the two-parameter fit by Eq. (10).

$$\frac{m_A}{p_z^2 a_{\text{eff}}^4} \alpha = b_- \frac{\sinh[2\eta_*]}{\sin^2[s_0 \ln(c_- |a_{\text{eff}}| \kappa_*)] + \sinh^2[\eta_*]}, \quad (10)$$

with two fitting parameters $b_- = 143$ and $c_- = 0.528$. We can see that when $\eta_* \ll 1$, the three-body recombination rate exhibits sharp resonant peaks at $a_{\text{eff}} \kappa_* = -1.89 e^{n\pi/s_0}$, which are clear signatures of the confinement-induced Efimov resonances.

Unlike the Efimov parameter κ_* , we cannot determine the width parameter η_* because it involves the complicated short-range physics, but we can estimate η_* to be very small in our system. The size of the Efimov trimers is typically given by the confinement length l . In order for the Efimov trimer to decay into the deeply bound dimers, the three bound atoms have to come within the range of an interatomic potential $r_0 (< l)$. Its probability $\sim (r_0/l)^{4.39}$ for $m_A/m_B = 6.67$ [19] multiplied by the typical energy scale of the short-range physics $\sim r_0^{-2}$ provides the order-of-

magnitude estimate of the width of the Efimov trimer. The width parameter is therefore found to be $\eta_* \sim (r_0/l)^{2.39} \ll 1$ and scales with respect to l . The small value of η_* sharpens the characteristic features in the three-body recombination rate such as the destructive interferences at $a_{\text{eff}} > 0$ and the resonant peaks at $a_{\text{eff}} < 0$ as seen in Figs. 3 and 4.

Finally we note that all the qualitative arguments presented in this Rapid Communication hold for the ^{40}K - ^6Li mixture when ^{40}K is confined in 2D. However, the scaling factor in the 2D-3D mixed dimensions is $e^{\pi/s_0} = 1.78 \times 10^5$, and therefore it is possible that the confinement-induced Efimov resonances may not be observed in a range of the effective scattering length surveyed by experiments.

V. CONCLUSIONS

We have shown that the Fermi-Fermi mixture of ^{40}K and ^6Li in the 1D-3D mixed dimensions is a promising candidate to investigate the Efimov physics in fermions. Unlike other systems in a free space, the Efimov parameter is controlled by the external confinement potential and we can estimate the positions of the three-body resonances to be at $a_{\text{eff}}/l \approx -0.989 \times (22.0)^n$. An observation of the confinement-induced Efimov resonances in the three-body recombination rate at such predicted values of the effective scattering length will provide the first unambiguous evidence of the Efimov trimers in Fermi-Fermi mixtures.

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