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# Measurements of $\boldsymbol{K}_{e 4}$ and $\boldsymbol{K}^{ \pm} \rightarrow \pi^{0} \pi^{0} \boldsymbol{\pi}^{ \pm}$decays 

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The NA48/2 experiment at the CERN SPS collected in 2003 and 2004 large samples of the decays $K^{ \pm} \rightarrow \pi^{+} \pi^{-} e^{ \pm} \nu_{e}\left(K_{e 4}^{+-}\right), K^{ \pm} \rightarrow \pi^{0} \pi^{0} e^{ \pm} \nu_{e}\left(K_{e 4}^{00}\right)$ and $K^{ \pm} \rightarrow$ $\pi^{0} \pi^{0} \pi^{ \pm}$. From the $K_{e 4}^{+-}$form factors and from the cusp in the $M_{00}^{2}$ distribution of the $K^{ \pm} \rightarrow \pi^{0} \pi^{0} \pi^{ \pm}$events, the $\pi \pi$ scattering lengths $a_{0}^{0}$ and $a_{0}^{2}$ could be extracted. This measurement is a fundamental test of Chiral Perturbation Theory $(\chi P T)$. The branching fraction and form factors of the $K_{e 4}^{00}$ decay were precisely measured, using a much larger data sample than in previous experiments.

## 1 Introduction

The single-flavour quark condensate $\langle 0| \bar{q} q|0\rangle$ is a fundamental parameter of $\chi P T$, determining the relative size of mass and momentum terms in the expansion. Since it can not be predicted theoretically, its value must be determined experimentally, e.g. by measuring the $\pi \pi$ scattering lengths, whose values are predicted very precisely within the framework of $\chi P T$, assuming a big quark condensate [1], or of generalised $\chi P T$, where the quark condensate is a free parameter [2].

The $K_{e 4}^{+-}$decay is a very clean environment for the measurement of $\pi \pi$ scattering lengths, since the two pions are the only hadrons and they are produced close to threshold. The only theoretical uncertainty enters through the constraint [3] between the scattering lengths $a_{0}^{2}$ and $a_{0}^{0}$. In the $K^{ \pm} \rightarrow \pi^{0} \pi^{0} \pi^{ \pm}$decay a cusp-like structure can be observed at $M_{00}^{2}=4 m_{\pi^{+}}^{2}$, due to re-scattering from $K^{ \pm} \rightarrow \pi^{+} \pi^{-} \pi^{ \pm}$. The scattering lengths can be extracted from a fit of the $M_{00}^{2}$ distribution around the discontinuity.

## 2 Experimental setup

Simultaneous $K^{+}$and $K^{-}$beams were produced by 400 GeV energy protons from the CERN SPS, impinging on a beryllium target. The kaons were deflected in a front-end achromat in order to select the momentum band of $(60 \pm 3) \mathrm{GeV} / c$ and focused at the beginning of the detector, about 200 m downstream. For the measurements presented here, the most important detector components are the magnet spectrometer, consisting of two drift chambers before and two after a dipole magnet and the quasi-homogeneous liquid krypton electromagnetic calorimeter. The momentum of the charged particles and the energy of the photons are measured with a relative

[^0]

Figure 1: Topology of the $K_{e 4}$ decay.
uncertainty of $1 \%$ at 20 GeV . A detailed description of the NA48/2 detector can be found in Ref. [4].

## $3 \boldsymbol{K}^{ \pm} \rightarrow \boldsymbol{\pi}^{+} \boldsymbol{\pi}^{-} \boldsymbol{e}^{ \pm} \boldsymbol{\nu}_{\boldsymbol{e}}$

Analysing part of the 2003 data, $3.7 \times 10^{5} K_{e 4}^{+-}$events were selected with a background contamination of $0.5 \%$. The background level was estimated from data, using the so-called "wrong sign" events, i.e. with the signature $\pi^{ \pm} \pi^{ \pm} e^{\mp} \nu_{e}$, that, at the present statistical level, can only be background, since the corresponding kaon decay violates the $\Delta S=\Delta Q$ rule and is therefore strongly suppressed [5]. The main background contributions are due to $K^{ \pm} \rightarrow \pi^{+} \pi^{-} \pi^{ \pm}$events with $\pi \rightarrow e \nu$ or a pion mis-identified as an electron. The background estimate from data was cross-checked using Monte Carlo simulation (MC).

### 3.1 Form factors

The form factors of the $K_{e 4}^{+-}$decay are parametrised as a function of five kinematic variables [6] (see Fig. 1): the invariant masses $M_{\pi \pi}$ and $M_{e \nu}$ and the angles $\theta_{\pi}, \theta_{e}$ and $\phi$. The matrix element

$$
T=\frac{G_{F}}{\sqrt{2}} V_{u s}^{*} \bar{u}\left(p_{\nu}\right) \gamma_{\mu}\left(1-\gamma_{5}\right) v\left(p_{e}\right)\left(V^{\mu}-A^{\mu}\right)
$$

contains a hadronic part, that can be described using two vector ( $F$ and $G$ ) and one axial ( $H$ ) form factors [7]. After expanding them into partial waves and into a Taylor series in $q^{2}=$ $M_{\pi \pi}^{2} / 4 m_{\pi^{+}}^{2}-1$, the following parametrisation was used to determine the form factors from the experimental data [8, 9]:

$$
\begin{aligned}
F & =\left(f_{s}+f_{s}^{\prime} q^{2}+f_{s}^{\prime \prime} q^{4}\right) e^{i \delta_{0}^{0}\left(q^{2}\right)}+f_{p} \cos \theta_{\pi} e^{i \delta_{1}^{1}\left(q^{2}\right)} \\
G & =\left(g_{p}+g_{p}^{\prime} q^{2}\right) e^{i \delta_{1}^{1}\left(q^{2}\right)} \\
H & =h_{p} e^{i \delta_{1}^{1}\left(q^{2}\right)} .
\end{aligned}
$$

In a first step, ten independent five-parameter fits were performed for each bin in $M_{\pi \pi}$, comparing data and MC in four-dimensional histograms in $M_{e \nu}, \cos \theta_{\pi}, \cos \theta_{e}$ and $\phi$, with 1500 equal population bins each. The second step consisted in a fit of the distributions in $M_{\pi \pi}$ (see Figs. 3,2), to extract the (constant) form factor parameters. The $\delta=\delta_{0}^{0}-\delta_{1}^{1}$ distribution was fitted with a one-parameter function given by the numerical solution of the Roy equations [3], in order to determine $a_{0}^{0}$, while $a_{0}^{2}$ was constrained to lie on the centre of the universal band. The following


Figure 2: $\delta=\delta_{0}^{0}-\delta_{1}^{1}$ distribution as a function of $M_{\pi \pi}$. The points represent the results of the first-step fits, the line is fitted in the second step.


Figure 3: $F, G$ and $H$ dependence on $M_{\pi \pi}$. The points represent the results of the first-step fits, the lines are fitted in the second step.


Figure 4: Invariant mass distribution in logarithmic scale of the $K_{e 4}^{00}$ events selected from the 2003 data (crosses) compared to the signal MC (red) plus physical (yellow) and accidental (blue) background.
preliminary result was obtained:

$$
\begin{aligned}
f_{s}^{\prime} / f_{s} & =0.169 \pm 0.009_{\text {stat }} \pm 0.034_{\text {syst }} \\
f_{s}^{\prime \prime} / f_{s} & =-0.091 \pm 0.009_{\text {stat }} \pm 0.031_{\text {syst }} \\
f_{p} / f_{s} & =-0.047 \pm 0.006_{\text {stat }} \pm 0.008_{\text {syst }} \\
g_{p} / f_{s} & =0.891 \pm 0.019_{\text {stat }} \pm 0.020_{\text {syst }} \\
g_{p}^{\prime} / f_{s} & =0.111 \pm 0.031_{\text {stat }} \pm 0.032_{\text {syst }} \\
h_{p} / f_{s} & =-0.411 \pm 0.027_{\text {stat }} \pm 0.038_{\text {syst }} \\
a_{0}^{0} & =0.256 \pm 0.008_{\text {stat }} \pm 0.007_{\text {syst }} \pm 0.018_{\text {theor }},
\end{aligned}
$$

where the systematic uncertainty was determined by comparing two independent analyses and taking into account the effect of reconstruction method, acceptance, fit method, uncertainty on background estimate, electron-ID efficiency, radiative corrections and bias due to the neglected $M_{e \nu}$ dependence. The form factors are measured relative to $f_{s}$, which is related to the decay rate. The obtained value for $a_{0}^{0}$ is compatible with the $\chi P T$ prediction $a_{0}^{0}=0.220 \pm 0.005$ [10] and with previous measurements $[11,12]$.

## $4 K^{ \pm} \rightarrow \pi^{0} \pi^{0} e^{ \pm} \nu_{e}$

About $10,000 K_{e 4}^{00}$ events were selected from the 2003 data and about 30,000 from the 2004 data with a background contamination of $3 \%$ and $2 \%$, respectively. The background level was estimated from data by reversing some of the selection criteria and was found to be mainly due to $K^{ \pm} \rightarrow \pi^{0} \pi^{0} \pi^{ \pm}$events with a pion mis-identified as an electron (see Fig. 4). The branching fraction was measured, as a preliminary result from the 2003 data only, normalised to $K^{ \pm} \rightarrow$ $\pi^{0} \pi^{0} \pi^{ \pm}$:

$$
B R\left(K_{e 4}^{00}\right)=\left(2.587 \pm 0.026_{\text {stat }} \pm 0.019_{\text {syst }} \pm 0.029_{e x t}\right) \times 10^{-5}
$$

where the systematic uncertainty takes into account the effect of acceptance, trigger efficiency and energy measurement of the calorimeter, while the external uncertainty is due to the uncertainty


Figure 5: Left: $M_{00}^{2}$ of the selection $K^{ \pm} \rightarrow \pi^{0} \pi^{0} \pi^{ \pm}$data events. The arrow indicates the position of the cusp. Right: angle between the $\pi^{ \pm}$and the $\pi^{0}$ in the $\pi^{0} \pi^{0}$ centre of mass system. The points represent the data, the three curves, the MC distribution for different values of $k^{\prime}$
on the $K^{ \pm} \rightarrow \pi^{0} \pi^{0} \pi^{ \pm}$branching fraction. This result is about eight times more precise than the best previous measurement [13].

For the form factors the same formalism is used as in $K_{e 4}^{+-}$, but, due to the symmetry of the $\pi^{0} \pi^{0}$ system, the $P$-wave is missing and only two parameters are left: $f_{s}^{\prime} / f_{s}$ and $f_{s}^{\prime \prime} / f_{s}$. Using the full data sample, the following preliminary result was obtained:

$$
\begin{aligned}
f_{s}^{\prime} / f_{s} & =0.129 \pm 0.036_{\text {stat }} \pm 0.020_{\text {syst }} \\
f_{s}^{\prime \prime} / f_{s} & =-0.040 \pm 0.034_{\text {stat }} \pm 0.020_{\text {syst }},
\end{aligned}
$$

which is compatible with the $K_{e 4}^{+-}$result.

## $5 K^{ \pm} \longrightarrow \pi^{0} \pi^{0} \pi^{ \pm}$

From 2003 data, about 23 million $K^{ \pm} \rightarrow \pi^{0} \pi^{0} \pi^{ \pm}$events were selected, with negligible background. The squared invariant mass of the $\pi^{0} \pi^{0}$ system ( $M_{00}^{2}$ ) was computed imposing the mean vertex of the $\pi^{0} \mathrm{~s}$, in order to improve its resolution close to threshold. At $M_{00}^{2}=4 m_{\pi^{+}}^{2}$, the distribution shows evidence for a cusp-like structure (see Fig. 5) due to $\pi \pi$ re-scattering. Fitting the distribution with the theoretical model presented in Ref. [14] and using the unperturbed matrix element

$$
M_{0}=A_{0}\left(1+\frac{1}{2} g_{0} u+\frac{1}{2} h^{\prime} u^{2}+\frac{1}{2} k^{\prime} v^{2}\right),
$$

the following result was obtained [15], assuming $k^{\prime}=0$ [16]:

$$
\begin{aligned}
g_{0} & =0.645 \pm 0.004_{\text {stat }} \pm 0.009_{\text {syst }} \\
h^{\prime} & =-0.047 \pm 0.012_{\text {stat }} \pm 0.011_{\text {syst }} \\
a_{2} & =-0.041 \pm 0.022_{\text {stat }} \pm 0.014_{\text {syst }} \\
a_{0}-a_{2} & =0.268 \pm 0.010_{\text {stat }} \pm 0.004_{\text {syst }} \pm 0.013_{\text {theor }},
\end{aligned}
$$

where the $a_{0}-a_{2}$ measurement is dominated by the uncertainty on the theoretical model.
In a further analysis, evidence was found for a non-zero value of $k^{\prime}$ (see Fig. 5):

$$
k^{\prime}=0.0097 \pm 0.0003_{\text {stat }} \pm 0.0008_{\text {syst }},
$$

where the systematic uncertainty takes into account the effect of acceptance and trigger efficiency.

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