

IX. MULTIPATH TRANSMISSION

Prof. J. B. Wiesner
E. J. Baghdady

NARROW-BAND LIMITING

A study of the behavior of the plots relating the minimum requirement in discriminator bandwidth (to reject two-path FM interference) to the ideal limiter filter bandwidth used (Quarterly Progress Report, April 15, 1954, Fig. VIII-2) sheds some interesting light on the nature of a set of curves that may be associated with those plots. We refer here to the smooth curves that may be drawn to connect the values of minimum required $(BW)_{disc}/(BW)_{if}$ at the odd integral values of $(BW)_{lim}/(BW)_{if}$ used. These curves are identified as the smooth curves shown in Fig. IX-1 superimposed on the calculated broken-curve plots. We will call these smooth curves the envelopes of the calculated plots since Fig. IX-1 shows that as $(BW)_{lim}/(BW)_{if}$ grows large, the broken-curve plots tend to be bounded by the smooth curves.

If we make the definitions

$$x \equiv \frac{(BW)_{lim}}{(BW)_{if}}$$

$$y \equiv \frac{(BW)_{disc}}{(BW)_{if}} \text{ at odd values of } x$$

$$= \text{"envelope" value of } \frac{(BW)_{disc}}{(BW)_{if}}$$

then the semilogarithmic plots of Fig. IX-2 show that

$$\ln \left[\frac{1+a}{1-a} - y \right]$$

is linearly related to x . We recall that a is the ratio of the undesired to the desired signal amplitude at the input to the ideal limiter. Simple calculations show that the plots of Fig. IX-2 lead to the expression

$$y(a, x) = \frac{1+a}{1-a} \left[1 - \zeta(a) e^{-k(a)x} \right] \tag{1}$$

where values of $k(a)$ and $\zeta(a)$ are plotted in Figs. IX-3(a) and (b). Figure IX-3(a) shows $k(a)$ plotted against $\ln a$ and shows that the plotted points fit rather closely into a straight line given by

$$k(a) = -0.395 \ln a \tag{2}$$

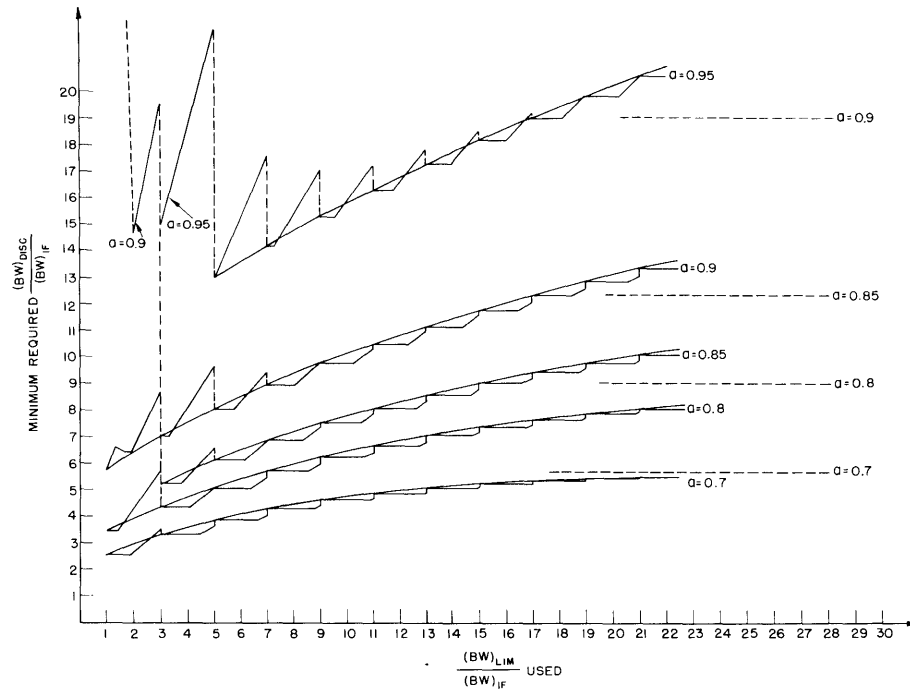


Fig. IX-1

Variation of the minimum discriminator bandwidth requirements with the limiter bandwidth used for various capture ratios, α . The broken curves connecting the calculated plots tend to be bounded by the superimposed smooth curves as the limiter bandwidth grows large.

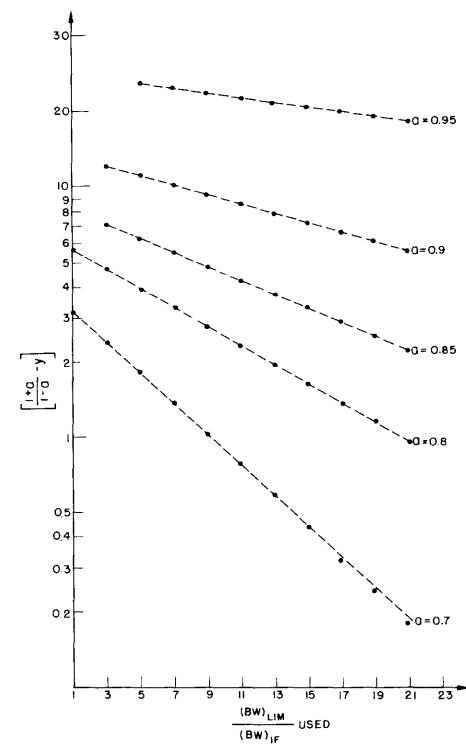


Fig. IX-2

Deviations from the asymptotic values (shown by the dashed horizontal lines in Fig. IX-1) of the discriminator bandwidth required at odd integral values of $(BW)_{lim} / (BW)_{if}$.

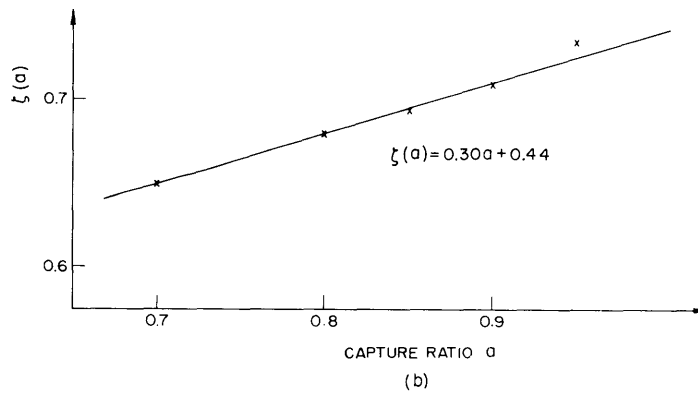
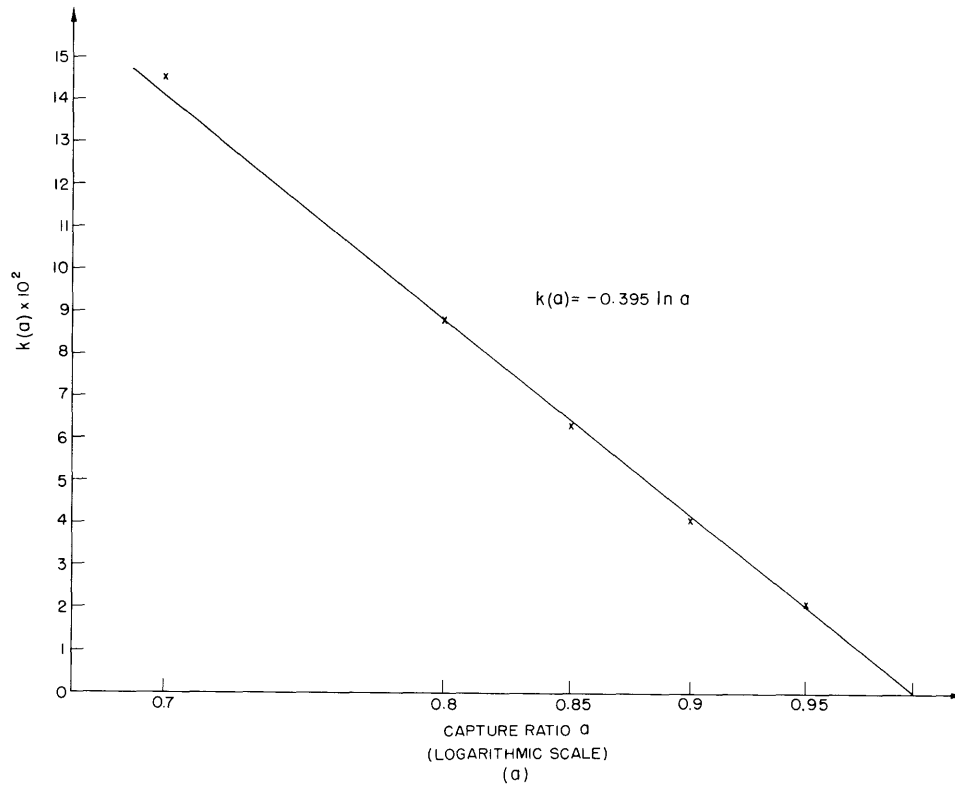


Fig. IX-3

Calculated values of the constants in Eq. 1, resulting from the plots of Fig. IX-2. The deviations of the plotted points from the straight line values may be attributed to small cumulative errors in computation.

(IX. MULTIPATH TRANSMISSION)

The plot of $\zeta(a)$ shown is based on $k(a)$ as given by Eq. 2, with the reasonable assumption that the small deviations of the calculated values of $k(a)$ from the values given by Eq. 2 may be attributed to small cumulative errors in the computations. Figure IX-3(b) shows that the plotted values of $\zeta(a)$ fit into a straight line given by

$$\zeta(a) = 0.30 a + 0.44 \quad (3)$$

Equation 1 may be normalized into the form

$$\begin{aligned} \psi(a) &\equiv y(a, x) / \left[\frac{1+a}{1-a} \right] \\ &= 1 - \zeta(a) e^{-k(a)x} \end{aligned} \quad (4)$$

which is plotted in Fig. IX-4. Clearly, the second term in the right member of Eq. 4 gives the fractional reduction in the minimum required discriminator bandwidth that results from passing the resultant two-path signal through an ideal limiter whose bandwidth is an odd multiple, x , of the bandwidth of the intermediate-frequency section.

Now, at the odd values of x , it is recalled that (1)

$$\frac{(BW)_{disc}}{(BW)_{if}} = 2 \frac{[\Delta\omega]}{(BW)_{if}} + 1 \quad (5)$$

where $[\Delta\omega]$ is the magnitude of the maximum deviation in the instantaneous frequency of the resultant signal at the output of the ideal limiter filter from the frequency of the desired carrier. It is clear from Fig. IX-1 that Eq. 5 also gives the value of $(BW)_{disc}/(BW)_{if}$ over the horizontal segment following the odd value of x for which $[\Delta\omega]$ is calculated. A combination of Eq. 1 and Eq. 5 yields

$$\begin{aligned} \frac{[\Delta\omega]}{(BW)_{if}} &= \frac{1}{2} (y-1) \\ &= \frac{a}{1-a} - \frac{1}{2} \left(\frac{1+a}{1-a} \right) \zeta(a) e^{-k(a)x} \end{aligned} \quad (6)$$

Further normalization yields

$$\begin{aligned} [\Delta\Omega] &\equiv \frac{[\Delta\omega]}{(BW)_{if}} \cdot \frac{1-a}{a} \\ &= 1 - \frac{1+a}{2a} \zeta(a) e^{-k(a)x} \end{aligned} \quad (7)$$

The second term on the right of Eq. 7 yields the fractional amount by which the magnitude of the maximum deviation in instantaneous frequency has been reduced by passing

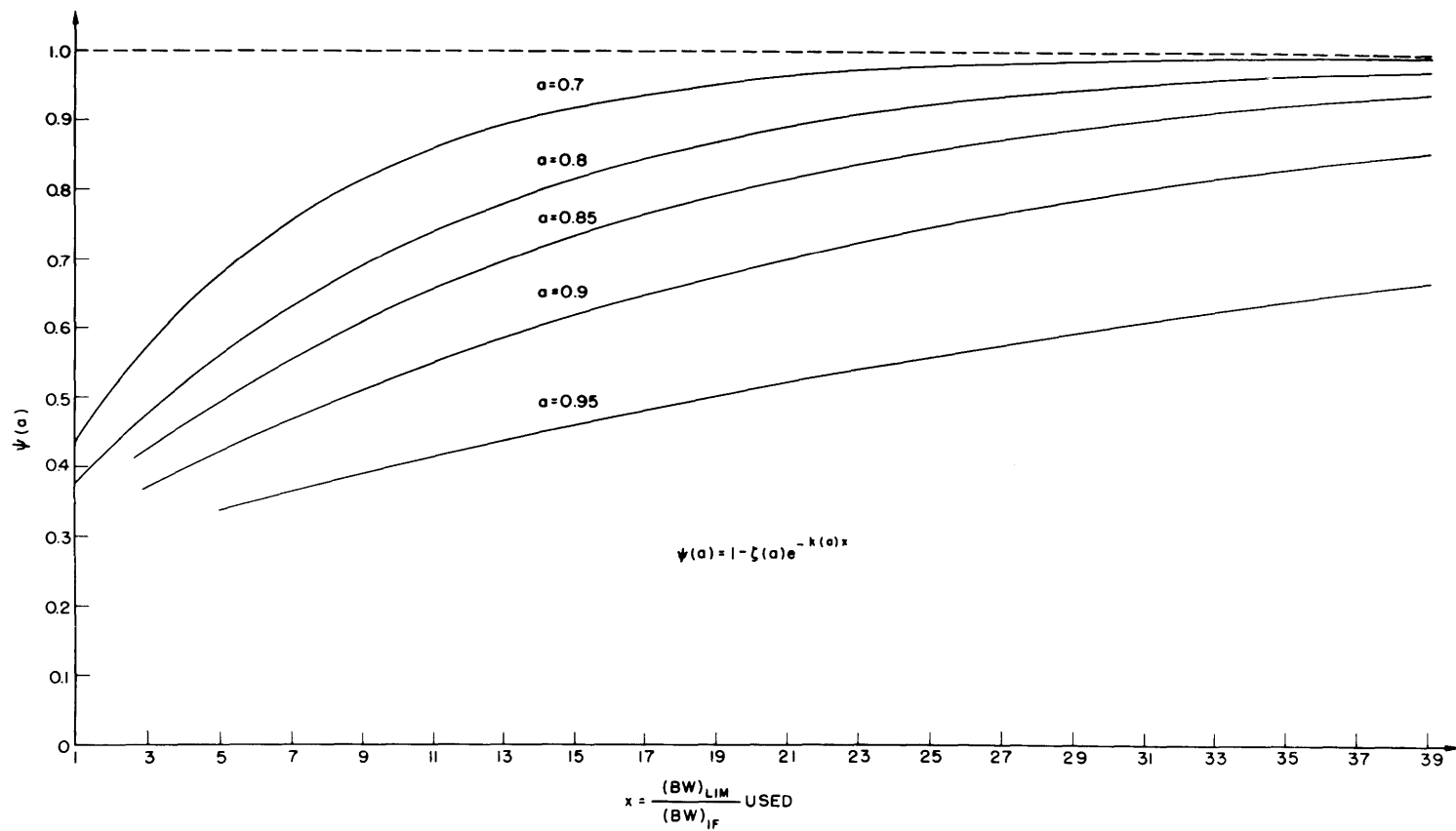


Fig. IX-4

Normalized plots of the envelopes of the broken curves in Fig. IX-1. $\psi(a)$ equals the normalized minimum required discriminator bandwidths at the odd integral values of x .

(IX. MULTIPATH TRANSMISSION)

the resultant two-path signal through an ideal limiter whose bandwidth is an odd multiple, x , of the bandwidth of the intermediate frequency.

Equation 1 is a useful supplement to the calculated plots of Fig. IX-1. It can be used to extrapolate the correct values of the minimum required discriminator bandwidth when the values of limiter bandwidth used are odd integral multiples, x , of the i-f bandwidth, that lie beyond the range covered by the calculated plots. At other values of x , Eq. 1 will yield an approximate answer. Equation 1 may also be used for the same purpose when the capture ratio, a , is required to assume a value that lies in the range $0.7 \leq a \leq 0.95$, but is not covered by the plots of Fig. IX-1. The indicated restriction in the range is given because the behavior of $\zeta(a)$ outside this range is not completely obvious from the plot of Fig. IX-3(b).

E. J. Baghdady

References

1. E. J. Baghdady, M. Sc. Thesis, Department of Electrical Engineering, M.I.T., August 1953.