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Scaling Phenomena in Gravity from QCD

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Abstract

We present holographic arguments to predict properties of strongly coupled gravitational systems in terms of weakly coupled gauge theories. In particular we relate the latest computed value for the Choptuik critical exponent in black hole formation in five dimensions, $\gamma_{5D} = 0.412 \pm 1\%$, to the saturation exponent of four-dimensional Yang-Mills theory in the Regge limit, $\gamma_{BFKL} \simeq 0.409552$.

To Pere Pascual, in memoriam

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1 Introduction

The realization of the holographic principle [1] in terms of a duality between gravity and a conformal field theory [2] allows, in principle, the study of gauge and gravitational dynamics beyond their perturbative regimes. However, most of the applications of the gauge/gravity correspondence so far have dealt with extracting gauge dynamics at large 't Hooft coupling from weak gravity. This is because the study of strong gravity effects involves backgrounds where the curvature becomes of the order of the string scale at some point. Thus, the worldsheet dynamics of the corresponding string theory becomes strongly coupled. This makes it difficult to obtain reliable results to be compared with the gauge theory calculations.

Because of this, it is of interest to find a window of gravitational phenomena that, using holography, would allow a description in terms of weakly coupled gauge dynamics. Generically, critical behavior is a very robust physical phenomenon and independent of many details of the dynamics involved.

Some years ago [3] the existence of critical behavior in black hole formation was discovered in numerical simulations of gravitational collapse of a free massless scalar field (see [4] for a review). Consider a family of scalar field initial conditions labeled by a real number p such that for small values of this parameter the collapse of this initial configuration leaves behind flat Minkowski space-time, while for large p a black hole is formed. Choptuik [3] found that there is a critical value p^* such that a black hole is formed when $p > p^*$ while the field disperses if $p < p^*$. Moreover, for supercritical initial conditions close to p^* the radius of the black hole horizon r_0 scales as

$$r_0 \sim (p - p^*)^\gamma, \tag{1}$$

where the critical exponent γ is independent of the particular family of initial conditions chosen. Numerical simulations showed that in four dimensions $\gamma \approx 0.372$. This scaling behavior is associated with departures from linear evolution when considering initial data close to the critical solution. In D space-time dimensions the black hole mass is related to the radius of the horizon by $M_{\mathrm{BH}} \sim r_0^{D-3}$. Thus, the critical scaling of this quantity is given by $M_{\mathrm{BH}} \sim (p - p^*)^{(D-3)\gamma}$.

We believe this is precisely the type of strong gravitational phenomenon accessible, in principle, to a gauge theory calculation at weak coupling. Next we have to identify the right

gauge theory framework. For this we take into account that the production of black holes at threshold involves strong gravitational fields as well as large velocities. This suggest the Regge domain of large s but weak coupling in the gauge dual. This is dominated by the exchange of a BFKL pomeron [5] (see [6] for review).

It is known that in the one-pomeron approximation the total cross section grows with the energy as $\sigma_{\text{tot}}(s) \sim s^{\alpha(0)}$, where $\alpha(0) > 1$ is the intercept of the Regge trajectory. Due to the power-like growth with s, this amplitude eventually violates the Froissart-Martin unitarity bound, $\sigma_{\text{tot}}(s) \leq \frac{\pi}{m_\pi^2}(\log s)^2$. It is expected that when the energy increases, saturation effects inside the hadron lead to a departure from the linear evolution for the BFKL kernel. This leads to a decrease of the total cross section making it compatible with unitarity requirements.

We conjecture that these two phenomena, the onset of nonlinear effects in critical gravitational collapse and in the evolution of the BFKL kernel, are dual and lead to the same critical exponents. Namely, we propose that the saturation threshold in pomeron physics is a holographic description of ("zero mass") black hole formation in gravitational collapse in five dimensions. The string regime we describe corresponds to curvatures of the order of the string scale but with small string coupling constant. In this sense the BFKL pomeron at weak coupling provides the dual description of the strongly coupled worldsheet string dynamics. A BFKL calculation of the critical exponent gives a result in surprisingly good agreement with numerical simulations of five-dimensional gravity

$$
\left.\begin{array}{l}
\gamma_{\text{5D}} = 0.408 \pm 2\% \quad [7] \\
\gamma_{\text{5D}} = 0.412 \pm 1\% \quad [8]
\end{array}\right\}, \qquad \gamma_{\text{BFKL}} = 0.409552. \tag{2}
$$

In the last section we present some heuristic arguments supporting this result.

Recently, a holographic interpretation of the pomeron was proposed in [9] according to which the BFKL Hamiltonian is identified with the Laplacian for a type of metric perturbation in the five-dimensional gravitational dual (see also [10] for other studies of the QCD pomeron in the context of the AdS/CFT correspondence). Using this duality they obtained an expression for the BFKL eigenvalue $\chi_0(\nu)$ in the strong 't Hooft coupling regime, $g^2 N \gg 1$. Notice, however, that our aim here is not to use gravity to obtain results for QCD at strong coupling. Rather, we identify phenomena in perturbative QCD providing reliable information on strong gravitational effects in five dimensions. Given the wildly different mathematical and physical tools leading to the remarkable agreement shown in Eq. (2), we are tempted to believe that a mere coincidence is unlikely, and that our arguments provide evidence for the

validity of the holographic hypothesis in a region hitherto unexplored.

2 Criticality in black hole formation

Numerical simulations of the gravitational collapse of a family of scalar field configurations show the appearence of a scaling law for the radius of the black hole formed. This implies the existence, in infinite dimensional phase space, of a codimension one critical hypersurface separating initial data leading to black hole formation from those where the scalar field disperses leaving behind flat space at late times.

Interestingly, the critical solution acting as an attractor for all initial data at the critical surface has interesting properties [3]. Physically it can be pictured as describing the formation of a black hole at "zero mass" and therefore presents a naked singularity at the origin. Moreover, this critical solution has the property of discrete self-similarity (DSS). This means that if we denote by $Z_*(t, r)$ any of the components of the metric or the scalar field, one finds the following symmetry

$$Z_*(t, r) = Z_*(e^\Delta t, e^\Delta r). \tag{3}$$

From the numerical analysis the "echoing" period Δ is extracted to be $\Delta \approx 3.44$. Here we take the convention that the black hole forms as $t \to 0^-$.

For initial conditions slightly away from the critical surface the critical solution $Z_*(t, r)$ acts as a transient attractor. The dynamical evolution drives the system to an "echoing regime" where the metric is close to the critical one and approximately self-similar near the origin. Eventually, the solution is repelled from the critical surface to evolve either to Minkowski space-time or to form a black hole.

The scaling in the black hole horizon can be understood as a consequence of the dynamical instability of the critical solution, i.e. the existence of a repulsive direction in phase space that takes the metric away from the critical one [11]. Indeed, introducing a fiducial length scale ℓ_0 it is possible to define coordinates (τ, ζ) by

$$\tau \equiv \log\left(-\frac{t}{\ell_0}\right), \qquad \zeta \equiv \log\left(-\frac{r}{t}\right) - \xi_0(\tau), \tag{4}$$

where $\xi_0(\tau)$ is periodic with period Δ. DSS acts now by discrete translations in the τ coordinate, $(\tau, \zeta) \to (\tau + \Delta, \zeta)$. Notice that in this coordinates the black hole forms at $\tau \to -\infty$.

For slightly supercritical solutions with $p \gtrsim p^*$ we can write $Z_p(\tau, \zeta)$ as a perturbation of the critical solution

$$Z_p(\tau, \zeta) \sim Z_*(\tau, \zeta) + \alpha_1(\tau, \zeta)(p - p^*)e^{\lambda_1 \tau} + \ldots \tag{5}$$

$\lambda_1 < 0$ is the eigenvalue associated with the repulsive direction and the dots stand for the terms associated with the other (positive) eigenvalues whose contribution is exponentially suppressed for large negative τ. The existence of a growing mode leads to a departure from the linear analysis. This happens for a value $\tau_*(p)$ of the (dimensionless) τ coordinate which in turn defines the scale

$$t_*(p) = \ell_0 e^{\tau_*(p)} \sim \ell_0 (p - p^*)^{-\frac{1}{\lambda_1}}. \tag{6}$$

As explained in [11] $t_*(p)$ is the only dimensionful quantity in the problem. It sets the scale of the black hole apparent horizon and as a consequence the scaling (1) is obtained. The critical exponent is determined by the negative eigenvalue λ_1 by

$$\gamma = -\frac{1}{\lambda_1}. \tag{7}$$

A more detailed analysis [11, 12] shows that in the case of critical solutions with DSS there are periodic wiggles superimposed to the scaling law (1)

$$r_0 \sim (p - p_*)^\gamma e^{f[\log (p - p^*)]}, \tag{8}$$

where $f(x)$ is a periodic function. On general grounds it can be proved that the period of this function is Δ/γ. Depending, however, on the particular model under study shorter periodicities compatible with this one are possible. For example, in the critical collapse of a massless scalar field the period of the wiggles is halved to $\Delta/(2\gamma)$ [11, 12].

The analysis presented here is completely general. In the collapse of a massless scalar field both the critical exponent for the black hole size γ and the period of the echo Δ have been computed in several dimensions [7, 8]. Specially interesting for our later discussion are the results for five-dimensional gravity which, as mentioned above, give a value $\gamma_{5D} = 0.408 \pm 2\%$ for the critical exponent with an echoing period $\Delta_{5D} = 3.19 \pm 2\%$ [7] (the values found in Ref. [8] are $\gamma_{5D} = 0.412 \pm 1\%$ and $\Delta_{5D} = 3.10 \pm 0.1$). In the following section we show how a QCD calculation leads to the result (2).

3 Black hole critical exponents from pomeron physics

In the previous section we have described the kind of strong gravitational physics that we want to capture using a weakly coupled gauge theory. Now we look for the appropriate gauge theory framework to provide a dual description of these phenomena.

The scattering amplitude of two hadrons in the one-pomeron exchange can be written as [5, 6]

$$\mathcal{A}(s,t) = s \int \frac{d^2 k_1 d^2 k_2}{(\vec{k}_1 - \vec{q})^2 \vec{k}_2^2} \Phi_1(\vec{k}_1, \vec{q}) f(\vec{k}_1, \vec{k}_2, s, \vec{q})_{\text{BFKL}} \Phi_2(\vec{k}_2, \vec{q}) \tag{9}$$

where $t \simeq -\vec{q}^{\,2}$, $\Phi_{1,2}(\vec{k}_{1,2}, \vec{q})$ are the so-called impact factors that encode the information about the coupling of the pomeron to the colliding hadrons and $\vec{k}_1$, $\vec{k}_2$ and $\vec{q}$ are two-dimensional transverse momenta.

In what follows we consider the simplest case of zero momentum transfer, i.e. very diffractive scattering. This seems to be enough to capture the dual s-wave gravitational collapse with spherical symmetry. In the case of zero momentum transfer, $t = 0$, and large s, the BFKL kernel in the leading $\log s$ approximation and at weak coupling can be written as

$$f(\vec{k}_1, \vec{k}_2, s, \vec{0})_{\text{BFKL}} = \frac{1}{2\pi^2 \sqrt{\vec{k}_1^2 \vec{k}_2^2}} \int_{-\infty}^{\infty} d\nu \, e^{\overline{\alpha}_s \chi_0(\nu) \log\left(\frac{s}{\vec{k}^2}\right) + i\nu \log\left(\frac{\vec{k}_1^2}{\vec{k}_2^2}\right)} \tag{10}$$

where $\overline{\alpha}_s = g^2 N/(4\pi^2)$, $\vec{k}$ is a characteristic transverse momentum scale and $\chi_0(\nu)$ is given in terms of digamma functions by

$$\chi_0(\nu) = 2\psi(1) - \psi\left(\frac{1}{2} - i\nu\right) - \psi\left(\frac{1}{2} + i\nu\right). \tag{11}$$

In order to establish the holographic map it is convenient to introduce the quantities

$$y = \overline{\alpha}_s \log\left(\frac{s}{\vec{k}^2}\right), \qquad \tau_i = \log\left(\frac{\vec{k}_i^2}{\vec{k}^2}\right) \quad (i = 1, 2) \tag{12}$$

as well as $\eta = \frac{1}{2} - i\nu$. Then we can define

$$\Psi(y, \tau_1 - \tau_2) \equiv \pi \vec{k}_2^2 f(\vec{k}_1, \vec{k}_2, s, \vec{0})_{\text{BFKL}} = \int_{\frac{1}{2} - i\infty}^{\frac{1}{2} + i\infty} \frac{d\eta}{2\pi i} e^{y\chi_0(\eta) - \eta(\tau_1 - \tau_2)} \tag{13}$$

where now $\chi_0(\eta) = 2\psi(1) - \psi(\eta) - \psi(1 - \eta)$. The function $\Psi(y, \tau)$ satisfies the imaginary time Schrödinger equation

$$\frac{\partial}{\partial y} \Psi(y, \tau) = \widehat{H} \Psi(y, \tau), \tag{14}$$

where the Hamiltonian is given by

$$\widehat{H} = \chi_0 \left(-i \frac{\partial}{\partial \tau} \right). \tag{15}$$

From this expression we see that the eigenvalues of this Hamiltonian associated with the plane-wave eigenfunctions $\psi_\nu(\tau) = \frac{1}{\sqrt{2\pi}} e^{i\nu\tau}$ are given by $\chi_0(\nu)$. Eq. (14) governs the evolution of the scattering amplitude to higher energies in the BFKL regime. In the holographic map proposed in Ref. [9], the Hamiltonian (15) is identified with the Laplacian associated with the h_{++} perturbations of the metric on the gravity side, with the coordinate τ identified with the holographic direction.

The BFKL amplitude (9) leads to violations of unitarity due to its growth at large s. In the variables (y, τ) this corresponds to an exponential increase of the amplitude with y. This exponential growth is the one *we conjecture* to correspond, on the gravity side, to the growing mode associated to the black hole formation at threshold.

In order to extract the critical exponent, we evaluate the integral in Eq. (13) using a saddle point approximation. We calculate the saddle point at a value of $\tau = \tau_c(y)$ that makes the leading exponential equal to one [13, 14]. Then, the equations determining both the saturation scale $\tau_c(y)$ and the saddle point value η_c are

$$y\chi_0'(\eta_c) = \tau_c(y), \qquad y\chi_0(\eta_c) = \eta_c\tau_c(y). \tag{16}$$

These equations determine the value of η_c to be

$$\eta_c\chi_0'(\eta_c) - \chi_0(\eta_c) = 0, \qquad \Longrightarrow \qquad \eta_c = 0.627549, \tag{17}$$

while the critical value of τ as a function of the rapidity y is given by

$$\tau_c(y) = \chi_0'(\eta_c)y. \tag{18}$$

At this saddle point the function $\Psi(y, \tau)$ becomes

$$\Psi(y, \tau) \approx \frac{e^{-\frac{\tau^2}{2\chi_0''(\eta_c)y}}}{\sqrt{2\pi\chi_0''(\eta_c)y}} e^{\eta_c[\tau_c(y) - \tau]}. \tag{19}$$

A more detailed calculation of the saturation scale $\tau_c(y)$ leads to logarithmic corrections to the right hand side of Eq. (18) but does not change the leading linear behavior [14].

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Physically speaking the variable $\tau = \tau_1 - \tau_2$ gives the logarithm of the quotient between the characteristic length scales of the two colliding hadrons as probed by the pomeron. For a given value of τ, saturation (and perturbative violations of unitarity) occurs for a value of the rapidity y such that $\tau_c(y) = \tau$. This value of y, where the linear BFKL evolution breaks down, is given by Eq. (18). In terms of the variables $\vec{k}_1$, and $\vec{k}_2$ it is

$$e^y = \left(\frac{|\vec{k}_1|}{|\vec{k}_2|} \right)^{\frac{2}{\chi_0'(\eta_c)}}. \tag{20}$$

According to the conjecture stated above, the exponent in this expression should correspond to the Choptuik exponent for critical black hole formation in five dimensions. A numerical evaluation gives

$$\gamma_{\text{BFKL}} = \frac{2}{\chi_0'(\eta_c)} = 0.409552, \tag{21}$$

in remarkable agreement with the gravitational computations as shown in Eq. (2).

4   Discussion and concluding remarks

We have presented a prescription to calculate the five-dimensional Choptuik critical exponent from the QCD pomeron. The role played by unitarity in the BFKL computation confirms the importance of this property in black hole dynamics (it has been suggested in a different context [15] that black hole production could lead to a saturation of the Froissart-Martin bound).

We should stress that the proposed duality is a gravity/CFT correspondence since, as it is proved in Ref. [16], there is a CFT structure encoded in the BFKL Hamiltonian with conformal weights determining the dependence on the holographic direction. Moreover, our analysis is robust with respect to supersymmetry, since supersymmetric effects only show up in the next-to-leading order [17]. This is because fermion loops do not contribute in the leading logarithm approximation to the diagrams involved in the BFKL pomeron analysis. It is also important to keep in mind that because the BFKL pomeron resums the leading logarithm contributions to all orders in perturbation theory the result automatically scales with the 't Hooft coupling $\overline{\alpha}_s \sim g^2 N$. Therefore it gives the leading large-N contribution even if we work at finite N. Thus, planarity is generated by the leading log s approximation.

Following the holographic map for pomeron physics suggested in [9] it would be interesting to study the properties of the bulk critical geometry from the BFKL Hamiltonian at weak coupling. On the other hand, as a marginal comment we just mention that a naive computation of the critical exponent from the strong coupling eigenvalue $\chi_0(\nu)$ derived in [9] leads to the value $\gamma = \frac{1}{2}$. Very likely this corresponds to the mean field approximation capturing thermal nucleation of black holes in a box.

Apart from the exponent γ, the solution to the Einstein equations describing critical gravitational collapse is characterized by an additional symmetry. It is either discretely or continuous self-similar, i.e. it has a discrete or continuous conformal symmetry. We are currently investigating the QCD analog of this phenomenon, and our findings will be reported elsewhere [18].

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References

[1] G. 't Hooft, *Dimensional reduction in quantum gravity,* `arXiv:gr-qc/9310026`.
L. Susskind, *The World As A Hologram,* J. Math. Phys. **36** (1995) 6377 [`arXiv:hep-th/9409089`].

[2] J. M. Maldacena, *The large N limit of superconformal field theories and supergravity,* Adv. Theor. Math. Phys. **2** (1998) 231 [`arXiv:hep-th/9711200`].

E. Witten, *Anti-de Sitter space and holography,* Adv. Theor. Math. Phys. **2** (1998) 253 [`arXiv:hep-th/9802150`].

S. S. Gubser, I. R. Klebanov and A. M. Polyakov, *Gauge theory correlators from non-critical string theory,* Phys. Lett. B **428** (1998) 105 [`arXiv:hep-th/9802109`].

O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, *Large N field theories, string theory and gravity,* Phys. Rept. **323** (2000) 183 [`arXiv:hep-th/9905111`].

[3] M. W. Choptuik, *Universality And Scaling In Gravitational Collapse Of A Massless Scalar Field,* Phys. Rev. Lett. **70** (1993) 9.

[4] C. Gundlach, *Critical phenomena in gravitational collapse,* Phys. Rept. **376** (2003) 339 [`arXiv:gr-qc/0210101`].

[5] L. N. Lipatov, *Reggeization Of The Vector Meson And The Vacuum Singularity In Non-abelian Gauge Theories,* Sov. J. Nucl. Phys. **23** (1976) 338

E. A. Kuraev, L. N. Lipatov and V. S. Fadin, *The Pomeranchuk Singularity In Non-abelian Gauge Theories,* Sov. Phys. JETP **45** (1977) 199

I. I. Balitsky and L. N. Lipatov, *The Pomeranchuk Singularity In Quantum Chromodynamics,* Sov. J. Nucl. Phys. **28** (1978) 822

[6] J. Forshaw and D. A. Ross, *Quantum Chromodynamics and the Pomeron,* Cambridge 1997.

S. Donnachie, G. Dosch, P. Landshoff and O. Nachtmann, *Pomeron Physics and QCD,* Cambridge 2002.

[7] E. Sorkin and Y. Oren, *On Choptuik's scaling in higher dimensions,* Phys. Rev. **D71** (2005) 124005 [`arXiv:hep-th/0502034`].

[8] J. Bland, B. Preston, M. Becker, G. Kunstatter and V. Husain, *Dimension-dependence of the critical exponent in spherically symmetric gravitational collapse,* Class. Quantum Grav. **22** (2005) 5355. [`arXiv:gr-qc/0507088`].

[9] R. C. Brower, J. Polchinski, M. J. Strassler and C. I. Tan, *The pomeron and gauge/string duality,* `arXiv:hep-th/0603115`.

[10] M. Rho, S. J. Sin and I. Zahed, *Elastic parton parton scattering from AdS/CFT,* Phys. Lett. **B466** (1999) 199 [`arXiv:hep-th/9907126`].

R. A. Janik and R. Peschanski, *High energy scattering and the AdS/CFT correspondence,* Nucl. Phys. **B565** (2000) 193 [`arXiv:hep-th/9907177`].

R. A. Janik and R. Peschanski, `Minimal surfaces and Reggeization in the AdS/CFT correspondence`, Nucl. Phys. **B586** (2000) 163 [`arXiv:hep-th/0003059`].

R. C. Brower, S. D. Mathur and C. I. Tan, *From black holes to pomeron: Tensor glueball and pomeron intercept at strong coupling,* `arXiv:hep-ph/0003153`.

R. A. Janik, *String fluctuations, AdS/CFT and the soft pomeron intercept,* Phys. Lett. B **500** (2001) 118 [`arXiv:hep-th/0010069`].

C. I. Tan, *Pomeron and AdS/CFT correspondence for QCD,* `arXiv:hep-ph/0102127`.

R. C. Brower and C. I. Tan, *QCD hard scattering from string/gauge duality,* Nucl. Phys. Proc. Suppl. **119** (2003) 938.

C. I. Tan, *String/gauge duality and soft-hard pomeron,* Acta Phys. Polon. B **36** (2005) 711.

[11] C. Gundlach, *Understanding critical collapse of a scalar field,* Phys. Rev. **D55** (1997) 695 [`arXiv:gr-qc/9604019`].

[12] S. Hod and T. Piran, *Fine-structure of Choptuik's mass-scaling relation,* Phys. Rev. **D55** (1997) 440 [`arXiv:gr-qc/9606087`].

[13] L. V. Gribov, E. M. Levin and M. G. Ryskin, *Semihard Processes In QCD,* Phys. Rept. **100** (1983) 1.

E. Iancu and L. D. McLerran, *Saturation and universality in QCD at small x,* Phys. Lett. **B510** (2001) 145 [`arXiv:hep-ph/0103032`].

A. H. Mueller, *Parton saturation at small x and in large nuclei,* Nucl. Phys. **B558** (1999) 285 [`arXiv:hep-ph/9904404`].

[14] A. H. Mueller and D. N. Triantafyllopoulos, *The energy dependence of the saturation momentum,* Nucl. Phys. **B640** (2002) 331 [`arXiv:hep-ph/0205167`].

S. Munier and R. Peschanski, *Geometric scaling as traveling waves,* Phys. Rev. Lett. **91** (2003) 232001 [`arXiv:hep-ph/0309177`].

S. Munier and R. Peschanski, *Traveling wave fronts and the transition to saturation,*

Phys. Rev. **D69** (2004) 034008 [`arXiv:hep-ph/0310357`].

S. Munier and R. Peschanski, *Universality and tree structure of high energy QCD,* Phys. Rev. **D70** (2004) 077503 [`arXiv:hep-ph/0401215`].

[15] S. B. Giddings, *High energy QCD scattering, the shape of gravity on an IR brane, and the Froissart bound,* Phys. Rev. **D67** (2003) 126001 [`arXiv:hep-th/0203004`].
H. Nastase, *The soft pomeron from AdS-CFT,* `arXiv:hep-th/0501039`.
K. Kang and H. Nastase, *Heisenberg saturation of the Froissart bound from AdS-CFT,* Phys. Lett. B **624** (2005) 125 [`arXiv:hep-th/0501038`].

[16] L. N. Lipatov, *The Bare Pomeron In Quantum Chromodynamics,* Sov. Phys. JETP **63** (1986) 904.

[17] A. V. Kotikov and L. N. Lipatov, *NLO corrections to the BFKL equation in QCD and in supersymmetric gauge theories,* Nucl. Phys. **B582** (2000) 19 [`arXiv:hep-ph/0004008`].

[18] L. Álvarez-Gaumé, C. Gómez, A. Sabio Vera and M. A. Vázquez-Mozo, in progress.