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A Mini–Landscape of Exact MSSM Spectra in Heterotic Orbifolds

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Abstract

We explore a "fertile patch" of the heterotic landscape based on a $\mathbb{Z}_6$-II orbifold with SO(10) and E_6 local GUT structures. We search for models allowing for the exact MSSM spectrum. Our result is that of order 100 out of a total 3×10^4 inequivalent models satisfy this requirement.

1 Introduction

Although there are only a few consistent 10D string theories, there is a huge number of 4D string compactifications [1, 2]. This leads to the picture that string theory has a vast landscape of vacua [3]. The (supersymmetric) standard model (SM) corresponds to one or more possible vacua which *a priori* might not be better than others. To obtain predictions from string theory one can employ the following strategy: first seek vacua that are consistent with observations and then study their properties. Optimistically, one might hope to identify certain features common to all realistic vacua, which would lead to predictions. Even if this is not the case, one might still be able to assign probabilities to certain features, allowing one to exclude certain patches of the landscape on a statistical basis. However, realistic vacua are very rare. For instance, in the context of orientifolds of Gepner models, the fraction of models with the chiral matter content of the standard model is about 10^{-14} [4, 5]. The probability of getting something close to the MSSM in the context of intersecting D–branes in an orientifold background is 10^{-9} [6, 7], even if one allows for chiral exotics. In this study, we show that certain patches of the heterotic landscape are more "fertile" in the sense that the analogous probabilities are at the percent level.

We base our model scan on the heterotic $\mathrm{E}_8 \times \mathrm{E}_8$ string [8, 9] compactified on an orbifold [10–16]. Our study is motivated by recent work on an orbifold GUT interpretation of heterotic string models [17–19]. We focus on the $\mathbb{Z}_6$-II orbifold, which is described in detail in [17, 19, 20]. The search strategy is based on the concept of "local GUTs" [20–23] which inherits certain features of standard grand unification [24–26]. Local GUTs are specific to certain points in the compact space, while the 4D gauge symmetry is that of the SM. If matter fields are localized at such points, they form a complete GUT representation. This applies, in particular, to a **16**–plet of a local SO(10), which comprises one generation of the SM matter plus a right–handed neutrino [26, 27],

$$\mathbf{16} \;=\; (\mathbf{3},\mathbf{2})_{1/6} + (\overline{\mathbf{3}},\mathbf{1})_{-2/3} + (\overline{\mathbf{3}},\mathbf{1})_{1/3} + (\mathbf{1},\mathbf{2})_{-1/2} + (\mathbf{1},\mathbf{1})_{1} + (\mathbf{1},\mathbf{1})_{0}\;, \tag{1}$$

where representations with respect to $\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(2)_\mathrm{L}$ are shown in parentheses and the subscript denotes hypercharge. On the other hand, bulk fields are partially projected out and form incomplete GUT multiplets. This offers an intuitive explanation for the observed multiplet structure of the SM [20–23]. This framework is consistent with MSSM gauge coupling unification as long as the SM gauge group is embedded in a simple local GUT $G_\mathrm{local} \supseteq \mathrm{SU}(5)$, which leads to the standard hypercharge normalization.

We find that the above search strategy, as opposed to a random scan, is successful and a considerable fraction of the models with SO(10) and E_6 local GUT structures pass our criteria. Out of about 3×10^4 inequivalent models which involve 2 Wilson lines, $\mathcal{O}(100)$ are phenomenologically attractive and can serve as an ultraviolet completion of the MSSM.

2 MSSM search strategy: local GUTs

It is well known that with a suitable choice of Wilson lines it is not difficult to obtain the SM gauge group up to U(1) factors. The real challenge is to get the correct matter spectrum and the GUT hypercharge normalization. To this end, we base our strategy on the concept of local GUTs. An orbifold model is defined by the orbifold twist, the torus lattice and the gauge embedding of the orbifold action, i.e. the gauge shift V and the Wilson lines W_n. We consider only the gauge shifts V which allow for a local SO(10) or E_6 structure. That is, V are such that the left–moving momenta p (we use the standard notation, for details see e.g. [18–20]) satisfying

$$p \cdot V \;=\; 0 \mod 1\;, \quad p^2 \;=\; 2 \tag{2}$$

are roots of SO(10) or E_6 (up to extra group factors). Furthermore, the massless states of the first twisted sector T_1 are required to contain **16**–plets of SO(10) at the fixed points with SO(10) symmetry or **27**–plets of E_6 at the fixed points with E_6 symmetry.

Since these massless states from T_1 are automatically invariant under the orbifold action, they all survive in 4D and appear as complete GUT multiplets. In the case of SO(10), that gives one complete SM generation, while in the case of E_6 we have $\mathbf{27} = \mathbf{16} + \mathbf{10} + \mathbf{1}$ under SO(10). It is thus necessary to decouple all (or part) of the **10**–plets from the low energy theory.

The Wilson lines are chosen such that the standard model gauge group is embedded into the local GUT as

$$G_{\mathrm{SM}} \;\subset\; \mathrm{SU}(5) \subset \mathrm{SO}(10) \text{ or } \mathrm{E}_6\;, \tag{3}$$

such that the hypercharge is that of standard GUTs and thus consistent with gauge coupling unification. The spectrum has certain features of traditional 4D GUTs, e.g. localized matter fields form complete GUT representations, yet there are important differences. In particular, interactions generally break GUT relations since different local GUTs are supported at different fixed points. Also, gauge coupling unification is due to the fact that the 10D (not 4D) theory is described by a single coupling.

Our model search is carried out in the $\mathbb{Z}_6$-II orbifold compactification of the heterotic $\mathrm{E}_8 \times \mathrm{E}_8$ string, which is described in detail in [19, 20]. In this construction, there are 2 gauge shifts leading to a local SO(10) GUT [28],

$$\begin{aligned} V^{\mathrm{SO}(10),1} &= \left(\tfrac{1}{3},\, \tfrac{1}{2},\, \tfrac{1}{2},\, 0,\, 0,\, 0,\, 0,\, 0\right) \quad \left(\tfrac{1}{3},\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0\right)\;, \\ V^{\mathrm{SO}(10),2} &= \left(\tfrac{1}{3},\, \tfrac{1}{3},\, \tfrac{1}{3},\, 0,\, 0,\, 0,\, 0,\, 0\right) \quad \left(\tfrac{1}{6},\, \tfrac{1}{6},\, 0,\, 0,\, 0,\, 0,\, 0,\, 0\right)\;, \end{aligned} \tag{4}$$

and 2 shifts leading to a local E_6 GUT,

$$V^{\mathrm{E}_6,1} = \left(\tfrac{1}{2},\, \tfrac{1}{3},\, \tfrac{1}{6},\, 0,\, 0,\, 0,\, 0,\, 0\right) \quad (0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0,\, 0)\;,$$

$$V^{\mathrm{E}_6,2} = \quad \left(\tfrac{2}{3},\, \tfrac{1}{3},\, \tfrac{1}{3},\, 0,\, 0,\, 0,\, 0,\, 0\right) \quad \left(\tfrac{1}{6},\, \tfrac{1}{6},\, 0,\, 0,\, 0,\, 0,\, 0,\, 0\right). \tag{5}$$

We will focus on these shifts and scan over possible Wilson lines to get the SM gauge group. The $\mathbb{Z}_6$-II orbifold allows for up to two Wilson lines of order 2 and for one Wilson line of order 3 (cf. [29,19,20]).

The next question is how to get 3 matter generations. The simplest possibility is to use 3 equivalent fixed points with **16**–plets [21] which appear in models with 2 Wilson lines of order 2. If the extra states are vectorlike and can be given large masses, the low energy spectrum will contain 3 matter families. However, this strategy fails since all such models contain chiral exotic states [20]. In the case of E_6, it does not work either since one cannot obtain $G_{\mathrm{SM}} \subset \mathrm{SU}(5) \subset \mathrm{SO}(10) \subset \mathrm{E}_6$ with 2 Wilson lines of order 2.

The next–to–simplest possibility is to use 2 equivalent fixed points which give rise to 2 matter generations. The third generation would then have to come from other twisted or untwisted sectors. The appearance of the third family can be linked to the SM anomaly cancellation. Indeed, the untwisted sector contains part of a **16**–plet. Then the simplest options consistent with the SM anomaly cancellation are that the remaining matter either completes the **16**–plet or provides vector–like partners of the untwisted sector. In more complicated cases, additional **16**– or $\overline{\mathbf{16}}$–plets can appear. The localized **16**– and **27**–plets are true GUT multiplets, whereas the third or "bulk" generation only has the SM quantum numbers of an additional **16**–plet. We find that the above strategy is successful and one often gets net 3 families. The other massless states are often vector–like with respect to the SM gauge group and can be given large masses consistent with string selection rules.

In our MSSM search, we focus on models of this type (although we include all models with 2 Wilson lines in the statistics). These are realized when 1 Wilson line of order 3 and 1 Wilson line of order 2 are present. We require that the spectrum contain 3 matter families plus vector–like states. Furthermore, we discard models in which the SU(5) hypercharge is anomalous. Although a non–anomalous hypercharge could be defined, typically it would not have the GUT normalization and thus would not be consistent with gauge unification.

3 Results

Let us now present our results for models with the SO(10) local structure. For each of the SO(10) shifts of Eq. (4), we follow the steps:

① Generate Wilson lines W_3 and W_2.

② Identify "inequivalent" models.

③ Select models with $G_{\mathrm{SM}} \subset \mathrm{SU}(5) \subset \mathrm{SO}(10)$.

④ Select models with three net $(\mathbf{3}, \mathbf{2})$.

⑤ Select models with non–anomalous $\mathrm{U}(1)_Y \subset \mathrm{SU}(5)$.

⑥ Select models with net 3 SM families + Higgses + vector–like.

Our results are presented in table 1. The models with the chiral MSSM matter content are listed in [30].

criterion	$V^{\mathrm{SO}(10),1}$	$V^{\mathrm{SO}(10),2}$	$V^{\mathrm{E}_6,1}$	$V^{\mathrm{E}_6,2}$
② inequivalent models with 2 Wilson lines	$22,000$	$7,800$	680	$1,700$
③ SM gauge group $\subset$ SU(5) $\subset$ SO(10) (or E_6)	3563	1163	27	63
④ 3 net $(\mathbf{3}, \mathbf{2})$	1170	492	3	32
⑤ non–anomalous $\mathrm{U}(1)_Y \subset \mathrm{SU}(5)$	528	234	3	22
⑥ spectrum = 3 generations + vector-like	128	90	3	2

Table 1: Statistics of $\mathbb{Z}_6$-II orbifolds based on the shifts $V^{\mathrm{SO}(10),1}, V^{\mathrm{SO}(10),2}, V^{\mathrm{E}_6,1}, V^{\mathrm{E}_6,2}$ with two Wilson lines.

Before continuing further, we make a few comments. In order to obtain the models listed under points ③- ⑥, we generate all possible Wilson lines along the lines of Refs. [31] and [32]. However, due to the rapid growth in computing time, generating all inequivalent models is not possible using these tools. Thus the inequivalent models under point ② have been generated by exploiting symmetries of the gauge lattice along the lines discussed in [33]. Two models are considered "equivalent" if they have identical spectra with respect to non–Abelian gauge groups and have the same number of non–Abelian singlets. Thus, models differing only in U(1) charges are treated as equivalent. Further ambiguities arise in certain cases when $\mathrm{U}(1)_Y$ can be defined in different ways. In addition, some models differ only by the localization of states on the different fixed points. We know that these ambiguities occur and it is possible that in some cases Yukawa couplings are affected. Hence our criterion may underestimate the number of truly inequivalent models.

In the E_6 case, we consider the SM embedding

$$G_{\mathrm{SM}} \subset \mathrm{SU}(5) \subset \mathrm{SO}(10) \subset \mathrm{E}_6 \,. \tag{6}$$

Again, models with 2 Wilson lines of order 2 fail and the analysis proceeds similarly to the SO(10) case. These results are also presented in table 1.[1]

It is instructive to compare our model scan to others. In certain types of intersecting D–brane models, it was found that the probability of obtaining the SM gauge group and

[1] In the analysis of [34] looking at non-supersymmetric heterotic string vacua, about 10% of the models scanned contained the SM gauge group. Our result (step 3) is comparable.

three generations of quarks and leptons, while allowing for chiral exotics, is 10^{-9} [6, 7]. The criterion which comes closest to the requirements imposed in [6, 7] is ④. We find that within our sample the corresponding probability is 5 %.

In [4,5], orientifolds of Gepner models were scanned for chiral MSSM matter spectra, and it was found that the fraction of such models is 10^{-14}. In our set of models, the corresponding probability, i.e. the fraction of models passing criterion ⑥, is of order 1 %. Note also that, in all of our models, hypercharge is normalized as in standard GUTs and thus consistent with gauge coupling unification.

This comparison shows that our sample of heterotic orbifolds is unusually "fertile" compared to other constructions. The probability of finding something close to the MSSM is much higher than that in other patches of the landscape analyzed so far. It would be interesting to extend these results to other regions of the landscape where promising models exist [35–38] (see also [39]).

4 Towards realistic string models

The next step on the path towards realistic models is the decoupling of the vector–like extra matter $\{x_i\}$. The mass terms for such states are provided by the superpotential

$$W \;=\; x_i\,\bar{x}_j\,\langle s_a\,s_b\ldots\rangle\;, \tag{7}$$

where $s_a, s_b, \ldots$ are SM singlets. Some singlets are required to get large (close to M_{str}) VEVs in order to cancel the Fayet–Iliopoulos (FI) term of an anomalous U(1). The supersymmetric field configurations are quite complicated and generally there are vacua in which all or most of the SM singlets get large VEVs. This breaks many of the gauge group factors, such that the low energy gauge group can be G_{SM} up to a hidden sector,

$$G_{\mathrm{SM}}\times G_{\mathrm{hidden}}\;, \tag{8}$$

where the SM matter is neutral under G_{hidden}. Furthermore, if the relevant Yukawa couplings are allowed by string selection rules, this makes the vector–like matter heavy; thus it decouples from the low energy theory. We note that there are in general several pairs of Higgs doublets with a matrix of μ–like mass terms, for which we require only one small eigenvalue. [2]

Clearly, one cannot switch on the singlet VEVs at will. Instead one has to ensure that they are consistent with supersymmetry. Supersymmetry requires vanishing of the

[2] To get a pair of massless Higgs doublets usually requires fine-tuning in the VEVs of the SM singlets such that the mass matrix for the $(\mathbf{1},\mathbf{2})_{-1/2},(\mathbf{1},\mathbf{2})_{1/2}$ states gets a zero eigenvalue. This is the notorious supersymmetric μ–problem. The fine-tuning can be ameliorated if the vacuum respects certain (approximate) symmetries [40, 41].

F– and D–terms. The number of the F–term equations equals the number of complex fields s_a, therefore there are in general non–trivial singlet configurations with vanishing F–potential. The D–terms can be made zero by complexified gauge transformations [42] if each field enters a gauge invariant monomial [43]. Thus, to ensure that the decoupling of exotics is consistent with supersymmetry, one has to show that all SM singlets appearing in the mass matrices for the exotics enter gauge invariant monomials involving only SM singlets and carrying anomalous charge. In this letter, we assume that the relevant singlets develop large supersymmetric VEVs.

In the process of decoupling, the vector–like states can mix with the localized **16**– and **27**–plets (if it is allowed by the SM quantum numbers) such that the physical states at low energies are *neither* localized *nor* “true” GUT multiplets. Nevertheless, it is clear that whatever the mixing, in the end exactly 3 SM families will be left, if the mass matrices have maximal rank.

To show that the decoupling of exotics is consistent with string selection rules is a technically involved and time consuming issue. In order to simplify the task and to reduce the number of models, we first impose an additional condition. We require that the models possess a renormalizable top–Yukawa coupling as motivated by phenomenology. Then we consider only superpotential couplings up to order 8. Thus, the next two steps in our selection procedure are:

⑦ Select models with a heavy top.

⑧ Select models in which the exotics decouple at order 8.

First, we require a renormalizable $\mathcal{O}(1)$ Yukawa coupling $(\mathbf{3},\mathbf{2})_{1/6}\,(\overline{\mathbf{3}},\mathbf{1})_{-2/3}\,(\mathbf{1},\mathbf{2})_{1/2}$, i.e. one of the following types

$$U\,U\,U\,,\quad U\,T\,T\,,\quad T\,T\,T\,,\tag{9}$$

where U and T denote generic untwisted and twisted fields, respectively. The $U\,U\,U$ coupling is given by the gauge coupling, $U\,T\,T$ is a local coupling and thus is unsuppressed, while the $T\,T\,T$ coupling is significant only when the twisted fields are localized at the same fixed point. We discard models in which the above couplings are absent or suppressed.

In the next step ⑧, we select models in which the mass matrices for the exotics (cf. Eq. (7)) have a maximal rank such that no exotic states appear at low energies. Here, we consider only superpotential couplings up to order 8 and for this analysis we assume that all relevant singlets can obtain supersymmetric vevs.[3] We find that a significant fraction

[3]We also address to some extent the question of D–flatness. In many of the models, we find that all SM singlets enter gauge invariant monomials. A full analysis of this issue is deferred to a subsequent publication.

criterion	$V^{\mathrm{SO}(10),1}$	$V^{\mathrm{SO}(10),2}$	$V^{\mathrm{E}_6,1}$	$V^{\mathrm{E}_6,2}$
⑦ heavy top	72	37	3	2
⑧ exotics decouple at order 8	56	32	3	2

Table 2: A subset of the MSSM candidates.

of our models passes requirements ⑦ and ⑧ (see table 2 and for further details [30]). In particular, we identify 93 models that can serve as an ultraviolet completion of the MSSM in string theory.

To verify whether an MSSM candidate is consistent with phenomenology requires addressing several questions. The most important issues include

- realistic flavour structures,
- absence of fast proton decay,
- hierarchically small supersymmetry breaking.

A model that passes all of our criteria ③–⑧ and comes very close to the supersymmetric standard model has been presented in [20, 22]. In our scan, we obtain many comparable models. In what follows, we substantiate this statement by studying a specific example, leaving a complete survey for future work.

Example

The model is based on the gauge shift

$$V^{\mathrm{SO}(10),1} \;=\; \left(\tfrac{1}{3},\,-\tfrac{1}{2},\,-\tfrac{1}{2},\,0,\,0,\,0,\,0,\,0\right)\left(\tfrac{1}{2},\,-\tfrac{1}{6},\,-\tfrac{1}{2},\,-\tfrac{1}{2},\,-\tfrac{1}{2},\,-\tfrac{1}{2},\,-\tfrac{1}{2},\,\tfrac{1}{2}\right)\;. \tag{10}$$

where we have added an $\mathrm{E}_8\times\mathrm{E}_8$ lattice vector to simplify computations. The Wilson lines are chosen as

$$\begin{aligned} W_2 &= \left(\tfrac{1}{4},\,-\tfrac{1}{4},\,-\tfrac{1}{4},-\tfrac{1}{4},\,-\tfrac{1}{4},\,\tfrac{1}{4},\tfrac{1}{4},\,\tfrac{1}{4}\right)\left(1,\,-1,\,-\tfrac{5}{2},\,-\tfrac{3}{2},\,-\tfrac{1}{2},\,-\tfrac{5}{2},\,-\tfrac{3}{2},\,\tfrac{3}{2}\right)\;, \\ W_3 &= \left(-\tfrac{1}{2},\,-\tfrac{1}{2},\,\tfrac{1}{6},\tfrac{1}{6},\,\tfrac{1}{6},\,\tfrac{1}{6},\tfrac{1}{6},\,\tfrac{1}{6}\right)\left(\tfrac{10}{3},\,0,\,-6,\,-\tfrac{7}{3},\,-\tfrac{4}{3},\,-5,\,-3,\,3\right)\;. \end{aligned} \tag{11}$$

The standard SU(5) hypercharge generator is given by

$$\mathsf{t}_Y \;=\; \left(0,\,0,\,0,\,-\tfrac{1}{2},\,-\tfrac{1}{2},\,\tfrac{1}{3},\,\tfrac{1}{3},\,\tfrac{1}{3}\right)(0,\,0,\,0,\,0,\,0,\,0,\,0,\,0)\;. \tag{12}$$

The gauge group after compactification is

$$G_{\mathrm{SM}}\times\mathrm{SO}(8)\times\mathrm{SU}(2)\times\mathrm{U}(1)^7\;, \tag{13}$$

#	irrep	label	#	anti-irrep	label
3	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})_{1/6}$	q_i			
8	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{-1/2}$	ℓ_i	5	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{1/2}$	$\bar{\ell}_i$
3	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{1}$	$\bar{e}_i$			
3	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{-2/3}$	$\bar{u}_i$			
7	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{1/3}$	$\bar{d}_i$	4	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{-1/3}$	d_i
4	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{1/6}$	v_i	4	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{-1/6}$	$\bar{v}_i$
20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{1/2}$	s_i^+	20	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{-1/2}$	s_i^-
2	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{1/2}$	$\tilde{s}_i^+$	2	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{-1/2}$	$\tilde{s}_i^-$

#	irrep	label
4	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{0}$	m_i
2	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{2})_{0}$	m_i'
47	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{0}$	s_i
26	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{0}$	h_i
9	$(\mathbf{1},\mathbf{1};\mathbf{8},\mathbf{1})_{0}$	w_i

Table 3: Massless spectrum. The quantum numbers are shown with respect to $\mathrm{SU}(3)_\mathrm{C}\times\mathrm{SU}(2)_\mathrm{L}\times\mathrm{SO}(8)\times\mathrm{SU}(2)$, the hypercharge is given by the subscript.

while the massless spectrum is given in table 3.

Renormalizable Yukawa couplings involving the SM fields are shown in table 4. The top Yukawa coupling comes from the $U\,U\,U$ interaction $q_i\bar{\ell}_j\bar{u}_k$, which allows us to identify the right–handed top, the up–type Higgs doublet and the quark doublet of the third generation. (Here we denote the leptons and Higgses collectively by $\ell_i,\bar{\ell}_i$.) Other renormalizable interactions $q_i\,\ell_j\,\bar{d}_k$ and $\bar{e}_i\,\ell_j\,\ell_k$ can produce the down–type quark and lepton masses as well as lepton number violating interactions. What happens precisely depends on the form of the matrix of μ-like mass terms for the vector–like states and, thus, on the vacuum configuration. We note that, due to the absence of the $\bar{u}_i\bar{d}_j\bar{d}_k$ operator, the proton is stable at this level.

coupling	$q_i\,\bar{\ell}_j\,\bar{u}_k$	$\bar{u}_i\,\bar{d}_j\,\bar{d}_k$	$q_i\,\ell_j\,\bar{d}_k$	$\bar{e}_i\,\ell_j\,\ell_k$
#	1	0	4	4

Table 4: Renormalizable interactions involving the SM fields.

The model has three generations of SM matter plus vector–like exotics. Once the SM singlets s_i get VEVs, the gauge group reduces to

$$G_\mathrm{SM}\times G_\mathrm{hidden}\;, \tag{14}$$

where $G_\mathrm{hidden}=\mathrm{SO}(8)\times\mathrm{SU}(2)$. At the same time, the vector–like states get large masses. We have checked that the rank of all the mass matrices is maximal, such that the exotics do decouple (assuming all singlets acquire supersymmetric vevs). Below we present most of them. An entry s^n indicates that the coupling appears first when n singlets are involved. Each entry usually contains many terms and involves different singlets as well as coupling strengths.

$$\mathcal{M}_{d\bar{d}} = \begin{pmatrix} s^6 & s^6 & s^3 & s^6 & s^6 & s^1 & s^1 \\ s^6 & s^6 & s^3 & s^6 & s^6 & s^1 & s^1 \\ s^3 & 0 & 0 & s^3 & 0 & s^6 & s^6 \\ s^6 & s^3 & 0 & s^6 & s^3 & s^6 & s^6 \end{pmatrix}, \qquad \mathcal{M}_{\ell\bar{\ell}} = \begin{pmatrix} s^3 & s^1 & s^1 & s^1 & s^1 \\ s^1 & s^3 & s^3 & s^3 & s^3 \\ s^1 & s^3 & s^3 & s^3 & s^3 \\ s^1 & s^3 & s^3 & s^3 & s^3 \\ s^1 & s^3 & s^3 & s^3 & s^3 \\ s^1 & s^3 & s^6 & s^6 & s^3 \\ s^4 & s^2 & s^6 & s^2 & s^2 \\ s^4 & s^2 & s^6 & s^2 & s^2 \end{pmatrix},$$

$$\mathcal{M}_{m'm'} = \begin{pmatrix} s^1 & s^5 \\ s^5 & s^1 \end{pmatrix},$$

$$\mathcal{M}_{v\bar{v}} = \begin{pmatrix} s^5 & s^5 & s^5 & s^5 \\ s^5 & s^5 & s^5 & s^5 \\ s^6 & s^6 & s^1 & s^5 \\ s^6 & s^6 & s^5 & s^1 \end{pmatrix}, \quad \mathcal{M}_{mm} = \begin{pmatrix} 0 & s^5 & s^6 & s^6 \\ s^5 & 0 & s^6 & s^6 \\ s^6 & s^6 & 0 & s^5 \\ s^6 & s^6 & s^5 & 0 \end{pmatrix}.$$

Similarly, the mass matrices for $s_i^\pm$ and $\tilde{s}_i^\pm$ have a maximal rank. The $d\bar{d}$ mass matrix is $4{\times}7$ such that there are 3 massless $\bar{d}$ states. The $\ell\bar{\ell}$ mass matrix is $8{\times}5$, so there are 3 lepton doublets. By choosing a special vacuum configuration one can reduce the rank of the $\ell\bar{\ell}$ mass matrix to 4 such that there is a pair of massless Higgs doublets. (This is just the supersymmetric "μ–problem"). Thus we end up with the exact MSSM spectrum.

We have checked that the required vacuum configuration is D–flat. That is, one can assign large VEVs to the singlets without inducing the D–terms. Since the number of the F–term equations equals the number of the field variables, there are generally non–trivial solutions to $F = 0$. Then, using complexified gauge transformations, one can make the F– and D–terms vanish simultaneously. Such supersymmetric vacua would correspond to isolated solutions in field space. Although we expect such solutions to exist, their explicit form remains undetermined and will be studied elsewhere.

Finally, the model allows us to define a suitable $B-L$ generator which leads to the standard charges for the SM matter,

$$\mathtt{t}_{B-L} = \left(1,\,1,\,0,\,0,\,0,\,-\tfrac{2}{3},\,-\tfrac{2}{3},\,-\tfrac{2}{3}\right)\left(2x-\tfrac{1}{2},\,\tfrac{1}{2},\,0,\,x,\,x,\,0,\,0,\,0\right), \tag{15}$$

with arbitrary x. An interesting feature is that the spectrum contains a pair of fields which have $B-L$ charges ± 2. If $B-L$ gauge symmetry is broken by VEVs of these fields, the matter parity (or family reflection symmetry [44,45]) $(-1)^{3(B-L)}$ is conserved and proton decay is suppressed.

5 Conclusion

We have analyzed the heterotic $\mathrm{E}_8 \times \mathrm{E}_8$ string compactified on a $\mathbb{Z}_6$-II orbifold, allowing for up to two discrete Wilson lines. Employing a search strategy based on the concept of local GUTs, we have obtained about 3×10^4 inequivalent models. Almost 1 % of these models have the gauge group and the chiral matter content of the MSSM. This result shows that orbifold compactifications of the heterotic string considered here correspond to a particularly fertile region in the landscape and the probability of getting something close to the MSSM is significantly higher than that in other constructions.

The most important outcome of our scan is the construction of $\mathcal{O}(100)$ models consistent with the MSSM gauge group and matter content, amended by a hidden sector. A detailed phenomenological analysis of these models is in progress.

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