VII. MICROWAVE TUBE RESEARCH

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A. EQUIVALENT CIRCUIT FOR A THREE-TERMINAL PAIR NETWORK

The study of microwave amplifiers of the beam type requires at times (1) an equivalent circuit representation of nonreciprocal three-terminal pair networks. The equivalent circuit is useful if it leads to mathematical simplifications.

A simple equivalent circuit for the most general lossless nonreciprocal three-terminal pair network can be constructed with the aid of a lossless three-terminal pair gyrator. The gyrator is defined by the impedance matrix Z.

$$Z = \begin{bmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{bmatrix}$$

The transfer impedances x, y, and z are real quantities.

The most general, lossless, nonreciprocal, three-terminal pair network has an impedance matrix of the form (2)

$$Z = j \begin{bmatrix} X_{11} & X_{12}e^{j\phi_{12}} & X_{13}e^{j\phi_{13}} \\ X_{12}e^{-j\phi_{12}} & X_{22} & X_{23}e^{j\phi_{23}} \\ X_{13}e^{-j\phi_{13}} & X_{23}e^{-j\phi_{23}} & X_{33} \end{bmatrix}$$

The X's are all real quantities. The matrix has thus 9 adjustable parameters. The equivalent circuit shown in Fig. VII-1 also has 9 adjustable parameters. It includes, besides the gyrator and lumped reactances, ideal two-terminal pair phase shifters (2) characterized by the matrix of generalized circuit parameters (3)
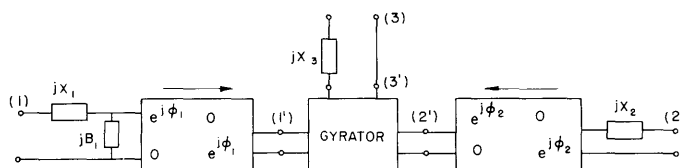

Fig. VII-1
The equivalent circuit.

$$\begin{bmatrix} e^{j\phi} & 0 \\ 0 & e^{j\phi} \end{bmatrix} \tag{1}$$

The arrows above the ideal phase shifters in Fig. VII-1 point from the input to the output terminals of the phase shifters. The voltage and current at the input of a phase shifter with the matrix (Eq. 1) lead the voltage and current at the output by a phase angle ϕ.

Below, the elements of the equivalent circuit are given in terms of the 9 parameters of the three-terminal pair network:

$$X_1 = X_{11} \quad x = X_{12} \quad y = X_{13}$$

$$\phi_1 = \phi_{13} + \frac{1}{2}\pi \quad \phi_2 = \phi_{13} - \phi_{12}$$

$$z = -X_{23} \sin(\phi_{23} + \phi_{12} - \phi_{13}) \quad B_1 = \frac{X_{23}}{X_{12}X_{13}} \cos(\phi_{23} + \phi_{12} - \phi_{13})$$

$$X_2 = X_{22} - \frac{X_{12}X_{23}}{X_{13}} \cos(\phi_{23} + \phi_{12} - \phi_{13})$$

$$X_3 = X_{33} - \frac{X_{13}X_{23}}{X_{12}} \cos(\phi_{23} + \phi_{12} - \phi_{13})$$

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References

1. Quarterly Progress Report, Research Laboratory of Electronics, M.I.T., Oct. 15, 1954.
2. H. A. Haus, Paper to be published in the Journal of Applied Physics.
3. E. A. Guillemin, Communication Networks (John Wiley and Sons, Inc., New York, 1935) VII, p. 138.

B. THE MINIMUM NOISE FIGURE OF LONGITUDINAL BEAM AMPLIFIERS

Some general relations were derived concerning the minimum noise figure of microwave amplifiers of the beam type. A general model of a microwave amplifier with arbitrary noise smoothing schemes was taken as a starting point. The model assumes that the amplifier can be split into an input and an output section. The rf coupling between the input and output section is provided solely through the electron beam. A klystron, a velocity jump amplifier, and a traveling-wave tube with a severed helix are all special cases of this model. The interaction between the beam and the rf circuit of such
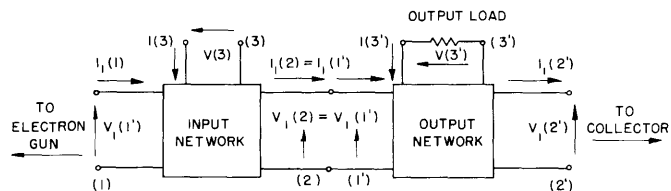
OUTPUT LOAD

Fig. VII-2

The equivalent circuit of a microwave amplifier.

amplifiers can be represented by an equivalent circuit shown in Fig. VII-2; that is, by two linear nonreciprocal passive three-terminal pair networks connected in cascade. (The reader is referred to reference 1 for the definitions and concepts used in this section.) The terminals marked (1), (2), (1'), and (2') are the "beam terminals." The voltage and current at those terminals are the ac beam voltage and beam current. The terminals marked (3) and (3') are the input and output terminals of the amplifier.

A nonreciprocal passive three-terminal pair network is characterized by 9 complex quantities. However, for our purpose it is sufficient to characterize the output network by two complex quantities, a and b. They are defined as follows: When an ac beam voltage $V_1(1')$ and an alternating beam current $I_1(1')$ are applied to the terminal pair 1' of the output circuit, the voltage across the output load is because of linearity

$$V(3') = aV_1(1') + bI_1(1') \tag{1}$$

The subscript "1" of the voltage and current indicates small signal beam voltages and currents.

One of the relations derived concerns the general form of the minimum noise figure of the amplifier described above. The following definition of the noise figure is used:

$$F = 1 + \frac{\text{Noise power output per unit frequency from beam noise}}{\text{Noise power output per unit frequency from input circuit noise}} \tag{2}$$

The above definition is applicable if the input circuit noise and the beam noise are uncorrelated. The denominator of Eq. 2 is proportional to kT, where T is the temperature of the input circuit.

The numerator of Eq. 2 can be evaluated by techniques of four-terminal network transformations of beam noise. The Thévenin impedance of the input circuit is connected across the terminal pair 3 of Fig. VII-2 with the result that the three-terminal pair input network can be treated as a two-terminal pair network so far as its action upon the beam noise is concerned. Under the assumption that a lossless four-terminal beam transducer between the cathode and the amplifier is optimized for lowest noise figure, we find for the noise figure the general expression

$$F_{min} = 1 + \frac{2\pi}{kT} |K| \left[S_o - \frac{K}{|K|} \Pi_o \right] \qquad (3)$$

where

$$S_o = \left(\Phi_o \Psi_o - \Lambda_o^2 \right)^{1/2}$$

S_o and Π_o are two parameters of the beam noise at a plane beyond the potential minimum in the electron gun. K is a constant dependent upon the characteristic parameters of the amplifier. The constant K can be greater or less in magnitude than unity. It is of great interest to find the dependence of K upon the particular amplifying structure used. This has been done under the assumption that the input three-terminal pair network of Fig. VII-2 is lossless. The equivalent circuit for the three-terminal pair network of the preceding section has been used. It is found that two cases have to be distinguished

Case A: $\text{Re}(ab^*) < 0$ \qquad (4)

Inequality 4 can be interpreted as follows: Eq. 1 shows that a modulation on the beam at the point (1') with a small signal impedance

$$\frac{V_1(1')}{I_1(1')} = - \frac{b}{a} \qquad (5)$$

does not deliver any power to the output circuit. According to inequality 4, the critical impedance 5 must have a positive real part for case A to apply. A long helix terminated in a matched load is a known example of such an output structure.

If case A applies, it can be shown that the minimum possible value of K, K_{min}, is

$$K_{min} = 1 \qquad (6)$$

Case B: $\text{Re}(ab^*) > 0$

The critical impedance 5 for case B has a negative real part. A cavity with a long gap (a transit angle θ in the range $0 < \theta < 2\pi$) is a known example of such a structure.

If case B applies

$$K < 1 \qquad (7)$$

and indeed can be zero or become negative.

It is obviously of great interest to determine whether any useful amplifier structures exist with a K appreciably less than unity. This question has been investigated under the additional assumption that the output three-terminal pair network of Fig. VII-2 is lossless. Under this assumption it was possible to show that the available power gain G of the amplifier is limited by the magnitude of K by

$$G \leqslant \frac{1}{1 - K} \qquad (8)$$

Thus, a structure with a K appreciably less than unity and a correspondingly low noise figure would be useless as an amplifier because of its low gain. It should be emphasized that relations 6, 7, and 8 have been obtained under the restrictive assumptions that either the input three-terminal pair network or the input and output networks are loss-less. A generalization of the proof to include loss encounters great mathematical difficulties.

<div style="text-align: right">H. A. Haus</div>

References

1. Quarterly Progress Report, Research Laboratory of Electronics, M.I.T., April 15, 1954, p. 32.

C. BEAM NOISE MEASUREMENTS

Some measurements were made on a gun of higher perveance than was heretofore used. This gun had a convergence half angle of 13°, a perveance of 0.35×10^{-6}, and produced a beam about 0.08 inch in diameter. The heater construction was such as to produce an observable magnetic field at the cathode surface (a few gauss). As with the previous gun, a strong growing noise wave was observed; however, when viewed on an oscilloscope, it fluctuated violently at 60 cps. This was soon traced to the heater. It was observed that the growing noise wave might vary by about 10 db, depending upon the polarity of the heater current, and that this bore a definite relation to the direction of the main focusing field; reversing one required reversing the other to produce a given effect. Previous measurements had indicated that the main focusing field of several hundred gauss produced a leakage field of a few gauss at the cathode, outside the magnetic field.

In order to pin this down more specifically, an auxiliary winding was wrapped around the steel tube surrounding the gun. With this it was possible to produce fields of a few gauss at the cathode, and with the proper polarity the growing noise wave could be eliminated. Subsequent tests, when the gun was removed from the system, indicated that the noise was reduced when the two fields at the cathode were aiding, and the noise was enhanced when they were opposing.

A more careful experiment is being prepared in order to get a better picture of this phenomenon.

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D. THE INTERNALLY COATED CATHODE

Previous investigation of internally coated cathodes made from thin (less than 0.005 inch) nickel with holes approximately 0.015 inch in diameter has shown that the electrons
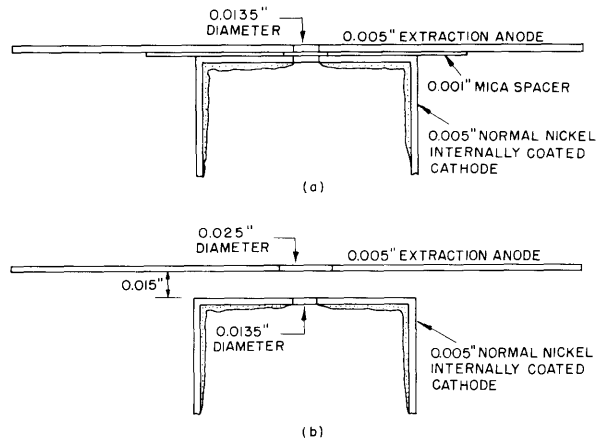
Fig. VII-3

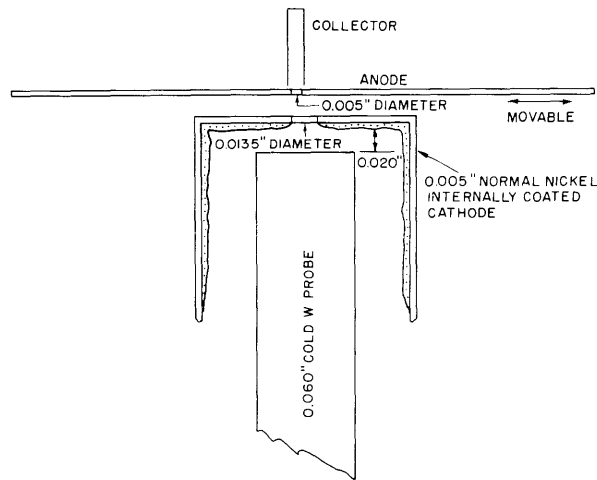Simple electron sources with internally cooled cathodes.

Fig. VII-4

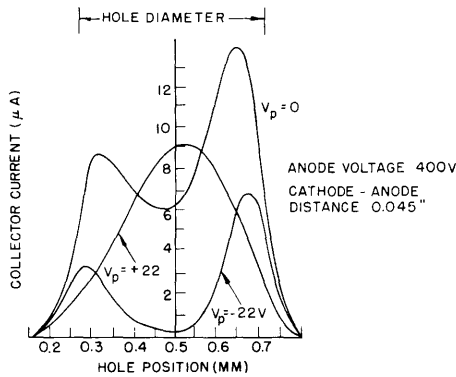Tube allowing determination of regions of emission.

Fig. VII-5

Current density across beam a few hole
diameters in front of the cathode.

originate from the coating and are drawn through the hole by the applied anode voltage. The more field that can be made to penetrate the hole, the greater the emission. If such cathodes, then, are to be used as electron sources, we must have "extraction" anodes similar to the simple guns shown in Fig. VII-3(a) and (b).

The source shown in Fig. VII-3(a) has produced current densities of approximately 1 amp/cm^2 at moderate temperatures and less than 100 volts with about 10 per cent interception current. The source in Fig. VII-3(b) shows no interception current, and the construction and operation are not so critical. The presence of the extraction anode necessarily produces a lens combination that is divergent, and a strong field on the outside of this anode is necessary to remove this divergence. A magnetic field might then hold together a hollow beam, or a strongly convergent lens might produce a crossover point to be imaged by conventional means in a cathode-ray tube.

Investigation of the cathode shown in Fig. VII-4 with a small movable anode hole backed by a collector gives us an insight into the origin of the electrons by letting us see the effect of the probe voltage on the amount and pattern of the emission. The significant results are illustrated in Fig. VII-5. Here, the sampled current through the anode hole to the collector is plotted as a function of the distance the anode hole has moved across a diameter of the cathode hole. The results of this experiment seem to be the same as those of a previous similar experiment; the electrons originate behind the hole, but the lens action of the cathode hole is uncertain.

During this investigation, one cathode showed a surprising increase in emission (about ten times) after the coating had been bombarded by electrons of approximately 50 volts from an emitter inside the cavity. This state of activity occurred instantly and has persisted. Possible reasons for this phenomenon were the production of free barium by the bombardment or excessive temperature of the coating (without excessive temperature of nickel).

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