II. MICROWAVE GASEOUS DISCHARGES

Prof. S. C. Brown
Prof. W. P. Allis
O. T. Fundingsland
D. H. Looney
J. J. McCarthy
K.-B. Persson
A. V. Phelps
D. J. Rose

A. THE STEADY-STATE DISCHARGE IN HYDROGEN

Measurements of the maintaining electric field in the microwave hydrogen discharge have been made, using techniques reported earlier (1). Data are shown in Fig. II-1 for a parallel plate geometry. The variables are as follows: p is the pressure in mm Hg;

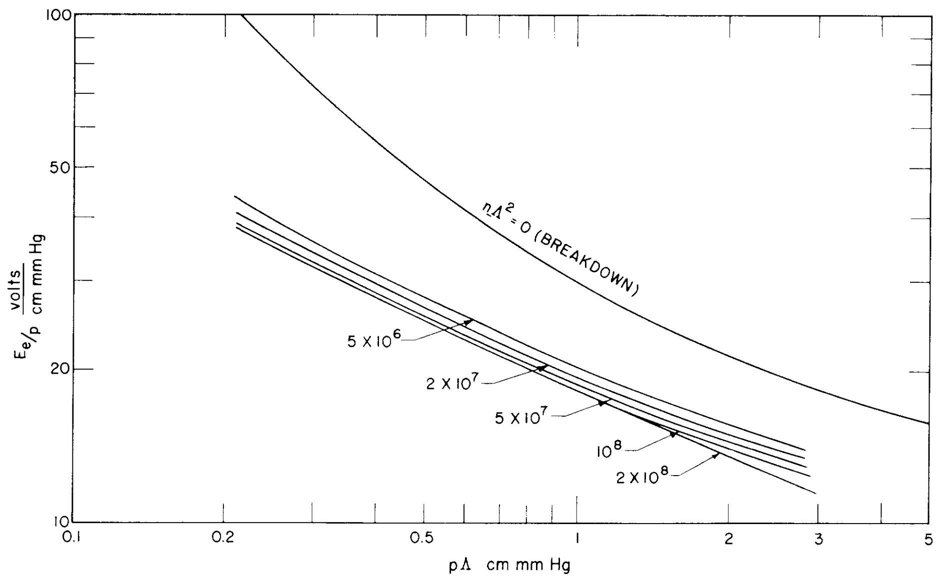

Fig. II-1 Contours of constant $n_-\Lambda^2$ cm^{-1} at the center of a parallel plate tube, for hydrogen, on the E_e/p - $p\Lambda$ plane.

the diffusion length $\Lambda = L/\pi$ for this case, and L is the parallel plate spacing; $E_e = E/\sqrt{1 + (\omega^2/\nu_c^2)}$, where E is the rms electric field required to maintain the discharge, ω its radian frequency, and ν_c the collision frequency of electrons with gas atoms; n_- is the electron density/cm^3. Breakdown data are also shown for comparison.

A theory of the discharge has been developed, similar to that of MacDonald and Brown (2) for breakdown. Except when noted below, the notation is that of reference 2. If the d-c space charge E_s is included in the Boltzmann transport equation, the distribution function F in velocity and configuration space may be written as

$$\nu_q F_o^o - \frac{2m}{Mv}\frac{\partial}{\partial u}(uv\nu_c F_o^o) = \frac{v}{3}\left\{\frac{1}{u}\frac{\partial}{\partial u}u\left[\vec{E}_s \cdot \vec{F}_1^o + \frac{\vec{E}_1 \cdot \vec{F}_1^1}{2}\right] - \nabla \vec{F}_1^o\right\} \qquad (1)$$

$$\vec{F}_1^o = \frac{v}{\nu_c}\left(\frac{\partial}{\partial u}\vec{E}_s F_o^o - \nabla F_o^o\right) \qquad (2)$$

$$\vec{F}_1^1 = \frac{v\nu_c}{\nu_c^2 + \omega^2} \frac{\partial}{\partial u} \vec{E}_1 F_o^o \tag{3}$$

where ν_q is the frequency of excitation plus ionization, and $\vec{E}_1$ is the peak value of the a-c applied field. All terms in F_o^1 and higher order can be shown to be negligible. It is justifiable to neglect the term $\vec{E}_s \cdot \vec{F}_1^o$ compared to $(\vec{E}_1 \cdot \vec{F}_1^1)/2$; in this case, Eq. 2 and Eq. 3 can be substituted in Eq. 1, and F_o^o separated into an energy function f(u) times a space function $n_-(x, y, z)$, provided $\vec{E}_s$ is proportional to $\nabla n_-/n_-$. That is, if

$$\vec{E}_s = -u_s \frac{\nabla n_-}{n_-} \tag{4}$$

then

$$\nabla^2 n_- - \frac{1}{\Lambda^2} n_- = 0 \tag{5}$$

and

$$\nu_q f - \frac{2m}{Mv} \frac{d}{du} (uv\nu_c f) = \frac{v}{3} \left\{ \frac{1}{u} \frac{d}{du} \frac{uv}{\nu_c} E_e^2 \frac{df}{du} - \frac{vu_s}{\nu_c \Lambda^2} \frac{df}{du} - \frac{v}{\nu_c \Lambda^2} f \right\} \tag{6}$$

where E_e is defined above. Normalization gives

$$\int_0^\infty f \, 4\pi v^2 \, dv = 1 \quad .$$

The variable u_s must correspond to the actual space charge present, and is obtained by solution of the diffusion equation in the plasma (3). Equation 4 is exact at high electron density (the ambipolar limit); it is in serious error at low densities only, in which case, u_s itself is small and its effect in Eq. 6 is negligible. It is readily seen, since $df/du < 0$, that the effect of the space charge is to oppose the flow of electrons by diffusion, represented by the last term in the equation.

In this approximation, then, the energy distribution function is not a function of position, and the principal effect of the space charge is merely to form a potential well for the electrons inside the cavity. The approximation is valid as long as the electron drift current is small compared to the random current.

Equation 6 is then solved by methods similar to those of reference 2. As in breakdown, the steady-state condition implies that production of new electrons just balances all of the losses, which exist here only by diffusion in the space charge field. By integration of either Eq. 2 or Eq. 6, this loss is seen to be

$$\frac{D_s}{\Lambda^2} = \frac{D_- - u_s \mu_-}{\Lambda^2} \tag{7}$$

where

$$D_- = \int_0^\infty \frac{v^2}{3\nu_c} f\, 4\pi v^2 dv \tag{8}$$

$$\mu_- = \frac{e}{m} \int_0^\infty \frac{1}{\nu_c} f\, 4\pi v^2 dv$$

$$= \frac{e}{m\nu_c} \qquad \text{(for constant } \nu_c\text{)} \ . \tag{9}$$

Thus the equation to be solved is

$$\frac{\Lambda^2 \displaystyle\int_{u_i}^\infty \nu_i f\, 4\pi v^2 dv}{D_- - u_s\mu_-} = 1 \tag{10}$$

which is an implicit equation in u_s, E_e/p and $p\Lambda$. It is then possible to plot contours of constant u_s, D_s, or D_-/μ_- on the (E_e/p) - $p\Lambda$ plane.

Finally, electron density must be determined as a function of these variables. The nonlinear current equations treated in reference 3 have been solved approximately to yield D_s as a function of $n_-\Lambda^2$ along contours of constant D_-/μ_-. Such a plot is shown in Fig. II-2.

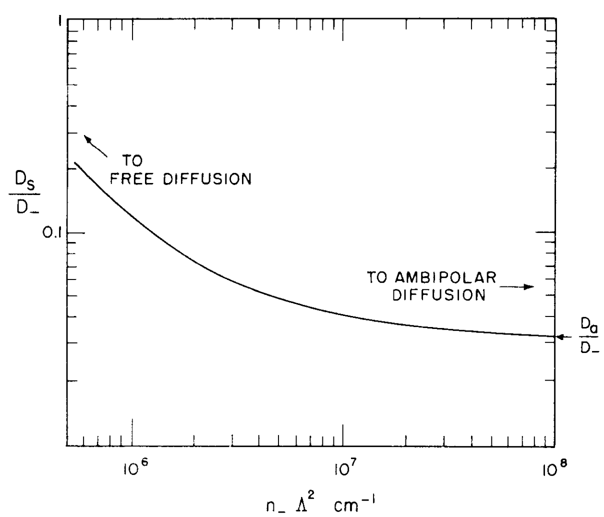

Fig. II-2 Space charge diffusion coefficient D_s/D_- as a function of density for hydrogen. $D_-/\mu_- = 3$, $D_+/\mu_+ = 0.0259$, $\mu_-/\mu_+ = 31.6$.

The theoretical computations are now being carried out. The value of ν_c for hydrogen is taken as $4.85 \times 10^9 p$ sec^{-1} at 300°K and is obtained by measurement of the discharge conductance and susceptance (1). This value is thought to represent more closely an "averaged" ν_c for hydrogen rather than that of $5.93 \times 10^9 p$ formerly reported. Excitation and ionization probabilities are recorded by Druyvesteyn and Penning (4); mobility of the positive hydrogen ion is assumed (5) to be $\mu_+ = (1.15 \times 10^4)/p$ cm^2/volt-sec at 300°K; also $D_+/\mu_+ = 0.026$ volt for thermal ions. As far as computation has proceeded, theory and experiment agree within about 10 percent, which is approximately the experimental error.

References

1. Quarterly Progress Report, Research Laboratory of Electronics (January 15, 1950).
2. A. D. MacDonald, S. C. Brown: Phys. Rev. 76, 1634 (1949).
3. Quarterly Progress Report, Research Laboratory of Electronics (October 15, 1948).
4. M. J. Druyvesteyn, F. M. Penning: Rev. Mod. Phys. 12, 87 (1940).
5. A. M. Tyndall: The Mobility of Position Ions in Gases, appendix (Cambridge, 1938).

B. COLLISION CROSS-SECTIONS

From the work of Ramsauer (1), Brode (2), and others it is well known that the cross-section P_c for elastic collisions between electrons and gas molecules varies anomalously with electron energy and also depends upon the nature of the gas. Published experimental data do not include values of P_c for energies below 0.5 ev, and slopes of the P_c vs. energy curves for various gases show marked differences at the low energy limits of previous measurements. Academic interest in extending measurements of P_c to thermal energies is enhanced by the requirements of other experiments in ionized plasmas where P_c is an important parameter in the application of distribution theory and in determining experimental limits for certain techniques.

An attempt is in progress to extend measurements of P_c into the lower energy region by observing electrons in thermal equilibrium with the gas molecules in an ionized plasma. The ratio (σ_r/σ_i) of real to imaginary parts of the complex conductivity in a microwave resonant cavity can be determined at any time during the post-discharge plasma decay by transient standing-wave techniques, as described in Research Laboratory of Electronics Technical Report No. 140. From this ratio (σ_r/σ_i) the collision cross-section can be calculated by distribution theory for cases in which P_c is given by a simple power of the electron velocity. The average electron energy decay from pulse discharge level to equilibrium with the gas occurs rapidly. It should therefore be possible to determine values of P_c at several low energy values by controlling the ambient temperature of the gas.

The integral expression for the complex conductivity of an ionized gas which is macroscopically neutral and in thermal equilibrium with the gas has been obtained by Margenau (3) for cases in which the collision probability P_c is given by a simple power of the electron velocity. Solutions are given for the two cases where P_c is assumed to be either constant or inversely proportional to the electron velocity. If we assume the collision frequency $\nu_c = vpP_c$ and $P_c = av^h$, it is convenient for experimental applications to plot $1/\gamma_h \left| \sigma_r/\sigma_i \right|$ against the parameter γ_h (see Fig. II-3) where

$$\gamma_h = \frac{apa^{h+1}}{\omega}$$

$$a = \text{most probable velocity} = \sqrt{\frac{2kT}{m}}$$

$$\omega = \text{radian frequency.}$$

For ν_c = constant, h = 1 and for constant P_c, h = 0.

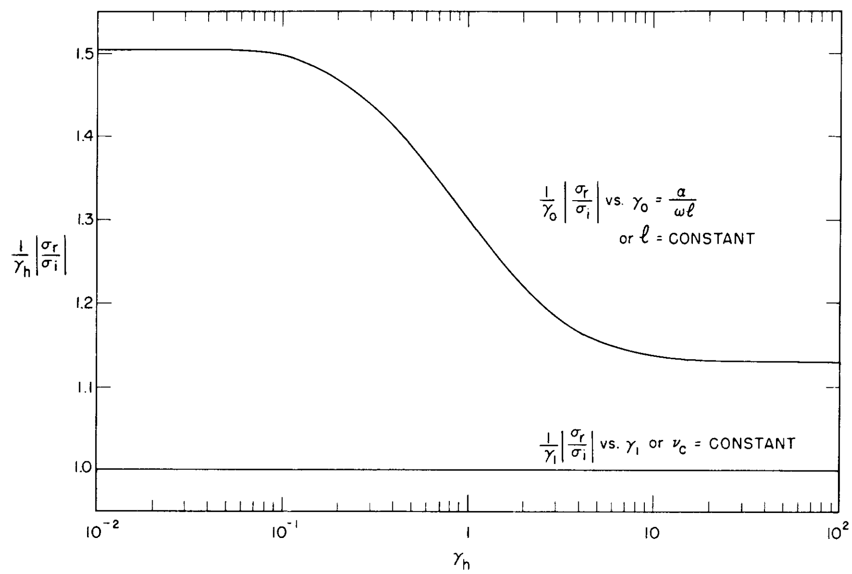

Fig. II-3 Curve for $1/\gamma_h \left|\sigma_r/\sigma_i\right|$ vs. γ_h for ℓ = constant and ν_c = constant.

It can be shown that for small electron densities the changes in conductance and susceptance of the cavity are, respectively,

$$\Delta g = \frac{\beta}{Q_u'} = \frac{\beta n \sigma_r}{\epsilon_o \omega}$$

$$\Delta b = \frac{2\beta\Delta\omega}{\omega} = \frac{\beta n \sigma_i}{\epsilon_o \omega}$$

where β = coupling factor obtained from measurements on the empty cavity, ω = angular frequency, n = electron density, ϵ_o = permittivity of free space, and Q_u' = unloaded Q of cavity when electrons are present.

The ratio of real to imaginary parts of the conductivity is thus given by

$$\frac{\sigma_r}{\sigma_i} = \frac{\omega}{Q_u' \cdot 2\Delta\omega} = \frac{\omega\Delta g}{2\beta\Delta\omega} = \frac{\lambda_o \Delta g}{2\beta\Delta\lambda}$$

where λ_o = resonant wavelength of the empty cavity, and $\Delta\lambda$ = change in resonant wavelength of the cavity with electrons present. The methods for determining Δg, $\Delta\lambda$, and β from standing-wave ratio and phase curve measurements are described in R.L.E. Technical Report No. 140.

Preliminary measurements in helium at 20 mm Hg pressure have yielded values of $P_c \approx 15$, which agree with the value obtained by extrapolating Brode's curve. In these calculations it was assumed that P_c = constant or h = 0. The fact that the conductivity ratio is independent of electron density was verified by comparing measurements at 1 ms, 5 ms, and 10 ms during the plasma decay.

References

1. Ramsauer and Kollath: Handbuch der Physik XXII, p. 243 (1933).
2. R. B. Brode: Rev. Mod. Phys. 5, 257 (1933).
3. H. Margenau: Phys. Rev. 69, 508 (1946).