## IX. MISCELLANEOUS PROBLEMS

A. ETLECTRONIC DIFFERENTIAL ANALYZER

Prof. Henry Wallman
R. Masrtmann-Moe

A new main gate generator has been designed and built. It employs a cathode follower output with cathode resistor center-tap grounded, and a floating power supply. A delayed feedback circuit is used to get the pulse tops flat within 20 mv ( 0.07 percent). This is still not quite satisfactory with the present method of obtaining initial condition voltages, as the percentage deviation from flatness increases with decreasing initial condition voltage setting. New potentiometers emplojing a grounded center tap are expected to clear this trouble.

The long-time drift of the power supplies has been investigated. The negative (-110 volts) supplies have an initial drift rate of one volt per hour, final voltage being reached after about 5 hours. The positive ( +250 volts) supplies have no appreciable drift. Minor alterations to the negative supplies are hoped to reduce the drift to more tolerable levels. All supplies show occasional jumps of about one-tenth volt due to mode changes in the voltage regulator tubes (VRI05). The drifts of the supply voltages cause the zero settings on the function-generator-multiplier units to change, as these units employ d-c amplifiers without feedback.

Five new function-generator-multiplier units have been tested and some minor changes made to obtain better linearity of the magnetic channel amplifiers, and to make the zero settings more easily accomplished.

One function-generator-multiplier unit has been sent to Evans Signal Laboratories, Belmar, N.J. for use with the analyzer being constructed there with this analyzer as model.

Mr. Robert H. Cannon, Mechanical Engineering Department, M.I.T., continues to use the analyzer on a speedboat design problem which requires the use of every component of the machine. The problem also required the addition of more function-generator-multiplier units to the machine, so as to enable two waveforms to be generated, and two multiplications to be performed simultaneously. About 200 different conditions of the boat construction have been studied, and about 500 pictures taken.

## B. ANALOG DEVICES FOR NETWORK PROBLEMS

Prof. E. A. Guillemin
R. E. Scott
R. H. Pantell

## 1. An Automatic Impedance-Function Analyzer

A new buffer amplifier and integrator unit has been designed and built for the automatic impedance-function analyzer. A block diagram of the system is shown in Figure IX-1.


Fig. IX-1 Block diagram of buffer amplifier and integrator.
The system is a novel one which uses a-c amplifiers throughout, but still retains the full duty cycle. This is accomplished by adding the d-c component to the output. For the magnitude, this involves merely the addition of a direct voltage to the output or alternatively the setting of the zero db line. The method of obtaining the phase is slightly more complicated, since an integrated constant, or a sweep wave, must be added to the output. The amount of the sweep wave which must be added is determined from the fact that the phase curve must be flat at very low frequencies and at very high frequencies.

In the magnitude position, the outputs from the two commutators are identical. The filters remove the 1000 -cycle hash, and the output is averaged and displayed on the cathode ray tube. In the phase position, the outputs of the two commutators are slightly different. One of them is inverted and then added to the other. The result is the difference between the two commutator voltages. This difference is the derivative of the phase with respect to frequency. It is integrated and a sweep wave is added to it. The final result is the phase.

For stability and accuracy, active integrators, adders and inverters have been used. The general scheme is well known and is illustrated in Fig. IX-2.


INVERTER


ADDER


INTEGRATOR

Fig. IX-2 The inverters, adders and integrators.

The requirements on the amplifier are stringent. It must have a high gain and a large bandwidth but it must not break into oscillations when large amounts of feedback are applied. In addition the output impedance must be low. A design which satisfies these requirements is shown in Fig. IX-3.


Fig. IX-3 The basic amplifier.

The output impedance of this amplifier is about 100 ohms. The amplification is about -1000. The high-frequency phase shift is controlled by means of the 0.02-ufd shunting capacitor at the grid of the first 6J6 tube. The low-frequency phase shift is controlled by means of the coupling capacitor between the 6AG5 and the 6J6.

Some experimental results of magnitude, delay and phase for singletuned circuits are shown in Fig. IX-4.


Fig. IX-4 Experimental results. Quadratic factor $=\frac{T^{2} s^{2}+2 \zeta T s+1}{s^{2}} \quad$.
Left, log magnitude vs. $\log$ frequency; center, negative phase derivative vs. log frequency; right, negative phase vs. $\log$ frequency (final value $=0$ ).

## 2. The Electronic Isograph

a. Theory

A problem frequently encountered in various engineering problems is the solution of polynomials of the fourth, fifth, sixth degree or higher. The process of solution can be quite tedious. Thus there is the apparent desirability for a device that would obtain the roots of a polynomial in a rapid, simple manner. The electronic isograph performs this function.

To comprehend the construction and performance of the isograph, express the polynomial:

$$
\begin{equation*}
0=a_{0}+a_{1} x+a_{2} x^{2}+a_{3} x^{3}+\ldots+a_{n} x^{n} \tag{1}
\end{equation*}
$$

in polar form. Let

$$
\begin{equation*}
x=r(\cos \theta+j \sin \theta) \tag{2}
\end{equation*}
$$

The polynomial then becomes

$$
\begin{equation*}
0=\sum_{k=0}^{n}\left(a_{k} r^{k} \cos k \theta+j a_{k} r^{k} \sin k \theta\right) \tag{3}
\end{equation*}
$$

In essence, the isograph produces a summation of sine and cosine terms of multiple frequencies with the proper coefficient values in accordance with Eq. (3). The sine terms, which have been shifted $90^{\circ}$ out of phase with the cosine terms, are placed on the vertical plates of a CRO; the cosine terms are placed on the horizontal plates of a CRO. In this manner, a plot of the polynomial is viewed on the oscillograph. Variation of the parameter $r$ causes the plot to pass through the origin thereby indicating that the setting of $r$ is a root of the polynomial.

## b. Design

Multiple frequency square waves are obtained from flip-flop circuits that operate at integral sub-multiples of a master crystal oscillator as in Fig. IX-5.

The isograph illustrated in Fig. IX-5 is capable of handing polynomials up to the sixth degree.

A square wave is a composite of a series of sinusoids, where the fundamental component is at the frequency of the square wave. Therefore, a high$Q$ circuit tuned to the frequency of the square wave yields the desired sinusoid.

The magnitude of cos $k \theta$ must be equal or proportional to $r^{k}$. This is

## (IX. MISCETLLANEOOS PROBLEMSS)



Fig. IX-5 Block diagram of multiple-frequency square wave synchronization.
realized if the output from the flip-flop circuit is clipped at the value of $r^{k}$ (Fig. IX-6). As indicated in Fig. IX-6, the powers of $r$ are obtained from ganged potentiometers. All the resistors move as a unit, and so each rheostat is set at the same relative position. Hence if the first resistor is $1 / 2$ of $E_{b}$ (for $r=1 / 2$ ), the second resistor is $1 / 2$ of $1 / 2$ or $r^{2}$; and so on. In this manner the powers of $r$ are obtained by the movement of one shaft.


Fig. IX-6 Block diagram of square wave clipping.

The value of $a_{k}$ is introduced by means of a voltage divider. Since $a_{k}$ may be either positive or negative, a double-throw switch is used to connect the circuit to the cathode or plate resistor respectively of a cathode follower.

With the polynomial plotted on a CRO, and $r$ adjusted so that the plot passes through the origin it is then essential to find the $\theta$ for which the polynomial is zero (Fig. IX-7). The fundamental frequency component is

fed into a variable phase shifter and subsequently into a clipper and peaker. This resultant pulse can thus be placed at any point along the time axis. The setting of the phase shifter indicates the desired value of $\theta$ when the pulse occurs coincident with the passing of the polynomial plot through the origin.

By varying $r$ continuously, a plot of the polynomial for all values of the variable appears simultaneously on the oscillograph. Applying a voltage to the Z-axis at the proper time interval will indicate the region of the polynomial for which $x$ is positive and real. This information is necessary to predict the response of numerous electrical and mechanical systems. Mechanical isographs are now in operation which produce multiple frequency square waves by rotating brushes. The latter are limited to a maximum frequency, thereby precluding a simultaneous plot of all values of the variable without the use of a long-persistence screen. In this respect, the electronic isograph is superior.

