

DFP-435/12/06
CERN-PH-TH/2006-246
December 2006

Infrared Sensitive Physics in QCD and in Electroweak Theory ¹

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Abstract

I recall the main ideas about the treatment of QCD infrared physics, as developed in the late seventies, and I outline some novel applications of those ideas to Electroweak Theory.

arXiv:hep-ph/0612067 v1 6 Dec 2006

¹To appear in the volume *String Theory of Fundamental Interactions*, published in honour of Gabriele Veneziano in his 65-th birthday, M. Gasperini and J. Maharana editors (*Lecture notes in Physics*, Springer, Berlin/Heidelberg, 2007).

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1 Infrared Sensitive Observables

The high energy physics of elementary particles, as described by the Standard Model, gives particular emphasis to states constructed out of *massless* partons or leptons, because of either the original gauge symmetry, or of the QCD chiral symmetry. This in principle introduces a number of problems because of the existence of mass singularities in gauge theories – that is, of infrared and collinear divergences due to the initial or final states being massless. Of course, physical states yield finite cross-sections because of QCD confinement, or of Electroweak symmetry breaking, or of QED coherent states. However, a remnant of the mass singularities of the problem is that the cross-section, besides being dependent on energy and momentum transfers of the process at hand, may also depend on energy through large logarithmic variables, involving some infrared sensitive mass parameters.

In QCD, avoiding large parameters is vital for the perturbative description of hard processes, characterized by probe(s) with large momentum transfer(s) Q and by a supposedly small coupling. Therefore, the cross-section must be infrared safe, i. e., sufficiently inclusive in order to cancel the mass singularities according to the KLN and/or Bloch-Nordsieck (BN) theorems [1], [2]. As a consequence, fully inclusive processes are truly perturbative, while the inclusive processes in which some partons of virtuality Q_0 are looked at (in the initial or final state) show anomalous dimensions [3]. However, observables in which soft emission is suppressed (e.g., at the boundary of the phase space) or emphasized (e.g., of multiplicity type) are infrared sensitive [4], and still contain parametrically large logarithms of infrared origin, because of an incomplete cancellation of virtual corrections with real emission.

The above observation raises a problem for quite interesting observables (like p_T -form factors and jet multiplicity distributions), but indicates also how to solve it because we know that the infrared behaviour is largely universal due to the QED factorization theorem [1] and generalizations thereof. This fact triggered, in the late seventies, a number of seminal papers dealing with factorization of the collinear behaviour [5], form factor resummation [6], pre-confinement [7], jet evolution [8] and multiplicities [9]. It also appeared that one could describe in full the final state [10] at the level of partons with offshellness Q_0 much smaller than Q but still large with respect to Λ , the QCD scale, thus providing a ground for event generators [11].

All the above papers are largely based on factorization theorems for various hard processes, and gradually introduce generalized renormalization group techniques in order to predict the logarithmic dependence on the infrared sensitive parameters at leading-logarithm anomalous dimension level, extended, by further analysis [12], to the subleading ones. The factorization properties are in turn dependent on the cancellation of truly infrared divergent contributions for all such processes, which requires a generalized Bloch-Nordsieck theorem to be valid in QCD, as better established in the eighties [13]. In fact, the BN theorem states that a cross-section which is inclusive over soft *final* states is also infrared safe, irrespective of the fixed, possibly degenerate initial state. In this form, the theorem is not automatically valid, because the nonabelian nature of QCD allows degenerate initial states in a multiplet, which have different charges and thus in general different cross-sections for the *same* momentum configuration. This spoils the cancellation of virtual corrections with real emission when summing over final soft states, unless an average over initial colour is performed in order to restore the BN theorem. Fortunately, this averaging is automatic because of QCD confinement, which allows only colour singlet asymptotic states.

The ideas above have been refined over the years in QCD, leading to an approximate treatment of coherence effects by angular ordering in jet evolution [14], and to a more general treatment of subleading logarithms in form factor calculations [15]. Recently, they have also led to a new interesting development in Electroweak Theory. Naïvely, one would say that in

the latter case the infrared structure is irrelevant because of the spontaneously broken gauge symmetry, which provides a mass for weak bosons and for fermions. However, with the advent of TeV scale accelerators, we shall soon have access to energies which are much larger than the symmetry breaking scale (say, the W mass) which may act as *infrared* cutoff and thus give rise to parametrically large infrared logarithms in the energy dependence, in addition to the ones of collinear origin. That this is indeed the case was first remarked in the late nineties [17] and soon applied to inclusive observables [18]. The failure of the BN theorem is due again to the nonabelian nature of electroweak theory, where now *no averaging* over flavour is possible, because the initial state consists of electrons, protons, and so on, each of them having a nontrivial weak isospin charge. This also means that double logarithms depending on the electroweak scale affect most cross-sections which are apparently infrared safe, so that electroweak radiative corrections are enhanced, sometimes comparable to QCD ones, and to be carefully evaluated in a unified way.

My purpose in this note is to outline, in a few examples, how the novel ideas of the seventies allow to understand the physics of large logarithms for both QCD and Electroweak Theory, thus turning a potential problem into a powerful tool. They also lead to a precise calculational framework for the logarithmic energy dependence, for which I refer to the reviews already mentioned [4], [14], and to further dedicated papers [15], [16].

2 QCD Form Factors, Multiplicities, Preconfinement

Form Factors An early consequence of the understanding of infrared and collinear behaviours in QCD was the remark [6] – [10] that observables where real emission is suppressed are sensitive to the (square of) the partons’ Sudakov form factor. The latter is evaluated, at leading logarithmic level, by an evolution equation in μ^2 (the parton virtuality) which is derived by a dispersive argument [4], [6], or by applying [19] Gribov’s generalization of the Low theorem [20] as follows:

$$\frac{d \log F_a(Q^2, \mu^2)}{d \log \mu^2} = C_a \frac{\alpha_s(\mu^2)}{2\pi} \log\left(\frac{Q^2}{\mu^2}\right), \quad (1)$$

where $C_a = C_F, C_A$ is the Casimir charge of parton $a = q, g$. Note that $\mu^2 > Q_0^2$ plays the role of cutoff for an infrared divergent anomalous dimension, so that F_a shows an exponential suppression which, in the frozen α_s limit, involves two logarithms per power of α_s , one of collinear type and the other of infrared origin. In the case of physical observables, the cutoff on μ^2 should be replaced by a parameter which regulates real emission, like Q^2/N for the parton PDFs at large moment index N , or $1/B^2$ for impact parameter distributions. The outcome is the characteristic large- N dependence of PDFs for DIS and for the Drell-Yan processes and the corresponding p_T -distributions.

For instance, the DIS structure function $F_N(Q^2)$ allows real emission up to gluon momentum fraction $z < 1/N$, and this regulates the anomalous dimension of (1) in the form

$$F_N(Q^2) \simeq \exp\left[-\frac{C_F}{\pi} \int_{Q_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \alpha_s(\mu^2) \log \text{Min}\left(\frac{Q^2}{\mu^2}, N\right)\right]. \quad (2)$$

We can see that the anomalous dimension becomes finite and of $\log N$ type for $\mu^2 < Q^2/N$, while the “exclusive” limit is reached for $N = Q^2/Q_0^2$, in which case Eq.(2) reduces to $F_q^2(Q^2, Q_0^2)$, where Q_0 is the minimal quark virtuality.

Multiplicities Actually, the idea underlying Refs. [7], [10] is to describe outgoing hadronic jets in semi-inclusive form, at the level of partons of virtuality $Q_0 > \Lambda$, the decay products of the latter being summed over. Here a problem of consistency arises, because Q_0 is a somewhat arbitrary scale, and hadronic distributions should be independent of it. Fortunately, two important properties help. Firstly, multiplicity distributions show a *factorized* Q -dependence with respect to the Q_0 dependence and, secondly, *preconfinement* holds, namely the average mass of “minimal” colour singlets connected to a $q - \bar{q}$ pair is of order Q_0 , much smaller than Q . This means that jet evolution can be viewed in two steps, a perturbative QCD evolution from Q down to Q_0 (of order Λ) and a hadronization process at scale Q_0 . Thus, the virtue of factorization and preconfinement is that the conversion into hadrons does not affect the Q -dependence, and occurs at a much lower scale.

Of course, the infrared analysis is essential in order to derive the above properties. Factorization of multiplicity distributions is argued for by resumming the double-log Feynman- x dependence of jet distribution functions in the soft region, which eventually leads to a *finite* anomalous dimension with a singular α_s -dependence [9, 10] of type $\gamma_0 \simeq \sqrt{\frac{N_c \alpha_s}{2\pi}}$ [21], [19]. Correspondingly, the average hadronic jet multiplicity has the behaviour

$$\bar{n}(Q^2) \sim \exp \int_0^t dt \gamma_0(\alpha_s(t)) \simeq \exp \sqrt{\frac{2N_c}{\pi b} \log \frac{Q^2}{\Lambda^2}}, \quad (3)$$

and thus grows more rapidly than any power of $\log(Q^2/\Lambda^2) = t$.

The behaviour (3) is remarkably different from the one of QED radiation, essentially because of the gluon charge, implying that the QCD jet evolution is a branching process, leading to a cascade, rather than a bremsstrahlung process off one leg, as in QED. Correspondingly, strong correlations of the final soft partons are present, leading to an approximate KNO scaling of “exclusive” n -parton emission probabilities, which for a gluon jet have the form [4]

$$\frac{\sigma_n}{\sigma_{jet}} \simeq \frac{1}{\bar{n}} \exp[-\frac{1}{2}(\log \frac{n}{\bar{n}})^2], \quad (n \ll \bar{n}). \quad (4)$$

This result shows that the the approximate proportionality of the σ_n s in a gluon jet to the corresponding form factor (1) still holds, at double-log level, as for the electron in QED, but their relationship to the average multiplicity (3) - in the frozen α_s limit - is quite different from QED because of the QCD cascade.

Preconfinement On the other hand, preconfinement [7] follows from a veto on the possible final states which are allowed in the minimal colour singlets in which, by definition, a $U(3)$ colour line connects a quark of offshellness Q_0 to the corresponding antiquark. Because of factorization, and of the veto, the inclusive mass distribution of minimal singlets being produced in a jet of mass up to Q is independent of Q and is instead sensitive to the quark form factor, as follows [7], [10]

$$\frac{M^2 d\sigma}{\sigma_{jet} dM^2} \sim F_q^2(M^2, Q_0^2), \quad (5)$$

so that its average mass is of order Q_0 . Therefore, the conversion of partons into hadrons can occur by an interaction of partons which are close in phase space, leading to the so-called *local* parton-hadron duality [22], and to the possibility of building event generators with relatively simple hadronization models [11], [23].

3 Inclusive Electroweak Double Logarithms

The infrared physics outlined above relies on the BN cancellation of virtual and real emission singularities, which in QCD occurs because of the colour averaging in the initial state, as remarked above. Therefore, the form factor behaviour of type (1) shows up only if some veto uncovers the “exclusive” limit of the given hard process. On the other hand, in Electroweak (EW) theory the BN theorem *fails* because of the flavour charges of the accelerator beams. For instance, the total cross-section for e_+e_- annihilation into hadrons is an infrared safe observable from the QCD standpoint, but carries nevertheless EW double logarithms, embodied into an enhanced effective coupling

$$\alpha_{eff}(s) = \frac{\alpha_W}{4} \left(\log \frac{s}{M_W^2} \right)^2, \quad (6)$$

which is of order 0.2 in the TeV energy range and leads, therefore, to sizeable corrections, of the same order as QCD ones. Besides the expected collinear logarithm, the expression (6) carries an additional one, of infrared origin, due to the violation of the BN theorem.

The analysis of such inclusive double logarithms [18] involves form factors of type (1), where now μ^2 is cutoff by the EW scale $M_W^2 \simeq M_Z^2 = M^2$ and the Casimir C_a refers to the isospin I representation $a = I = 0, 1, \dots$ in the t -channel of the lepton-antilepton overlap matrix. For instance, the combinations $\sigma_{e-\nu} \pm \sigma_{e-e_+}$ correspond to $I = 0$ ($I = 1$), so that

$$\sigma_{e_+e_-}(s, M^2) \simeq \frac{1}{2}(\sigma_0 - \sigma_1 F_1(s, M^2)) \simeq \frac{1}{2}(\sigma_0 - \sigma_1 \exp(-2\frac{\alpha_{eff}(s)}{\pi})), \quad (7)$$

where σ_0 corresponds to the isospin averaged cross section and has therefore no double logarithms, while the antisymmetric combination σ_1 is damped by the $I = 1$ form factor, with $C_1 = 2$. We note that, because of the optical theorem, the inclusive form factor is not squared, though referring to a physical cross-section in the crossed channel. Note also that in this example $\sigma_1 > 0$, because the neutrino cross-section is larger, and therefore the $\sigma_{e_+e_-}/\sigma_0$ ratio *increases* in the TeV energy range towards its high-energy limit, which is provided by the flavour average.

The above description can be generalized, by collinear factorization, to single logarithmic level and to a generic overlap matrix involving leptons and partons in the initial states, thus coupling the EW and QCD sectors of the Standard Model. The result of this procedure is a set of evolution equations in μ^2 which are similar to the DGLAP equations [24], except that evolution kernels exist in the channels with $I \neq 0$ also, and are infrared singular or, in other words, depend on a logarithmic cutoff, much as in Eq.(1). For instance, in the evolution of lepton densities f_l and boson densities f_b , the $I = 0$ evolution kernels coincide with the customary DGLAP splitting functions P_{ba} , while the $I = 1$ ones involve the cutoff dependent virtual kernels

$$P_f^V = \delta(1-z) \left(-\log \frac{Q^2}{\mu^2} + \frac{3}{2} \right), \quad P_b^V = \delta(1-z) \left(-\log \frac{Q^2}{\mu^2} + \frac{11}{6} - \frac{n_f}{6} \right). \quad (8)$$

The corresponding evolution equations have the form

$$-\frac{df_a^1}{d \log \mu^2} = \frac{\alpha_W}{2\pi} f_a^1 P_a^V + \text{regular terms}, \quad (9)$$

and have been described in fully coupled form in [25]. Here I just notice that Eq.(9) shows a Sudakov behaviour similar to (1) and is consistent with Eq.(7) after taking into account the antilepton evolution, which doubles the virtual kernel.

The presence of inclusive double logarithms in spontaneously broken gauge theories remains an intriguing subject. It is mostly an initial state effect and, as such, it is present for any final states of the same class (e.g., flavour blind) and strongly depends on the accelerator beams. Leptonic accelerators maximize it, while hadronic ones (like LHC) provide some partial average on the initial partonic flavours, thus decreasing it. But the effect appears also if the flavour charges are looked at in the final state instead of the initial state, for instance in gluon fusion processes in which some W s are observed [26]. Furthermore, the effect occurs whenever the soft boson emission mixes several degenerate states having different hard cross-sections. Non abelian theories have it because of the nontrivial multiplets, but also a broken abelian theory shows it whenever the mass eigenstates are not charge eigenstates [27]. An example of the latter type is the mixing of the Higgs boson with the longitudinal gauge boson occurring in a $U(1)$ theory. The Standard Model shows both kinds of effects and, given their magnitude in Eq.(6), I think that the coupled evolution equations of parton-lepton distribution functions [25] deserve by now a quantitative study at the TeV scale.

Perhaps, the most important lesson to be learned from several decades of investigation of infrared sensitive high-energy physics is that, even at the level of hard processes, the fundamental interactions look much more intertwined, due to the large time nature of asymptotic states which possibly increases their effective couplings. By the same token, because of the large times involved, factorization theorems are at work and allow a good understanding of the infrared dynamics. It remains true, however, that a unified treatment of all degrees of freedom is needed already at Standard Model level – that is, even before discovery of a possible short-distance unification.

Acknowledgements

It is a pleasure to thank Gabriele, as a collaborator and as a friend, for sharing over many years and subjects the excitement of long discussions and, sometimes, of real understanding. I also warmly thank old and new teams on this subject, in particular Stefano, and Paolo and Denis, for various updates of the picture presented here. I am finally grateful to the CERN Theory Division for hospitality while this work was being completed, and to the Italian Ministry of University and Research for a PRIN grant.

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