

## XI. MICROWAVE COMPONENTS

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### A. STRIP TRANSMISSION SYSTEM

The microstrip transmission system will, in general, support two free modes. These are a perturbed TEM mode, and a perturbed, surface-wave E mode. Both modes have been observed experimentally to be excited with any of the couplings designed to date (1).

In order to improve this system, it is desired to excite mainly the perturbed TEM mode. Therefore, an investigation was started to determine the field distribution of each of these two modes (2).

M. Schetzen

### References

1. Quarterly Progress Report, Research Laboratory of Electronics, M. I. T., Jan. 15, 1953, p. 53
2. Quarterly Progress Report, Research Laboratory of Electronics, M. I. T., April 15, 1953, p. 86

### B. HIGH-Q MICROWAVE RESONATOR

The object of this project is to design, construct, and test a microwave resonator having a very high  $Q$ . Preliminary work was done by P. H. Rose (1).

A lead cavity was designed to operate at a frequency of approximately 3000 Mc/sec in the  $TE_{011}$  mode. This cavity will be cooled below the lead superconducting transition temperature ( $7.26^\circ$  K). The cavity test rig is shown in Fig. XI-1. A double-walled tank is evacuated by a two-stage vacuum system (fore pump and diffusion pump).

The cavity inside the tank is first precooled with liquid nitrogen and then further cooled by liquid helium. The cavity is excited by magnetic probes which are connected to the vacuum-tight coaxial lines.

The loaded  $Q$  of the cavity will be measured by the decrement method (2). A block diagram of the measuring setup is shown in Fig. XI-2. The unloaded  $Q$  will be determined by auxiliary measurements and calculations.

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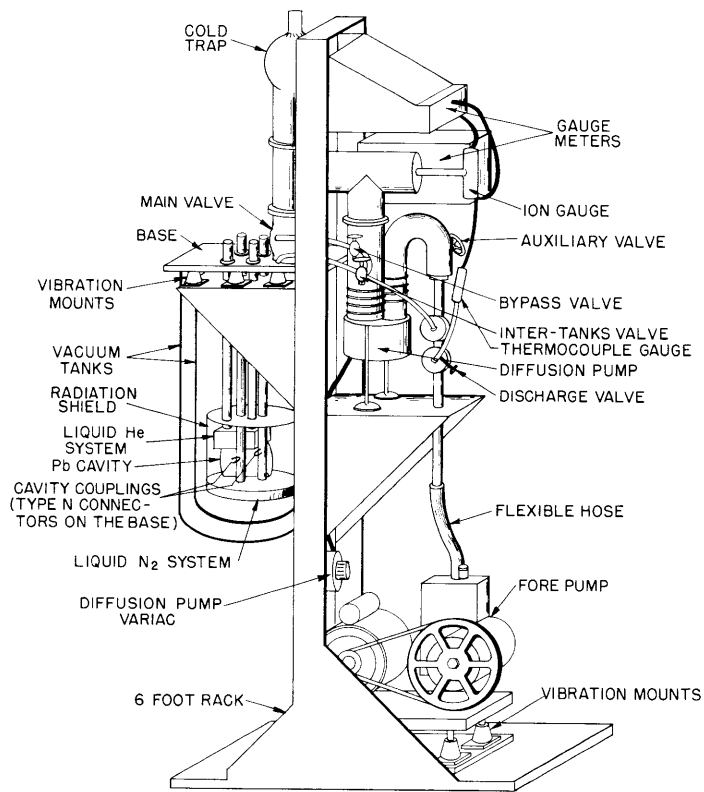


Fig. XI-1  
Cavity test rig.

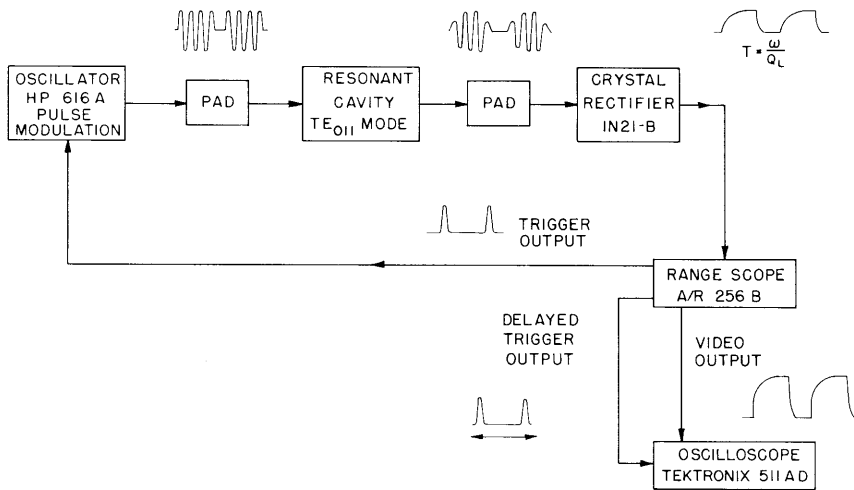


Fig. XI-2  
High-Q measurement block diagram.

## References

1. Quarterly Progress Report, Research Laboratory of Electronics, M. I. T., July 15, 1953
2. C. G. Montgomery: Technique of Microwave Measurements, Radiation Laboratory Series, Vol. 11, McGraw-Hill, New York, 1947

## C. FERRITES AT MICROWAVE FREQUENCIES

## 1. A Variational Principle for Cavities Filled with a Ferrite

Consider a cavity with perfectly conducting walls to be completely filled with a ferrite. Then the following can be shown to be a variational principle for the magnetic field configuration:

$$\left[ \omega_n^2 \right] = \frac{\int_v (\nabla \times \mathbf{H}) \cdot (\nabla \times \mathbf{H}^*) dv}{\epsilon \int_v \mathbf{H} \cdot \vec{\mu} \cdot \mathbf{H}^* dv} \quad (1)$$

where  $\mathbf{H}$  is the magnetic field,  $\mathbf{H}^*$  its conjugate,  $\epsilon$  the permittivity,  $\vec{\mu}$  the permeability tensor of the ferrite, and the integrals are over the volume of the cavity.

Furthermore, the extremal values of  $\left[ \omega_n^2 \right]$  equal the square of the resonance frequencies of the ferrite-filled cavity as implied by the notation. Thus Eq. 1 can be used in obtaining an approximate solution for either  $\mathbf{H}$  or  $\omega_n$ .

## 2. A Variational Principle for Waveguides Containing Ferrites

Consider a cylindrical waveguide, not necessarily circular, with perfectly conducting walls, and completely filled with a ferrite. Then the following can be shown to be a variational principle for the magnetic field at cutoff:

$$\left[ \omega_c^2 \right] = \frac{\int_s (\nabla \times \mathbf{H}) \cdot (\nabla \times \mathbf{H}^*) ds}{\epsilon \int_s \mathbf{H} \cdot \vec{\mu} \cdot \mathbf{H}^* ds} \quad (2)$$

where the integrals are over the cross section of the waveguide. Again, extremal values of  $\left[ \omega_c^2 \right]$  equal the square of the cutoff frequencies.

Variational principles for more general cases, of less immediate interest, have also been obtained.

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### 3. Some Considerations Relating to Boundary-Value Problems of the Electromagnetic Field

A critical study of certain methods of solving boundary value problems of the electromagnetic field has been made, with special regard to some new arguments advanced recently by Teichmann and Wigner (1, 2). While there appear to be certain errors in reference (2), they do not alter the validity of the main argument advanced by the authors: that is, that the set of normal modes usually used for the solution of cavity problems is certainly not complete. Two major conclusions have been drawn from the critical study of these matters:

a. Treatments already known, which employ the Green's function for solution of the boundary value problem, do contain essentially the same results as those described in reference (2).

b. Slater's treatment of the boundary-value problem in terms of normal modes (3) requires some modification to bring it into agreement with the others.

The required modification of Slater's analysis rests upon two points, which (in notation consistent with Slater's (3)) are:

a. A change in the boundary conditions on the irrotational modes  $F_a$ , so that  $F_a$  is redefined as follows:

$$\begin{array}{ll} 1. F_a = 1/k_a \nabla \psi_a & 3. \psi_a = 0 \text{ on } S \\ 2. (\nabla^2 + k_a^2) \psi_a = 0 & 4. (\partial \psi_a)/(\partial n) = 0 \text{ on } S' \end{array} \quad (3)$$

b. The introduction of a set of irrotational magnetic modes  $G_a$  defined by:

$$\begin{array}{ll} 1. G_a = 1/k_a \nabla \phi_a & 3. \phi_a = 0 \text{ on } S' \\ 2. (\nabla^2 + k_a^2) \phi_a = 0 & 4. (\partial \phi_a)/(\partial n) = 0 \text{ on } S \end{array} \quad (4)$$

With these changes, the impedance or admittance calculations for cavities containing isotropic substances can be performed by procedures which parallel closely those of reference (3). For example, the admittance of an empty cavity is determined by letting  $S'$  vanish and expressing the fields in the cavity in terms of expansions in  $E_a$ ,  $H_a$ , and  $G_a$ .

As a result of the clarification of the matters referred to above, it has been possible to extend Slater's method to the problem of cavities filled with anisotropic media (ferrites, for example). It is convenient in this connection to introduce magnetic current density  $J_m$  into the general analysis, with the result that it simply appears in the equations in a manner which is dual to that of the electric current density  $J_e$ . Thus the cavity fields are expressed in terms of  $E_a$ ,  $H_a$ ,  $F_a$ , and  $G_a$ .

## 4. Thin Ferrite Post in a Rectangular Waveguide

Consider a waveguide of rectangular cross section containing a very thin cylindrical post of ferrite, as shown in Fig. XI-3. Assume that only the fundamental mode of the guide will propagate (with electric field parallel to the post). Under these conditions, the magnitude of the reflection coefficient of the fundamental mode can be shown to be given, to a first approximation, by

$$|R| = \frac{k^2 S_0}{a\gamma} \left\{ \chi_e \sin^2\left(\frac{\pi d}{a}\right) + \chi \left[ \left(\frac{\gamma}{k}\right)^2 \sin^2\left(\frac{\pi d}{a}\right) + \left(\frac{\pi}{ka}\right)^2 \cos^2\left(\frac{\pi d}{a}\right) \right] - \kappa \frac{\gamma\pi}{k^2 a^2} \sin\left(2\pi \frac{d}{a}\right) \right\} \quad (5)$$

where  $k^2 = \omega^2 \mu_0 \epsilon_0$ ,  $S_0$  = the cross-sectional area of the ferrite,  $\chi_e$  = electric susceptibility,  $\gamma$  = the propagation constant of the fundamental mode and

$$\text{tensor magnetic susceptibility} = \begin{bmatrix} \chi & -j\kappa & 0 \\ j\kappa & \chi & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The expression (Eq. 5) giving the reflection coefficient is nonreciprocal. Indeed, if the direction of the steady magnetic field applied to the ferrite is reversed, or if the direction of propagation of the incident wave is changed,  $\kappa$  changes sign and the value of  $R$  is altered.

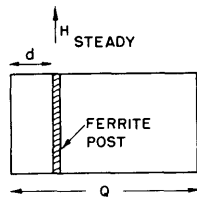


Fig. XI-3  
Thin ferrite post in  
rectangular waveguide.

Note that the maximum effect of the ferrite occurs when the latter is located at a distance  $a/4$  from either of the side walls. Two posts symmetrically placed will double the nonreciprocal effect if the respective steady magnetic fields are in opposite directions. With the fields in the same direction, the nonreciprocal effect is cancelled.

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References

1. T. Teichmann: J. Appl. Phys. 23, 701, 1952
2. T. Teichmann, E. P. Wigner: J. Appl. Phys. 24, 262, 1953
3. J. C. Slater: Microwave Electronics, Van Nostrand, New York, 1950, Chap. 4
4. Quarterly Progress Report, Research Laboratory of Electronics, M.I.T., July 15, 1953, p. 115