X. MISCELLANEOUS PROBLEMS

A. ELECTRONIC DIFFERENTIAL ANALYZER

Prof. Henry Wallman
Dr. A. B. Macnee
R. Maartmann-Moe

Engineering of the computing elements of the electronic differential analyzer is continuing. The differential analyzer and the more recently developed Fourier Transformer have been used in the solution of various problems. The analyzer has also been used to evaluate solutions of Schlömilch's Integral Equation.

1. Computing Elements

The entire differential analyzer exclusive of the output oscilloscope has been mounted in two standard relay racks. The units are so interconnected that the analyzer can be turned on or off by throwing two switches.

2. Differential Equations

Mr. L. Biedenharn has used the analyzer to investigate solutions of the simultaneous equations

$$\frac{d^2U}{dt^2} + fU = -\lambda W \quad , \qquad (1)$$

$$\frac{d^2W}{dt^2} + \left(f - \frac{6}{t^2}\right) W = -\lambda U \qquad (2)$$

where f and λ are constants.

3. Integral Equations

The Fourier Transformer has been used by Mr. E. Callahan and Mr. N. H. Knudtzon during the past quarter. Mr. Callahan used the analyzer to obtain Fourier series expansions of a number of periodic functions. Bot. Mr. Knudtzon and Mr. Callahan used the analyzer to obtain cosine transforms of experimentally measured auto-correlation functions.

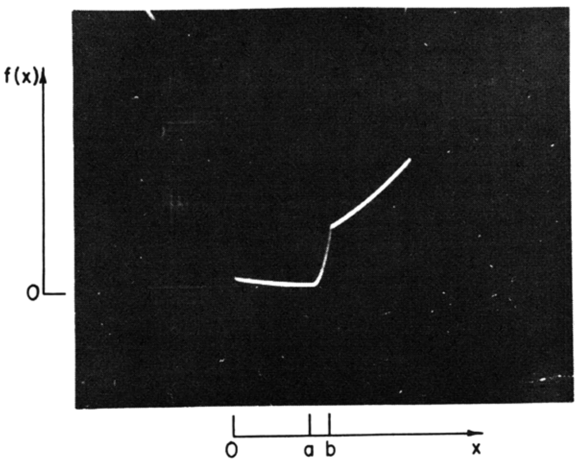
During the past quarter Mr. A. Soltes of the Cambridge Field Station and Mr. J. Turtora of the Camp Evans Signal Corps Laboratory visited this group for a two-week period. During his visit Mr. Soltes used the analyzer computing elements to solve Schlömilch's Integral Equation

$$f(x) = \frac{2}{\pi} \int_0^{\pi/2} F(x \sin \theta)\, d\theta \quad . \qquad (3)$$

This equation relates the dynamic and static characteristics of a detector for sinusoidal inputs. It has the solution (1)

$$F(x) = f(0) + x \int_0^{\pi/2} f'(x \sin \beta)\, d\beta \qquad (4)$$

which has been evaluated with the electronic differential analyzer computing elements. A typical dynamic characteristic f(x) and the static characteristic solution F(x) are given in Figure X-1.



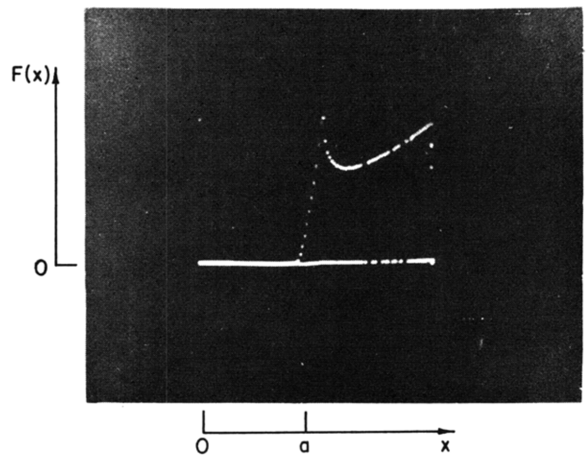


Fig. X-1 Observed solution of Schlömilch's Integral Equation.

B. AN AUTOMATIC IMPEDANCE-FUNCTION ANALYZER

Prof. E. A. Guillemin
R. E. Scott

The fundamental laws of linear passive networks are well known. The mechanical difficulties of applying them to a given problem, however, may be enormous. The primary purpose of the automatic impedance-function analyzer is to reduce this labour. In many cases, however, the machine goes beyond this and provides the possibility of a new philosophy of network design, with the approximation problem as its core.

The central importance of the approximation problem in network theory has only lately been recognized. Dr. Guillemin has shown, for example, that any impedance characteristic may be represented by an RC network, providing that the approximation problem is solved in a certain fashion. The machine which is described here provides a logical extension of this philosophy to other types of networks. For example, the machine makes it possible to design a given impedance using coils of a specified "Q".

1. Construction of the Mark 4 Machine

The Mark 4 machine has been completed with the exception of a possible integrator for the phase function. The machine consists of four basic components:

a. A conducting sheet of Teledeltos paper.
b. Probes which introduce currents into the paper.
c. Probes which measure voltages in the paper.
d. A commutator which displays these voltages on the face of a cathode ray tube.

The sheet of Teledeltos conducting paper is 53 inches by 15.8 inches. It rests upon a layer of cork and is held at each end by clamps. This sheet is an analogue for the logarithmic frequency plane which is formed by applying the transformation $W = \mathrm{Log}_e Z$ to the usual complex frequency plane. The advantages of this transformation are that it removes any difficulties with the finite size of the analogue, and that it allows a direct utilization of a great deal of the standard gain-phase network theory (also called log-db theory). For convenience in using the device a graph of the original frequency variable has been printed upon the logarithmic plane.

There are 72 probes to introduce currents into the plane. They are constructed of steel needles embedded in a polystyrene holder and held in place by nickel sleeves which are spot welded to the steel. They represent the poles and zeros of the impedance function. Four distinct types are necessary: positive and negative currents for poles and zeros, and half-value positive and negative currents for the poles and zeros on the real axis. This distinction is made necessary because only the upper half-plane is used in the analogy.

The voltage probes are permanently fixed along the center of the plane, along the imaginary axis. They are in two rows of 300 probes covering three decades of the frequency domain. Physically they consist of phonograph needles pressed through a bakelite mount, and soft-soldered at the butt end to bare copper lead-off wires. They are spaced 1/10 inch apart each way. The logarithm of the magnitude of the impedance function is obtained by taking the sum of the voltages on the two separate rows of probes. The phase is obtained by integrating the difference of the voltages on the two separate rows. As an alternative the paper may be shifted and either row may be used separately for the magnitude function.

The commutator is a mechanical one consisting of two separate sets of 100 brass contacts. They are flush-mounted in bakelite and the rotating members which contact them are silvered springs. To prevent the rotating contacts from wearing the bakelite and becoming dirty it has been found necessary to lubricate them with vaseline. The commutator will operate satisfactorily for periods of 4 to 8 hours at a speed of 1200 rpm. Then it must be cleaned and regreased with vaseline.

2. Method of Use

The machine has just been completed and exhaustive tests have not yet been made. However, some idea of its usefulness may be obtained from the following. It was desired to explore the possibility of approximating a series resonant circuit by means of low "Q" elements. The circuit is shown in Figure X-2.

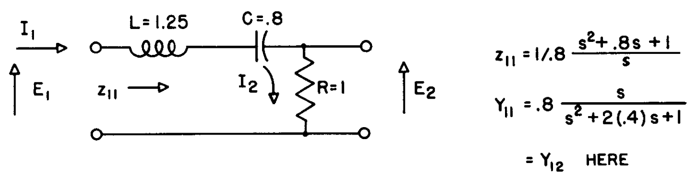


Fig X-2 Diagram of circuit for producing resonant behavior with low "Q" elements.

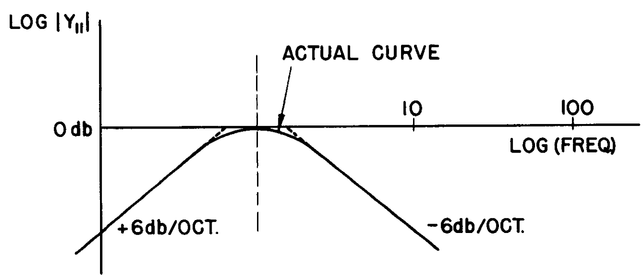


Fig. X-3 The logarithm of the magnitude of the impedance, Y_{11}, sketched against the frequency.

Using the machine this characteristic was matched with poles and zeros on the real axis. The function obtained was (neglecting the constant multiplier)

$$Y(s) = \frac{s(s+5)(s+.2)}{(s+1)^4}$$

This is not positive real and cannot be realized as a driving point admittance. However, it can be realized as a transfer admittance by standard methods.

The scope of the machine will be investigated more fully in the near future. In particular, attention will be given to the following three problems:

a. The design of networks to use elements of a particular type.

b. The design of networks to have a particular form, e.g., ladder networks, double-tuned circuits, etc.

c. The related transient response for various networks giving the same magnitude for their transfer impedances.

The transient response could be obtained in several ways. If a somewhat faster commutator were available it could be integrated directly from the steady state phase and amplitude responses on Dr. Macnee's Fourier Transform Machine. Alternatively it can be inferred from the position of the poles and the residues at the poles. The residues can be obtained experimentally by removing the pole and measuring the voltage at the same point.

C. ELECTRONIC-POTENTIAL MAPPING

Dr. S. Goldman
W. F. Santelmann, Jr.
W. E. Vivian

This is the final report on the project. R.L.E. Technical Report No. 121, "Electronic Mapping of the Electrical Activity of the Heart and Brain", will give a detailed story of the history of the project, the techniques developed and the results obtained. A summary of the results is given here.

In this project an attempt has been made to supplement and extend the techniques of electrocardiography and electroencephalography by introducing area displays. In these area displays, the two dimensional distribution of electrical potential across the surface of the chest or the skull is shown on the screen of a cathode-ray tube. The change in light intensity on the surface of the screen is proportional to the instantaneous potential at the corresponding point on the body. Then by watching the screen of the cathode-ray tube, or motion pictures of it shown in slow motion, one can follow the electrical activity on the surface of the chest or the skull, which in turn gives important information about the electrical activity of the heart and brain.

In this project it has been standard practice to use 16 pickup electrodes (2). In the case of the chest for example, these are distributed as shown in Figure X-4. In order to display these potentials on the screen of a cathode-ray tube it is necessary to send them through a commutating, synchronizing, and scanning system. A block diagram of the system finally developed is shown in Figure X-5.

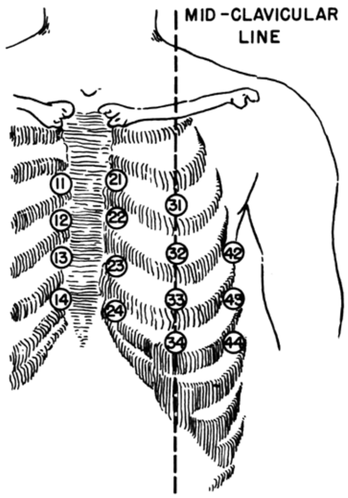


Fig. X-4 Location of pick-up electrodes on the chest.

As a result of pictures of the electrical activity of normal and abnormal hearts taken with this equipment it appears that this apparatus is a powerful tool for the study of the physiology of the normal heart and for

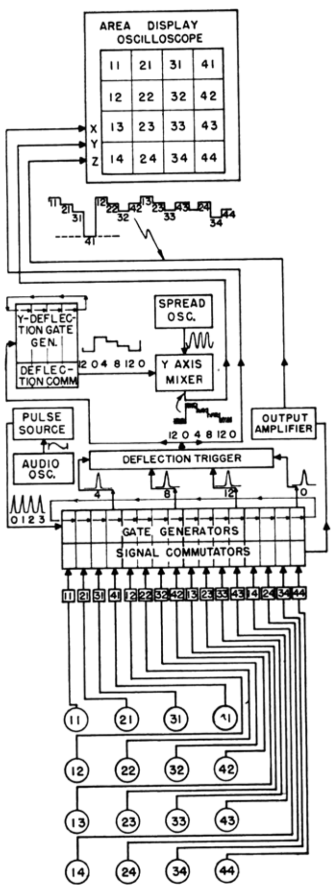


Fig. X-5 Block diagram of the electronic circuit commutator system.

diagnosis of disease in the abnormal heart. A report of the findings in a series of patients with abnormal hearts will be published in the American Heart Journal.

In the case of the brain, pictures taken with this equipment have shown for the first time that the alpha rhythm of electroencephalography consists of traveling waves in the individual lobes of the brain (3). It is expected that this apparatus will be useful both in studies of the physiology of the normal brain and in the differential diagnosis and localization of pathological conditions of the brain.

Starting July 1, 1949, Dr. S. Goldman will continue the development and application of this apparatus at Syracuse University.

D. HYDROGEN ARC ULTRA-VIOLET SOURCE

H. W. Wyckoff

For such purposes as monochromator illumination and ultra-violet microscopy, a high-intensity, long-life, small-volume, wide-angle source of the ultra-violet continuum, 2000-3500 Å, with no spectral lines is much needed. In 1941, A. J. Allen (4) reported a low-voltage arc in hydrogen, at 6 mm Hg with a ten-ampere capacity running at about 60 volts d-c, which far surpasses any tube which is yet commercially available. We have tried to duplicate his reported performance and improve on it.

In his tube, the arc passes from an oxide-coated nickel screen cathode through a 1/8-inch diameter hole in the wall of a copper cylinder surrounding it and strikes the anode. We use these same principles. Our modification omits the inner cylinder, and includes a central baffle, an insulated cathode, a wider angle of view, a larger cathode placed at least 1/2-inch from the constriction, higher-melting-point metals, and a side port for observation. This design is shown in Figure X-6.

No lifetime was mentioned in Allen's paper, but in a recent personal communication he indicated that 100 to 200 hours was not unreasonable. An error, or rather an unexplained diagrammatic representation in the paper, led us to build our first tubes in such a way that they had lifetimes of only about 25 hours. The cathode should be rotated 90° on its long axis from the way it is drawn in Allen's paper. One tube closely approximating his design ran 500 hours at 10 amperes before an accident terminated the run. At this time, the light output was down and the voltage at 10 amperes was up to 110, due to separate causes.

The diameter of the constriction has generally been kept at 1/8 inch. In arbitrary units, the output of the 500-hour tube was initially 0.8, on

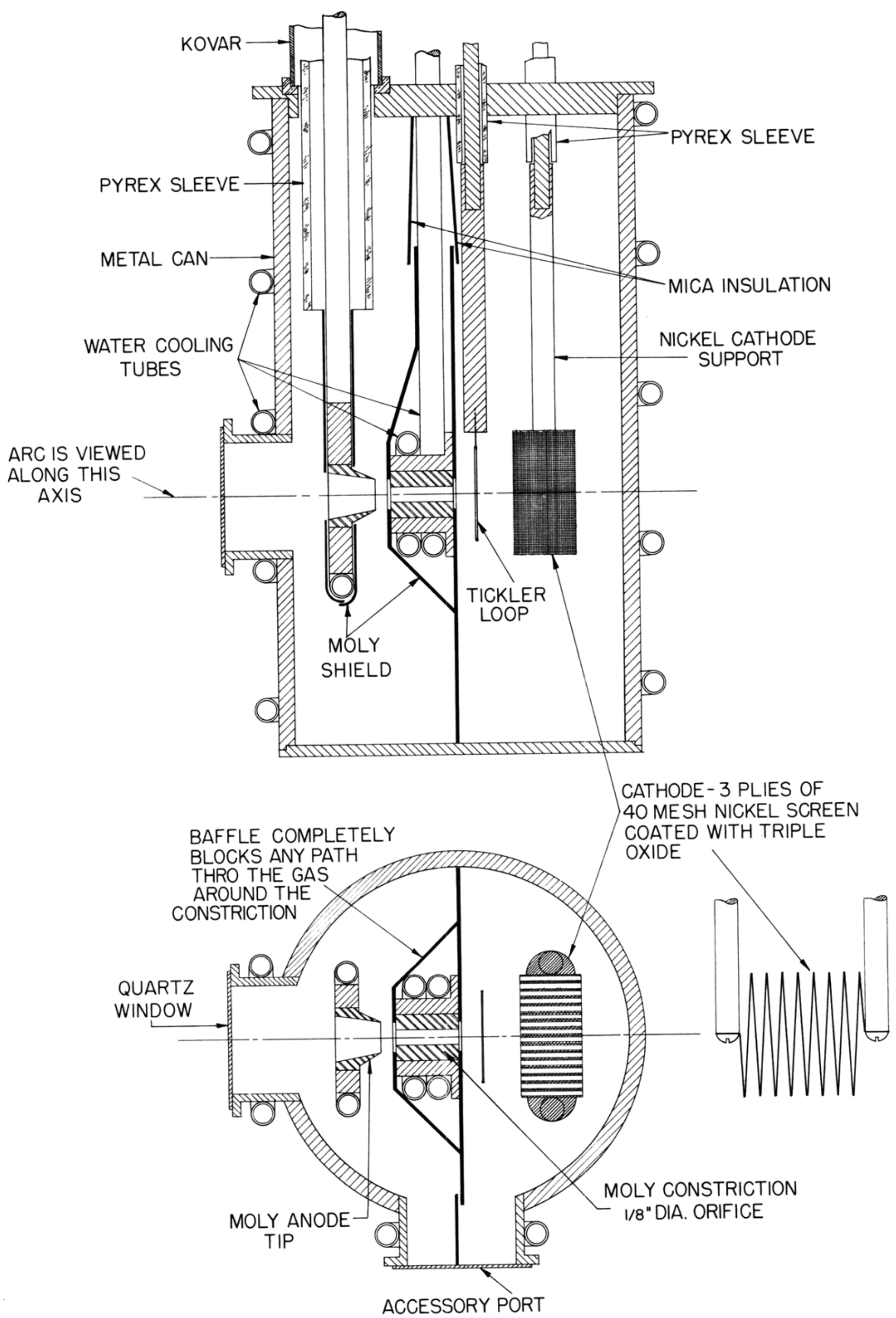


Fig. X-6 Hydrogen arc ultra-violet source.

axis. The highest on-axis intensity at the same ten-ampere current was 1.63, obtained in a tube with a 1/2 inch long × 1/8 inch diameter constriction. At 20 amperes, the output of the latter was increased to 2.3. Output is approximately proportional to the square root of the current. A quarter-inch-long constriction gave intensities only 20 per cent less than the half-inch one, but the output dropped off sharply when the length was further decreased. A tube with a constriction of rectangular cross-section 1/8 inch × 1/4 inch and 7/8 inch along the arc axis had an on-axis output about the same as the 1/2 inch tubular constriction. Pressure was not found to be very important so far as output was concerned.

Roughly comparable measurements on the Hanovia small-angle, high-voltage hydrogen arc gave on-axis intensities of 0.9 at its maximum current and when new. From the geometry of the Hanovia tube, it is obvious that at 1 1/2° off-axis, less than half of the source is visible. Measurements on tubes, such as that diagrammed, indicated a constant intensity up to 7° off the axis. Thus the total light flux is at least 2300 per cent greater than that of the Hanovia lamp.

The arc behaves and appears like a normal arc with a hot cathode but with insufficient thermal emission to carry the whole current. The constriction, with respect to current-voltage characteristics, behaves qualitatively like a Langmuir cold probe.

Many types of cathode have been tried, including coiled-coil oxide-filled tungsten, pure tungsten, indirectly heated oxide-coated-nickel cylinders, and sintered thoria-tungsten directly heated.

Starting techniques employed include "floating" the constriction, adding an insulated tickler ring, and the use of chokes or booster transformers in the d-c line. The life of the long-constriction-tubes (as determined by ability to strike an arc at about 100 volts) was always short, despite any of the above-mentioned starting tricks.

Further work will proceed in the direction of establishing an optimum practical source, and will be carried on in the Biology Department at M.I.T.

References

(1) E. T. Whittaker and G. N. Watson, "A Course of Modern Analysis", Cambridge University Press (England, 1945) pp. 229-230.

(2) S. Goldman, W. E. Vivian, C. K. Chien and H. N. Bowes, Science, 108, 2817 (Dec. 24, 1948) p. 720-723.

(3) S. Goldman, W. F. Santelmann, Jr., W. E. Vivian and D. Goldman, Science, 109, 2838 (May 20, 1949) p. 524.

(4) A. J. Allen, J. Opt. Soc. Am. 31, (March, 1941) pp. 268-370.