The Radio Frequency Quadrupole (RFQ)

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Abstract

In this paper we explain the function and the beam dynamics of a Radio Frequency Quadrupole (RFQ) in the context of the CAS School for Small Accelerators.

1 Introduction

When the RFQ was invented in the 1970s it represented the ‘missing link’ to high power beam. It is the machine that allowed an efficient preparation of a high-intensity low-energy hadron beam for acceleration in a conventional Drift Tube Linac, boosting the efficiency of the transfer between a source and a DTL accelerator from 50% to more than 90%.

The history of the RFQ starts in 1970 when Kapchinskij and Teplyakov proposed the idea of the Radio Frequency Quadrupole [1]. In 1974 there is the first experimental test of the innovative idea of Kapchinskij and Teplyakov at the USSR Institute for High Energy Physics in Protvino: a 148.5-MHz RFQ accelerated 100-keV protons to 620 keV with an efficiency of 50%. Only in 1977 the RFQ concept is published in the western world raising a strong interest in Los Alamos National Laboratory (USA). Los Alamos takes the decision to test the RFQ principle for possible application in development of high-current low-emittance beams. In parallel a strong effort is put in the developments of computer codes for RFQ design. In 1979 the Proof-of-principle experiment (P.O.P) starts at Los Alamos: on the 14 of February 1980 a 425 MHz RFQ accelerates a 100-keV proton beam to 640 keV with an efficiency of 90%, as predicted by the codes. This success opened the way a fast spreading of RFQ in the world: nearly every major laboratory opted for substituting a ‘old style’ pre-injector with a much more compact and efficient RFQ. Nowadays hundreds of RFQ accelerators are operating in the world.

2 RFQ basic principle

The Radio Frequency Quadrupole is a linear accelerator which focuses, bunches and accelerates a continuous beam of charged particles with high efficiency and preserving the emittance. The focusing as well as the bunching and acceleration are performed by a Radio Frequency (RF) electric field.

2.1 RFQ resonating mode

Let us recall Maxwell’s equation for E and B field [2]:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}=0$$

In free space the solution of the above equation shows that electromagnetic fields are of the transverse electro magnetic,TEM, type: the electric and magnetic field vectors are perpendicular to each other and to the direction of propagation. In a bounded medium, like for example a cavity, the solution of the equation must satisfy the boundary conditions:

$$\vec{E}_{//} = \vec{0} \quad \text{and} \quad \vec{B}_{\perp} = \vec{0}$$

therefore only either Tranverse Electric (TE) or Transverse Magnetic (TM) modes are possible.

In TE modes the electric field is perpendicular to the direction of propagation whereas in TM modes the magnetic field is perpendicular to the direction of propagation. In a cylindrical cavity they are denoted as $\mathrm{TE}_{\mathrm{nml}}$ and $\mathrm{TM}_{\mathrm{nml}}$, respectively, where the index n,m,l refer to the azimuthal, radial and longitudinal component.

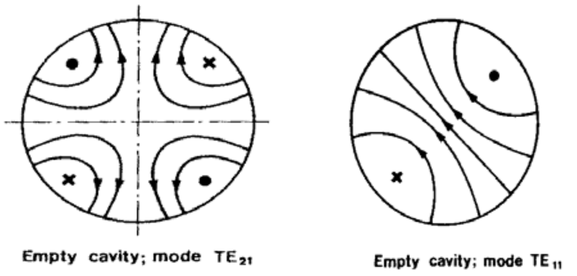

Fig. 1: First two TE modes in a cylindrical cavity: dipole mode TE_{110} and quadrupole mode TE_{210}

Figure 1 shows the first two TE modes in a cylindrical cavity. The cut is perpendicular to the direction of propagation of a beam, the lines represent the electric field, the dots and the cross the points where the magnetic field, parallel to the direction of propagation, enters/exits from the paper.

A RFQ is composed of a cavity loaded with four electrodes and therefore forced to resonate in the TE_{210} mode. A sketch is shown in Fig. 2.

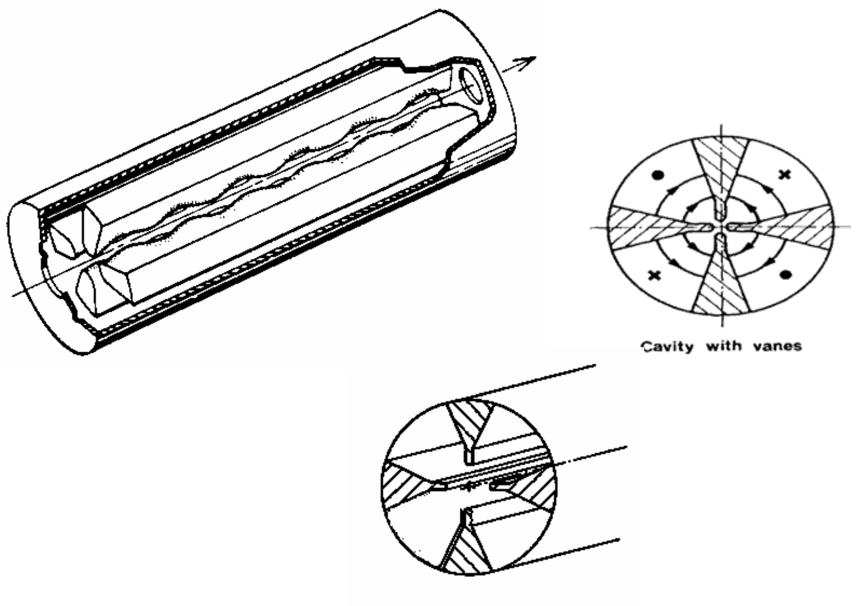

Fig. 2: Sketch of an RFQ cavity—courtesy T. Wangler

There are two main possible configurations for achieving the TE_{210} mode: the so called ‘four vane’ structure or the so called ‘four-rod’ structure. Other structures (split coaxial, double H) are beyond the scope of this lecture. In the four vane structure, of which a picture can be seen on Fig. 3 and the electrical equivalent in Fig. 4, each vane is a resonator with the capacitance concentrated between the vane tips and the inductance in the inter-vane space. The resonating frequency is inversely proportional to the cylinder dimensions and therefore is advantageous for frequency higher than 200 MHz, because at lower frequency the diameter of the tank becomes too big. The vane tips are machined by a computer controlled milling machine.

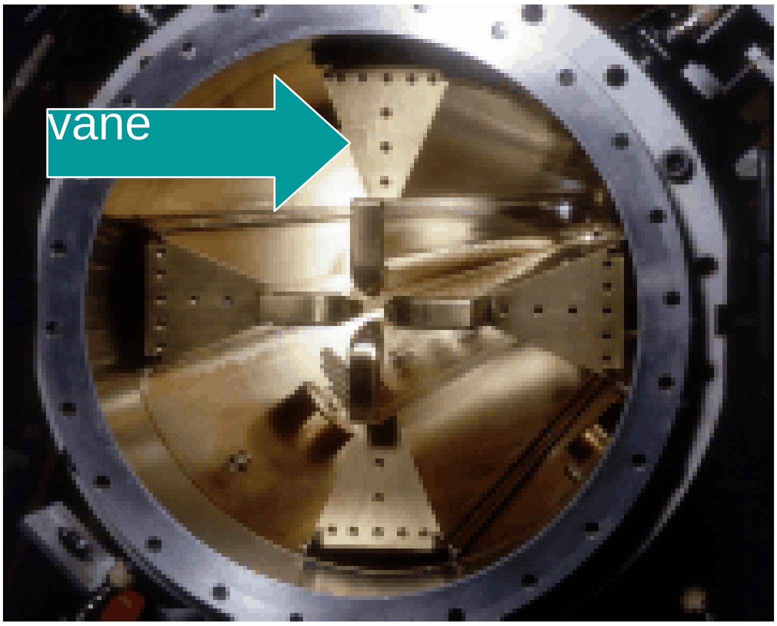

Fig. 3: CERN RFQ1

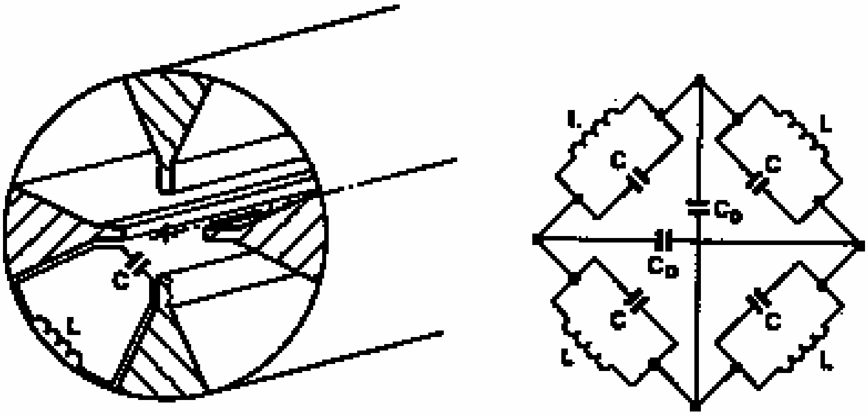

Fig. 4: Electrical equivalent for a four-vane structure

A four-rod structure is conversely composed of rods placed in the centre of the cylindrical cavity by holding bars, see Fig. 5. The capacitance is located between rods, inductance with holding bars. In this configuration each cell is a resonator and therefore the cavity dimensions are independent from the frequency. It is easy to machine with a lathe. Care must be put in the regulation of the end cells, it is generally less efficient than the 4-vane due to strong current in the holding bars.

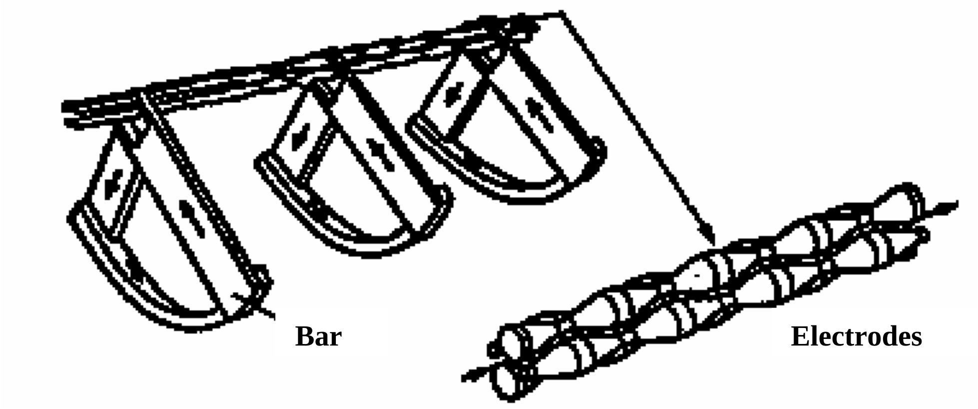

Fig. 5: Sketch of a four-rod structure

2.2 Field in an RFQ

The transverse field in a RFQ provides an alternating gradient focusing structure with period length equal to $\beta\lambda$, where β is the velocity of the particle and λ is the RF-wavelength. This can be more easily understood by looking at the sketch on Fig. 6. Let's assume that the top and bottom electrodes have positive polarity during the time DT1, DT3 … whereas the left and right electrodes have negative polarity during this time. A positively charged ion beam will pushed away from the top/bottom electrodes and pulled towards the left/right. During this times (DT1, DT3…) the RFQ behaves like a defocusing quadrupole. Conversely during the time intervals DT2, DT4 the RFQ behaves like a focusing quadrupole. No force is exerting on the beam at the zero crossing between positive and negative electric field. During any of the time interval DT the beam has traveled a distance equal to $\beta\lambda/2$ and therefore has seen a smooth focusing channel with period $\beta\lambda$.

Acceleration in an RFQ is achieved by periodically modulating the electrodes in the longitudinal direction as shown in Fig. 7. The periodic longitudinal modulation, which is 180 degress out of phase in the top/bottom electrodes with respect to the left/right pair, deforms the pattern of the pure TE mode by creating a longitudinal component proportional to the depth of the modulation. The synchronism between the increasing velocity of the particle and the longitudinal component can be controlled with the wavelength of the modulation.

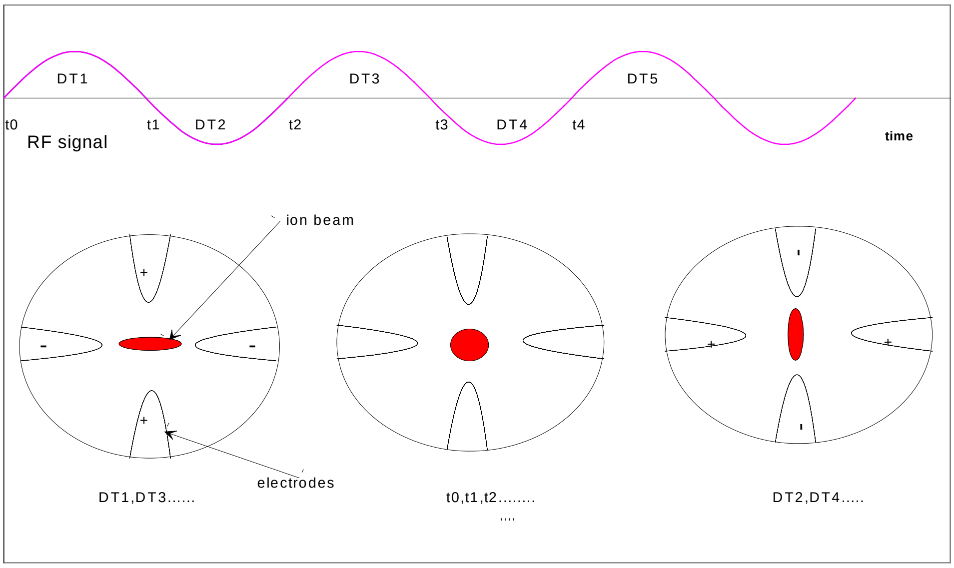

Fig. 6: Time-varying transverse focusing field in a RFQ

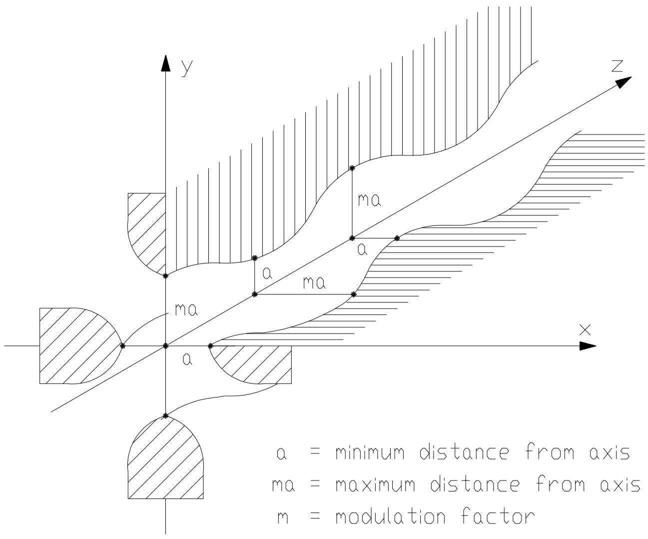

Fig. 7: Longitudinal modulation on the RFQ electrodes

2.3 Beam dynamics in the RFQ

The beam dynamics in an RFQ is fully determined by the geometrical parameter of the electrode structure, in particular the *aperture* determines the focusing strength and the acceptance; the depth of the *modulation* determines the field available for acceleration; and the *distance between the peaks and the trough* of the modulation determines the synchronicity between the field and the particles. Those geometrical quantities, shown in Fig. 7, can be related to the two fundamental parameters of the RFQ, the focusing strength (B) and the accelerating efficiency (E_0T) which are defined as follows:

$$B=\left(\frac{q}{m_0}\right)\left(\frac{V}{a}\right)\left(\frac{1}{f^2}\right)\frac{1}{a}\left(\frac{I_0(ka)+I_0(mka)}{m^2 I_0(ka)+I_0(mka)}\right)\partial$$

$$E_0T=\frac{m^2-1}{m^2 I_0(ka)+I_0(mka)}\cdot V\frac{2}{\beta\cdot\lambda}\frac{\pi}{4}$$

and are linked by the relation

$$\left(\frac{I_0(ka)+I_0(mka)}{m^2 I_0(ka)+I_0(mka)}\right)+\frac{m^2-1}{m^2 I_0(ka)+I_0(mka)}\cdot I_0(ka)=1\,.$$

The quantities involved are: a = bore radius, β,γ= relativistic parameters, c = speed of light, f = rf frequency, $I_{0,}1$ = zero, first-order Bessel function, k = wave number, λ = wavelength, m = electrode modulation, m_0 = rest, q = charge, r = average transverse beam dimension, r_0 = average bore, V = vane voltage.

The beam dynamics design in the RFQ results, in first approximation, in determining the values of the above parameters all along the structure, i.e., from the input energy to the final energy. The first guideline is that the transverse phase advance including RF defocusing space charge must be below 90º at all time. This condition can be expressed as follows:

$$\sigma_{\mathrm{t}} = \sqrt{\frac{B^2}{8\pi^2} - \frac{\pi q E_0 T \sin(-\varphi)\lambda}{m_0 c^2 \beta\gamma^3} - \frac{3 Z_0 q I \lambda^3 (1-f)}{8\pi m_0 c^2 \gamma^3 r_{\mathrm{x}} r_{\mathrm{y}} r_{\mathrm{z}}}}$$

and $0 < \sigma < 90^{\circ}$

With φ is the synchronous phase Z_0 is the free-space impedance (376.73 Ω), I is the beam current, $f(p)$ is a geometrical factor, p is the ratio of the transverse beam dimensions, r is the average beam dimension and x,y,z refers to the two transverse and longitudinal planes. The three terms in the expression of σ represent the quadrupole focusing, the RF defocusing and the space charge defocusing.

In the longitudinal plane the principle of phase stability requires that the synchronous phase φ is between –90 º and 0º.

The RFQ has several functions; the first one is to bunch an incoming continuous beam to prepare it for acceleration. The bunching is done adiabatically over several cells in order to maximize the capture and minimize the emittance. When the beam is fully bunched, the phase is gradually tapered up to bring the beam to the final energy. All the time the transverse parameters are set as to keep a constant transverse phase advance per cell, by compensating the variation of the RF defocusing term and the space charge term during the process of bunching and acceleration.

A summary of the RFQ functions and the corresponding parameter setting in order of increasing energy is reported in Table 1.

Table 1: RFQ functions and corresponding parameter settings

Radial matching–to adapt the beam to a time-varying focusing system		
–	–	Aperture smoothly brought to the average value
Shaping–to give the beam a longitudinal structure		
Taper phase to –80º,–60º	Start modulation	Aperture such that focusing is constant
Bunching–to bunch and begin acceleration		
Taper phase to –30º, –20º	Modulation to max	Aperture such that focusing is constant
Acceleration–to bring the beam to the final energy		
Constant phase	Constant modulation	Constant aperture
Output matching–to adapt the beam to the downstream user's need		
–	–	–

The main difference when designing RFQs comes from the space charge effects. For high intensity beam, when the space charge term dominates the emittance term in the equation of motion, it is important to start acceleration almost at the same time of shaping and bunching in order to compensate for the increased space charge defocusing during longitudinal compression. A RFQ designed for a high intensity beam will result in a longer structure than one designed for the corresponding low intensity beam and, generally, will produce a higher output longitudinal emittance.

Table 2 shows the main difference in the design strategy for high- and low-intensity beam.

Table 2: Difference between a high-intensity and a low-intensity RFQ

	Space-charge dominated	**Emittance dominated**
RMS	–	–
Shaper	Shaping and acceleration	Over many cells w/o acceleration
Pre-buncher	Not used	Fast bunching
Gentle buncher	Bunching and acceleration	Complete the bunching (almost no energy increase up to here)
Booster	Not used	Fast transition to accelerating phase
Accelerator	Beam bunched around $\varphi = -35,-30$	Beam strongly bunched ($\varphi = -20,-15$)

3 RFQ highlights

The RFQ is the accelerator that has bridged the gap between a proton/hadron source and a conventional (TM mode) accelerator. Its strong point are the electric focusing which allows to accept low energy beam; and the adiabatic bunching which preserve beam quality and allows high capture. These two features combined have increased the efficiency of the very first phase of pre-acceleration from 50% to more than 90%, also in presence of strong space charge. Besides, as the transverse and longitudinal dynamics are machined in the electrode microstructure, the RFQ is very easy to operate in routine runs of an accelerator complex.

Acknowledgements

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