

IX. MISCELLANEOUS PROBLEMS

A. ELECTRONIC DIFFERENTIAL ANALYZER

Staff: Professor H. Wallman
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During the past quarter the electronic differential analyzer has been expanded to permit the solution of higher-order equations. Linear equations with constant coefficients of orders through four have been solved; in addition, a number of linear equations with variable coefficients have been investigated. A few examples are given below.

Linear Differential Equations with Constant Coefficients. The computing units now available consist of four integrators, two adders, and one multiplier or function generator. No particular difficulties have been encountered in the solution of linear, constant-coefficient differential equations of order through four. The solution of a typical fourth-order differential equation whose characteristic equation has two pairs of complex conjugate roots is shown in Fig. IX-1. In this example one pair of roots has negative real parts and one pair has positive. Thus one term in the solution shown is increasing with time while the other damps out with time.

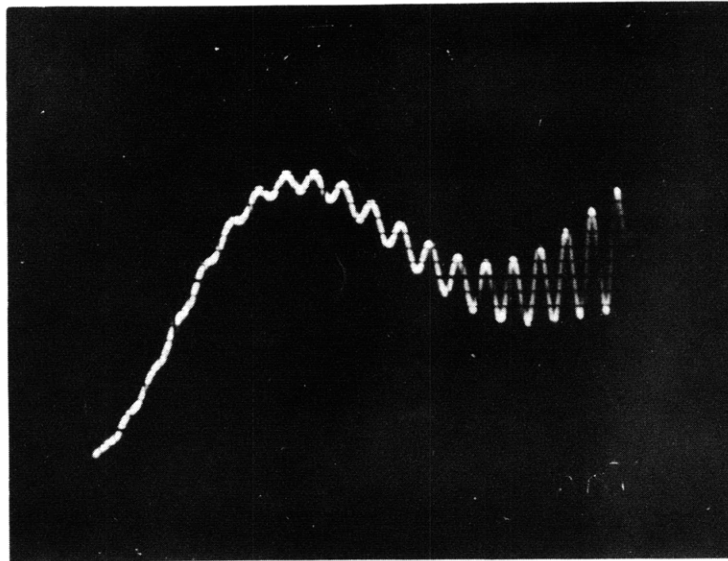


Fig. IX-1. Solution of a fourth-order differential equation with constant coefficients.

Linear Differential Equations with Variable Coefficients. A number of linear equations with variable coefficients of the form

$$\frac{d^2y}{dt^2} + F(t)y = 0 \quad (1)$$

have been solved. A block diagram of the differential analyzer setup for the solution of this equation is shown in Fig. IX-2.

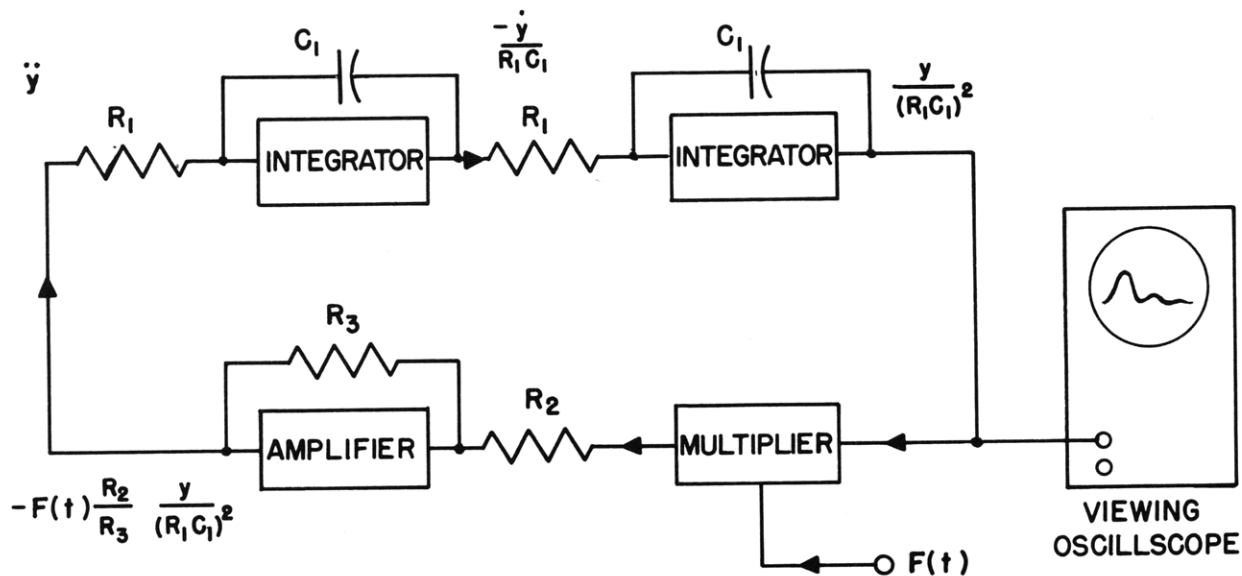


Fig. IX-2. Block diagram of electronic differential analyzer setup for solution of the equation $\frac{d^2y}{dt^2} + F(t)y = 0$

A solution of Eq. (1) for the case $F(t) = t^2$ is shown in part (a) of Fig. IX-3. For this equation the analytic solution happens to be known and is

$$y = t^{\frac{1}{2}} J_{1/4}(t^2/2);$$

a graph of the analytically obtained solution is given in part (b) of Fig. IX-3.

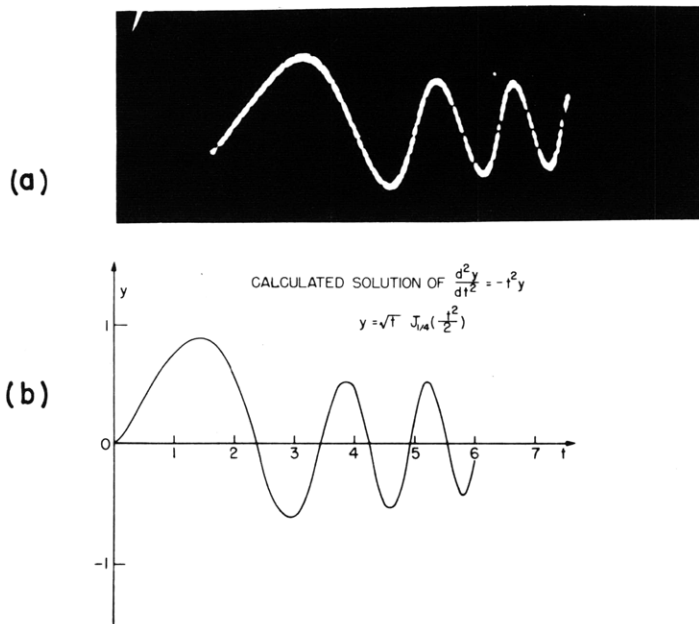


Fig. IX-3. Plot of y versus t for the equation $\ddot{y} + t^2y = 0$, (a) as observed on the differential analyzer and (b) as calculated from $y = \sqrt{t} J_{1/4}(t^2/2)$.

If

$$F(t) = \omega_0^2 (1 + \epsilon \sin \omega_m t), \quad (2)$$

Eq. (1) becomes the well-known Mathieu equation, particular solutions of which are the Mathieu functions. Figure IX-4 shows plots of a typical Mathieu function versus time as (a) observed on the cathode-ray tube, and (b) photographed from the Jahnke and Emde Tables.

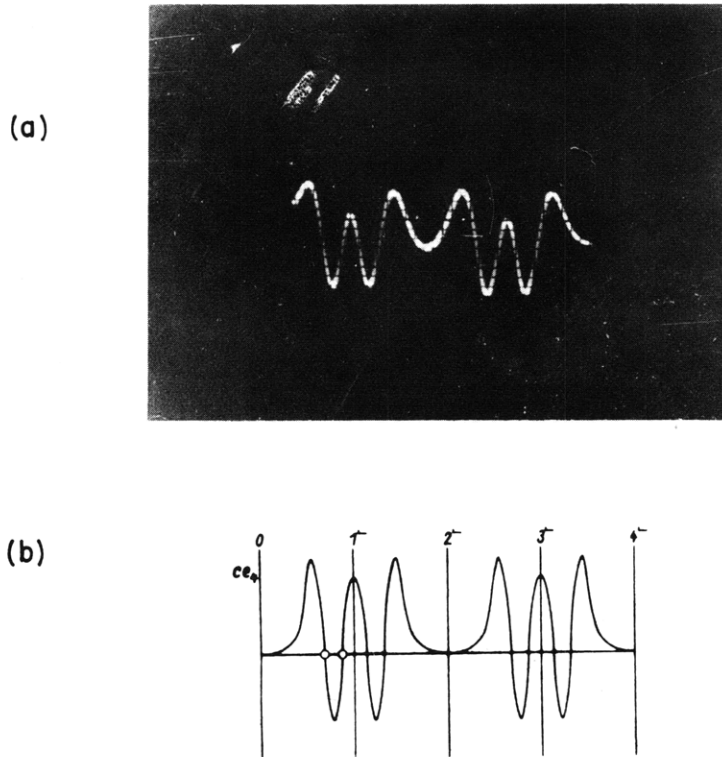


Fig. IX-4. Observed and calculated solutions of the Mathieu equation, $\ddot{y} + \omega_0^2 (1 + \epsilon \sin \omega_m t) = 0$.

B. AN IMPEDANCE FUNCTION ANALYZER

Staff: Professor H. Wallman
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This project is concerned with the design, construction, and testing of a machine for the analysis of impedance functions in the complex frequency domain. The device will consist of a conducting sheet of Teledeltos paper to represent the complex plane, a set of probes carrying plus and minus direct currents to represent the poles and zeros, and a means of displaying on a cathode-ray tube the voltage along the imaginary axis of the plane. This voltage can be made to represent either

- (a) the logarithm of the amplitude of the impedance function, or
- (b) the phase function of the network.

The machine can be used in several ways:

(1) It can be used to synthesize a desired curve of the logarithm of the magnitude of the impedance vs. frequency. To accomplish this the desired curve is plotted on the face of the cathode-ray tube and the poles and zeros are moved around in the plane until the desired curve is obtained within the required degree of approximation.

(2) In these transient-synthesis cases in which the desired phase-vs.-frequency curve as well as the logarithm of the amplitude-vs.-frequency curve are known, the whole factored complex form of the system function can be synthesized.

(3) The machine can be used to analyze a given transfer or driving point impedance if the poles and zeros are known. The magnitude and the phase-vs.-frequency curves can then be plotted directly by the machine.

(4) The machine introduces a new and helpful method of thinking about networks in general.

Theory. The analogy between the potential in a conducting plane and the logarithm of the magnitude of an impedance function was discussed in the last progress report. For this analogy to be exact, it is necessary to have an infinite plane. It is possible to make corrections for the finite size of the plane but a more useful procedure is based on the properties of conformal mapping. Because of symmetry, it is necessary to use only a half plane. This half plane can be mapped conformally upon a circular region as in the Smith Chart, or upon a rectangular strip by means of a logarithmic transformation. In the latter case the strip remains infinite but the log-frequency scale on it corresponds closely to the frequency spectrum, and this correspondence is useful in a practical problem.

Determination of the phase-vs.-frequency curve depends on the fact that the phase and the logarithm of the amplitude of an impedance function are conjugate potential functions. The rate of change of phase with respect to frequency is therefore equal to the rate of change of logarithm of the amplitude with respect to the real variable. This rate of change is approximately obtained from two parallel sets of probes along the imaginary axis. A simple summation then gives the phase.

Experiment. Three models of the device have been built to date. The first two have been described in the last progress report. The third is an improved version of the first two. It consists of a circular plane of

conducting Teledeltos paper of 16-inch diameter. One hundred pick-up probes are spaced uniformly along the imaginary axis. The voltages at these probes are fed to a mechanical commutator, and this procedure allows each voltage to be displayed in turn upon the face of a cathode-ray tube. The sequence of one hundred voltages is displayed in approximately one twenty-fifth of a second. With a linear sweep this displays satisfactory plots of the logarithm of the amplitude of the impedance vs. the frequency. No attempt has been made as yet to remove the restriction on the finite size of the plane or to obtain the phase-vs.-frequency curves.

C. PROTON-VELOCITY METER

Staff: L. D. Smullin
P. Lally

Since the last progress report the stabilized klystron and Klystron amplifier have been completed. The 30-Mc/sec, i-f amplifier with a balanced waveguide mixer has been tested, and a reaction ratio of about 25 db between a signal entering the local-oscillator arm and one entering the signal-arm has been obtained. Since the device is to operate only one frequency, it is hoped that a stable rejection ratio of 30 to 40 db can be achieved.

Currently, the major effort is being placed on the development of a diode (silicon crystal) modulator to produce either phase- or amplitude-modulation, to be used in the comparison arm as the "synchronous local oscillator".

A crystal-controlled microwave frequency calibrator is being assembled as a piece of test equipment. It consists of a 2C51 50-Mc/sec crystal oscillator-tripler putting 150 Mc/sec into a 6AK5 buffer amplifier, which in turn drives a silicon crystal. The harmonic output of the crystal is of the order of 10^{-7} watts at 300 Mc/sec, which is strong enough to be used for calibration.

D. ELECTRONIC POTENTIAL MAPPING

Staff: Dr. S. Goldman
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W. E. Vivian

The gate circuit commutator mentioned in the last progress report has been working very well and is now being used exclusively.

Studies are being made of the area distribution of potential across the surface of the chest for interpretation of the action of the human heart. The frame frequency (frequency of scanning a complete

picture) which is usually used is around 250 per second. Motion pictures of the activity have been taken. Since these pictures show only 15 frames per second some of the rapid activity is lost in photography. As a sample, the accompanying photographs, (see Fig IX-5) show four consecutive frames taken from the reel around the time of the QRS interval during the heart beat. The lower right-hand quadrant of the pictures corresponds to the location of the left ventricle. The difference in the intensity-area distribution in the region in successive frames, pictures the activity of the right ventricle during the QRS interval. The interpretation of the oscilloscope patterns has just begun.

Construction of equipment for the display of skull potential patterns is also in progress.

A comprehensive report on the circuits used is in progress and should be available at the time of the next quarterly report.



Fig. IX-5. Frames from motion picture reel of the heart beat.

E. PHYSICAL LIMITATIONS OF R-F RADIATING SYSTEMS

Staff: Professor L. J. Chu
Dr. M. V. Gerrillo
Dr. M. Loewenthal

A report entitled "Physical Limitations of Omnidirectional Antennas", RLE Technical Report No. 64, has been published. This report has also been submitted to the Journal of Applied Physics to be considered for publication. A second report on the physical limitations of pencil-beam antennas is in preparation.

F. MATHEMATICAL PROBLEMS

Staff: R. M. Redheffer

As noted in the last progress report, the co-ordinates for which Laplace's equation separates when the solution has the form $RXYZ$, R being fixed, are essentially the well-known cases or their inversions, and the wave equation so separates only in the standard cases giving $R = 1$.

We have now found the condition for partial separation, the solution with co-ordinates (u,v,w) being $R(u,v,w)S(u,v)Z(w)$, for both the wave

and Laplace equations, and have shown that co-ordinates for partial or complete separation of the Laplace equation have the group property under inversion.

We have investigated in detail the difference between separation of the equation and of the solution, which questions appear not to be clearly treated in the literature. Separation of the equation in one step is proved essentially equivalent to existence of a two-parameter family $RX_{ab}Y_aZ_b$ plus $\partial X/\partial a = \partial X/\partial b$; equivalent to $RX_{ab}Y_{ab}Z_b$; and the general case $RX_{ab}Y_{ab}Z_{ab}$, for which the equation itself does not separate, leads to still other conditions. In each case we have the necessary and sufficient conditions on R and the linear element of the space, which conditions show how to get R by inspection.

This work was done in collaboration with N. Levinson, and a joint paper by Levinson, Bogert, and Redheffer has been submitted to the Quarterly of Applied Mathematics.

G. SCATTERING OF ELECTROMAGNETIC WAVES. MATRIX METHODS

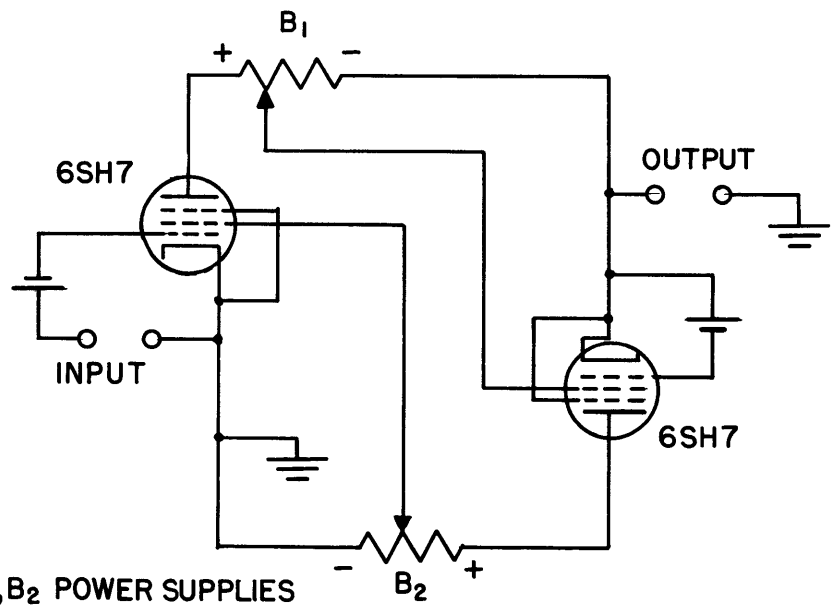
Staff: Professor H. Mueller
Dr. N. G. Parke

This research, last described in the Quarterly Progress Report, April 15, 1948, has been completed. A summary of this work will appear as RLE Technical Report No. 70, "Matrix Algebra of Electromagnetic Waves", dealing with the mathematical aspects of an algebraic technique developed during optical research, but which appears to be generally applicable to any class of linear systems for which there exists a transfer function. The algebra incorporates Wiener's generalized harmonic analysis in a natural manner and thus allows a uniform treatment of the statistical aspects of waves and of the instruments through which they pass. In particular it furnishes a basis for answering questions of coherence and depolarization, i.e., the statistical aspects of superposition.

H. A D-C AMPLIFIER

Staff: R. Parshad

A low-drift d-c amplifier has been built using two separate B-supply voltages and two triodes or pentodes. Figure IX-6 shows the connections for the pentode case. An improvement of greater than 50:1 in stability can be realized by this connection as compared with conventional single-supply circuits. The voltage gain obtained was 2,200.



B₁, B₂ POWER SUPPLIES
 VOLTAGE GAIN 2200

Fig. IX-6. High-gain d-c amplifier.