II MICROWAVE PHYSICS

A MICROWAVE SPECTROSCOPY

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Introduction In view of the widespread interest that has rapidly arisen in this field and the variety of research projects being carried on, it is felt that a brief summary of the present status of the art in addition to the usual project description would be helpful

Spectroscopy in the microwave region involves the study of vibrational and rotational transitions of polar molecules in the gaseous state That is, the atoms of a molecule may vibrate in several different fashions, and likewise the molecule may rotate with various allowed values of angular velocity, when the molecule changes from one vibrational or rotational state to another, the energy difference between the states divided by Planck's constant "h" corresponds to a frequency characteristic of the transition When the molecule is subjected to an electromagnetic field of this particular frequency, it will absorb energy from the field, detection of the absorption is the basis of microwave spectroscopy

In principle the apparatus is simple A source of microwave electromagnetic energy is passed through a cell containing the gas to be studied and is detected As the source is varied in frequency, a plot can be made of the detector output with and without gas in the chamber, any difference in the output then being due to absorption by the gas The frequency of the absorption the frequency interval over which absorption occurs, and the magnitude of the absorption are thus obtained at once Further measurements may then be made to find how these quantities vary with the pressure and temperature of the gas and with a static electric or magnetic field added

Details of several methods for making these measurements will be discussed later in the report

Specific Applications There are applications for the results of measurements of absorption of electromagnetic waves in gases A study of the absorption in atmospheric gases, such as oxygen and water vapor, is obviously of interest to determine practical ranges of communication equipment at various operating frequencies

What is not so immediately apparent, perhaps, is the wealth of information to be gained that is of direct interest to chemists and physicists This can be illustrated best by considering reports published in the literature on several specific molecules.

OCS Let us begin, not chronologically, but with experiments[1] on a molecule that illustrate the results most simply Carbonyl sulphide is a linear triatomic molecule

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[1] W E Good, T W Dakin and D K Coles, Letter to Editor, Phys Rev 70 560 (1946)

arranged as O-C-S It will rotate about an axis perpendicular to the line connecting the atoms, but according to quantum mechanics, with only certian discrete values of angular velocity allowed, that is with only certain energy levels allowed The molecule will be accelerated from one angular velocity to another (i e , make a transition to a higher energy level) and thus absorb energy from the electromagnetic field when the impressed frequency is

$$f = 2(J + 1) \frac{h}{8\pi^2 I} \text{ cycles/sec}$$

where J is an integer which can have any value from 0,1,2,   ,

h = Planck's constant,

I = moment of inertia of the molecule about its center of mass

This formula is derived in any quantum mechanics text, and as given here neglects centrifugal stretching of the molecule

In the experiment, the frequency of one line (about 1 25 cm, for J = 1) was predicted from the formula by knowing the approximate moment of inertia The reverse procedure thus allows an accurate calculation of the moments of inertia of molecules

Now suppose an istope of oneof the atoms is substituted in the molecule, the moment of inertia will be different and so will the absorption frequency As the shift in the absorption frequency is simple to observe at microwave frequencies two different moments of inertia of the molecule are obtained for atoms of different mass From these two values the inter-atomic distances can be calculated

So far just the measurements of the frequencies of absorption lines have resulted in obtaining the moment of inertia of a molecule and the inter-atomic distances These results point out that microwave spectroscopy is a general tool in the analysis of molecular structure by providing more accurate values than may be available or by confirming doubtful structures expecially for molecules whose x-ray analysis may present difficulty

Furthermore since each molecule possesses certain characteristic absorption frequencies, the method is potentially able to identify the constituents of a mixture of polar gases by measurement of absorption frequencies and to give the percentage of each present by measurement of the intensity of the absorption lines

From the fact that an isotope of one of the atoms in a molecule causes a new absorption line to appear, it is also apparent that the microwave spectroscopy possesses potentialities as a crude type of mass spectrograph to determine the percentage of isotopes

In the report on OCS[1], data were given on the Stark effect of the molecule In the Stark effect a static electric field is superimposed on the gas, this static field splits the energy levels of the molecules and allows new transitions to take place This effect shows up in an apparent splitting of the original single absorption line as the static electric field is increased Since the "splitting" of the absorption line involves

[1] Good, Dakin and Coles, loc cit

the dipole moment of the molecule, the dipole moment is found simply by measuring the frequency separation between the "split" lines Microwave spectroscopy is especially suitable for such measurements because of the tremendous resolution available, and since the accuracy is not affected by the purity of the gas For example, a shift of 1 Mc/sec is readily measurable at 25,000 Mc/sec

This parameter, the dipole moment, in addition to the others already mentioned, is of great assistance in determining the arrangement of the atoms in a polar molecule

The equation previously given for the absorption frequencies was an approximation in that it neglected an additional small term which arises because of certrifugal stretching of the molecule as it rotates By finding the difference between the actual absorption frequency and the frequency given by the simplified equation above for a series of rotational lines, this extra term is calculated In this way the manner in which the forces between the atoms vary as the atoms are moved apart, (i e , as the molecule is stretched) is determined and thus the vibrational frequencies of the molecule can be calculated This process provides another check on the theory of molecular structure

$\underline{NH_3}$ Actually in point of time, the first molecule studied at microwave frequencies was ammonia[1], and it has provided the strongest absorptions observed to date The absorption mechanism is rather unusual in that it results from the vibration of the nitrogen atom through the plane of the three hydrogen atoms This single vibration line is split into a number of components because of the rotational levels of the molecule This splitting can be calculated so that investigations of these lines provide accurate data to check these theoretical calculations[2]

The hyperfine structure, reported by Good[3], accompanying some of these lines had not been predicted It has only recently been explained by Van Vleck and Wilson of Harvard as due to the electric quadrupole moment of the nitrogen nucleus The discovery was again a result of the very high resolution available in the microwave region and adds another kind of information obtainable by microwave spectorscopy

$\underline{O_2}$ An exhaustive set of measurements being made on the oxygen molecule will illustrate other applications For instance, it will obviously yield practical data for communication range calculations, and will, moreover, provide a detailed check on Van Vleck's general theory of absorption for all polar molecules This work is discussed in detail later in the report

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[1] C E Cleeton and N H Williams Phys Rev 45, 234 (1934)

[2] H Sheng, E F Barker and D M Dennison Phys Rev 60, 786 (1941)

[3] W E Good Phys Rev 70, 213 (1946)

H_2O Only one absorption line of water vapor has been investigated to date, but this line is of considerable importance to communication since it lies at 1 34 cm [1-3] Besides being of practical importance the data in the several reports published provide another check on the Van Vleck formula

Equipment and Methods A number of various equipments is necessary to study microwave absorption spectra effectively First, apparatus for disclosing absorptions is necessary, it should be characterized by ease of operation and rapidity in data taking over a wide frequency range Essentially it should disclose exact frequencies of absorption, and the order of magnitude of the intensity of the line Second, possibly, there should be apparatus that will conveniently allow the study of the effect of electric and magnetic field and temperature on a known absorption line. And a third type of apparatus should give a precise value of the absorption

There are at present three pieces of apparatus of the last type operating at frequencies around 9000 Mc/sec, 24000 Mc/sec and 50,000 Mc/sec The first is capable of operating over a frequency band 12 per cent wide, the second two operate over bands approximately 30 per cent wide

These equipments are essentially of two different types Two are bridge spectrometers and are shown schematically in Fig 1 Power from a microwave source is divided between two transmission paths Then it is recombined and detected With one path evacuated, the phase shifter and attenuator in the other arm are adjusted to balance so that the detector registers no output The evacuated arm is then filled with the gas to be studied and the unbalanced power due to attenuation in the gas is registered by the detector

An audio balance bridge can also be made in this fashion Thus instead of balancing the actual r-f signals, an audio modulation on the carrier may be balanced after detection Here again the bridge is unbalanced because of the absorption in the gas, and the absolute intensity of absorption may be calculated

The third apparatus uses the change in resonant frequency of a cavity as an absorbing gas is introduced At present the gas being studied is caesium, and instead of evacuating and refilling the cavity with caesium vapor for each measurement, a simpler method is employed The particular transition observed in caesium may be split into 14 components by a magnetic field The resonant frequency of any one of these components is a function of the applied magnetic field For a measurement, the cavity is tuned near the caesium resonance, and then by changing the external magnetic field, (of the order of 100 gauss) the frequency of each line in turn is caused to coincide with the cavity frequency The effect of the line resonance on the cavity Q and tuning

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1 G E Becker, S H Autler Phys Rev 70 300, 307 (1946)

2 R H Dicke, R Kyle, R L Beringer, A B Vane Phys Rev 70 340 (1946)

3 C H Townes, F R Merritt, Letter to Editor, Phys Rev 70 558 (1946)

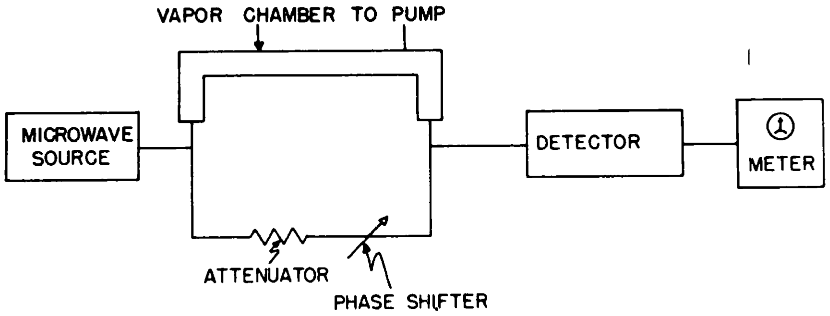

Figure 1 Bridge Spectrometer

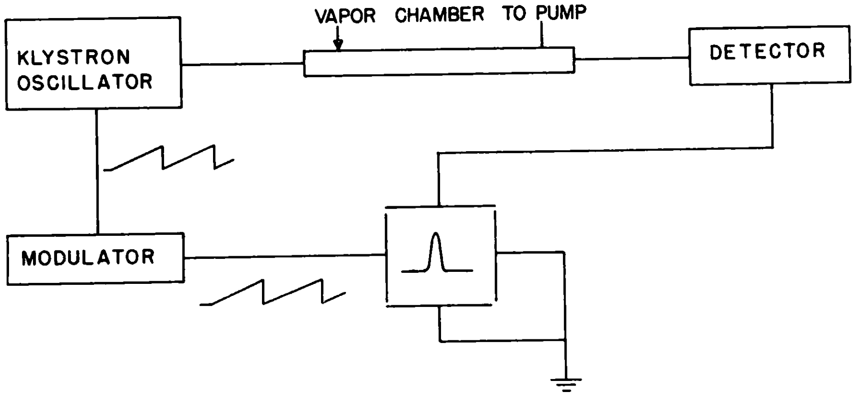

Figure 2 Frequency-modulated Spectroscope

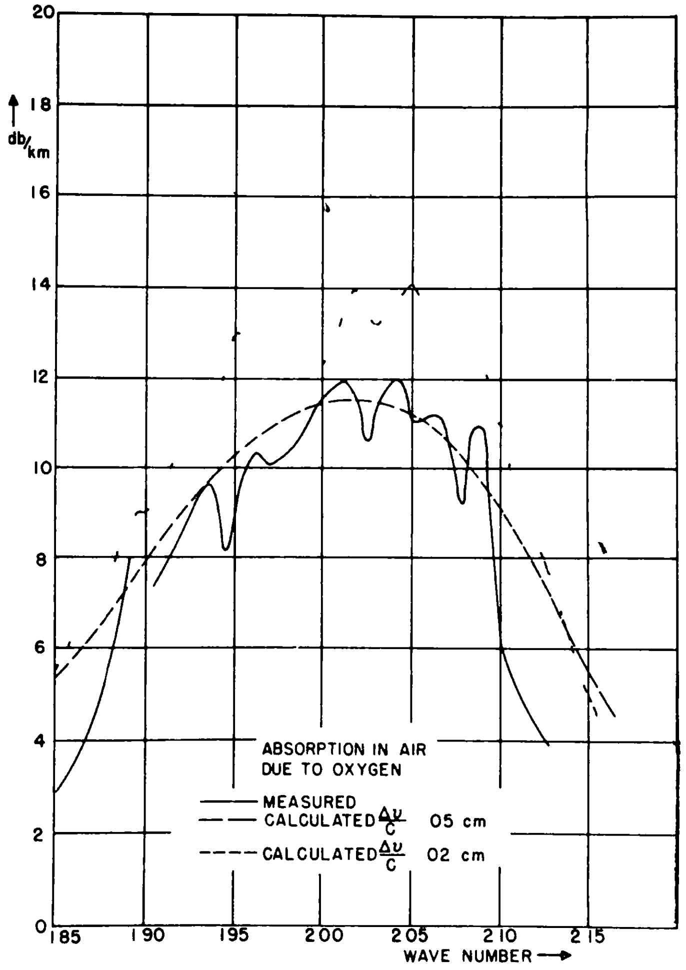

Figure 3 Experimental data taken with oxygen at 80 cm Hg pressure reduced to atmospheric values

are indications of the line absorption and intensity Similar experiments can be performed to study the Stark Effect by substituting an electric field for the magnetic field

An example of the first type of apparatus mentioned above for rapid searching for absorption lines is the frequency-modulated spectroscope in operation at the laboratory This apparatus is shown in block diagram in Figure 2 The long transmission path is made of waveguide operating in its lowest mode at about 9000 Mc/sec It is of course operable at higher frequencies and it is intended to equip this apparatus for use at frequencies up to 60,000 Mc/sec

A microwave source is modulated in frequency at a 20 cps rate This modulated carrier is sent down the long waveguide path filled with an absorbing gas The power is then detected and is displayed on a cathode ray oscillograph Variations in output of the detector due to the absorption in the gas are thus observed

Equipment for studying the effect of electric and magnetic fields and temperature is only now in the process of development It is expected that these studies will eventually be a fruitful field of research

In short, the study of the spectrum of a gas proceeds as follows Rough calculations are used to indicate possible absorption lines in the microwave region This portion of the spectrum is then examined in the frequency-modulated spectroscope and the absolute frequencies and line widths are accurately determined The gas is then measured with a bridge or cavity spectrometer to determine absolute intensity of absorption Temperature, Stark and Zeeman studies finally are used to verify the transition causing the absorption measured, and to calculate the dipole moment Any or all of these data are of great use in determining the molecular structure in case this is unknown, or in evaluating structure parameters, e g , inter-atomic distances, quadrupole moment, etc

1 Microwave-frequency Bridge

Staff M W P Strandberg
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Absorption measurements on oxygen and carbonyl sulfide are being made in the frequency range 40,000 to 60,000 Mc/sec with a radio frequency bridge circuit A general discussion of bridge circuits has been given above

In this case the source of power is a QK-140 reflex klystron operating in the region of 9 to 11 mm wavelength This power is fed to a silicon crystal and the second harmonic is used for the actual bridge measurements The receiver is a superheterodyne also using for a local oscillator the second harmonic of a QK-140 oscillator Frequency stabilization and measurement are performed at the fundamental frequency

Measurements on oxygen at 80 cm Hg pressure are almost completed, and are shown in Figure 3 Plotted on the same graph are the Van Vleck calculations based on atmospheric band spectra data In spite of the inaccuracy of his fundamental data, the experimental and calculated curves show extremely good agreement The line breadth facto $\frac{\Delta \nu}{c}$ cm^{-1}, it is interesting to note, is about 04 cm^{-1} if calculated from the oxygen mean free path

Further measurements will be made at very low pressure to assign precise frequencies to the individual lines in the oxygen band

Measurements are also being made on the rotation lines of carbonyl sulfide in this frequency region These data will yield factors for the centrifugal stretching in the molecule, but as yet these data have not been reduced

2 Audio-frequency Bridge Method

Staff C I Beard

Method This system is designed to accomplish both functions of searching and intensity measurement Since the system was described in detail in the Final Report of June 1946, only a brief summary is given at this time

For searching, the apparatus is run as follows The r-f oscillator output is split into two waveguide branches, one of which is the gas chamber Each branch is terminated by a crystal detector, one of which is in a reversed-polarity holder The crystal outputs are subtracted, amplified, and connected to the vertical plates of a scope The oscillator is swept by a sawtooth voltage which also provides the sweep for the detecting scope so that absorption versus frequency is presented on the scope Searching is carried out at about 20 microns pressure of gas, since at this pressure the absorption lines are only a few megacycles per second wide and stand out sharply on the scope

When a line is found by this searching method, the system is changed to an audio-frequency bridge as follows the oscillator is frequency stabilized and is square-wave modulated at 1000 cycles/sec The two crystal outputs are now applied to two different amplifiers, subtracted, and the difference signal further amplified before it reaches the detector A lock-in detector is now used to increase the sensitivity A phase shifter and an attenuator in one of the amplifiers are adjusted so that the difference signal is zero with no gas in the waveguide With gas admitted, a difference signal exists which is amplified and read to give the intensity of the absorption The absorption is then given by the expression,

$$\alpha = \frac{\lambda_a}{\lambda_g} \frac{10^4}{L} \log_{10}\left(1 - \frac{\Delta P}{P_0}\right) \mathrm{db/km}$$

where α is the absorption in db/km,

λ_α is the wavelength in free space,

λ_g is the guide wavelength,

ΔP is the difference power between the two arms,

P_o is the power from one arm alone,

and L is the length of the absorbing guide in meters

Characteristics of Method The system has been operated for searching from 1 15 cm to 1 8 cm by means of three oscillator tubes No attempt has yet been made to extend the lower wavelength range Efforts are being made at present to extend the longer wavelength range to nearer the waveguide cutoff of 2 13 cm by doubling the output of a modified Shepherd-Pierce tube through a crystal

One of the factors limiting the measurement of weak absorption lines is the instability of the oscillator and its power supply Tests are now being made to improve power supply regulation For intensity measurements of lines which are only a few megacycles per second wide at low pressures, it is also essential that the oscillator be stabilized The power output of the oscillators over the wide frequency range being covered drops so low that the abbreviated i-f strip generally used in Pound's stabilizing circuit possesses insufficient gain The full complement of stages in the i-f strip will therefore be necessary

The length of the waveguide gas chamber has been increased to 5 5 meters so that absorptions of 1 db/meter can be measured

<u>Program</u> In cooperation with Dr G W King and R M Hainer of Arthur D Little, Inc , and P C Cross of Brown University the system is being used to investigate the absorption spectra predicted by them for a series of triatomic assymmetric rotators, H_2O, HDO, D_2O, H_2S, HDS, and D_2S

The H_2O line at 22,267 Mc/sec and the HDO line at 22,309 Mc/sec have been checke to within 2 Mc/sec of the values reported by another laboratory[1] Measurements of their intensities are awaiting the improved oscillator power supply and a revamped i-f strip for the stabilization circuit With D_2O vapor in the waveguide a search has been made from 1 34 cm to 1 8 cm without finding any D_2O lines. The attempt to extend the long wavelength range of the system is for the purpose of locating a predicted D_2O line

3 <u>Caesium Spectrum</u>

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The work on the cavity spectroscope operating around 9200 Mc/sec is at present directed toward making a container for the caesium vapor As has been described above, the measurements required the caesium to be contained in a resonant cavity At present work is progressing on the fabrication of a quartz liner to be inserted in an ordinary 9200 Mc/sec cavity Although the effect has been detected some four or five times, many problems concerning line breadth, line shape, and pressure effects remain to be settled

4 <u>Nuclear Magnetic Moment of Hydrogen</u>

Staff R B Lawrance

Three oscillators covering the range 1260 to 1550 Mc/sec have been completed and one of these has been delivered to the Molecular Beam Group for their experiment on atomic hydrogen They have completed an atomic hydrogen generator which will be used in the present cavity type measurement Substantially all equipment with the exception of the test cavity has been completed or will be soon, the latter will be designed after preliminary tests on one of the stabilizing cavities

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[1] Townes, Merritt, <u>loc cit</u>

5 Frequency-modulated Spectroscope

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By employing a sweep technique that is in general use in the field of microwave spectroscopy, a frequency-modulated spectroscope has been assembled The transmission path consists of 16 meters of 9000 Mc/sec waveguide The detector is a superheterodyne with automatic frequency control on the local oscillator

At present the apparatus has been operating in the region 23,000 to 25,000 Mc/sec Frequencies of the ammonia lines appearing in the region have been accurately measured

The separation of the hyperfine structure of some ammonia lines due to the electric quadrupole moment has also been measured and reported[1] The carbonyl sulfide rotational line in this region has been studied and further work with sulphur isotopes in this molecule is planned

A search for absorption in the region 8,500 to 9,600 Mc/sec will also be made Components are under construction to allow the use of the spectroscope in the 40,000 to 60,000 Mc/sec region

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[1] Letter to the Editor, Phys Rev , to appear in a forthcoming issue

II B MOLECULAR BEAMS

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Description of Project The Final Report of June 1946 and the October Progress Report have described an atomic beam apparatus for magnetic resonance experiments In an experiment now projected, an arc source produces hydrogen atoms moving with thermal velocities Slits define a narrow beam of atoms which passes through a region where it is slightly deflected by the presence of a magnetic field having a strong gradient The atoms then pass through a transition region where there are two magnetic fields, one of which is homogeneous and constant with time, and the other oscillates at a microwave frequency Next the beam passes through a region where the magnetic field has a strong gradient similar to the first, and finally it passes near a Pirani gauge detector If the frequency of the oscillating magnetic field has the proper value, the atoms will reorient themselves in space in such a way that the second gradient causes a deflection which nullifies the first The atoms will then strike the detector only if the oscillating field has the correct frequency to cause these reorientations The experiment consists of a precise measurement of the frequencies at which transitions, that is reorientations, occur

The natural width of the r-f transition line is determined by the time the atoms spend in the homogeneous field In the present apparatus the homogeneous field is 10 cm long and atoms up to mass number 100 will be used, so that a minimum natural line width of several kilocycles (2000-3000 cps) will be present This means we must measure frequencies to an accuracy of about 300 cycles regardless of the frequency. Such requirements suggested the use of an interpolation oscillator of this precision in conjunction with very accurate spot frequencies throughout the band in question. At the highest frequency we will use, this requires a precision of one part in 10^{-7}.

Status Our first experiment will require frequencies from 1416-1864 Mc/sec The block diagram (Fig 1) shows the method to be used in measuring these frequencies and as seen from the figure, the equipment is about two-thirds complete

The photographs (Figs 2, 3, 4) show the apparatus as it looked after the first complete assembly Preliminary tests have now been carried out, which indicate

(a) A pressure of 5×10^{-7} mm of mercury can be reached, which should be adequate for beam experiments

(b) A hydrogen discharge can be maintained in the source end of the apparatus with the discharge tube previously described There is a pressure range from about 2 to 0 01 mm of mercury where the arc operates satisfactorily for hydrogen. A system for continuous pressure control is being devised.

(c) The transition magnetic field is homogeneous to 0 1 per cent over the region traversed by the beam This is the degree of accuracy desired for this experiment. Errors are due to nonparallelism of the pole faces

The Pirani described previously has been redesigned to permit simplified assembly and lineup, and also better electrical features Difficulties with mounting the fragil detector wires should soon be overcome

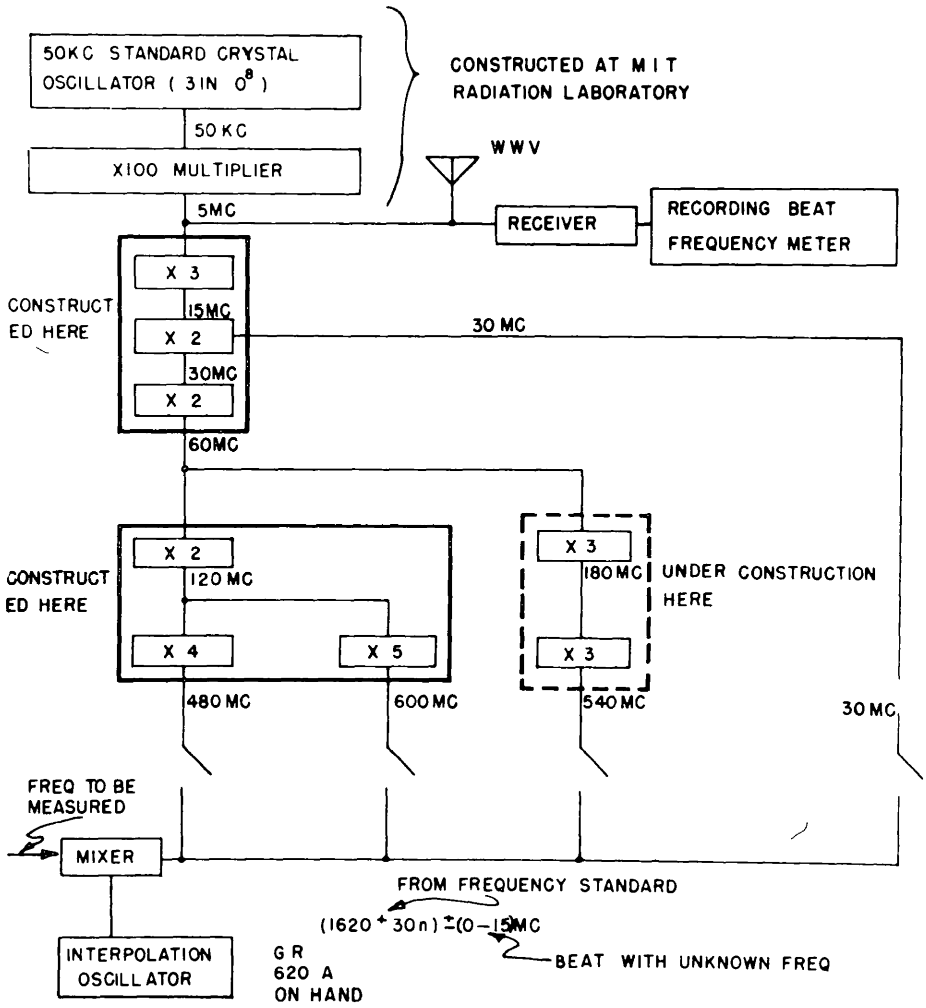

Figure 1 Block diagram of frequency standard

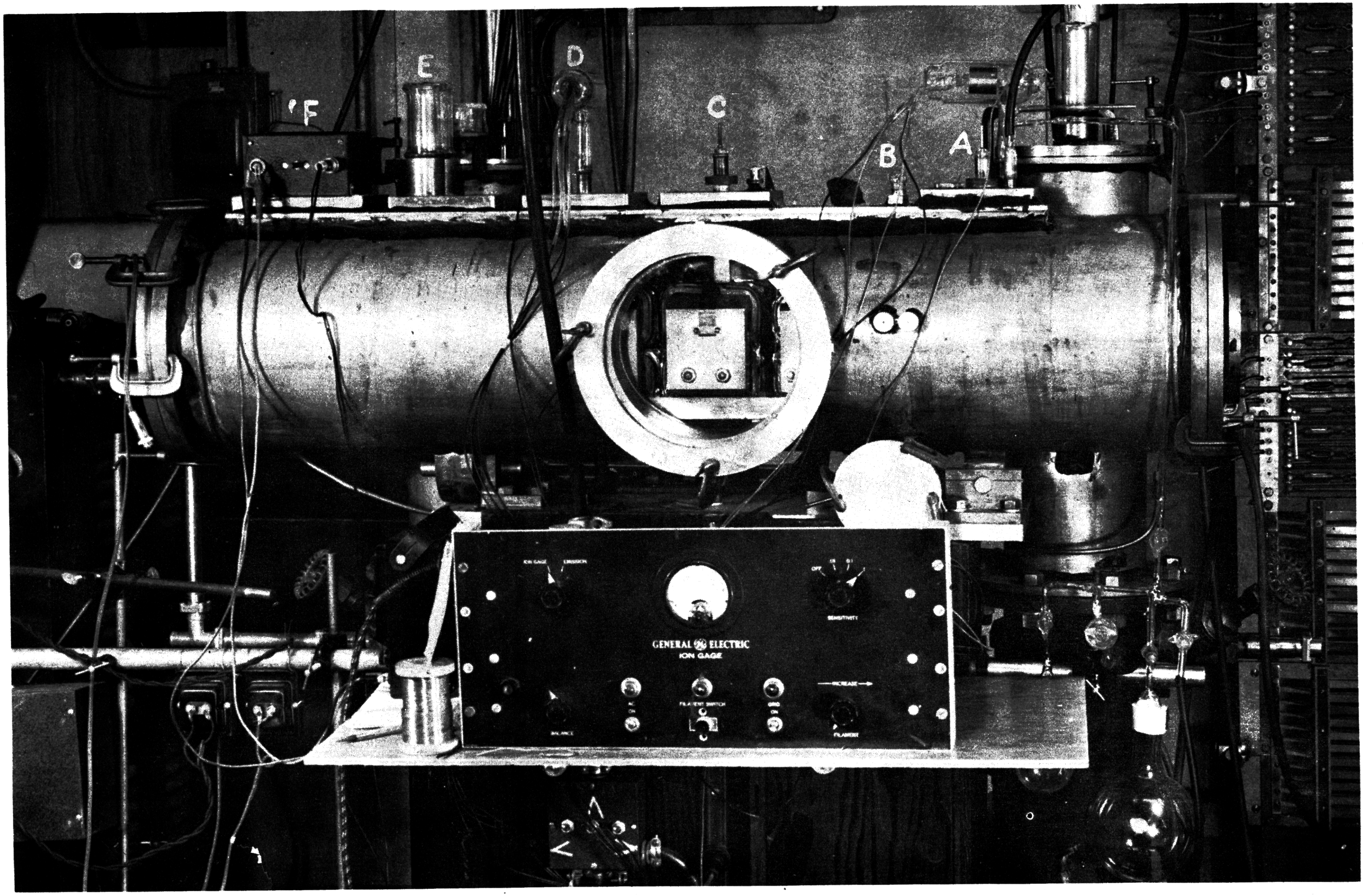

Figure 2. Atomic hydrogen apparatus, side view. (A) atomic hydrogen source, (B) beam shutter, (C) transition r-f field, (D) thermionic vacuum gauge, (E) obstacle wire, (F) Pirani detector.

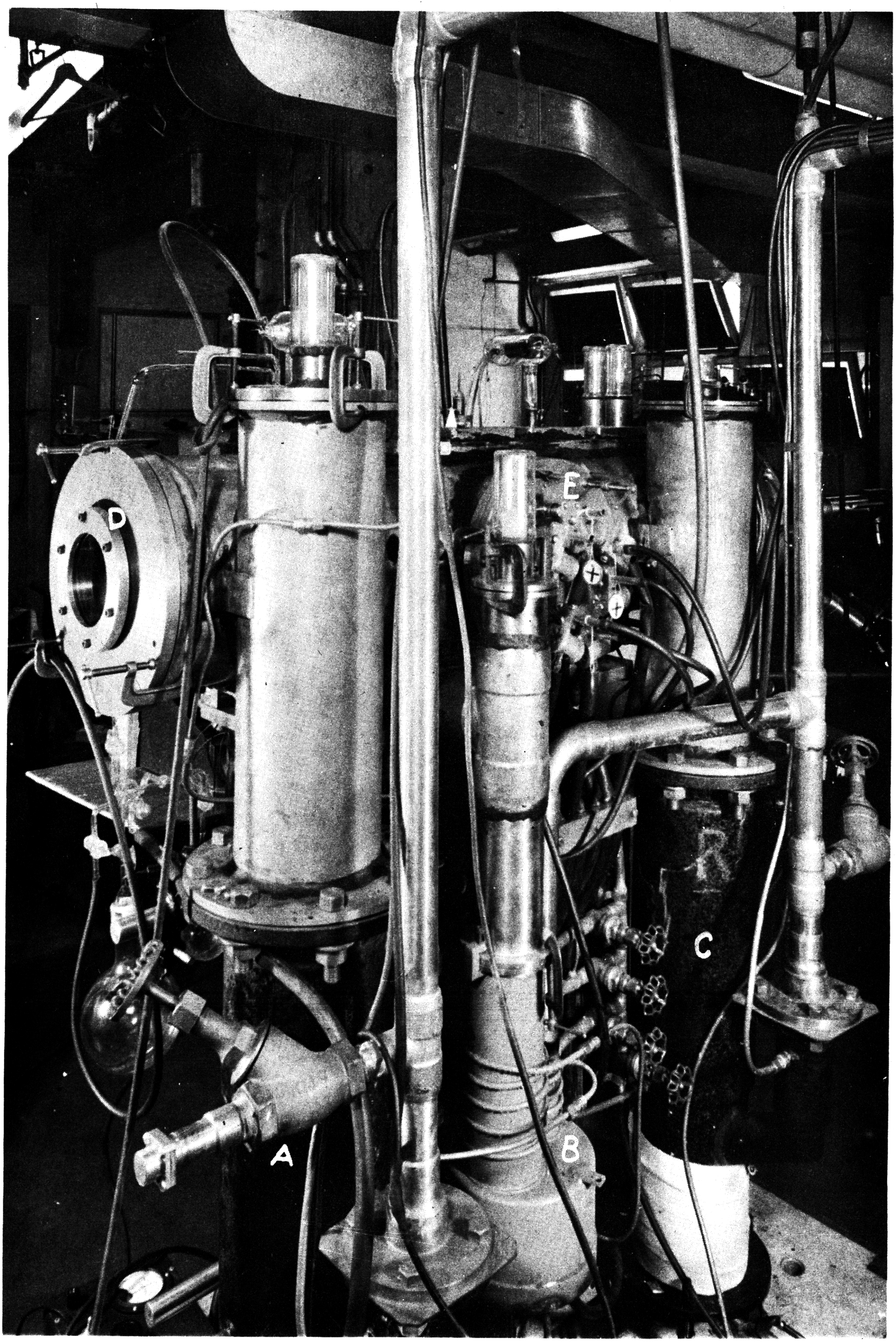

Figure 3. Atomic hydrogen apparatus, wide angle view. (A) source diffusion pump, (B) partition chamber diffusion pump, (C) detector chamber diffusion pump, (D) source chamber window, (E) magnet leads.

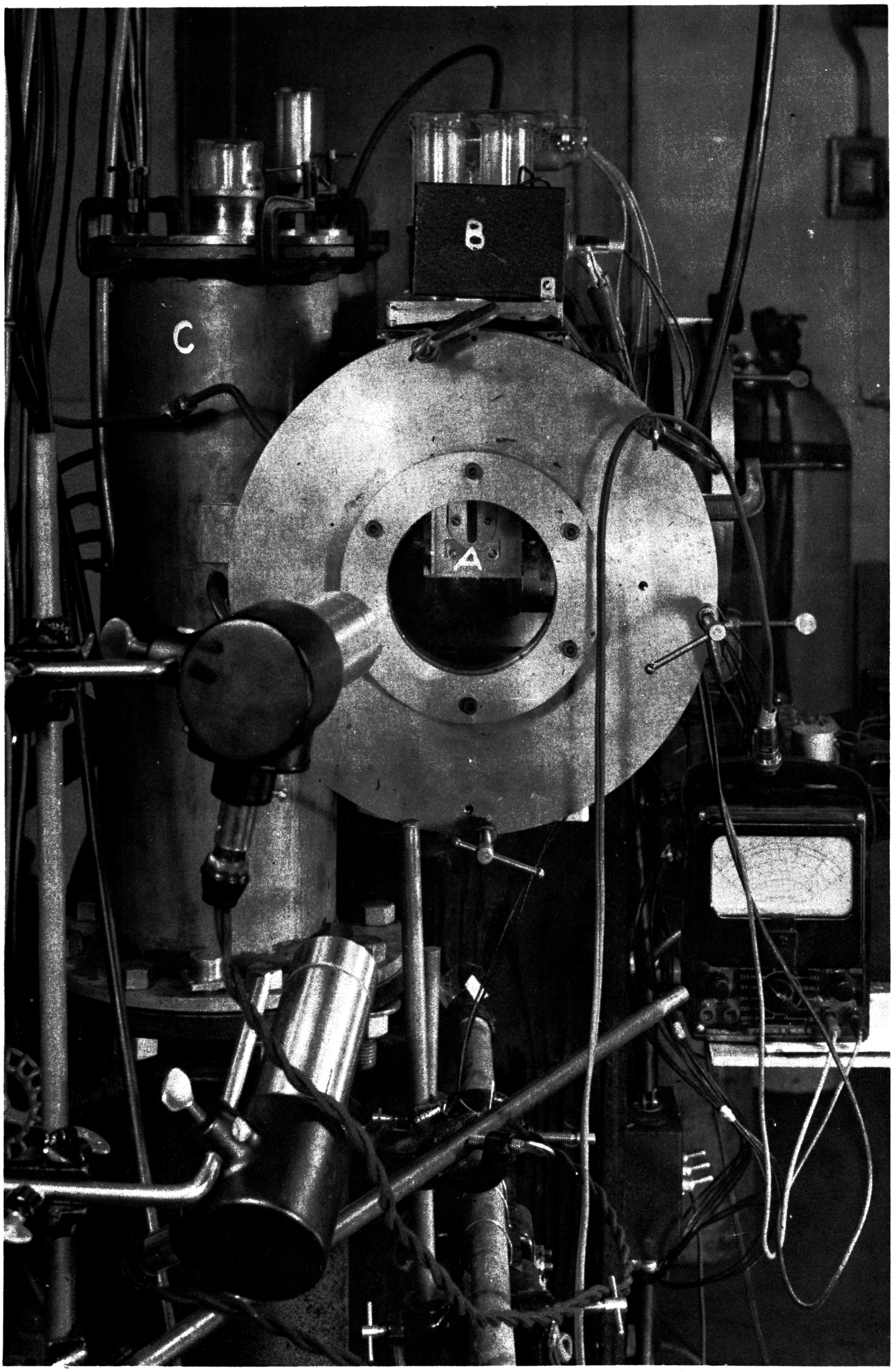

Figure 4. Atomic hydrogen apparatus, end view. (A) detector end window, showing Pirani gauge, (B) Pirani gauge trimming circuit, (C) detector end diffusion pump.

II C LOW PRESSURE GAS DISCHARGES

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1 Introduction

Electrical discharges in gas maintained by d-c power sources have received detailed study for many decades during which a large body of information and data have been accumulated The discharge problem as it has developed may be divided into three general aspects The first of these is the group of processes by which charged particles are produced in the discharge, such as ionization by collision, photoelectric emission, and secondary emission of electrons from a surface under positive ion bombardment The second is the opposite the processes by which charged particles are removed from the discharge, such as diffusion and recombination at the walls of the discharge vessel, volume recombination, and electron attachment to neutral molecules The third aspect is the properties of the discharge as an electric circuit element If a discharge is maintained by an a-c power source many of these processes undergo a shift of relative importance in which certain experimental advantages result

Some work has been done with discharges maintained by r-f power, but the energy distribution of the electrons and the ionization process remain nearly unchanged until a frequency of the order of the mean collision frequency of the electrons with the gas molecules is attained or exceeded At the pressures ordinarily used in gas discharges this requirement implies frequencies in the so-called microwave region, in which until recently sufficiently strong sources of power have not been available. Recent developments of microwave power sources therefore open a new field of investigation in the gas discharge problem It proves to be possible to cover completely the transition from d-c behavior to the new region of a-c behavior, with pressure variation within the range ordinarily used in gas-discharge experimentation

A notable advantage which enters into the high-frequency discharge is the fact that it may be maintained without the use of electrodes Phenomena at the cathode prove to be extremely important to the operation of a d-c glow or arc, and unfortunately the determining factors here include secondary emission phenomena which are poorly understood and difficult to control experimentally In the high-frequency discharge however there is no d-c migration of charged particles with or against the direction of the a-c field, the loss of ionization products being governed rather by ambipolar diffusion with recombination at the walls or by some volume process These processes seem to be more easily controlled and understood than cathode phenomena, and the experimental results are then more readily subject to interpretation. The electrodeless feature is further valuable for assuring gas purity The bombardment of metallic cathode surfaces in d-c discharge tubes is a troublesome source of gas contamination.

At the microwave frequencies found in the 10-cm and 3-cm band, ions, because of their mass are not measurably affected by the high-frequency power One of the great advantages of these studies therefore is that only electrons are observed This results in great simplification of interpretation and leads to the use in transient discharges of the high-speed electron phenomena so desirable in many applications

Electromagnetic Properties of an Ionized Medium The properties of gas discharges which appear directly in measurements as described in the following sections are predominantly electrical in character and it is therefore essential to know how they appear in the field equations It seems at present that a good approximation to discharge conditions may be had from the assumptions that there are no bound charges nor magnetic materials and that an electric field interacts sensibly only with the electrons the positive or negative ions being too massive to contribute a measurable current (The last item is true of alternating currents only, the conservation condition for direct currents requires that an equal amount of positive and negative charge reach opposite plates in unit time) The further assumption that the field giving rise to current flow is composed of the externally applied field only and is not due to local contributions of other charges holds when the electron and ion density is small

The two quantities, which appear in the field equations as contributions from the discharge are the complex conductivity $\sigma_c = \sigma_r + j\,\sigma_i$ which is the ratio of the conduction current density to the electric field, and the space charge density ρ also a complex number related to the conductivity in a way to be shown below The conductivity is the quantity which is characteristic of the discharge, but it is sometimes convenient to lump it with the displacement current of the space, thus defining an effective dielectric coefficient $\mathcal{E}_c$ as follows

$$\nabla \times H = \frac{\partial D}{\partial t} + J = (j\omega\epsilon_0 + \sigma_c)\,E = j\omega\epsilon_0\,\mathcal{E}_c E$$

where

$$\mathcal{E}_c = \mathcal{E}_r - j\mathcal{E}_i$$

and

$$\mathcal{E}_r = 1 + \frac{\sigma_i}{\omega\epsilon_0}$$

$$\mathcal{E}_i = \frac{\sigma_r}{\omega\epsilon_0}$$

Mks rationalized units are used, $\epsilon_0 = 10^{-9}/36\pi$ farad/meter, and ω is the radian frequency of the field $E = E_0 e^{j\omega t}$

The conductivity of a discharge may be calculated from kinetic theory if the energy distribution function of the electrons is known A considerable effort has been made in this direction, but the results cannot be given in detail here There are simple cases however One for which the conductivity is given below is that of a pure electron gas of density low enough to make electron interactions unimportant For this condition a simple application of Newton's Law to the electrons in the a-c field yields

$$\sigma_c = \frac{ne^2}{j\omega m} \qquad (1)$$

where e and m are the electronic charge and mass, respectively and n is the electron density Thus $\mathcal{E}_c$ is real and given by

$$\mathcal{E}_c = 1 - \frac{ne^2}{\epsilon_0 \omega^2 m} \qquad (2)$$

It is seen that for sufficiently large n or small ω $\mathcal{E}_c$ deviates markedly from unity, and may become negative Perhaps the most common evidence of this property is found in the ionosphere which reflects radio waves of a frequency such that $\mathcal{E}_c \leq 0$, but allows penetration of frequencies for which $\mathcal{E}_c > 0$ The condition for which $\mathcal{E}_c = 0$ may be designated as space resonance since $\mathcal{E}_c$ is zero at the frequency at which the free space displacement current and the electron drift current are equal in magnitude, and at this frequency space resonance occurs

As a second example the effect of collisions with gas molecules may be roughly accounted for in the following manner Collisions destroy the 90-degree phase relationship indicated in Eq (1) During the period when the electrons are recovering from the collisions and returning to their normal phase they absorb energy (on the average) from the field This energy is transferred to the heavier molecules during the collisions and eventually manifests itself as an increase in the temperature of the gas Thus, whereas with no collisions assumed, $\mathcal{E}_c$ was real it is now complex to account for the power loss If it is assumed that at each collision an electron loses its component of momentum in the direction of the electric field, and that the frequency of the collisions is unaltered by the presence of the applied field the real part of $\mathcal{E}_c$ is now the same as (2) and the imaginary part contains the effective conductivity σ_r

If ω_r is the angular frequency for space resonance, ν the average collision frequency, $\mathcal{E}_r$ and σ_r become

$$\mathcal{E}_r = 1 - \frac{(\omega_r/\omega)^2}{1 + (\nu/\omega)^2} \qquad (3)$$

$$\sigma_r = \frac{-(\omega_r/\omega)^2}{1 + (\nu/\omega)^2} \epsilon_0 \nu \qquad (4)$$

The average collision frequency is obtained from the energy distribution function of the electrons but the well-known value for the Maxwellian distribution is good enough for the purpose of the above approximate treatment.

The second factor entering the field equations from the discharge is the charge density Since a-c terms only are of interest here the a-c charge density, which differs in some respects from the d-c, is considered here The a-c electric field produces an a-c drift velocity of the electrons which will result in an alternating variation in space charge provided that there exists a concentration gradient of electrons But the a-c drift velocity may be obtained from the conductivity of the discharge The final result of using these facts is

$$\rho_0 = \frac{\sigma}{j\omega e} E_0 \frac{\nabla n}{n} \qquad (5)$$

where $\rho = \rho_0 e^{j\omega t}$

The quantity ρ_0 is thus seen to be a complex number also Under the conditions where the space charge is large, then, the phase of the electric field will vary from point to point

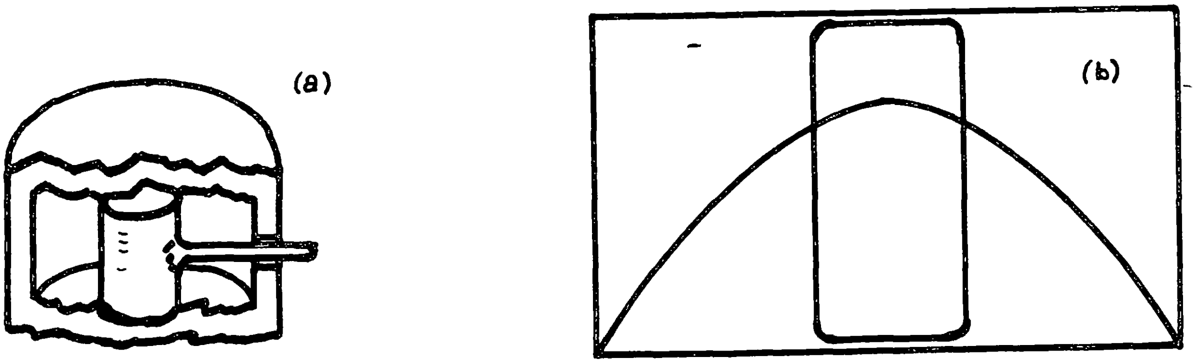
Methods of Measurement Two general methods of measuring the conductivity of discharge have been investigated The first and more thoroughly studied method is the use of a high-Q cavity containing the discharge to be studied. The conductivity is related to the cavity characteristics, that is, to the Q and resonant frequency (or equivalently, its input admittance) which are measurable The second is the use of a transmission line one of whose elements is the discharge The propagation characteristics which are again measurable are related to the discharge conductivity Two examples are (1) a coaxial line whose center conductor is an arc tube, and (2) a coaxial line whose free-space region is the region of gas discharge.

The characteristics of these two general methods supplement each other to cover a wide useful range. The cavity proves to be very sensitive to small electron densities while the transmission line is not badly perturbed by high electron densities Either method may be used for steady state or transient phenomena.

2 Steady State Low Current Density Discharge Characteristics

Experimental Arrangement A cylindrical glass tube filled with low-pressure gas was placed in a high-frequency electric field of sufficient strength to maintain a discharge in the gas The complex conductivity of the discharge as a function of power, or equivalently the volt-ampere characteristic of the discharge was measured The discharge in general filled the tube completely with the brightness greatest at the center falling off at the walls At the higher pressures (about 50 mm Hg) the glow tended to concentrate into a ball which either stayed at one end of the tube or oscillated visibly from one end to the other This effect at the high pressure end could be minimized with the appropriate shape of tube The experimental limit at the low pressure end (about 0 1 mm Hg) was set by the available power

The electric field was obtained by the excitation of a cylindrical resonant cavity in the TM_{010} mode with the E-field along the axis and maximum at the center. The cavity contained the tube of thin low-loss glass of smaller diameter filled with low pressure helium The tube diameter was confined generally to the region of nearly constant electric field in the center of the cavity, as shown in Fig 1 A block diagram of the apparatus is shown in Fig 2 The high-frequency power was delivered by a tunable 3000-Mc c-w magnetron (of the QK59-62 series) into a coaxial line leading to a variable power divider A directional coupler which samples a known proportion of the wave incident upon the attenuating pad isolating the cavity from the power source provided a signal to the thermistor bridge power meter The attenuation of the pad was calibrated and the pad was matched looking in both directions this assured a nearly matched line looking either toward the cavity or back toward the magnetron The power measurement thus provided the value of the power incident upon the cavity, the reflected power being nearly all dissipated in the pad The terminating admittance of the line for the load consisting of the cavity and its coupling loop, and the discharge inside the cavity, was measured with the aid of standard standing-wave technique[1] employing a slotted section and traveling probe leading to the detector The calibrated waveguide-beyond-cutoff-type attenuator on the detector was used to measure standing-wave ratio while the position of the minimum voltage point in the slotted line was measured with a scale mounted on the line The frequency was monitored with a cavity wavemeter capable of measuring frequency differences to high accuracy



CAVITY AND TUBE

ELECTRIC FIELD DISTRIBUTION

Figure 1 (a) Cylindrical TM_{010} cavity and discharge tube
(b) Electric field distribution in relation to discharge region

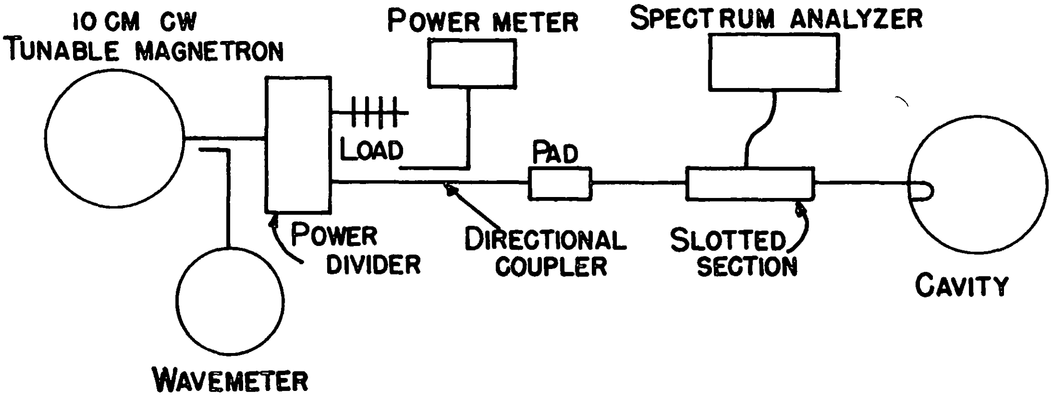


Figure 2 Block diagram of experimental arrangement

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[1] F J Gaffney "Microwave Measurements and Test Equipments", Proc IRE, 34 775 1946) This article is a good general reference regarding the microwave apparatus used in this experiment

Experimental Results The cavity input admittance is related to the discharge admittance through the calculated and measured constants of the cavity The data thus yield the discharge admittance Y_d as a function of power dissipated in the discharge the frequency of the a-c field and the pressure of the gas Since the frequency varies only within the resonance region of the cavity, the discharge admittance does not change appreciably as a result of this small amount of frequency variation and therefore only the admittance as a function of power and pressure need be considered

Although curves of Y_d as a function of power and pressure would give the information desired, it is more convenient to convert admittance and power to voltage and current since d-c curves are given in this way and more importantly since this presentation shows clearly the important characteristics of the discharge

Figure 3 shows how the voltage and imaginary component of current behave with the variation of the real component of current for a fixed pressure The voltage and phase angle are seen to be constant This behavior holds until the pressure is high enough to allow a large dark sheath to form (at about 50 mm Hg) or low enough so that the mean free path is considerably longer than the tube dimensions. The characteristic quantities describing the discharge behavior are thus breakdown voltage and the ratio of the imaginary to the real current (the tangent of the phase angle)

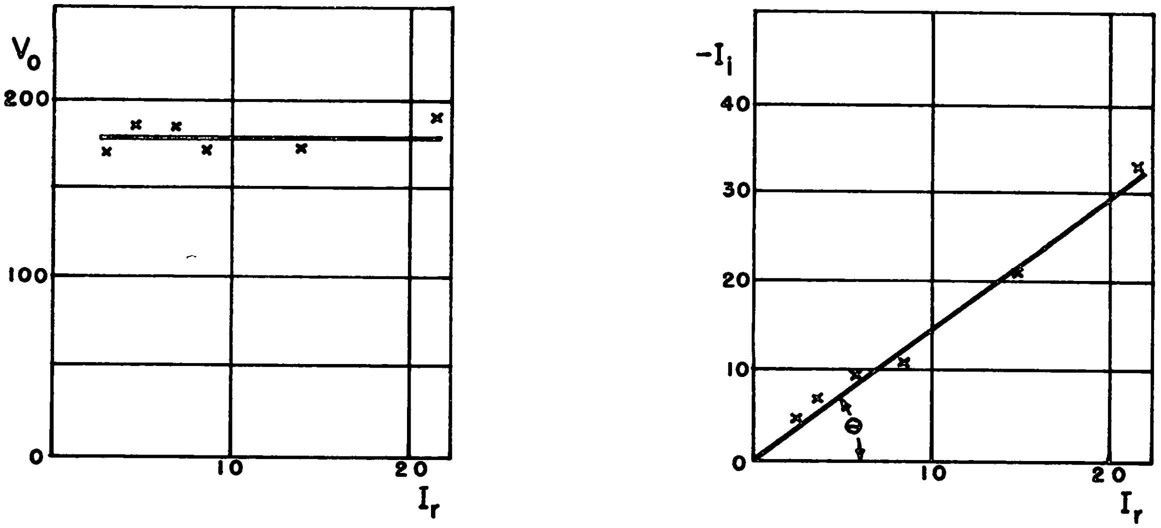

Figure 3 Typical voltage current and phase curves for a discharge at a fixed pressure

These two quantities are shown as a function of pressure in Fig 4 The breakdown voltage is seen to pass through a minimum similar to the d-c behavior, but the pressure at which this minimum occurs is not related to the tube dimensions in the same way as in the d-c discharge The 1/p rise on the low-pressure side is in accordance with theory The high-pressure side has not been explained theoretically, except by making use of experimental values or ionization rates taken from Townsendtype measurements The range of d-c experimental data however is insufficient to give the theoretical a-c curve completely

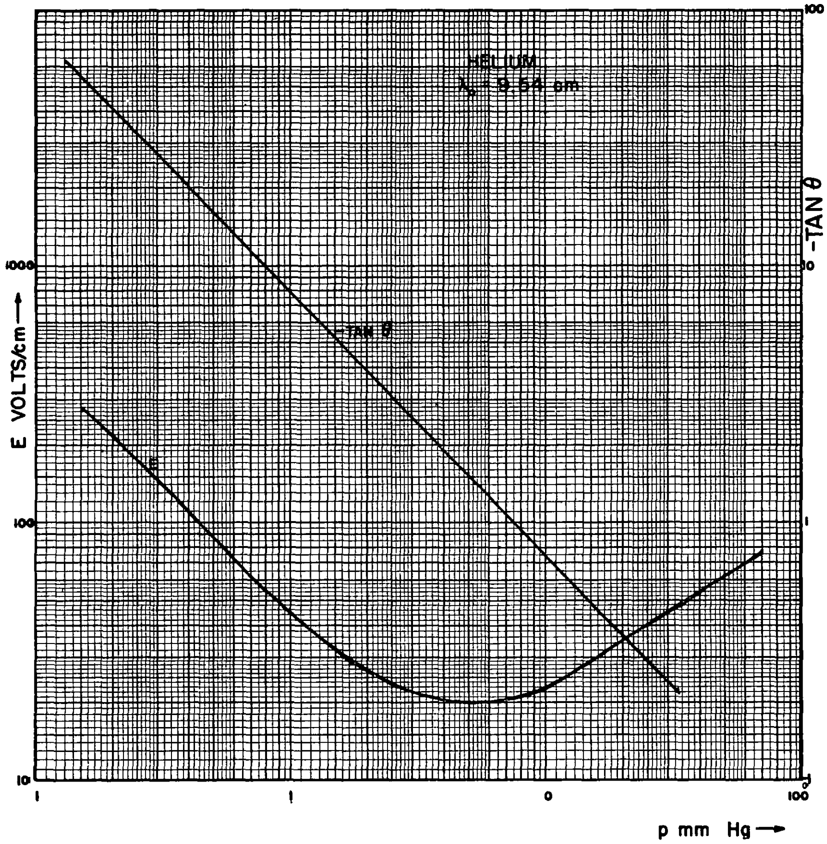

Figure 4 Breakdown voltage and the ratio of the imaginary to the real current plotted against pressure

The tangent of the phase angle curve has a 1/p variation over the complete range, this is related to the fact that the mean energy of the electrons is about the same at all pressures

3 Steady State High Current Density Experiments

Coaxial Systems Terminated by Discharges The techniques of measurement of the dielectric properties of materials of arbitrary Q at microwave frequencies are now well advanced Perhaps the best known method consists of measurement of the reflection from a sample of material enclosed in the end of a short-circuited coaxial transmission line or waveguide[1] Figure 5 shows how this procedure has been applied to measurement of the properties of gas discharges (Mechanical details are omitted) With the principal transmission mode (TEM) incident on the mica window the standing-wave pattern to the left of the window is determined by the conductivity σ_r and

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[1] von Hippel Jelatis and Westphal "The Measurement of Dielectric Constant and Loss with Standing Waves in Coaxial Wave Guides" Laboratory for Insulation Research M I T April 1943

dielectric coefficient $\mathcal{E}_r$ of the discharge and by its length L

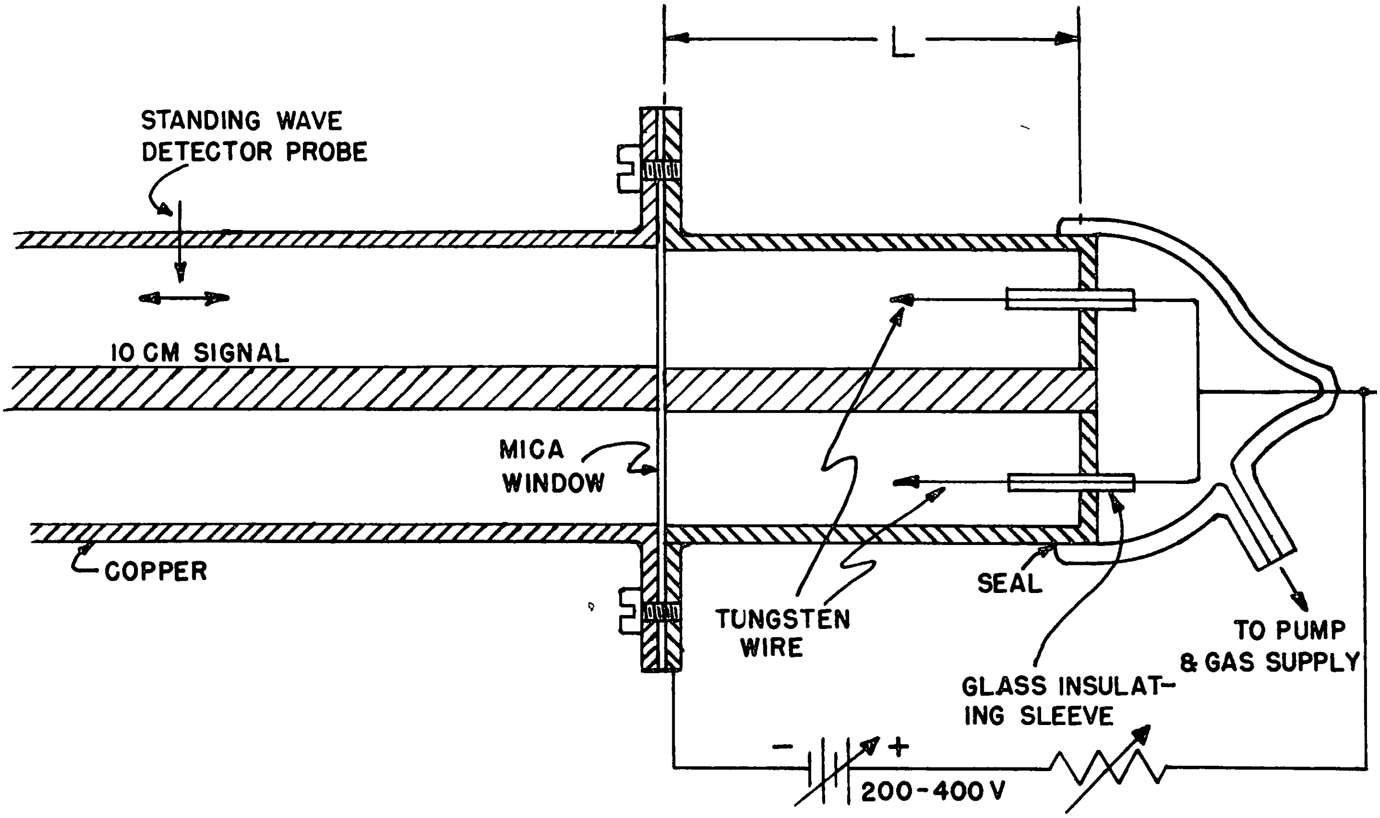

Figure 5 Cross section of coaxial line device to measure dielectric properties of gas discharges. The line in the present instance is standard 7/8" line with a characteristic impedance of 47 ohms. A glow discharge is maintained between the fine tungsten wires and the walls.

In the first preliminary measurements using the arrangement shown in Fig. 5 the wavelength was 10 cm and L was approximately $\lambda/4$. Three fine tungsten wires were used as the anode of a glow discharge, with a current of 15 ma to each electrode in commercial grade helium at a pressure of 5 mm Hg. (At this pressure the glow appeared by visual inspection to fill the enclosure uniformly, but the distribution of $\mathcal{E}_r$ and σ_r is undoubtedly still far from uniform. As the object of the first measurements was only to obtain orders of magnitude, however, the nonuniformity was ignored.) From the measured value of the impedance the results for $\mathcal{E}_r$ and σ_r were $\mathcal{E}_r = 0.90$ and $\sigma_r = 0.013$ mho/meter. If the formulas for n and ν given in the Introduction are applied, these values of σ_r and $\mathcal{E}_r$ yield

$$n = 1.7 \times 10^{10} \text{ electrons/cm}^3 \text{ and } \nu = 1.1 \times 10^{10} \text{ collisions/sec}$$

Although admittedly rough, these figures suggest that it should not be difficult to obtain much higher current densities and a wide range in $\mathcal{E}$, σ, n and ν.

Although the above discussion has referred to steady-state measurements, the method described is adaptable to measurement of the discharge properties as a function of time after the voltage, causing the discharge is switched off The discharge is pulsed periodically and the pulse starts a variable delay multivibrator, which "gates" an appropriate circuit of the standing-wave detector (such as a spectrum analyzer) for a time long enough to obtain a readable pulse on the oscilloscope screen, but short enough that the discharge properties do not change appreciably during the interval The standing-wave pattern and consequently $\mathcal{E}_r$ and σ_r may then be measured as a function of time following the cessation of the discharge This technique may possibly serve to extend the cavity resonator methods for measurement of diffusion and attachment to high electron densities

The next stages of development of the coaxial instrument consist of arranging the discharge to improve the uniformity of electron density and of adding the variable time measurements described above

<u>Coaxial Systems with Discharge as Center Conductor</u> A second method for measuring the dielectric properties of discharges involves the use of a coaxial transmission system in which the center conductor, instead of being the customary metal rod, is a gas discharge in a glass tube It will be found that this kind of system has a certain amount of frequency sensitivity, lying roughly between the broad-band coaxial system just described and the high-Q cavity resonator

If both inner and outer conductors of a coaxial line have infinite conductivity, the well known transmission properties are found All wavelengths are transmitted without attenuation, and down to a wavelength corresponding roughly to the average circumference of the inner and outer conductors the transmission occurs in a TEM mode with radial electric field and circular magnetic field Below this wavelength transmission may also occur with more complex field patterns of both TE and TM types and the propagation constant for each mode is a function only of geometry and wavelength

Let us, however consider the situation in which the conductivity of the outer conductor is so large that for practical purposes it may still be considered infinite, but the conductivity and dielectric coefficient of the center conductor are varied over a wide range As soon as the conductivity becomes finite the waves must of necessity all be of the TM type as the TE types require an angular component of magnetic field[1] which in turn causes longitudinal components of current and electric field, contradicting the original assumption of a TE field As long as the conductivity is high the perturbation of the TE to TM types will not be large however and the propagation constant may be calculated by the usual procedure in which the fields and surface currents are assumed to depart negligibly from those obtained with perfect conductors

As the conductivity decreases the above approximation finally fails and in the limit of zero conductivity the transmission has a pure dielectric center and is incapable of transmitting the TEM coaxial mode, but may support high order modes at sufficiently short wavelengths To cover this wide range of conditions we have found solutions of Maxwell's equations that will take into account the arbitrary

[1] Except for TE_0

properties of the center conductor

In the apparatus represented by Fig 6 when there is no arc in the center, L acts as a waveguide beyond cutoff but as the arc current increases from its minimum operating value the propagation constants α and β undergo a rapid change in a narrow current range, and approach the values they would have with a perfectly conducting center conductor

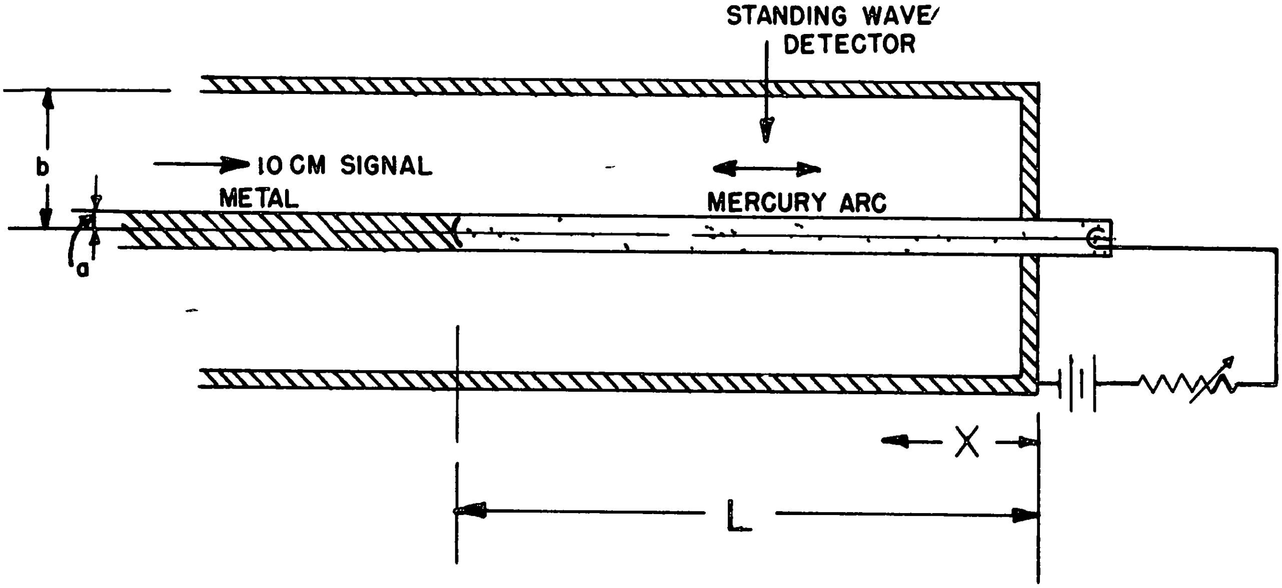

Figure 6 Simplified cross section of coaxial line device to measure dielectric properties of arcs

4 Transient Discharge Characteristics

Previous investigations of electron attachment and diffusion suffered from a common disadvantage The presence of metal surfaces in the measuring apparatus caused difficulty in maintaining high purity of gases and in some cases led to undesired interaction between the metal electrodes and the charged particles Also in attachment studies the necessary minimum electric field was too large for measurements to be obtained at thermal energies Microwave techniques, however, permit the development of a method for measuring attachment and diffusion at thermal energies without electrodes in the gas High purity of gases is thus obtained and the region between thermal energy (0 04 volts) and the point at which former experiments left off (≈0 2 volts) can be investigated

Experimental Method If a gas-filled glass bottle is placed inside a cavity resonant at 3000 Mc, and ions and electrons are generated in the gas by applying a high r-f field, the resonant frequency of the cavity will be changed Since the ions are so massive in comparison to the electrons their effect is negligible It can be shown that the resonant frequency shift is proportional to σ_1 the imaginary component of the complex conductivity of the cavity, σ_1 is a function of the electron density within the cavity and is given by

$$\sigma_1 = - \left[\frac{\ell e^2}{\sqrt{2 m \epsilon_2}} \quad (1 - 5/2 \frac{kT}{\epsilon_2}) \right] n \qquad (6)$$

where e and m are electronic charge and mass, respectively, ℓ is the electronic mean free path, $\epsilon_2 = 1/2\, m\, \omega^2 \ell^2$ ω is the applied radian frequency, k is Boltzmann's constant, T is the absolute temperature and n is the electron density

If a large number of electrons are initially produced in a gas they will disappear with time principally by diffusion or attachment As their density decreases the frequency shift they produce in the cavity also diminishes proportionally

Therefore in order to observe electron clean-up by attachment and diffusion it is sufficient to measure the frequency shift produced by the electrons as a function of time

The measuring apparatus is shown in a simplified block diagram in Fig 7 A discharge is initiated in the glass-filled gas bottle for one microsecond by the pulsed magnetron then turned off for 4000 µsec A small c-w signal from the McNally tube is set to a higher frequency than that of the cavity in the absence of electrons and used as a probe The time of maximum transmission through the cavity of this signal is observed by means of the calibrated scope time sweep The frequency of the probe signal is varied and the various times of peak transmission are recorded Care is taken to use a probe signal which imparts much less than thermal energy to the electrons so that their energy distribution is undisturbed. The frequency shift versus time curve which is obtained by this method determines the clean-up of the electrons

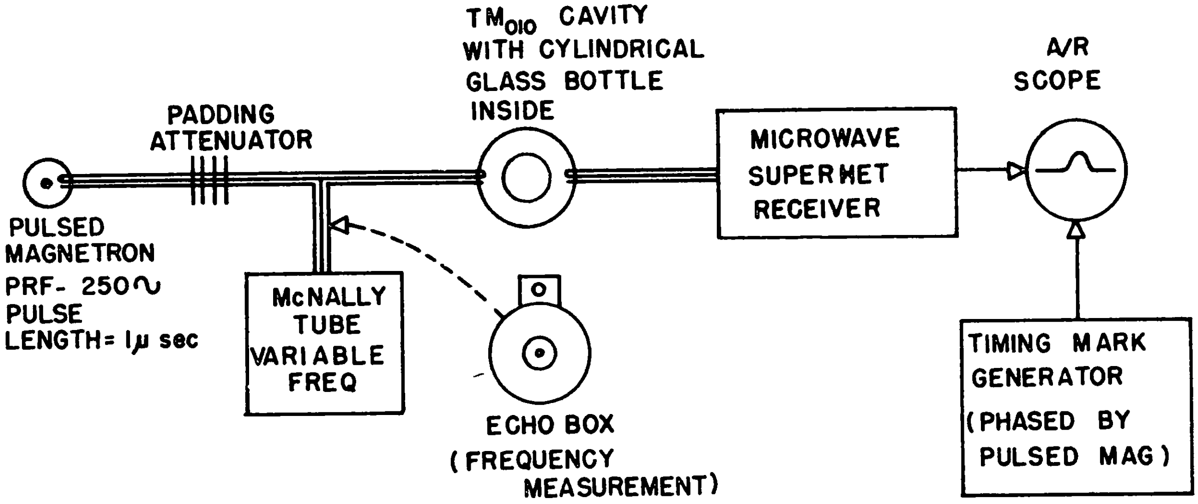

Figure 7 Simplified block diagram of transient discharge measurement apparatus

<u>Physical Considerations of Attachment and Diffusion</u> A few gases principally oxygen SO_2, H_2O vapor, and NO_2 exhibit the property of attachment If electrons are produced in these gases there is a certain probability that on collision with a gas atom or molecule the electron will be captured to form a negative ion This probability, expressed as the inverse of the average number

of collisions with gas atoms per attaching collision is known as the attachment probability h and is usually quite small (10^{-3} and less)

The removal of electrons due to attachment is an exponential process. For a Maxwellian distribution of electron energy (a good approximation at thermal energy) the time constant T C , of the process is related to the attachment probability by

$$T\,C = 1/2\sqrt{\frac{\pi}{2}}\sqrt{\left(\frac{m}{kT}\right)}\quad\frac{\ell}{h} \tag{7}$$

The process of diffusion of electrons in the presence of positive ions takes place in a manner known as ambipolar in which on the average, an electron cannot diffuse out without taking along a positive ion, therefore ambipolar diffusion is much slower than electron diffusion The theoretical evaluation of D_a, the ambipolar diffusion constant governing this process, cannot be obtained exactly The approximate result is

$$D_a \sim \frac{4\sqrt{2}}{3\sqrt{\pi}}\sqrt{\frac{kT}{M}}\,\ell_i \tag{8}$$

where M is the positive ion mass and ℓ_i the ionic mean free path

By solving the diffusion equation for the cylindrical container, the time constant T C , governing the process (also an exponential) is given by

$$R\sqrt{\frac{1}{D_a(T\,C)} - \left(\frac{\pi}{L}\right)^2} = 2.405 \tag{9}$$

where L and R are the length and radius of the cylindrical container

Experimental Results (a) Attachment If we evaluate Eq (7) for oxygen at 300°K

$$h = \frac{6.9 \times 10^{-3}}{T\,C\ (\mu\text{sec})\ \text{pressure (mm)}}$$

The behavior of oxygen has not yet been studied in detail but a curve of the attachment at 8.3 mm is given in Fig 8 The slope of the second half of the curve gives a value of $h = 1.3 \times 10^{-4}$ which is in fair agreement with the results of Margenau's measurement of thermal attachment of O_2 (RL Report 929) The bending of the curve has not as yet been sufficiently studied to permit us to draw any conclusions The bend may merely be due to the decay to thermal energy of the electrons, which greatly varies their attaching probability and hence their rate of clean-up

Experimental Results (b) Diffusion Evaluating D_a according to Eq (8) for helium and substituting it and the cylindrical glass bottle's diminisions in Eq (9) we find that the time constant of the diffusion process in helium is given theoretically as

$$T\,C = 315\ \mu\text{sec/mm(pressure)}$$

Experimental curves for helium are given in Fig 9 Here all curves have a slope of 450 μsec/mm of pressure This means that ambipolar diffusion is slower then that predicted by the theoretical derivation This deviation is to be expected since the calculations of D_a could be done theoretically only when $M_{ion} \ll M_{atom}$ whereas in this case $M_{ion} \approx M_{atom}$ The theoretical calculation therefore led to the prediction of too fast a diffusion so that the data deviate to the correct side of the theoretical result

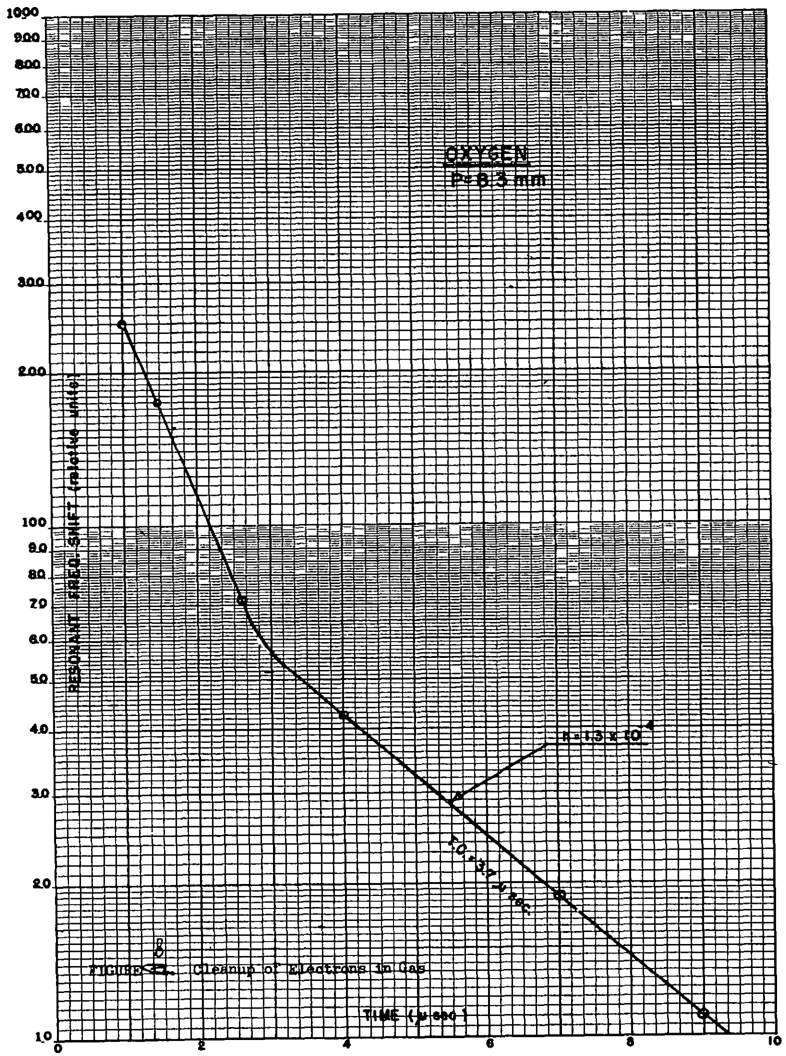

Figure 8 Cleanup of electrons in gas

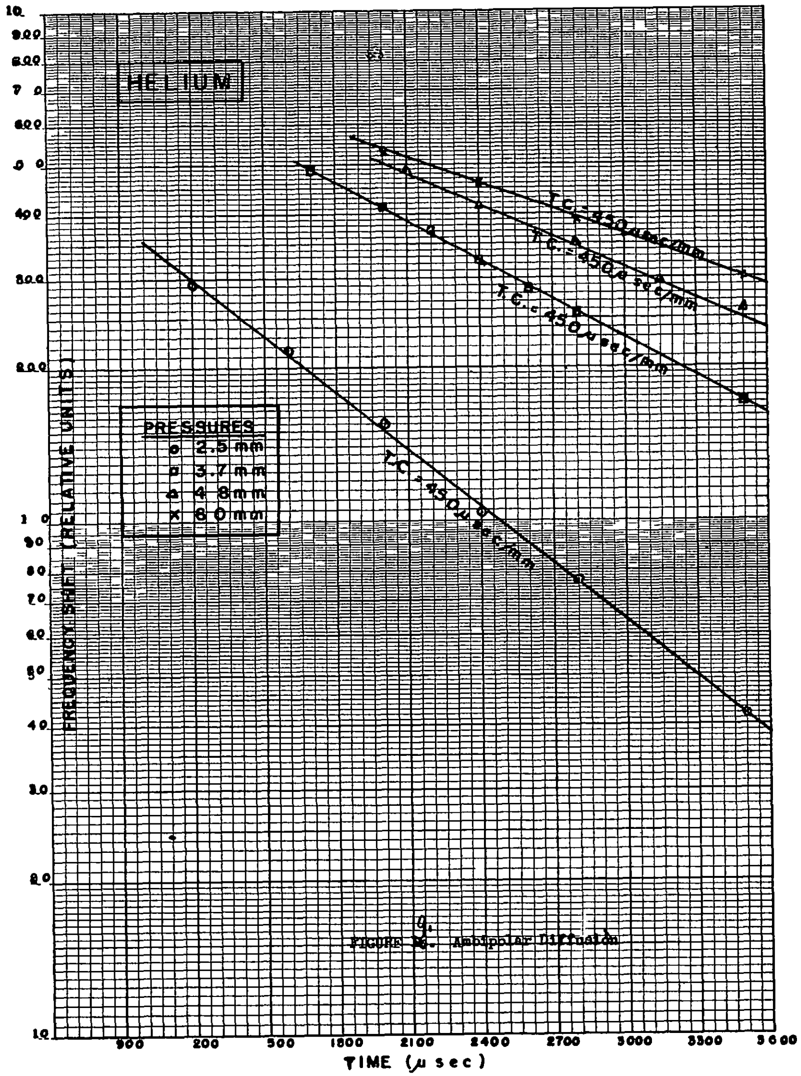

Figure 9 Ambipolar diffusion

5 Applications

Detectors of Ionizing Radiation (a) Studies of Geiger-Müller Counter Discharge The quenching of the discharge in a Geiger-Müller counter depends upon the modification of the inhomogeneous electric field in the neighborhood of the central wire by a positive ion space charge The counter cannot be resensitized until this space charge sheath has moved far from the anode. Thus the action of the counter is controlled by the positive ions Since these move relatively slowly it would be a great advantage to eliminate their action insofar as it is practical and deal with the swifter moving electrons Microwave technique offers some hope of achieving this result There are different conditions which are desired of counters for different purposes. Two of these are pulses of very short duration for high coincidence resolution, and high speed counting for detection of high intensity sources The first of these two conditions might be met by observing only the electrons in the d-c discharge of a counter and by leaving the positive ions to pass unnoticed. This has been undertaken in the following way

A Geiger-Müller counter was constructed in such a way that it was both an ordinary counter and a cavity resonant to 3-cm microwave power acting in a TE_{113} mode. It has been mentioned previously in this report that the electrons in a cavity may have two effects, one a frequency shift and the other, a lowering of the Q of the cavity The frequency shift is due to the free oscillation of the electrons in the r-f field and is therefore a low-pressure effect The lowering of the Q of a cavity by the electrons is due to their resistive loading and is a high-pressure effect The resonant frequency shift due to the presence of electrons in the cavity is the most sensitive method of detecting the presence of electrons in a cavity and it is the method so far used in this investigation

The experimental arrangement is the following The tube is connected as a Geiger-Müller counter to a standard counter amplifier This amplifier has sufficient gain so that the initial 0 2 μsec of the d-c pulse may be used as a trigger pulse for a single sweep on an A/R oscilloscope A c-w 3-cm probe signal is fed through the cavity counter at the resonant frequency of the cavity in the absence of electrons. When electrons are present in the counter, under the conditions to be discussed below, the shift in resonant frequency of the cavity is sufficient to cause a marked attenuation of the r-f signal The presence of the electrons is therefore detected by feeding a constant microwave signal into the cavity and connecting an output loop from the cavity through a crystal to the video amplifier of the A/R oscilloscope At a pressure of 5 mm Hg of helium the electron pulses appear on the oscilloscope screen to be of the order of 1 μsec duration

Reference to the discussion in Sec 2 and to Fig 4 shows that the imaginary component of the electron current becomes small, at 10 cm, above 1 cm Hg pressure Thus the frequency-shift method of detecting electrons becomes experimentally difficult above this pressure On the other hand, the ionization efficiency for detecting an entering ionizing particle is very low at this pressure for the standard size counter.

If one uses fundamental mode resonant cavities, the physical size is dictated by its resonant frequency A 3-cm TE_{113} cavity was originally chosen because of its similarity in size to standard Geiger-Muller counters The minimum in the E vs p curve of Fig 4 corresponds to the equality of the real and imaginary components of the current This is, of course, a function of the mean free time between collisions of electrons and the molecules of the gas If one goes from 10 cm, where these data were taken to 3 cm where the counter experiment is being performed the point where tan $\Theta = 1$ shifts to higher pressure is proportional to the change in mean free time, and hence frequency This does not, however, gain anything in over-all ionizing efficiency since the physical size of the cavity must also be decreased in the same ratio, so that an ionizing particle crossing the cavity counter would still have the same gas path The way one can increase the over-all ionizing efficiency lies in the direction of working in a high-mode cavity; i e one that is sufficiently large that many resonances occur in a small wavelength interval in the region of the operating wavelength In the high mode region cavity dimensions become very important and the success of the method depends upon holding manufacturing tolerances to narrow limits.

Detectors of Ionizing Radiation (b) Ultra-high Frequency Discharge Counters The Geiger-Muller counter discharge is controlled by the positive ion space charge, hence the speed with which it may count successive pulses depends on the mobility of the positive ions. In an attempt to devise a counter which depends only on electron mobility, work has been started on studying the possibility of using a high-frequency discharge for an ionization detector. With the knowledge gained in the studies discussed in Sec 2, it is not difficult to set a gas-filled resonant cavity to a condition where a discharge can be initiated by a small source (10 μc Co^{60}) The difficulty with this method lies in our incomplete knowledge of the behavior of the electrons after the discharge has been turned off. The studies of the electromagnetic properties of discharges allow us to determine the electron density in the discharges The study of the transient characteristics should allow us to predict that in attaching gases such as O_2 all electrons should be gone in a given length of time The result seems to be, however, that free electrons do not disappear as quickly as one would suppose At the present time, it is found that electrons remain in the cavity up to 100 milliseconds after the discharge has been turned off Hence until this behavior can be controlled, it cannot be used practically as a counter.

Microwave Power Stabilizer A gaseous discharge maintained by high-frequency power has many of the properties found in the d-c discharge Thus one is not particularly surprised to find that voltage across the discharge remains constant for large changes in current through the discharge Such a condition is clearly illustrated in Fig 3 If one places a tube of gas at a suitable pressure within a waveguide, a discharge will take place such that variations in the input power will be absorbed by the gas discharge and one has a stabilizer which will take care of very large variations of input power with negligible changes in output power No particular attempts have been made to study the detailed properties of this device, although use is made of it in our laboratory experiments when great stability is desired for accurate physical measurements

Microwave Power Modulators (a) High Current Density No attempt has been made within the Gas Discharge Group to study possible applications of the results which have been achieved Some of the phenomena however, have suggested obvious applications, for example with the arrangement of Fig 6 one observes very large changes in transmitted power through a coaxial line controlled by very small changes in the direct current carried by the mercury arc These properties suggest uses of this device as an r-f switch or modulator

Microwave Power Modulators (b) Low Current Density The use of a particular property of a low current density gaseous discharge as an r-f modulator was suggested by an attempt to devise a null method for measuring the properties of a gas discharge In standard rectangular waveguide the power is transmitted down the guide in the $TE_{0,1}$ mode, that is with the electric field across the short side of the guide. If a rectangular hole is cut in the top of the guide, and another piece of standard waveguide is soldered on in the form of a T, in such a way that both the short and long sides of the guides are mutually perpendicular, see Fig 10 theoretically no power will be transmitted from one guide to the other If a low pressure gas tube is placed at the junction of the two guides, a discharge may be maintained by a magnetron and the electrons in the discharge will be oscillating under the action of the applied high-frequency field If the discharge is placed in a suitably oriented magnetic field, the electrons will no longer oscillate in a manner dictated solely by the applied r-f field but will contribute a component of power in a direction along the perpendicular guide The amount of power transmitted down the perpendicular guide will depend both upon the current density of the discharge and upon the intensity and orientations of the magnetic field If a completely r-f discharge is used modulation of power in the side arm guide may be achieved by modulating the magnetic field A more practical method would seem to be to maintain a corona discharge with d-c electrodes, keep the magnetic field constant and modulate the d-c corona discharge In this way one could modulate amounts of high-frequency signals even in the case when the power is too small to maintain a discharge High r-f powers could be controlled by designing the discharge tube to be below the firing point for the r-f power, the d-c supply may then be used to control the discharge and therefore the r-f power to the perpendicular arm

Microwave Power Modulators (c) High Q A certain amount of interest has centered on the possibility of controlling high-Q devices with gas-discharge phenomena Such problems as the frequency modulation of a cyclotron is a case in point The use of these discharges in highly reactive circuits is subject to serious difficulties as seen from the results of the experiments illustrated in Fig 4 The tan Θ curve gives the relative contributions of the real and imaginary components of the current the data being taken at the 10-cm band Thus we see that for helium even at a pressure of 0 01 mm Hg the imaginary current is only 100 times the real current To control a device

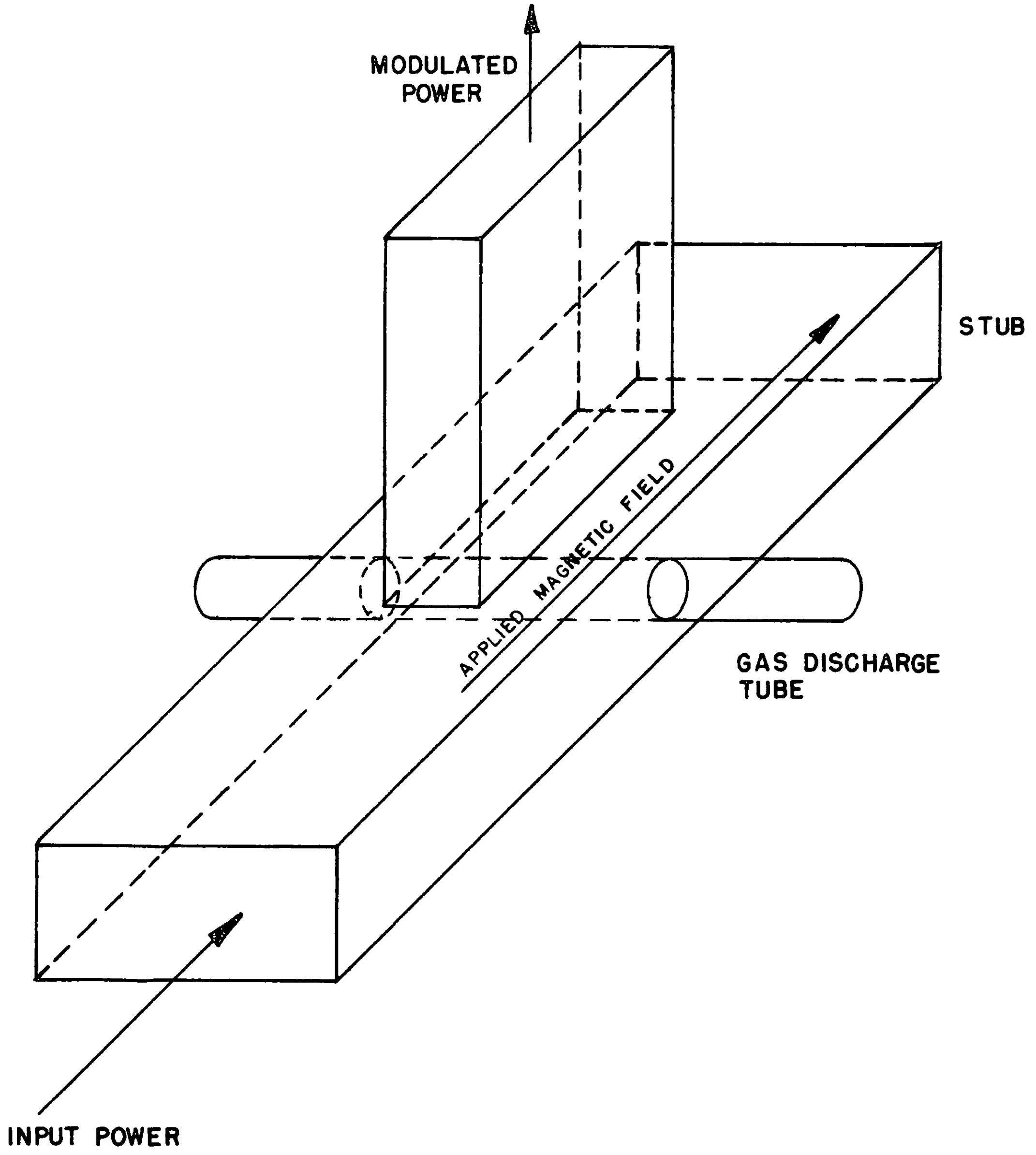

Figure 10 Suggested form for the waveguide construction of a low-current density r-f power modulator.

with a Q of 10,000 the ratio of inductive to resistive current would have to be at least this value, this would require an extrapolation of the curve to a pressure of 10^{-4} mm Hg at the 10-cm band

The characteristics of the curves in Fig 4 are controlled by the mean free time of the electrons in the discharge Thus if the wavelength is increased by a factor of ten, the mean free time will occur at a pressure of one tenth that shown in the figure If a device is to be operated at a wavelength of ten meters, Fig 4 predicts its behavior provided the pressure scale is divided by a factor of 100 This would mean that to get a Q of the discharge at ten meters to a value of 10,000, the discharge would be required to operate at a pressure of 10^{-6} mm Hg If one is to control this a-c behavior with a d-c discharge it is obvious that since the mean free path of an electron at 10^{-6} mm Hg pressure is several meters, one does not arrive at a practical device On the other hand, instead of using the reactive effect of a low-pressure discharge one may design a discharge, (maintained by either radio frequency or direct current--whichever is more convenient), which has a sufficiently high electron density to approach a good conductor and therefore act as an electronically controlled cavity wall This use would be subject to the disadvantage that the discharge would have to be all on or all off, that is, it would behave like a switch since varying the electron density would vary only the losses while the frequency of the cavity changed very slowly But the high Q could be obtained by sufficiently high current density A T-R switch is a simple example of this principle The coaxial control system suggested by the experiment described in Sec 3 is another example applicable to higher powers

This discussion serves as an indication of the practical information one may obtain from the experimental curves of the characteristics of the steady state high-frequency discharge

II D LOW TEMPERATURE RESEARCH

1 Helium Cryostat[1]

Staff Professor F Bitter
Professor S C Collins
Professor C F Squire
R P Cavileer

Equipment for producing liquid helium has been augmented by the purchase of a new liquifier, compressor, and gasometer from A D Little, Inc The new unit is very similar to the one built at M I T and in operation for the past eight months The main points of difference are (1) provision for effective pre-cooling with liquid nitrogen, (2) provision for liquefaction of any gas in an auxilary circuit, and transfer of liquified gas to an internal Dewar flask, (3) a new type of gasometer having a greater volume, and (4) a four-stage compressor which it is hoped will obviate the necessity for continuous external clean-up of oil vapors in the circulating helium

Some difficulty has been experienced in the trial runs of this equipment, presumably due to too tight a fit of the piston in the cylinder of one of the expansion engines

2 Properties of Germanium

Staff Professor C F Squire
Dr W E Henry

In the Final Report of June 1946 some measurements were reported on the electrical resistance of Germanium as a function of temperature from 300°K down to 2°K Since repeated measurements spaced over six months show consistent values, it appears that a Germanium resistance thermometer can be used as a reliable secondary one Such a thermometer has, in fact been used in the work on superconductivity reported in Section D3 We give now some additional results on the diamagnetism and also on the change of electrical resistance with magnetic field In Fig 1 the measured diamagnetic susceptibility is shown plotted against temperature For comparison the resistance curve formerly reported is plotted in without bothering to give the size of the ordinate in ohms One sees that as the temperature is lowered the resistance rises strongly and the diamagnetism decreases At temperatures of 80°K and above, we observe no dependence of diamagnetic susceptibility on the magnetic field At temperatures of 20°K and 14°K we observe field dependence very much like that reported by de Haas and van Alphen[2] for bismuth at these same low temperatures Measuring fields up to 31 5 kilogauss were used

We measured the change of electrical resistance with externally applied magnetic field at several temperatures and found the well-known relationship $\Delta R/R=AB^2$ to be in accord with the measurements Here ΔR is the change of resistance, R the resistance in zero field, A is a constant, and B is the magnetic field in gauss Figure 2 shows the plot of $\Delta R/R$ against B^2 for two temperatures

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[1] See Section IIIB for ultrasonic measurements in liquid helium

[2] Comm Leiden, 212a (1930), Proc Amsterdam Ac 33, 1106 (1930)

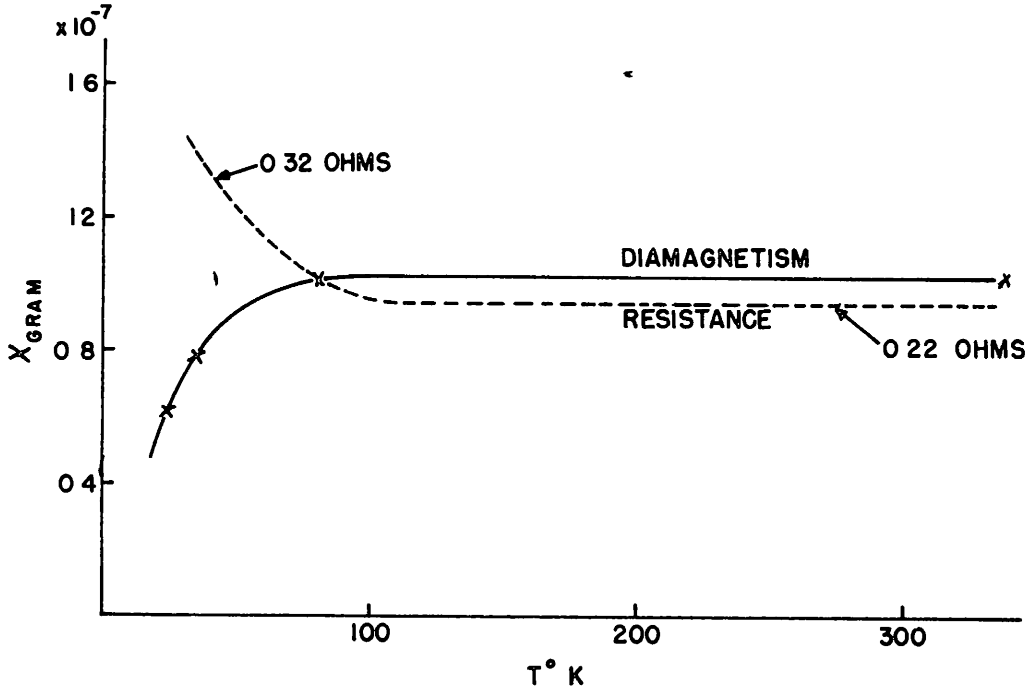

Figure 1 Measured diamagnetic susceptibility per gram of Germanium plotted against absolute temperature The broken curve indicates the variation of the resistance witn temperature

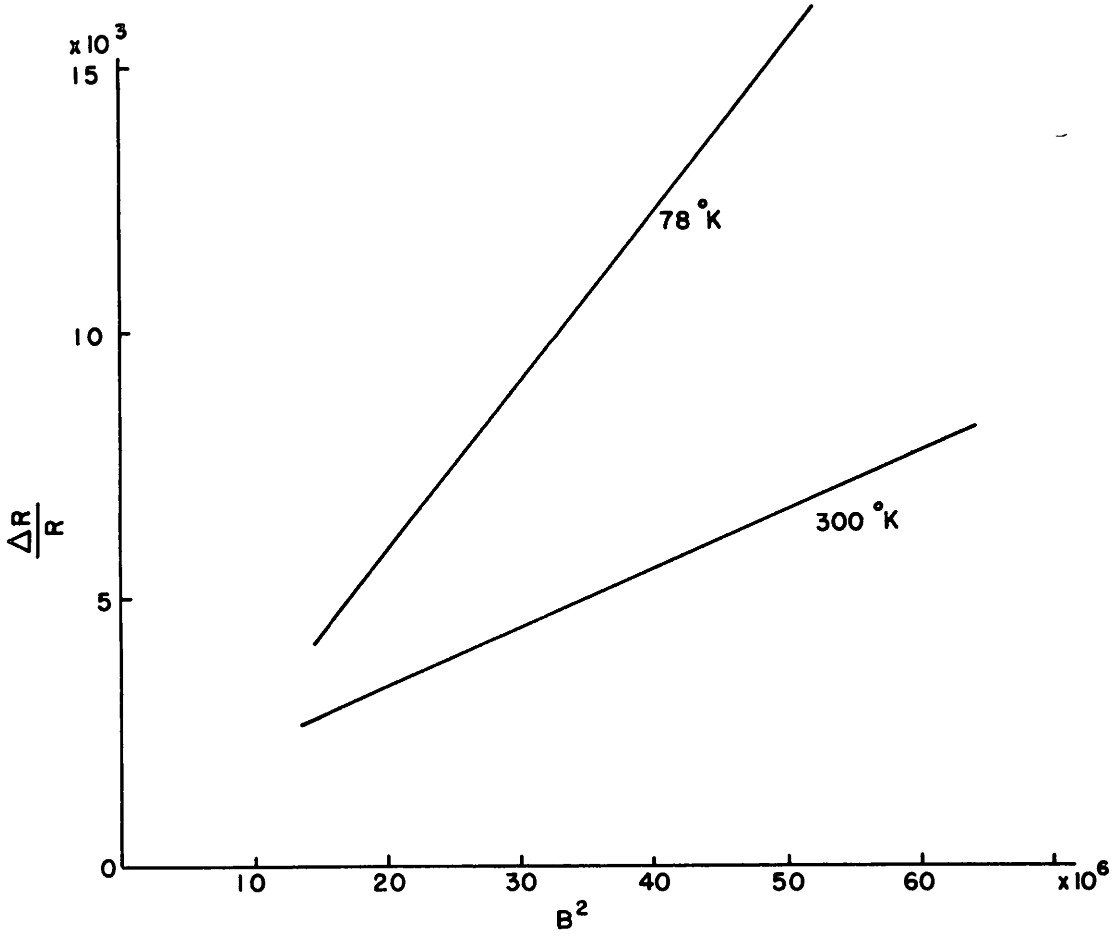

Figure 2 Change of resistance/resistance of Germanium plotted against the square of the magnetic field for two temperatures

We looked for a change in the expansion coefficient above and below the temperature where the resistance and diamagnetism seemed to show a marked change, but found no large first-order effect

The results described here agree with, and supplement, those made at Purdue[1] and at Bell Laboratories[2] No further work is contemplated on this subject at R L F

3 Superconductivity at 1.25 Cm

Staff Professor J C Slater
Dr J Halpern
J B Garrison
E Maxwell

Description of Project Since the last Quarterly Progress Report work has been directed towards obtaining semi-quantitative measurements on the superconductivity of lead at 1 25 cm These measurements are made by using an unbalanced microwave bridge with an f-m signal source This permits the presentation of reflected power vs frequency on an oscilloscope, from which a measurement of the power reflection coefficient at resonance is obtained From the data the fractional change in resistivity of the cavity can be computed without further knowledge of the cavity parameters, or if the window Q is known the unloaded Q can be calculated

Status Preliminary results indicate that the resistance of the lead at 1 25 cm in going from a temperature of 8°K to 2°K decreases by about a factor of 20 The corresponding change observed at 3 cm was about 1000 In comparing these results it should be noted that the purity of the lead used was different in each case, the lead used at 3 cm being the purer It is planned to repeat these experiments with Hilger lead.

4 Nuclear Resonance at Low Temperatures and Strong Magnetic Fields

Staff Professor F Bitter
N L Alpert
S T Lin
H L Poss

The development of equipment for measuring nuclear resonances is proceeding The motor-generator of the MIT Magnet Laboratory is being rewound and should be installed within a few months This equipment will make possible the application of specially constructed magnets to the nuclear resonance problems

5 Properties of Matter at Temperatures below 1° Absolute

a Theoretical Studies

Staff Professor L Tisza
J M Luttinger

The attempts aimed at the quantum mechanical generalization of the theory of dipole interaction (see Final Report of June, 1946 and last Quarterly Progress Report) are being continued

[1] Phys Review 69, 258 (1946)

[2] Bulletin of Am Physical Soc 21, 9 (Nov 1946)

b Experimental Studies

Staff Professor C F Squire

Equipment has just been completed which will permit substances to be cooled by pumped liquid helium to 1°K and it is expected to use the Bitter[1] magnets for cooling to 0 01°K This experimental program ties in closely with the theoretical work mentioned in Sec 5a We are interested in the dipole-dipole interaction of paramagnetic salts at very low temperatures

6 Theory of "Second Sound" in Helium II

Staff Professor L Tisza
J M Luttinger

It is well known that liquid helium II (the modification stable below the λ-point $T_0 = 2\ 19^\circ K$) has many unusual properties Among these are an abnormally high heat-conductivity, superfluidity, and the thermomechanical effect observed in the "helium fountain" It should be emphasized that this behavior cannot be described by the ordinary differential equations governing heat flow and capillary flow simply by assuming unusual values for the constants of these equations Instead, the underlying mechanisms have to be changed, and these lead to new differential equations The theory of the macroscopic behavior of helium II was first developed by L Tisza This theory was based on F London's hypothesis according to which helium II has in many respects the properties of a "condensed Bose-Einstein gas" It is a mixture of a superfluid and a normal fluid with additive densities $\rho = \rho_s + \rho_n$ A gaseous type of viscosity and an "osmotic pressure" P_n are associated with the normal fluid The theory led to several predictions regarding helium II, which were later verified by experiment The most important of these was that inhomogeneities of the temperature will propagate in the form of waves with the velocity,

$$u_2 = \left[\frac{\rho_s}{\rho} \frac{dP_n}{d\rho_n} \right]^{\frac{1}{2}} \tag{1}$$

Later, L Landau advanced a theory based on quantum hydrodynamics which, while apparantly very different from the one based on the Bose-Einstein theory, led to very similar results; in particular, to the existence of temperature waves which Landau called "second sound" For their velocity he obtained,

$$u_2 = \left[- \frac{\rho_s}{\rho_n} \frac{dT}{d(1/s)} \right]^{\frac{1}{2}} \tag{2}$$

where s is the specific entropy

[1] Rev of Scientific Instruments **10**, 373 (1939)

The question has been reopened with a view to clarifying the relation of the theories. Most of the above mentioned results could also be derived from very general assumptions by means of a quasi-thermodynamic method. The most important properties of helium II are reduced to the knowledge of the so-called anomalous fraction of the entropy. This may be obtained from the measurements of Kapitsa and can be represented by $s = s_0(T/T_0)^n$ with $s_0 = 0.40$ cal/gm°K and $n = 5.6$. The two expressions for u_2 prove to be identical, provided in (2) s is interpreted as the anomalous fraction of the entropy rather than the total entropy as was assumed by Landau. Using the empirical entropy function one obtains,

$$u_2 = 26 \sqrt{\frac{T}{T_0}\left[1 - \left(\frac{T}{T_0}\right)^{5.6}\right]} \text{ meter/sec} \qquad (3)$$

This temperature dependence of the second sound velocity is in good agreement with the measurements by Peshkov (results presented July, 1946 at the Cambridge, England, meeting) and by C. T. Lane, H. Fairbank, H. Schultz and W. Fairbank[1], who used an idea of Onsager's for the detection of second sound.

On the basis of his theory, Landau had concluded that u_2 increased monotonously with decreasing temperature to reach a maximum of about 100 meters/sec at absolute zero. This can be shown to be in contradiction with Nernst's theorem. Landau's results conflict also with the above mentioned measurements. Nevertheless, it seems probable that the hydrodynamical theory can be so modified as to conform with the requirements of the thermodynamic theory which thus admits two different molecular interpretations.

A detailed paper on this subject is being prepared for publication.

II. E. FERROMAGNETISM AT MICROWAVE FREQUENCIES

Staff: Dr. C. Kittel

With a view to interpreting the results of magnetic measurements on thin films at microwave frequencies, the theory was developed of the domain structure of ferromagnetic bodies whose smallest dimension is comparable with the thickness of the Weiss domains as found in crystals of ordinary size. Calculations of the domain boundary, magnetic and anisotropy energies of various domain configurations were made for thin films, small particles, and long needles of ferromagnetic material. For sufficiently small dimensions, the optimum structure consists of a single domain magnetized to saturation in one direction. This result implies unusual magnetic characteristics, such as have in fact been reported by a number of experimenters[2]. The critical dimensions for transition from a configuration with domain structure to a saturated configuration are estimated to be $\approx 3 \times 10^{-5}$ cm in films and $\approx 2 \times 10^{-6}$ cm in particles or grains. These estimates are based on typical values of the relevant material constants, and may be increased or decreased by a factor of ten for other values of the constants. A detailed report on this subject has been published.

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[1] Phys. Rev. 70, 431 (1946)

[2] A good review of the experimental literature is given in J. T. Allanson, J. Inst. Elec. Eng. 92, Part III, (1945); see also, W. Arkadiew, J. Phys. U.S.S.R. 9, 373 (1945)

[3] C. Kittel (Phys. Rev. 17 Dec 1946)