VII. MICROWAVE ELECTRONICS

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A. NOISE IN ELECTRON BEAMS

In the April 15, 1955, Quarterly Progress Report, it was shown that Pierce's model of a transverse field tube permits the definition of a small signal kinetic power flow in the beam of the same significance as the kinetic power flow defined by Chu for longitudinal beam tubes (1). The electromagnetic power extracted from the beam by a surrounding structure can be computed, on the basis of the small-signal assumptions, from the state of excitation of the beam at the entry into, and exit from, the structure. The electromagnetic power extracted from the beam is equal to the difference between the kinetic power leaving the structure and the kinetic power entering it. The combined system, the electron beam-microwave structure, can then be treated as passive (or lossless).

In this report we shall outline briefly how the minimum obtainable noise figure can be evaluated for a tube in whose electron beam a kinetic power can be defined. This work has been done jointly with F. N. H. Robinson, temporarily at Bell Telephone Laboratories (2).

The excitation in the beam with n degrees of freedom entering the amplifier structure can be expressed in terms of the normalized amplitudes a_1 to a_n of the modes of excitation in the beam. These modes are orthogonal with respect to their kinetic power flow. The normalized amplitudes of the incoming waves into the input and output of the amplifier will be denoted by a_{n+1} and a_{n+2}, respectively. The normalized amplitudes of the modes in the beam leaving the amplifier will be denoted by b_1 to b_n, the reflected waves at the input and output terminals of the amplifier by b_{n+1} and b_{n+2}. (See Fig. VII-1.) The action of the amplifier can be described in terms of a matrix S with $(n+2)^2$ elements which gives the b_k's in terms of the a_k's. The amplitudes a_k and b_k
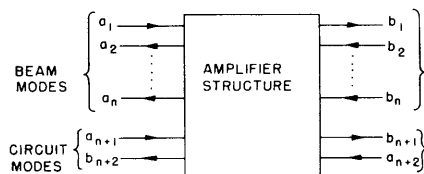

Fig. VII-1

The modes of the beam and amplifier (arrows indicate direction of power flow).

are arranged into column matrices a and b.   We have then

$$b = Sa \tag{1}$$

We shall confine ourselves to amplifiers with lossless microwave structures.   The lossy case is more complicated and does not add to our understanding (2).   The conservation of the sum of the kinetic and electromagnetic power then gives

$$\sum_{k=1}^{n+2} p_k |a_k|^2 = \sum_{k=1}^{n+2} p_k |b_k|^2 \tag{2}$$

where the p_k's will be referred to as the "parities" of the modes; p_k equals +1 if the power (kinetic or electromagnetic) carried by the mode is positive, and p_k equals -1 if it is negative.   The restriction of Eq. 2 imposes upon S the matrix relation (3)

$$S^+ P S = P \tag{3}$$

where P is a diagonal matrix formed of the p_k's, $P = \text{diag}(p_1, p_2, \ldots, p_{n+2})$, and $(S^+)_{jk} = S^*_{kj}$. S^+ is the Hermitian conjugate of S.

The noise in the beam is defined sufficiently, once all the power spectra

$$\lim_{\Delta f \to 0} \overline{a_j a_k^*} / 4\pi\Delta f \qquad\qquad j = 1, \ldots, n; \quad k = 1, \ldots, n$$

are given.   The a_j's are obtained from a Fourier analysis of the noise process over a period $T = 1/\Delta f$.   The bar indicates an ensemble average.   The noise parameters can be arranged into a matrix A.

$$A = \lim_{\Delta f \to 0} \overline{(aa^+)} / 4\pi\Delta f \tag{4}$$

where a^+ is the row matrix with the elements a_1^* to a_n^*.   The transformation of the noise by a general beam transducer characterized by the n-by-n matrix M leads the noise matrix A over into a noise matrix B.

$$B = M A M^+ \tag{5}$$

It has been shown by Robinson (2) that the B matrix can be diagonalized by a proper choice of M, the latter satisfying the condition of a lossless beam transducer

$$M^+ P M = P$$

where $P = \text{diag}(p_1, p_2, \ldots, p_n)$.

Equation 5 implies

$$A = N B N^+ \tag{6}$$

with $N = M^{-1}$, satisfying the condition $N^{+} P N = P$.

With M so chosen that B is diagonal, we have proved the following statement: Any noise process can be represented by a diagonal noise process (a noise process with zero correlation among the different beam modes) followed by a lossless beam transducer.

This discovery allows one, using a beam with a noise matrix A, to find in a simple way the amplifier which gives the lowest noise figure. The noise with the matrix A can be given in terms of the diagonal noise process B followed by the proper lossless beam transducer of matrix N. The beam transducer can be included in the amplifier with the matrix S so that a new structure, characterized by a matrix G, is formed.

$$G = \begin{pmatrix} N & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} S$$

The available power gain of the over-all amplifier, $|G_{n+2, n+1}|^{2}$, is equal to the available power gain before the inclusion of the beam transducer, $|S_{n+2, n+1}|^{2}$. If the noise power fed into the amplifier from the output end in the wave a_{n+2} is neglected, we find for the noise figure of the amplifier

$$F = 1 + \frac{4\pi}{kT} \frac{1}{|G_{n+2, n+1}|^{2}} \sum_{k=1}^{n} |G_{n+2, k}|^{2} B_{kk} \tag{7}$$

The matrix G has to satisfy the condition

$$G^{+} P G = P \tag{8}$$

This condition also implies (see ref. 3)

$$G P G^{+} = P$$

The n+2, n+2 element of the matrix relation given above reads

$$\sum_{k=1}^{n+2} |G_{n+2, k}|^{2} p_{k} = 1 \tag{9}$$

Noting that $p_{n+1} = p_{n+2} = +1$, we see from Eq. 9 that the existence of a large gain, $|G_{n+2, n+1}|^{2}$, requires at least one correspondingly large $G_{n+2, k}$ element of negative parity, $p_{k} = -1$.

The noise figure of Eq. 7 can be minimized in the following obvious way:

First, the mode of negative parity with the smallest value of B_{kk} is chosen. Let this be the jth mode. Then, all elements $G_{n+2, k}$ are made equal to zero, except $|G_{n+2, n+1}|^{2}$ (the available gain) and $G_{n+2, j}$. We have from Eq. 9

$$\left| \frac{G_{n+2,j}}{G_{n+2,n+1}} \right|^2 = 1 - \frac{1}{\left| G_{n+2,n+1} \right|^2}$$

and the minimum obtainable noise figure is

$$F = 1 + \frac{4\pi}{kT} B_{jj} \left(1 - \frac{1}{\left| G_{n+2,n+1} \right|^2} \right) \tag{10}$$

This minimization has the following physical interpretation.

The output power of the amplifier is gained at the expense of the kinetic power in the beam. An amplifier structure has to couple strongly to at least one mode in the beam with negative parity. The amplifier is optimized with regard to the noise figure when it is made to couple only to the "least noisy" mode of negative parity.

It is not hard to show that the minimization procedure described above, when applied to a longitudinal beam amplifier (n = 2), yields the familiar result (4)

$$F_{min} = 1 + \frac{2\pi}{kT} \left(1 - \frac{1}{gain} \right) (S - II)$$

H. A. Haus

References

1. L. J. Chu, Paper delivered at the Conference of the Professional Group on Electron Devices, Orono, Maine, June 1951.

2. F. N. H. Robinson and H. A. Haus, unpublished paper.

3. H. A. Haus and F. N. H. Robinson, Paper to be published in the Proceedings of the I.R.E.

4. H. A. Haus, Trans. I.R.E. (PGED) 1, 238 (1954).