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PATH SIMULATIONS OF THE LINAC2 TRANSFER LINE

S. Lanzone, A. Lombardi

Abstract

The purpose of this work is to evaluate the transverse emittance growth in a beam line in presence of space charge, dispersion and energy spread.
The Transfer Line of the Linac2 has been taken as an example, as measurement to confirm might be possible. In the original design, the dispersion has a very low value; the dispersion has been increased varying the quadrupoles between the bending magnets. Finally the bending magnets have been removed, nullifying the dispersion.
In order to investigate the influence of the energy spread, the gap voltage and phase have been changed, observing the emittance growth after the BHZ30.

Geneva, Switzerland
September 2006

Contents

Introduction

The purpose of this work is to evaluate the transverse emittance growth in a beam line in presence of space charge, dispersion and energy spread. The result of this work might have an impact on the design of the Linac4 to booster transfer line.
The Transfer Line of the Linac2 has been taken as an example, as measurement to confirm might be possible. The optics has been simulated with PATH [1] and TRACE3D [2]. In Fig. 1 a section of this line is plotted: the gap, that determinates the energy spread, two bending magnets BHZ20 and BHZ30 and six quadrupoles. For a more complete description see the appendix.

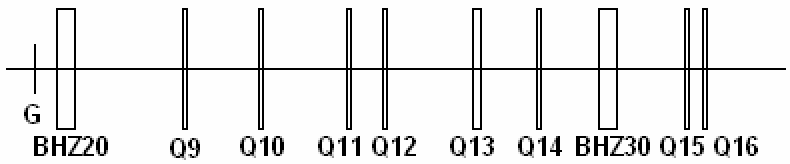


Figure 1: Section of the Transfer Line layout of the Linac2

The angles of the two bending magnets BHZ20 and BHZ30 are 16 deg and -22 deg, respectively. The first bending magnet introduce a term depending on the momentum (dispersion) that leads to an emittance increase, while the second lowers it to the initial value; the quadrupoles between the two magnets are used to control the dispersion [3].
Three configurations have been studied:

1) the baseline design, the dispersion has a very low value;
2) a high-dispersion line, the dispersion has been increased varying the quadrupoles between the bending magnets;
3) a zero-dispersion line, the bending magnets have been removed, nullifying the dispersion.

In order to investigate the influence of the energy spread, the gap voltage and phase have been changed, observing the emittance growth at the end of the line.

1. Nominal optics

The Transfer line of the Linac2, in the original design, has a very low dispersion at 0 current, and with I = 150 mA the dispersion is under 1 meter. In Fig. 2 there are the overlapping Trace3D plots of the line with I = 0 mA and I = 150 mA.

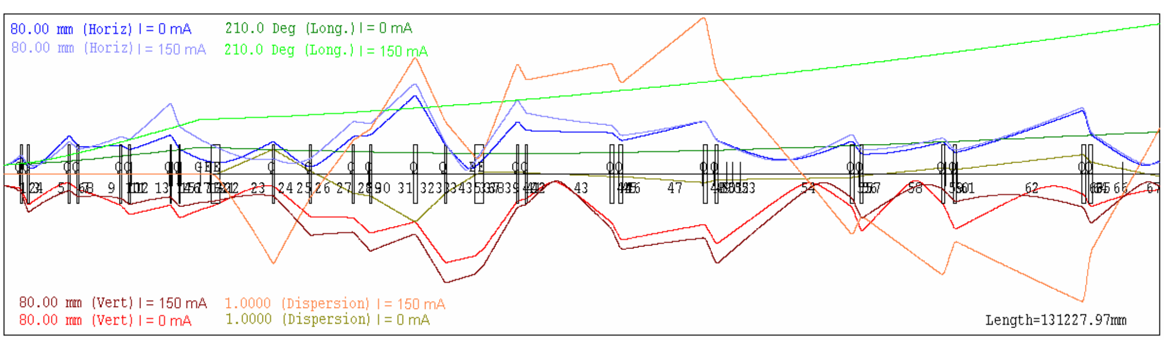
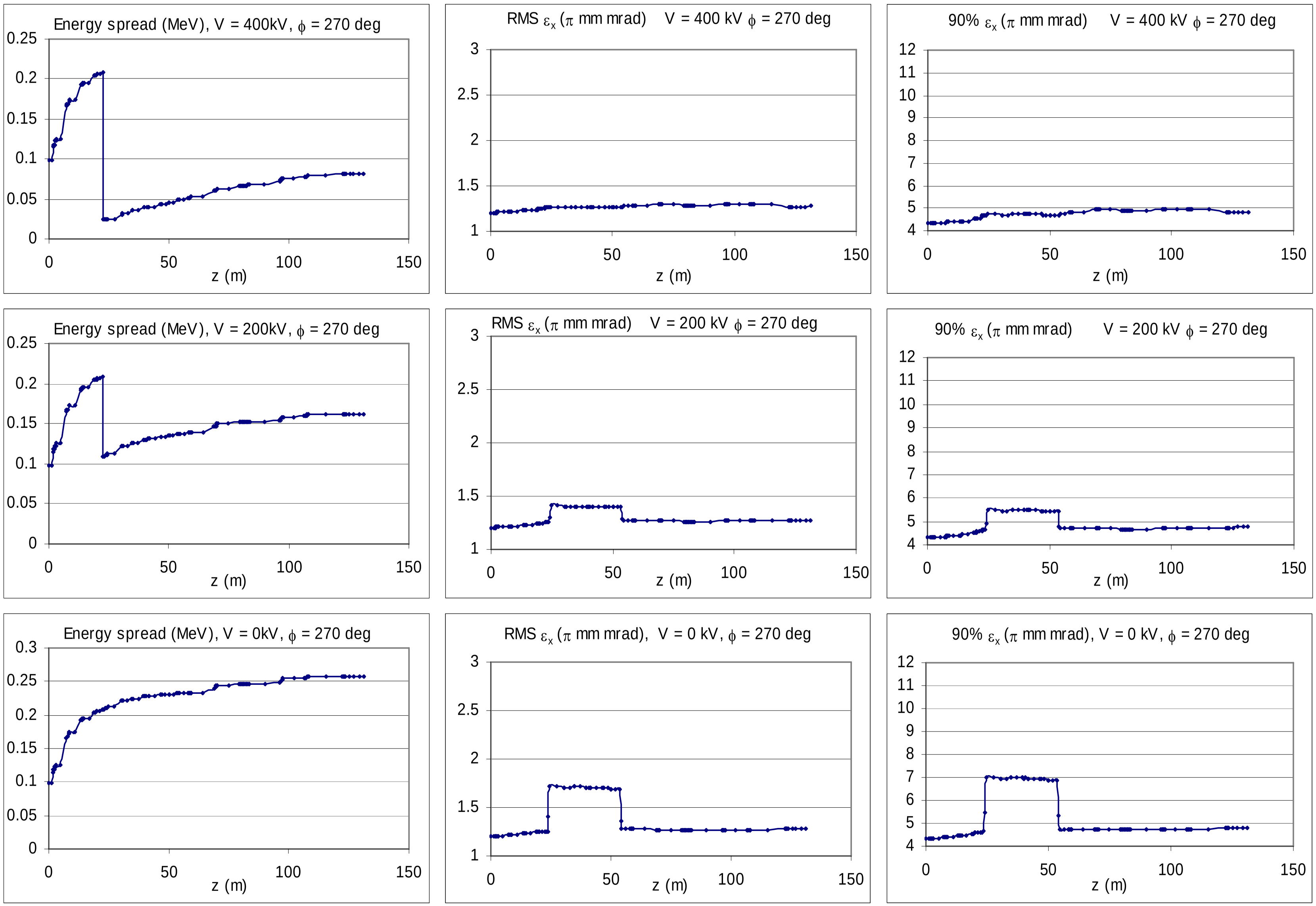


Figure 2: Beam size, phase and dispersion along the Transfer Line with Trace3D, I = 0 mA and I = 150 mA

In Fig. 3 the evolution of the RMS energy spread, the RMS and 90% x-emittance along the transfer line are plotted for several values of the voltage and the phase of the gap. The losses are always negligible.


Energy spread (MeV), V = 400kV, ϕ = 270 deg
RMS εx (π mm mrad) V = 400 kV ϕ = 270 deg
90% εx (π mm mrad) V = 400 kV ϕ = 270 deg
Energy spread (MeV), V = 200kV, ϕ = 270 deg
RMS εx (π mm mrad) V = 200 kV ϕ = 270 deg
90% εx (π mm mrad) V = 200 kV ϕ = 270 deg
Energy spread (MeV), V = 0kV, ϕ = 270 deg
RMS εx (π mm mrad), V = 0 kV, ϕ = 270 deg
90% εx (π mm mrad), V = 0 kV, ϕ = 270 deg
z (m)

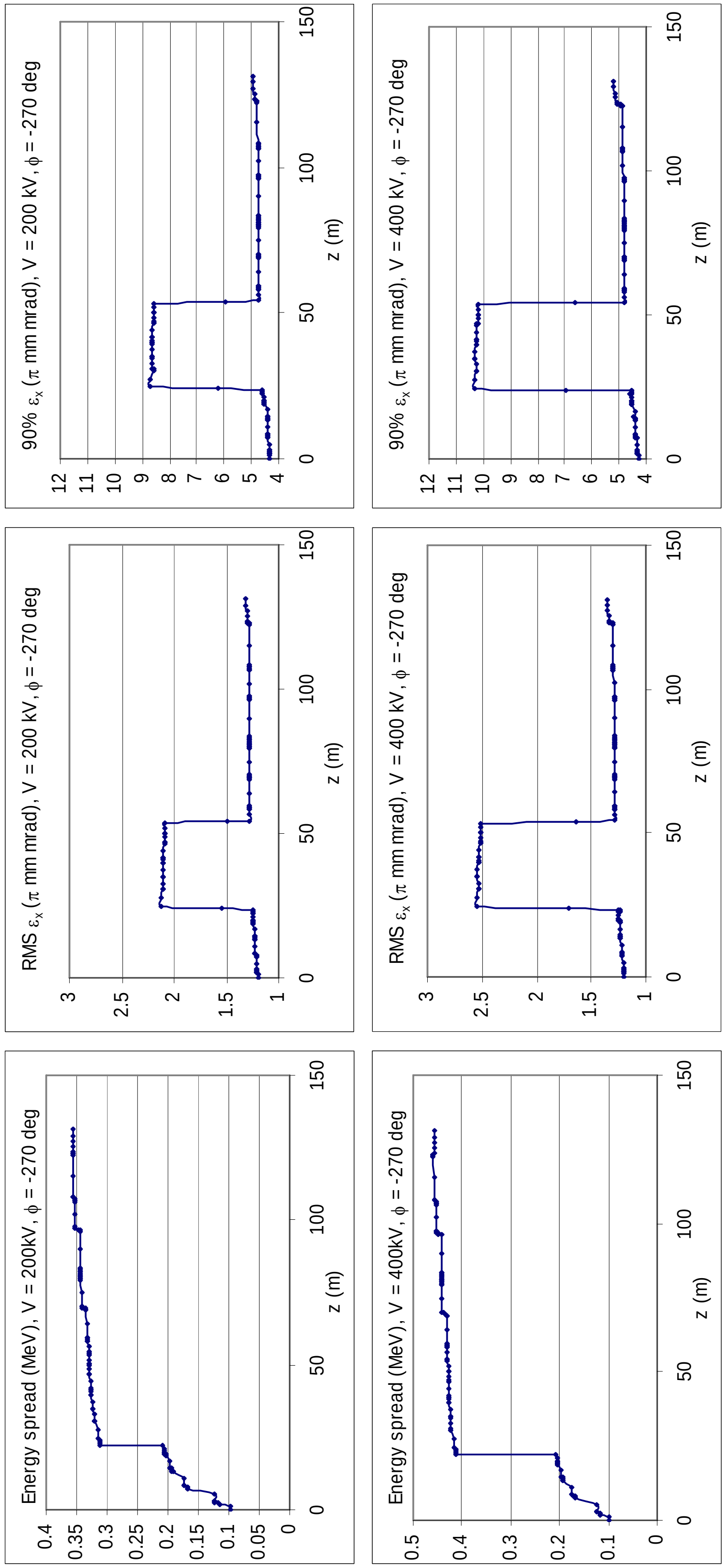


Figure 3: RMS energy spread, RMS and 90% normalized emittance along the Transfer Line, varying the cavity voltage and phase

The energy spread values at the output of the gap are reported in table 1.

Table 1: Energy spread at the gap output, varying the voltage and the phase

	V = 400 kV ϕ = 270 deg	V = 200 kV ϕ = 270 deg	V = 0 kV ϕ = 270 deg	V = 200 kV ϕ = -270 deg	V = 400 kV ϕ = -270 deg
energy spread σ_W	0.02 MeV	0.11 MeV	0.21 MeV	0.31 MeV	0.41 MeV

As the energy spread increase the emittance steps corresponding to the bending magnets and the difference between the initial and final value become higher, as reported in table 2.

Table 2: Percentage difference between the ε_x at the input of BHZ20 and the output of BHZ30

Energy spread σ_W	0.02 MeV	0.11 MeV	0.21 MeV	0.31 MeV	0.41 MeV
Δ (%)	-	0.8	1.09	2.25	3.52

In the following table there is the RMS x-emittance growth, for the different values of the energy spread.

Table 3: RMS emittance growth at the end of the transfer line and at the output of the 16th quadrupole

Energy spread σ_W	0.02 MeV	0.11 MeV	0.21 MeV	0.31 MeV	0.41 MeV
ε_x growth (%)	6.63	6.55	7.41	9.37	13.19

2. High dispersion

The gradient of the quadrupole between the two bending magnets have been increased of 5 %. This leads to a dispersion increment as we can see in Fig. 4 (the original design case in olive green, this new case in orange).

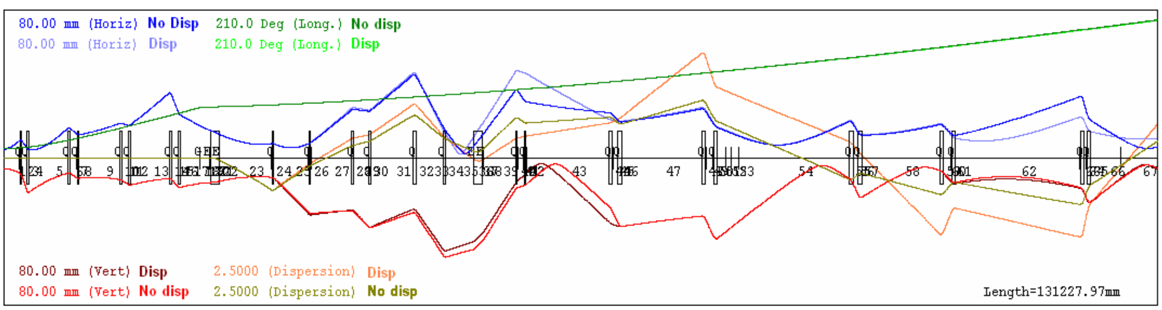
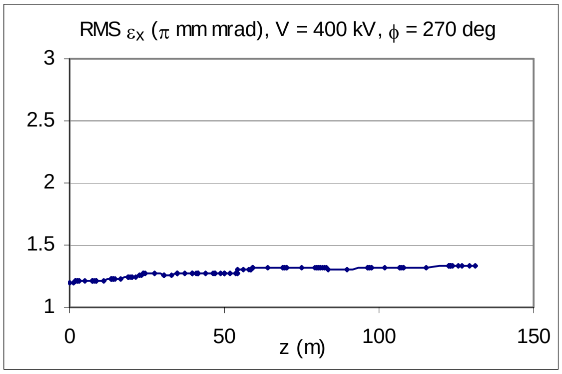


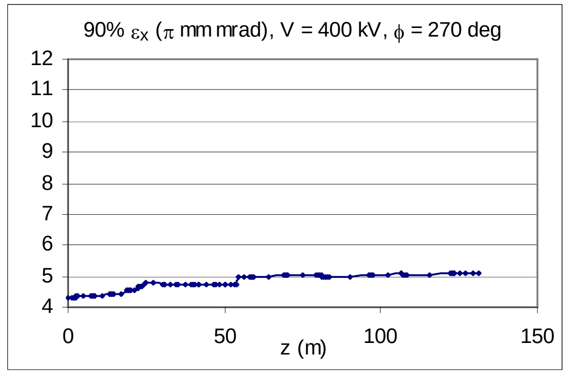
Figure 4: Beam size, phase and dispersion along the Transfer Line with Trace3D, with and without dispersion

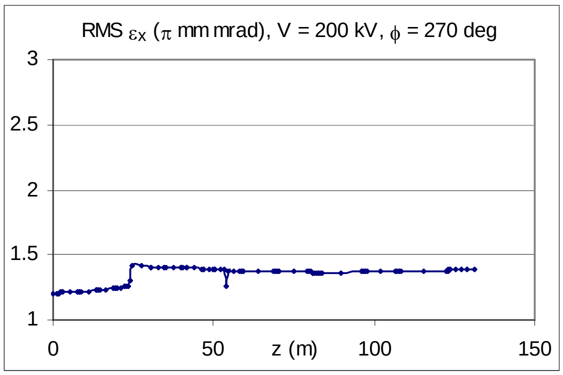
The line has been matched in order to keep a 100% transmittance with varying the energy spread. The changes have involved the quadrupoles QFN10, QDN20, QFW30, QDW40 and BI.QN30.

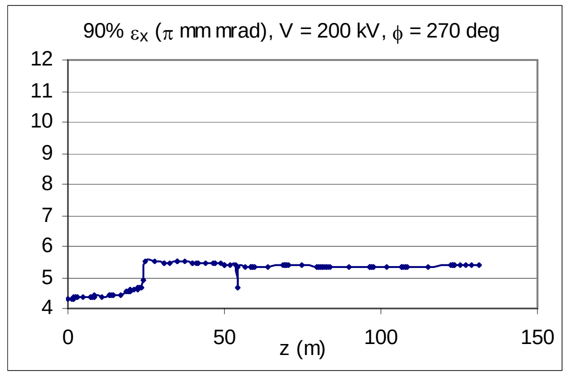
The x-emittance envelopes along the transfer line with the different values of the gap voltage and phase are reported in Fig. 5. The lowest point after the step due to the first bending magnet is internal to the second magnet.

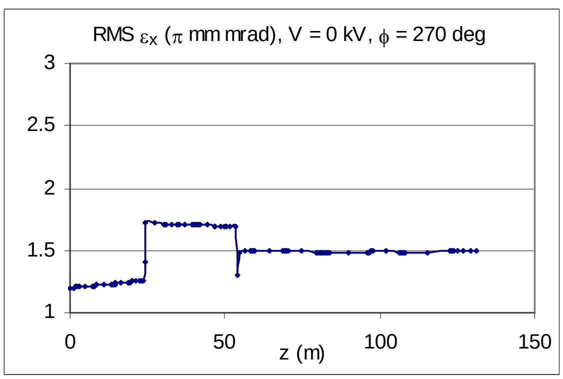
Because of the higher dispersion, the second bending isn't able to reinstate the emittance, so its final value is very different from the initial one. The difference between them and the emittance growth are reported in tables 4 and 5.

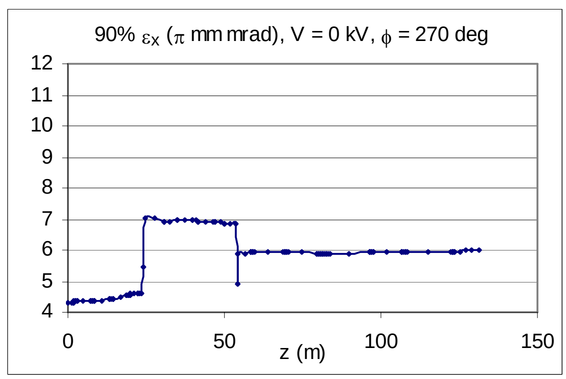

RMS εx (π mm mrad), V = 400 kV, ϕ = 270 deg
3
2.5
2
1.5
1
0
50
100
150
z (m)

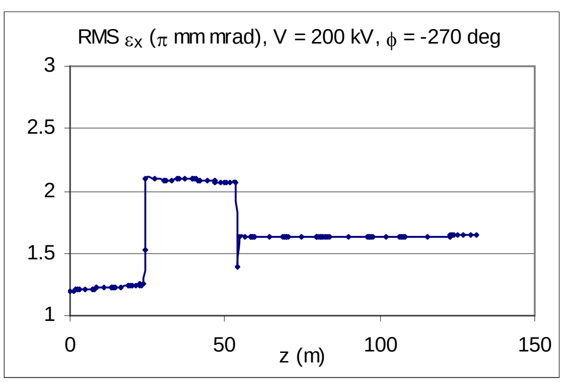

90% εx (π mm mrad), V = 400 kV, ϕ = 270 deg
12
11
10
9
8
7
6
5
4
0
50
100
150
z (m)

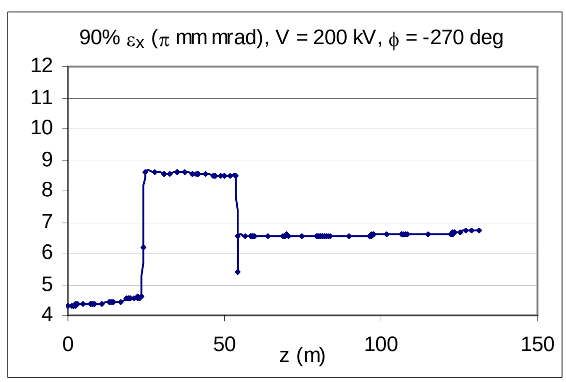

RMS εx (π mm mrad), V = 200 kV, ϕ = 270 deg
3
2.5
2
1.5
1
0
50
z (m)
100
150


90% εx (π mm mrad), V = 200 kV, ϕ = 270 deg
12
11
10
9
8
7
6
5
4
0
50
100
150
z (m)


RMS εx (π mm mrad), V = 0 kV, ϕ = 270 deg
3
2.5
2
1.5
1
0
50
100
150
z (m)


90% εx (π mm mrad), V = 0 kV, ϕ = 270 deg
12
11
10
9
8
7
6
5
4
0
50
100
150
z (m)


RMS εx (π mm mrad), V = 200 kV, ϕ = -270 deg
3
2.5
2
1.5
1
0
50
100
150
z (m)


90% εx (π mm mrad), V = 200 kV, ϕ = -270 deg
12
11
10
9
8
7
6
5
4
0
50
100
150
z (m)

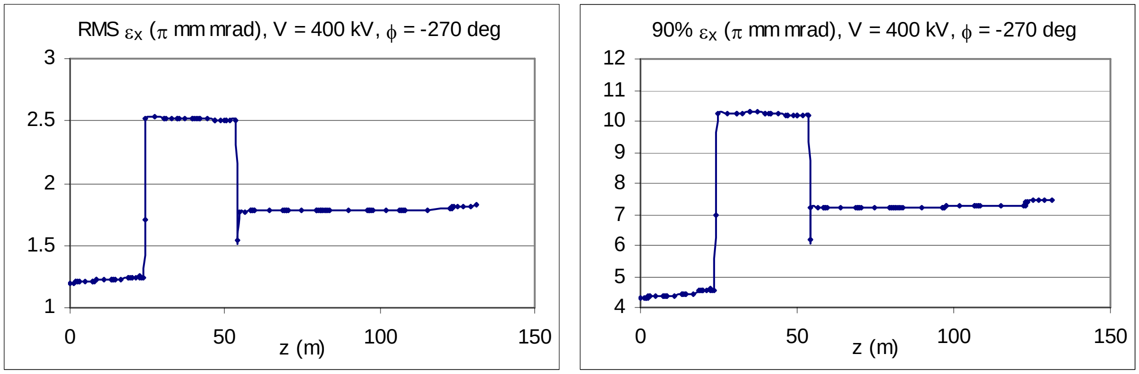


Figure 5: RMS and 90% normalized emittance along the Transfer Line with dispersion, varying the cavity voltage and phase

Table 4: Percentage difference between the ε_x at the input of BHZ20 and the output of BHZ30

Energy spread σ_W	0.02 MeV	0.11 MeV	0.21 MeV	0.31 MeV	0.41 MeV
Δ (%)	3.49	9.26	18.51	30.07	42.16

Table 5: RMS emittance growth at the end of the transfer line and at the output of the 16th quadrupole

Energy spread σ_W	0.02 MeV	0.11 MeV	0.21 MeV	0.31 MeV	0.41 MeV
ε_x growth (%)	11.49	15.4	25.12	37.79	51.90

3. No bending magnets

Removing the bending magnets, also the dispersion is removed, Fig. 6.

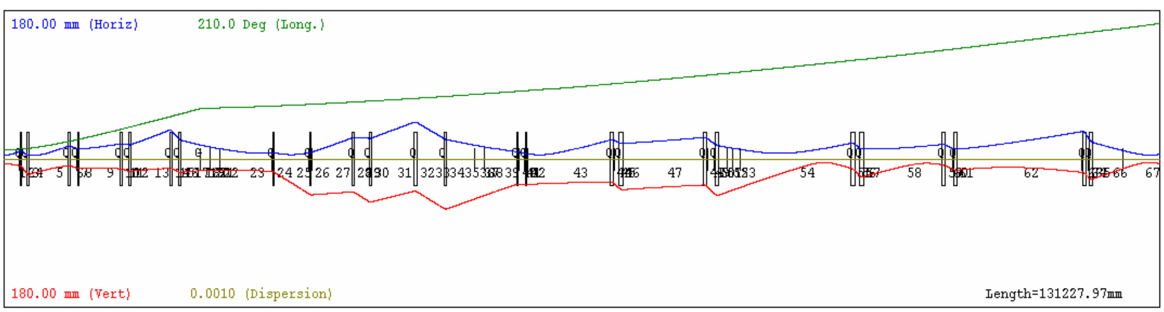
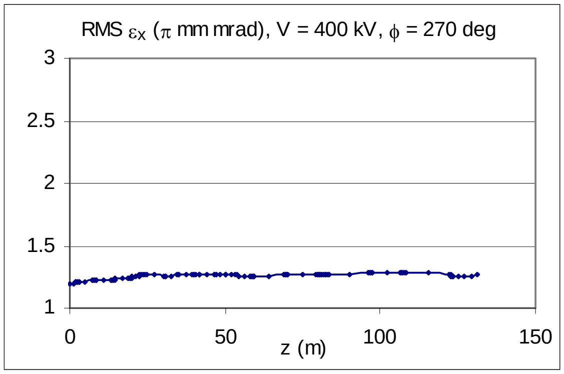
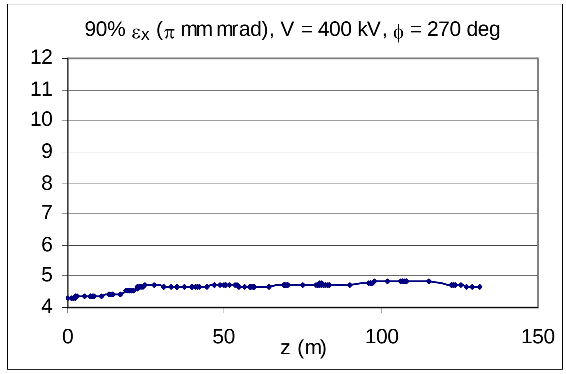
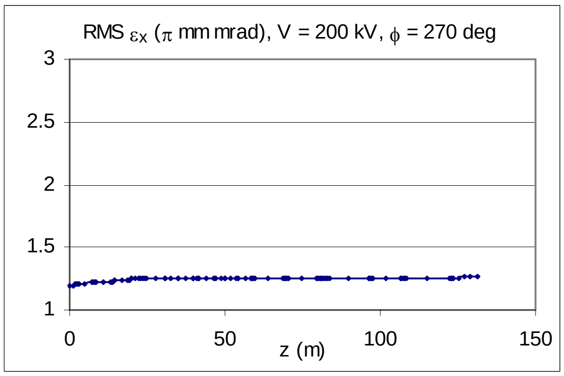


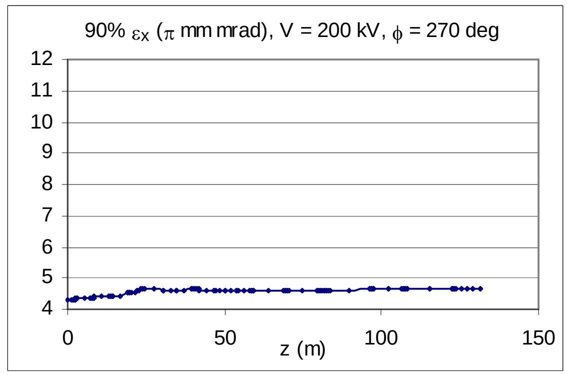
Figure 6: Beam size, phase and dispersion along the Transfer Line without bending magnets with Trace3D, I = 150 mA

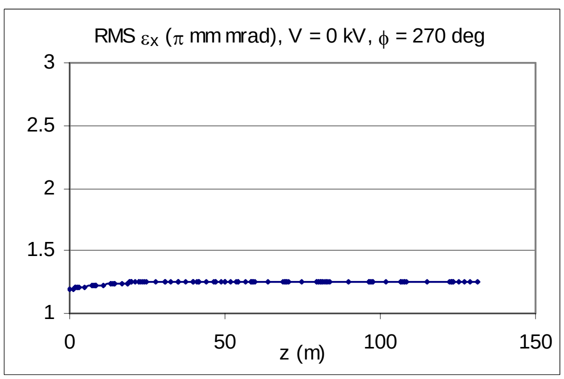
Again, the line has been matched in order to have no losses. The involved quadrupoles are QFW70, QDN75, QFN10, QDN20, QFW30 and QDW40.
The steps of the emittance disappear (Fig. 7), as expected, and the remaining emittance growth is due only to the space charge.

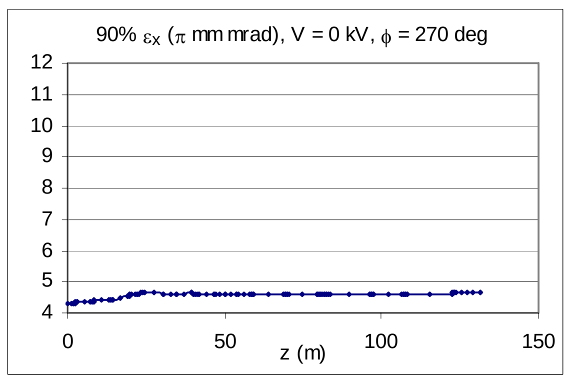

RMS ε_x (π mm mrad), V = 400 kV, ϕ = 270 deg
z (m)

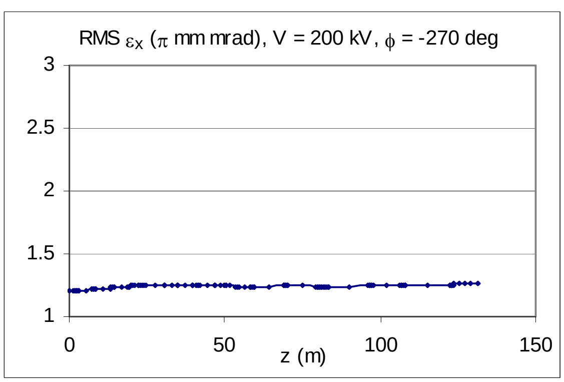

90% ε_x (π mm mrad), V = 400 kV, ϕ = 270 deg
z (m)

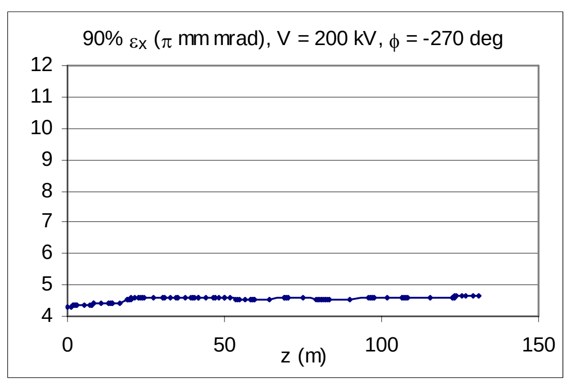

RMS ε_x (π mm mrad), V = 200 kV, ϕ = 270 deg
z (m)


90% ε_x (π mm mrad), V = 200 kV, ϕ = 270 deg
z (m)


RMS ε_x (π mm mrad), V = 0 kV, ϕ = 270 deg
z (m)


90% ε_x (π mm mrad), V = 0 kV, ϕ = 270 deg
z (m)


RMS ε_x (π mm mrad), V = 200 kV, ϕ = -270 deg
z (m)


90% ε_x (π mm mrad), V = 200 kV, ϕ = -270 deg
z (m)

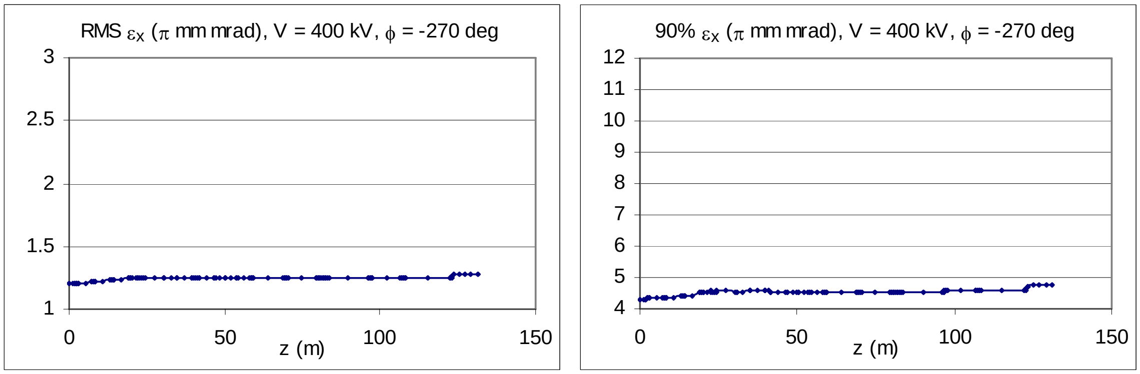


Figure 7: RMS and 90% normalized emittance along the Transfer Line without bending magnets, varying the cavity voltage and phase

In table 6 there are reported the ε_x growth.

Table 6: RMS x-emittance growth

Energy spread σ_W	0.02 MeV	0.11 MeV	0.21 MeV	0.31 MeV	0.41 MeV
ε_x growth (%)	5.21	5.13	5.00	5.26	6.93

Conclusion

In order to compare the three cases, low, high and no dispersion line, the obtained x-emittance growths are resumed in table 7, for the lowest and highest values of the energy spread.

Table 7: RMS emittance growth at the end of the line

	Order of D (m) †	σ_W = 0.02 MeV	σ_W = 0.41 MeV
High dispersive line	10	11.5%	51.9%
Low dispersive line *	1	6.6%	13.2%
No dispersion **	1E -20	5.2%	6.9%*

With a high value of the dispersion, in order to keep down the emittance growth, the energy spread has to be limited, while, with low or zero dispersion, it makes only a little difference.
In future a measurement could be done in order to confirm these results.

Appendix 1. The Linac2 Transfer Line

The Linac2 Transfer Line leads the 50 MeV protons coming out from the Linac2 to the PS Booster (for the layout see [4]).
As we can see in Fig 8, the beam (the red line) turns twice: the first one in order to join the PS room from the Linac2 tunnel end the second one to reach the Booster [5].

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* Nominal optics
** Without bending magnets
† Average values from the transfer matrixes

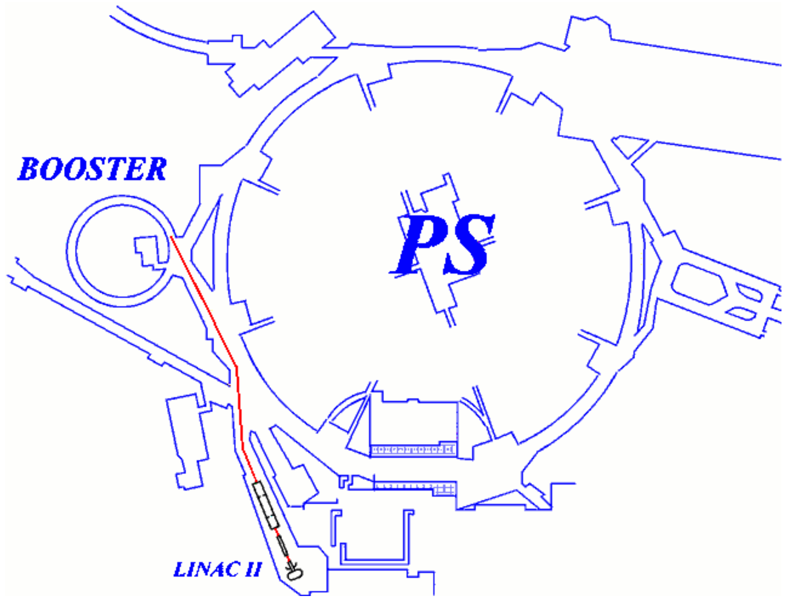

Figure 8: Beam path in the Linac2 Transfer Line

The line, formed by 26 quadrupoles and one buncher cavity, has been chosen in order to minimize the dispersion introduced by the two bending magnets needed to change the beam direction toward the booster. Along the line there are other two bending magnets that lead off the beam to the measurement lines.

Appendix 2. TRACE3D baseline Linac2 transfer line file: LTLTBBI_030504.t3d

```
&data
 er=   938.790, q= 1., w=    50.000, xi=  150.000,
 emiti(1)= 14.5,14.2, 2512.
 beami(1)=-0.190,1.342,0.103,2.48,0.134,0.066
 beamf(1)=  0.705,5.631,4.12,34.25
 mt=8,nc=4, mp=1,12,1,14,1,17,1,19
 freq=   202.560, pqext=  2.50, ichrom= 0,dispr=20.
 xm= 100.00, xpm= 100.0000, ym=  100.0, dpm=  60.0, dwm=   50.0, dpp=  60.0,
 n1=   1, n2=  67, smax=  10.0, pqsmax=   2.0, xc=  0.00,
 nt(  1)=  1, a(1,  1)= 1850.
 nt(  2)=  3, a(1,  2)= 4.6508    , 255.0
 nt(  3)=  1, a(1,  3)= 545.
 nt(  4)=  3, a(1,  4)=-3.6649    , 255.0
 nt(  5)=  1, a(1,  5)= 4430.
 nt(  6)=  3, a(1,  6)= 2.0719    , 255.0
 nt(  7)=  1, a(1,  7)= 745.
 nt(  8)=  3, a(1,  8)=-1.4633    , 255.0
 nt(  9)=  1, a(1,  9)= 4606.
 nt( 10)=  3, a(1, 10)= .6977    , 255.0
 nt( 11)=  1, a(1, 11)= 745.
 nt( 12)=  3, a(1, 12)=-1.1612    , 255.0
 nt( 13)=  1, a(1, 13)= 4396.
 nt( 14)=  3, a(1, 14)= 1.8636    , 255.0
 nt( 15)=  1, a(1, 15)= 745.
 nt( 16)=  3, a(1, 16)=-1.5396    , 255.0
 nt( 17)=  1, a(1, 17)= 2308.
```

```
 nt( 18)= 10, a(1, 18)= .000    , 270.0    , 1.000    , 1.000    ,0.0000
 nt( 19)=  1, a(1, 19)= 1102.
 nt( 20)=  9, a(1, 20)= 8.000    , 3588.    , 100.0    ,0.4500    , 2.800
 nt( 21)=  8, a(1, 21)= 16.00    , 3588.    ,0.0000
 nt( 22)=  9, a(1, 22)= 8.000    , 3588.    , 100.0    ,0.4500    , 2.800
 nt( 23)=  1, a(1, 23)= 5992.
 nt( 24)=  3, a(1, 24)= 1.4757    , 255.0
 nt( 25)=  1, a(1, 25)= 3941.
 nt( 26)=  3, a(1, 26)=-.7175    , 255.0
 nt( 27)=  1, a(1, 27)= 4616.
 nt( 28)=  3, a(1, 28)= .7174    , 255.0
 nt( 29)=  1, a(1, 29)= 1670.
 nt( 30)=  3, a(1, 30)=-.7174    , 255.0
 nt( 31)=  1, a(1, 31)= 4769.
 nt( 32)=  3, a(1, 32)= 0.6462    , 467.0
 nt( 33)=  1, a(1, 33)= 3025.
 nt( 34)=  3, a(1, 34)=-.6527    , 255.0
 nt( 35)=  1, a(1, 35)= 3231.
 nt( 36)=  9, a(1, 36)=-11.00    ,-2620.    , 100.0    ,0.4500    , 2.800
 nt( 37)=  8, a(1, 37)=-22.00    ,-2620.    ,0.0000
 nt( 38)=  9, a(1, 38)=-11.00    ,-2620.    , 100.0    ,0.4500    , 2.800
 nt( 39)=  1, a(1, 39)= 3686.
 nt( 40)=  3, a(1, 40)= 1.4773    , 255.0
 nt( 41)=  1, a(1, 41)= 738.
 nt( 42)=  3, a(1, 42)=-.7869    , 255.0
 nt( 43)=  1, a(1, 43)= 9395.
 nt( 44)=  3, a(1, 44)= .4520    , 461.0
 nt( 45)=  1, a(1, 45)= 539.
 nt( 46)=  3, a(1, 46)=-.6592    , 461.0
 nt( 47)=  1, a(1, 47)= 9090.
 nt( 48)=  3, a(1, 48)= .7376    , 461.0
 nt( 49)=  1, a(1, 49)= 839.
 nt( 50)=  3, a(1, 50)=-.7188    , 461.0
 nt( 51)=  1, a(1, 51)= 916.
 nt( 52)=  1, a(1, 52)= 659.
 nt( 53)=  1, a(1, 53)= 900.
 nt( 54)=  1, a(1, 54)= 12505.
 nt( 55)=  3, a(1, 55)= 1.1804          , 462.0
 nt( 56)=  1, a(1, 56)= 538.
 nt( 57)=  3, a(1, 57)=-1.3776          , 462.0
 nt( 58)=  1, a(1, 58)= 8854.
 nt( 59)=  3, a(1, 59)= .7544           , 462.0
 nt( 60)=  1, a(1, 60)= 838.
 nt( 61)=  3, a(1, 61)=-1.016           , 462.0
 nt( 62)=  1, a(1, 62)= 14179.
 nt( 63)=  3, a(1, 63)= 1.294           , 466.0
 nt( 64)=  1, a(1, 64)= 284.
 nt( 65)=  3, a(1, 65)=-1.406           , 466.0
 nt( 66)=  1, a(1, 66)= 3463.
 nt( 67)=  1, a(1, 67)= 4165.
&end
```

Appendix 3. TRACE3D high-dispersion Linac2 transfer line file

```
&data
 er=   938.790, q= 1., w=    50.000, xi= 150.000,
 emiti(1)= 14.5,14.2, 2512.
 beami(1)=-0.190,1.342,0.103,2.48,0.134,0.066
 freq=   202.560, pqext=  2.50, ichrom= 0,dispr=2.5
 xm= 80.00, xpm= 80.0000, ym= 80.0, dpm= 60.0, dwm=   200.0, dpp=  210.0,
 n1=   1, n2= 47, smax=  10.0, pqsmax=   2.0, xc=  0.00,  NP1= 1, NP2=67
mt = 8
nc=4
mp = 1,40,1,42,1,44,1,46
beamf =-1.7461,48.629,1.1227,70.744
 nt(  1)=  1, a(1,  1)= 1850.
 nt(  2)=  3, a(1,  2)= 4.6508    , 255.0
 nt(  3)=  1, a(1,  3)= 545.
 nt(  4)=  3, a(1,  4)=-3.6649    , 255.0
 nt(  5)=  1, a(1,  5)= 4430.
 nt(  6)=  3, a(1,  6)= 2.0719    , 255.0
 nt(  7)=  1, a(1,  7)= 745.
 nt(  8)=  3, a(1,  8)=-1.4633    , 255.0
 nt(  9)=  1, a(1,  9)= 4606.
 nt( 10)=  3, a(1, 10)= .6977    , 255.0
 nt( 11)=  1, a(1, 11)= 745.
 nt( 12)=  3, a(1, 12)=-1.1612    , 255.0
 nt( 13)=  1, a(1, 13)= 4396.
 nt( 14)=  3, a(1, 14)= 1.8636    , 255.0
 nt( 15)=  1, a(1, 15)= 745.
 nt( 16)=  3, a(1, 16)=-1.5396    , 255.0
 nt( 17)=  1, a(1, 17)= 2308.
 nt( 18)= 10, a(1, 18)= 0.400    , 270.0    , 1.000    , 1.000    ,0.0000
 nt( 19)=  1, a(1, 19)= 1102.
 nt( 20)=  9, a(1, 20)= 8.000    , 3588.    , 100.0    ,0.4500    , 2.800
 nt( 21)=  8, a(1, 21)= 16.00    , 3588.    ,0.0000
 nt( 22)=  9, a(1, 22)= 8.000    , 3588.    , 100.0    ,0.4500    , 2.800
 nt( 23)=  1, a(1, 23)= 5992.
 nt( 24)=  3, a(1, 24)= 1.549485    , 255.0
 nt( 25)=  1, a(1, 25)= 3941.
 nt( 26)=  3, a(1, 26)=-0.753375    , 255.0
 nt( 27)=  1, a(1, 27)= 4616.
 nt( 28)=  3, a(1, 28)= 0.75327    , 255.0
 nt( 29)=  1, a(1, 29)= 1670.
 nt( 30)=  3, a(1, 30)=-0.75327    , 255.0
 nt( 31)=  1, a(1, 31)= 4769.
 nt( 32)=  3, a(1, 32)= 0.67851    , 467.0
 nt( 33)=  1, a(1, 33)= 3025.
 nt( 34)=  3, a(1, 34)=-0.685335    , 255.0
 nt( 35)=  1, a(1, 35)= 3231.
 nt( 36)=  9, a(1, 36)=-11.00    ,-2620.    , 100.0    ,0.4500    , 2.800
 nt( 37)=  8, a(1, 37)=-22.00    ,-2620.    ,0.0000
 nt( 38)=  9, a(1, 38)=-11.00    ,-2620.    , 100.0    ,0.4500    , 2.800
 nt( 39)=  1, a(1, 39)= 3686.
 nt( 40)=  3, a(1, 40)= 0.785518    , 255.0
 nt( 41)=  1, a(1, 41)= 738.
 nt( 42)=  3, a(1, 42)=0.112126    , 255.0
 nt( 43)=  1, a(1, 43)= 9395.
```

```
 nt( 44)=  3, a(1, 44)= -0.193405    , 461.0
 nt( 45)=  1, a(1, 45)= 539.
 nt( 46)=  3, a(1, 46)=-0.137902    , 461.0
 nt( 47)=  1, a(1, 47)= 9090.
 nt( 48)=  3, a(1, 48)= .7376    , 461.0
 nt( 49)=  1, a(1, 49)= 839.
 nt( 50)=  3, a(1, 50)=-.7188    , 461.0
 nt( 51)=  1, a(1, 51)= 916.
 nt( 52)=  1, a(1, 52)= 659.
 nt( 53)=  1, a(1, 53)= 900.
 nt( 54)=  1, a(1, 54)= 12505.
 nt( 55)=  3, a(1, 55)= 1.1804          , 462.0
 nt( 56)=  1, a(1, 56)= 538.
 nt( 57)=  3, a(1, 57)=-1.3776          , 462.0
 nt( 58)=  1, a(1, 58)= 8854.
 nt( 59)=  3, a(1, 59)= 0.9   , 462.0
 nt( 60)=  1, a(1, 60)= 838.
 nt( 61)=  3, a(1, 61)=-1.016           , 462.0
 nt( 62)=  1, a(1, 62)= 14179.
 nt( 63)=  3, a(1, 63)= 1.294           , 466.0
 nt( 64)=  1, a(1, 64)= 284.
 nt( 65)=  3, a(1, 65)=-1.406           , 466.0
 nt( 66)=  1, a(1, 66)= 3463.
 nt( 67)=  1, a(1, 67)= 4165.
&end
```

Appendix 4. TRACE3D Linac2 transfer line without the bending magnets file

```
&data
 er=   938.790, q= 1., w=    50.000, xi= 150.000,
 emiti(1)= 14.5,14.2, 2512.
 beami(1)=-0.190,1.342,0.103,2.48,0.134,0.066
 freq=   202.560, pqext=  2.50, ichrom= 0,dispr=2.5
 xm= 80.00, xpm= 80.0000, ym= 80.0, dpm= 60.0, dwm=   200.0, dpp=  210.0,
 n1=   1, n2= 47, smax=  10.0, pqsmax=   2.0, xc=  0.00,  NP1= 1, NP2=67
mt = 8
nc=4
mp = 1,40,1,42,1,44,1,46
beamf =-1.7461,48.629,1.1227,70.744
 nt(  1)=  1, a(1,  1)= 1850.
 nt(  2)=  3, a(1,  2)= 4.6508    , 255.0
 nt(  3)=  1, a(1,  3)= 545.
 nt(  4)=  3, a(1,  4)=-3.6649    , 255.0
 nt(  5)=  1, a(1,  5)= 4430.
 nt(  6)=  3, a(1,  6)= 2.0719    , 255.0
 nt(  7)=  1, a(1,  7)= 745.
 nt(  8)=  3, a(1,  8)=-1.4633    , 255.0
 nt(  9)=  1, a(1,  9)= 4606.
 nt( 10)=  3, a(1, 10)= .6977    , 255.0
 nt( 11)=  1, a(1, 11)= 745.
 nt( 12)=  3, a(1, 12)=-1.1612    , 255.0
 nt( 13)=  1, a(1, 13)= 4396.
 nt( 14)=  3, a(1, 14)= 1.8636    , 255.0
 nt( 15)=  1, a(1, 15)= 745.
```

```
 nt( 16)=  3, a(1, 16)=-1.5396    , 255.0
 nt( 17)=  1, a(1, 17)= 2308.
 nt( 18)= 10, a(1, 18)= 0.400    , 270.0    , 1.000    , 1.000    ,0.0000
 nt( 19)=  1, a(1, 19)= 1102.
 nt( 20)=  9, a(1, 20)= 8.000    , 3588.    , 100.0    ,0.4500    , 2.800
 nt( 21)=  8, a(1, 21)= 16.00    , 3588.    ,0.0000
 nt( 22)=  9, a(1, 22)= 8.000    , 3588.    , 100.0    ,0.4500    , 2.800
 nt( 23)=  1, a(1, 23)= 5992.
 nt( 24)=  3, a(1, 24)= 1.549485    , 255.0
 nt( 25)=  1, a(1, 25)= 3941.
 nt( 26)=  3, a(1, 26)=-0.753375    , 255.0
 nt( 27)=  1, a(1, 27)= 4616.
 nt( 28)=  3, a(1, 28)= 0.75327    , 255.0
 nt( 29)=  1, a(1, 29)= 1670.
 nt( 30)=  3, a(1, 30)=-0.75327    , 255.0
 nt( 31)=  1, a(1, 31)= 4769.
 nt( 32)=  3, a(1, 32)= 0.51448878    , 467.0
 nt( 33)=  1, a(1, 33)= 3025.
 nt( 34)=  3, a(1, 34)=-0.70134589    , 255.0
 nt( 35)=  1, a(1, 35)= 3231.
 nt( 36)=  9, a(1, 36)=-11.00    ,-2620.    , 100.0    ,0.4500    , 2.800
 nt( 37)=  8, a(1, 37)=-22.00    ,-2620.    ,0.0000
 nt( 38)=  9, a(1, 38)=-11.00    ,-2620.    , 100.0    ,0.4500    , 2.800
 nt( 39)=  1, a(1, 39)= 3686.
 nt( 40)=  3, a(1, 40)= 0.55849402    , 255.0
 nt( 41)=  1, a(1, 41)= 738.
 nt( 42)=  3, a(1, 42)=-0.14971885    , 255.0
 nt( 43)=  1, a(1, 43)= 9395.
 nt( 44)=  3, a(1, 44)= 0.74508348    , 461.0
 nt( 45)=  1, a(1, 45)= 539.
 nt( 46)=  3, a(1, 46)=-0.62459288    , 461.0
 nt( 47)=  1, a(1, 47)= 9090.
 nt( 48)=  3, a(1, 48)= .7376    , 461.0
 nt( 49)=  1, a(1, 49)= 839.
 nt( 50)=  3, a(1, 50)=-.7188    , 461.0
 nt( 51)=  1, a(1, 51)= 916.
 nt( 52)=  1, a(1, 52)= 659.
 nt( 53)=  1, a(1, 53)= 900.
 nt( 54)=  1, a(1, 54)= 12505.
 nt( 55)=  3, a(1, 55)= 1.1804          , 462.0
 nt( 56)=  1, a(1, 56)= 538.
 nt( 57)=  3, a(1, 57)=-1.3776          , 462.0
 nt( 58)=  1, a(1, 58)= 8854.
 nt( 59)=  3, a(1, 59)= 0.7544          , 462.0
 nt( 60)=  1, a(1, 60)= 838.
 nt( 61)=  3, a(1, 61)=-1.016           , 462.0
 nt( 62)=  1, a(1, 62)= 14179.
 nt( 63)=  3, a(1, 63)= 1.294           , 466.0
 nt( 64)=  1, a(1, 64)= 284.
 nt( 65)=  3, a(1, 65)=-1.406           , 466.0
 nt( 66)=  1, a(1, 66)= 3463.
 nt( 67)=  1, a(1, 67)= 4165.
&end
```

Reference

[1] A. Perrin, J.-F. Amand, T. Muetze, "Travel User Manual"

[2] K. R. Crandall, D. P. Rusthoi, "TRACE 3-D Documentation", LA-UR-97-886, Los Alamos National Laboratory Report, May 1997

[3] K. Hanke, J. Sanchez-Conejo, R. Scrivens, "Dispersion Matching of a Space Charge Dominated Beam at Injection into the CERN PS Booster", CERN-AB-2005-032, presented at PAC2005 Knoxville, USA, 16–20 May 2005

[4] http://psop.web.cern.ch/psop/L2/

[5] http://jeroen.home.cern.ch/jeroen/psring/pscomplex.shtml