Short-Term Shocks, Reversion,
and Long-Term Decision-Making

by

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ABSTRACT

Many observers claim that discounted cash-flow methods lead to a neglect of long-term and strategic decision-making. Using modern asset pricing methods, we examine one possible reason for this problem. If the cash-flows being discounted have an increasing dependence on an uncertain variable that tends to revert to a long-term equilibrium path in the face of short-term shocks, and if this reversion is ignored, then the uncertainty in the cash-flows will be overestimated. If this uncertainty causes risk discounting, then the amount of risk discounting that is appropriate will also be overestimated, which will tend to result is a relative undervaluation of long-term alternatives .

We examine the implications of such an error for the comparative analysis of decision alternatives, including some involving an initial timing option. We use, as examples, decisions about production projects where the output price is the reverting variable. Where applicable, we look at two measures of what is meant by long-term: the operating duration of the project and the length of an initial timing option.

For the projects without options, the analysis is based on the relatively straightforward "risk discounting effect" already mentioned. Reversion tends to decrease long-term uncertainty, and, with it, long-term risk discounting, which increases the relative value of long-term alternatives.

Options complicate matters. The long-term decrease in uncertainty due to reversion tends directly to decrease long-term option values. Moreover, in addition to the original risk discounting effect and this "variance effect", there can be direct "future reversion effects" if the options involve a timing component or payoffs generated by cash-flows over a period of time. The overall influence can be a complicated mixture of the three different types of effects.

We use this classification scheme to analyse two sets of examples: investment timing options on an instantaneous production project (equivalent to at-the-money American options on the project output price), and "now-or-never" options, as well as investment timing options, on projects that differ in their operating lives. We find that a neglect of reversion leads to an undervaluation of at- or in-the-money options on projects with longer operating lives. This is primarily due to the risk discounting effect. Longer timing options on the same project tend to be relatively overvalued by a neglect of reversion if the operating life of the project is moderately long, and undervalued if the project is instantaneous and currently at the money. The first is primarily due to variance and future-reversion effects. The second is primarily due to risk-discounting and future-reversion effects.

Because parts of the economy may be influenced by short-term shocks in the presence of long-term equilibrium, these results suggest a reexamination of those aspects of analyses in the "real options" literature that depend on the use of non-reverting models.

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1. INTRODUCTION

Many observers of managerial processes have come to the conclusion that discounted cash-flow methods lead to a damaging neglect of long-term and strategic investments (e.g., Hayes and Garvin 1982, MacCallum 1987, Dertouzos et al. 1989). One proposed remedy is to put less weight on financial analysis and more on managerial intuition. Myers (1984), as a financial economist, has countered that the problem is inappropriate financial analysis rather than financial analysis in general. He suggests that improper accounting for risk in potential cash-flows frequently leads in an evaluation to the use of discount rates that are too high. This results in the relative undervaluation of typical long-term alternatives. He suggests also that organisations may underestimate, or neglect, the value of options stemming from the decisions that they make. Because the creation of future options is often the essence of a strategic decision, and because there tend to be more options created with longer term investments, the undervaluation of future options would also induce a bias against strategic or long-term decision alternatives.

In this paper, we use modern asset pricing methods to examine one possible reason for overdiscounting. If the cash-flows being discounted have an increasing dependence on an uncertain variable that tends to revert to a long-term equilibrium path in the face of short-term shocks, and if this reversion is ignored, then the uncertainty in the cash-flows will be overestimated. If this uncertainty causes risk discounting, then the amount of risk discounting that is appropriate will also be overestimated.

We show how to classify the effects of such reversion on asset value, and also show, in some examples, the implications of ignoring it. Our examples include both "now-or-never" decisions about a production project and choices that involve a project timing option. The reverting variable in these examples is the project output price. For some examples, the measure of "long-term vs. short-term" is the operating duration of the project; for others, it is the length of the timing option. Throughout we use a set of valuation models designed for ease of calculation and usefulness for managers.[1]

[1] This research is part of a larger program on the use of modern asset pricing theory in decision analysis. Jacoby and Laughton (1991,1992) provide a description of the approach that does not require an extensive background in financial economics. Laughton and Jacoby (1991a, b) show an application to

The situations that we examine satisfy three conditions. First, the investing organisation is a price-taker in the output market, so that the price is an underlying exogenous variable. Second, uncertainty in future output prices is the only uncertainty underlying the decisions to be made, and this uncertainty results in positive risk discounting in the valuation of claims to any fixed future output. Third, the structure of the potential production opportunity, at the time it is undertaken, is independent of when it is undertaken.

The first two conditions are imposed so that we may focus on a simple specific model. The third condition is imposed so that the effects of reversion are not confounded with those resulting from a direct dependency on time of the project cash-flows themselves.

Output price reversion has a straightforward effect on "now or never" decisions about project alternatives provided there are no operating options to be considered. The stronger the reversion, the less is the uncertainty in long-term revenues when compared to short-term. With less uncertainty, the risk discounting for long-term revenues is reduced in relation to the discounting for short-term revenues. Therefore, the neglect or underestimation of reversion will provide a bias against project alternatives with larger long-term revenues, even if the valuation is done properly in other respects. Moreover, the use of a single discount rate to value, on a now-or-never basis, project alternatives with different operating lives may introduce a bias against long-term investments in situations where there is reversion in the project output price.

If options are built into the project alternatives being considered, the effects of output price reversion are more complex.

As we have already stated, because reversion tends to decrease long-term price uncertainty, it decreases the amount of risk discounting in, and raises the value of, any long-term output price bond (which is a claim to a cash-flow proportional to a long-term output price). Therefore reversion tends to increase the value of claims to cash-flows that increase with long-term prices, including call-like options, and to decrease the value of claims to cash-flows that decrease with price, including put-like options. This

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managerial flexibility. The position of the method within the wider finance literature is laid out in Laughton (1988).

effect may be called the "risk-discounting effect".

Less uncertainty also tends to depress directly the value of long-term options of any type. This may be called the "variance effect", which reinforces the risk-discounting effect for put-like options and mitigates, if not overwhelms, it for call-like options.

Finally, the reversion of future term structures for central tendencies of the price can have direct effects on asset values. These may be called "future-reversion effects". They exist for American options, for which the timing of the exercise of the option is optional itself, and may exist for options, the payoffs of which are generated by cash-flows that would occur over a period of time. The details of these effects on an option can depend on whether the option is in-, at- or out-of-the money now and whether the reversion is to prices where the option would be in-, at-, or out-of-the money in the future.

In Section 2, we set up the class of price models that we shall examine. To maintain consistency with our general evaluation framework, we restrict the analysis to price processes that have a lognormal structure; this condition allows a nonstochastic discounting framework to be applied to the valuation of the related output price bonds. We also wish to facilitate the valuation of project options, such as the initial timing option considered in this paper. Therefore, we examine price models that result in a simple state space for the option analysis. In particular, we use models where the state space is one-dimensional and indexed simply by the contemporaneous output price.

The output price models to be used are each specified using a process for the expectation of the prices, where the key feature is an exponentially decaying term structure of expectation volatilities.[2] New information has a greater impact on expectations for the prices that will occur a year or two in the future than on expectations for prices that will occur in 10 or 20 years. Moreover, the proportional drift in the resulting process for the price itself has a term that is logarithmic in the price, which shows the reversion forces at work in the price itself. Finally, the pattern of future conditional term structures of price medians shows the reversion directly.

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[2] A specification close to ours was outlined by Treynor and Black (1976) in an early paper applying modern asset pricing concepts to project evaluation. A similar specification, for the uncertainty in short-term risk-free interest rates, has been used in the valuation of derivative securities of treasury bonds (Turnbull and Milne 1991).

In Section 3, we show the effects of different levels of reversion on the valuation of options on projects where production and sales occur at one and the same time. These are equivalent to American call options on the output price. We examine options that are currently at-the-money. These simple examples allow us to focus on the effects of reversion without the cash-flow complexity of multiperiod projects. The comparisons are based on the presumption that, while managers have some knowledge of the output price uncertainties over the medium term, and of the appropriate valuation of these risks, they may mis-specify the long term. The results can be summarised a follows. Conflicting risk-discounting and variance effects give conflicting results for the European options. However, the early exercise premium for an American option is also influenced by a future-reversion effect. For the at-the-money options examined, the early exercise premium increases with the degree of reversion, and the net result is that neglecting reversion gives a bias against longer-term investment timing options.

Section 4 demonstrates how reversion can affect the evaluation of options on projects of different operating length. We show how a bias against long-term alternatives might occur, in a now-or-never analysis of the projects, through the use in the valuation of the longer project of an organisational discount rate based on the valuation of the shorter project. The results provide a clear example of the bias inherent in single-rate discounting methods. We also show, to continue the analysis begun in Section 3, what happens if reversion is neglected in an otherwise correct valuation of these projects in the presence of an initial timing option. In the examples that we have examined, the bias remains against the project with a longer operating duration, even if that bias is mitigated somewhat by the timing option. However, when considering the same operating project, longer timing options tend to be relatively *overvalued* if reversion is neglected, in contrast to the results in Section 3 for options on single-period projects. We show how the two effects are connected for the set of examples considered.

In Section 5, we conclude and suggest areas of future work.

2. THE PRICE MODEL AND VALUATION METHOD

2.1. The Price Process

The output price model can be formulated in terms of a process for the evolution over time of the price expectations. The process is based on the approximation that the information needed to determine the revision of future expectations is provided by the most recent unanticipated revision in the expectation of the current price.

For any given period, s to s+ds, the revision of expectations for all times at or after s+ds is determined by a single normal random variable, dz_s, which is normalised to have zero expectation and variance that is the length of the time period, ds.[3] This variable represents information coming from the output market during the period just after time s, in the form of the final movement in the expectation for the price at the end of that period. It is independent of the other dz's, because each dz represents new information at a different time.

The revision of each price expectation is modelled to be proportional to the expectation of that price at the beginning of the period and to the normalised information for that period. Thus, given the expectation at the beginning of the period s for the price that will occur at time t, $E_s(P_t)$, the change over that period in the expectation of that price is taken to be of the form

$$d_sE(P_t) = E_s(P_t)\, \sigma_{s,t}\, dz_s, \tag{1}$$

where the proportionality constant, $\sigma_{s,t}$ is called the volatility of the expectation of the price at time t in the period beginning at time s.

We can think of the pattern of volatilities in any given period as an influence function which reflects the relative effect of information arriving during that period on expectations of prices at different times. If, for a given time s, $\sigma_{s,t}$ is constant for all t, then a shock at s has the same proportional influence on

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[3] The specialization to one piece of information in each period is for simplicity. For a model with two types of information, incorporating long-term decay and cyclical effects of new information, see Laughton (1988). Such a model would result in a state space for the analysis of future options that is much more complex than the single dimensional state space considered here.

expectations for a price far in the future as for one near at hand. If $\sigma_{s,t}$ declines in the term, t - s, of the price, then the influence of new information arriving at s is "decaying" as one looks farther into the future.

It is plausible that, in many aspects of the economy, a development now is less and less relevant to the state of the economy, the farther out in the future we look. In effect, information becomes stale-dated. This will happen, for example, for prices in markets that are influenced by long-term forces of supply and demand, which limit the length of time that an exceptionally "low "or "high" price can be sustained. After a short shock, the price tends to revert to some "normal" long-term equilibrium path,[4] perhaps determined by the long-run marginal cost or (in the case of a cartelised commodity) the long-run profit-maximising price sought by cartel managers. The greater is this reversion tendency, the greater is the decay in the effect of new information on future prices.

In the applications of this paper, two restrictions are placed on the form of the volatility in a future price expectation (Jacoby and Laughton 1991, 1992).

First, the volatility at any future time s must be modelled as known with certainty at the time of the analysis: it may not vary according to the state of the economy at the time s. This allows a simple nonstochastic discounting model, described in Section 2.2, to be used for valuation.

Second, the decay in the volatility term-structure is expressed by an exponential form,

$$\sigma_{s,t} = \sigma_s \exp\left[-\gamma (t - s)\right] , \tag{2}$$

where γ is the rate of decay. The amount of reversion may also be measured by the half-life of the decay process, which is related to the decay rate by

$$H \equiv \ln(2) / \gamma . \tag{3}$$

Laughton and Jacoby (1992) show that a one-dimensional state space, indexed by the contemporaneous price, occurs only for price models from a slightly more general class of processes with multivariate

[4] Pindyck and Rubinfeld (1991, pp.462-5) have found possible instances of this type of behaviour. They reject no-reversion models for oil and copper prices on the basis of Dickey-Fuller unit root tests using more than 100 years of data.

lognormal probability measures.

Finally, for convenience, the short-term volatility, σ_s, is held to be constant for all s, and denoted as σ. Therefore

$$d_sE(P_t) = E_s(P_t)\,\sigma\,\exp\left[-\gamma(t-s)\right] dz_s. \tag{4}$$

There is a corresponding process in the price itself. This is the form for this type of model commonly used in "real options" work (e.g., Pindyck 1991, Brennan and Schwartz 1985). It is derived in Laughton and Jacoby (1992) to be

$$d_tP = \left[\alpha_t + \frac{1}{2}\sigma^2 - \gamma \log\left(\frac{P_t}{M_0(P_t)}\right)\right] P_t\,dt + \sigma\,P_t\,dz_t\ , \tag{5}$$

where $M_0(P_t)$ is the current median of the price at t, and α_t is the growth rate at term t in the current term structure of price medians

$$\alpha_t \equiv \frac{\partial_t M_0(P_t)}{M_0(P_t)}\ . \tag{6}$$

The current price medians are related to the current expectations by

$$M_0(P_t) = E_0(P_t)\,e^{-1/2\,\mathrm{var}_{0,t}}\ , \tag{7}$$

where $\mathrm{var}_{0,t}$ is the current variance of the logarithm of that price,

$$\mathrm{var}_{0,t} = \frac{\sigma^2}{2\gamma}\left(1 - e^{-2\gamma t}\right). \tag{8}$$

As can be seen in this formulation of the price model, the contemporaneous price is, as required, a sufficient state variable for the price dynamics. The price reversion is evident in the logarithmic term, involving $P_t / M_0(P_t)$, in the proportional rate of expected price change in Equation 5.

There is an integrated formulation of the model, which is also derived in Laughton and Jacoby

(1992), given by the conditional term structure of price medians in each future price state and the covariances at each time of the price logarithms. The median at time s, if the price at time s is P, of the price at time t is given by

$$M_s(P_t \mid P_s = P) = M_0(P_t)\left(\frac{P}{M_0(P_s)}\right)^{\exp(-\gamma\,(t-s))} , \tag{9}$$

and the covariance at time s, in any state at that time, of the prices at time t_1 and t_2 is given by

$$\mathrm{cov}_{s,t_1,t_2} = \frac{\sigma^2}{2\gamma}\left[e^{-\gamma|t_1-t_2|} - e^{-\gamma(t_1+t_2-2s)}\right] . \tag{10}$$

Notice that the medians in Equation 9 show the effects of reversion, through the behaviour of the second factor in the formula, which is a shock factor giving the effect on the conditional medians of the information arriving between times 0 and s. If there is reversion ($\gamma > 0$), the power in the shock factor approaches 0 as the term (t-s) becomes larger, so that this factor approaches 1, and the conditional future medians revert to the original medians. If there is no reversion ($\gamma = 0$), the power is 1, independent of the term, and the proportional changes to medians are the same for all terms.

2.2. The Mechanics of Derivative Asset Valuation[5]

The underlying assets of the derivative asset valuation are the set of output price bonds. The current term structure of their values is calculated as

$$V_0(P_t) = E_0(P_t)\exp\left(-\int_0^t ds\ \mu_{s,t}\right) , \tag{11}$$

where $V_0(P_t)$ is the current value of the bond maturing at time t. The expected rate of return at time s for the bond maturing at time t, $\mu_{s,t}$, is taken to be the sum of the risk-free rate, r, modelled to be constant for

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5 For a more extensive treatment of this valuation model, and the compromises between operational feasibility and "best" valuation theory, see Jacoby and Laughton (1991, 1992).

convenience, and a risk premium that is proportional to the amount of volatility at time s in the expectation of the price at time t, $\sigma_{s,t}$.[6] The proportionality constant, which is called the price of risk and denoted by ϕ, is also held constant over time for ease in presentation. It is presumed to be positive, so that there is risk discounting in the valuation of the output price bonds. Using the expression for $\sigma_{s,t}$ in Equation 2, and remembering that σ_s has been modelled to be a constant denoted as σ, the expected return at time s for the bond maturing at time t is

$$\mu_{s,t} = r + \phi\,\sigma \exp(-\gamma(t-s)). \tag{12}$$

The structure of forward prices implicit in these price bond values is the term structure of expectations of the prices with respect to their risk-adjusted measure (Cox, Ingersoll, and Ross 1985, Jacoby and Laughton 1991,1992). This measure may be labelled by the state at which it is defined, and each state is determined by its time s, and the realised level of the price at that time, P_s, which we denote by P. We denote the risk-adjusted measure for the state (s, P) by $dm_s(P_{\geq s} \mid P_s = P)$, and use it in the computation at time s of the value of any asset with cash flows (denoted by the index CF and occurring at time t) that are contingent only on the then future prices:

$$V_s(\text{Asset} \mid P_s = P) = \sum V_s(\text{CF} \mid P_s = P), \tag{13}$$

where

$$V_s(\text{CF} \mid P_s = P) = \exp(-r(t_{CF} - s)) \int dm_s(P_{\geq s} \mid P_s = P)\, X_{CF}(P_{\geq s}), \tag{14}$$

and where X_{CF} indicates the functional dependence of the cash-flow amount for the cash-flow CF on the

[6] Valuation with known expected returns having this structure may occur, for example, in Cox-Ingersoll-Ross (1985) economies where there is a representative agent with known impatience and logarithmic risk aversion, and where the structure of the production opportunity set is state-independent. The valuation of projects in this paper may viewed as partial equilibrium exercises in an economy that has these characteristics, at least to a good approximation for purposes of financial market price determination.

time series of the future prices.[7] Value calculations can also be based on Black-Scholes-Merton boundary problems that correspond to the integrals in Equation 14 (Laughton and Jacoby 1991a, b).

2.3 Comparative Statics

In constructing the comparative statics for the study of alternative assumptions about reversion, we presume that managers think about the uncertainty and riskiness within the time span of the principal revenue flows of typical new investments. For example, a manager who is familiar with projects that produce most of their output within a 10-year window after the start of the project will tend to think about price conditions over this particular horizon. We summarise this medium-term view by the conditions in the centre of the productive life, 5 years into the future. We presume then that managers focus on a medium-term "reference time" of 5 years, which we denote t_{ref} = 5 years, and hold constant this impression of the variance and discounting for the price at that time under alternative models of reversion. This presumption about managerial behaviour makes a bias against the long-term more difficult to demonstrate than would the assumption of a very short-term reference time.[8]

Under this framework, our comparisons need to give equivalent risk treatment to cash flows at t_{ref} by keeping fixed the current probability distribution of the price at that time, and the risk discounting in the price bond with that maturity. To achieve the desired probability distribution, different degrees of reversion are accompanied by adjustments to the short-term volatility, σ. We also preserve the current risk-discounting of the price bond maturing at t_{ref}, by an adjustment in the price of risk ϕ. For example, if $\sigma = 0.1$ and $\phi = 0.4$ in annual terms in the absence of reversion, then, for t_{ref} = 5 years and H = 3 years, the corresponding short-term volatility and price of risk are 0.160 and 0.421 respectively. The procedure for

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[7] The risk-adjusted measure for this model is multivariate lognormal. As noted, the risk-adjusted price expectations are the forward prices, which are the true expectations discounted for risk. The associated covariances (that is, the covariances of the price logarithms) are the same as for the true probability measure, and are given in Equation 10.

[8] If the focus is on short-term uncertainty and discounting, then t_{ref} = 0. For an exploration of this case, see Laughton and Jacoby (1992).

making these adjustments is presented in the Appendix.

Figure 1 shows the current term structure of the current price distributions under these two models if the current term structure of price medians is flat at \$20. The medians are shown by the central dotted line. The other lines show 0.9 and 0.1 fractiles. The effect of the change from a model with no reversion ($H = \infty$), shown by the solid lines, to a model with reversion ($H < \infty$), shown by the dashed lines, is to pull in the distant tails of the probability distribution of the output price, and to fatten the fractiles for $t < t_{ref}$. The "knowledge" of the long term distribution embodied in the reversion process keeps the price closer to the current term structure of medians, and indeed causes the total amount of uncertainty to approach a constant in the long term.

A similar figure for the fractiles of any conditional measure in the future would show, reflecting the results in Equation 9 for the conditional medians, that the fractiles in the no-reversion model are shifted up or down by the ratio of the realised price at the conditioning time to the original median of that price, while the fractiles in the model with reversion would tend, at long terms, to revert to the current fractiles.

3. EVALUATION OF PROJECTS WITH ONE OPERATING TIME

Because so many factors come into play if cash-flows occur over several years, we begin with examples of projects where production and sales take place both at the same single time. The timing options for these projects are simply American call options on the output price. We compare price models where the current term structure of medians of the price is held constant. This is like keeping the "base case" the same in different scenario analyses.

To reveal the mechanisms by which reversion influences value, we build up the American option value out of three component parts. We express the American value as the value of an European call option of equivalent length plus a premium for the possibility of early exercise. Using put-call parity, we express the value of the European call in two additive parts: the current value of the "call obligation" (which is the current value of price bond less the current value of a risk-free claim to the exercise price), and the value of the European put option.

In Figures 2 to 6, we show the effects of different degrees of reversion on these components of the value of American call options of different maturities. In these comparisons, the risk-free rate, r, is 0.03 per year, and the current price medians are all \$20, as is the exercise price. The short-term volatility and price of risk in the no-reversion price model are 0.1 and 0.4 in annual terms.

Figure 2 shows the value of the call obligations. The value of the obligation with a term of t_{ref} = 5 years is, by construction, independent of the degree of reversion. The obligation value is negatively (positively) related to the degree of reversion for maturities less (more) than t_{ref}. This is simply due to the risk-discounting effect, which reflects the greater (lesser) overall uncertainty, given higher degrees of reversion, in prices at times less (more) than t_{ref}.[9]

A pattern is also observable in the value of the European put options, shown in Figure 3. Two influences are at work here. First, we may define a put obligation as a short position in the call obligation. Reversion, through the risk-discounting effect, increases the short-term (less than t_{ref}) put obligation values and decreases the long-term values. This tends to increase (decrease) the short-term (long-term) put option values. Because reversion increases (decreases) the price variance for terms shorter (longer) than t_{ref}, it also tends to increase (decrease) short-term (long-term) put values directly through the variance effect. The two effects reinforce each other for puts, resulting in a relative overvaluation of long-term puts if reversion is ignored.

For the European call option, the risk-discounting effect and the variance effect counteract each other, and no clear pattern exists. The call obligation value decreases (increases) with greater reversion for short (long) terms, while the put option value does the opposite. At a maturity of t_{ref}, the values for different degrees of reversion are equal by construction. Figure 4 shows the results of adding these two values to form the European call option values for our set of examples.

· The effect of reversion on early exercise is shown in Figure 5. In contrast to the classic Black

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9 This effect is accompanied by a decrease in the price expectations with more reversion. More reversion reduces the variance of the logarithm of the price, which decreases the factor that relates the price expectation to the price median (Equation 7). For the given level of the price of risk and the volatility of short-term price expectations, the risk discounting effect always dominates. When the risk discounting or "value" effect is mentioned, the reader may assume that this small influence on the expectations is taken into account.

Scholes case (current price bond values constant over all maturities) the rate-of-return shortfall is non-zero, even when $H = \infty$. Early exercise commands a premium in all the examples, because there is always a positive expected shortfall. Moreover, the premium rises with the degree of reversion. This occurs because positive fluctuations in the potential option payoff are more likely to be short-lived, which increases the value of the right to exercise early in the face of any given positive fluctuation. This effect also tends to lower the exercise boundary, which increases the probability of early exercise. These are future-reversion effects. In the long term, however, reversion decreases the probability than any given positive fluctuation will occur, counteracting the effects of these fluctuations. This is a variance effect. The future-reversion effect dominates for the parameters examined. We have not explored the limits of the parameter range over which this dominance holds.

Summing the European call option value and the early exercise premium gives the value of the American call, shown in Figure 6. The future-reversion and risk-discounting effects dominate the variance effect so that the value increases with reversion, more so the longer the term of the option.

Recall that the call obligations and call options are examples of very simple projects that involve production and sales at a single time. Our examples are constructed so that they would have zero value if the production and sales were to occur now. Price reversion affects the value of these projects if production and sales must or can occur in the future. In particular, a neglect of reversion results in a relative bias, because of a risk discounting effect, against those projects where production and sales must be undertaken at some time in the long-term future. Because of a complex combination of risk-discounting, variance and future-reversion effects, there is also a bias against long-term timing options on such projects.

4. PROJECTS WITH MANY PERIODS OF OPERATION

Now we consider the effects of reversion on projects where production and sales occur at more than one time. We use the same class of price models, taking the half-life of the reversion to be $H = 3$ years, and consider the errors introduced if this information is not properly incorporated into the "now-or-never" evaluation of a mutually exclusive pair of projects or the evaluation of mutually exclusive options to undertake these projects.

The salient difference between the projects is that one has an operating life of L = 10 years, and the other has an operating life of L = 20 years. Initial project costs are taken to be known with certainty: $100M for the 10-year project and $150M for the 20-year project. Each has a known, constant stream of annual operating costs of $11.75M per year. For each project, the annual output, which begins one year after initial investment, is constant at 1.405M units. With these parameters, the 10-year project, if undertaken immediately, would currently have zero value, while the 20-year project would have negative value.

4.1 Now-or-Never Projects

The results of the analysis of the projects on a now-or-never basis are shown in Table 1. To achieve the correct valuation, the analysis must account properly for the difference in project operating life and for the degree of reversion in the prices. Failure to do so will introduce bias into the comparison between the long-lived and the short-lived project. The table shows the correct valuation and the result of two such errors. The left part of the first line in the table shows the true valuation of the two projects if H = 3 years. The value of the 10-year project is $3.62M, which is less than the value of $17.02M for the longer alternative. While either projects is better than nothing, the long-term alternative is preferred to the short-term.

An appropriate discount rate for each project may be defined as the single constant annual discount rate which, if used in discounting the cash-flows in the median price scenario, yields the correct value of the project.[10] The project discount rates thus defined are on the second line. The annual discount rate for the 10-year project is 0.093. The discount rate of the 20-year project is lower, at 0.075, because reversion in the prices tends to lower the volatility in the revenue of the out years and thus the risk-discounting in the overall revenue stream. This effect can be seen in the pattern of discount rates for the valuation of the revenue stream in each project considered in isolation: 0.063 for the shorter project

[10] We use this discount rate to compare with a standard scenario-based discounted cash-flow analysis based on the median price scenario.

and only 0.053 for the longer.

Now consider what happens if the difference in discounting is not taken into account and the longer project is discounted at the rate appropriate for the shorter project. This result is shown on the fourth line of the table. The value of the longer project would then be set to -\$3.76M, or \$20.78M less than its true value. Faulty analysis in this case would incorrectly suggest that the shorter project be chosen and that taking the longer project, with its negative "value", would be worse than doing nothing.

In this situation, however, the bias would not occur if there were no reversion. As the right hand side of the table shows, without reversion, the short-term project, which would have zero value under these circumstances, would be preferred to the long-term which would be worth -\$19.67M. Moreover, the use of a short-term discount rate would bias the value of the long-term project upwards, to -\$11.83M.[11] In this case, price reversion is an essential ingredient in the demonstration of bias against the long-term that arises from using the same discount rate for long-term as well as short-term projects.

Finally, the neglect of reversion would introduce bias against the long-term in an otherwise correct valuation of this pair of projects. With reversion, taking the 20-year project is the best alternative, while, without reversion, it is the worst.

4.2 Project Timing Options

Now, we consider the relative value of the projects, if, after one is chosen, it may be begun in any year up to a maturity (or relinquishment) time T, where T is allowed to vary from 0 (the now-or-never evaluation) to 10 years. As is stated in the Introduction, we presume that we are dealing with projects for which the time for starting does not affect the profile of production and sales or the project costs relative to that start time.[12] The project value is denoted $V_0(L,T)$. Once again, the analysis of the effects of reversion

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[11] The discount rate for the long-term project is higher because there is no reversion to counteract the effects of the greater operating leverage that exists for the longer term cash-flows.

[12] While this is done primarily so that the effects of different presumptions about output price reversion are not confounded with effects of a direct time-dependence of the cash-flows, many potential nonrenewable resource developments do at least approximately satisfy this condition.

assumes that the correct half-life is H = 3 years. The correct results are compared to the results of an analysis that incorrectly assumes that $H = \infty$.

This valuation applies a two-method approach to the valuation of the investment timing option developed by Laughton and Jacoby (1991 a, b). The first stage of the analysis is to use Equations 11 to 14 to calculate, for each project under each price model, the value of the project in each possible starting state. Because both the cash-flow model and the underlying output price model are stationary with respect to the starting time, this value is independent of the starting time, but it does depend on the output price at the starting time, which we call the starting price. Figure 7 presents the value of each of the two projects at the time it is initiated, as a function of the starting price, for price models with and without reversion. This function for each project under each price model is called the "value function" of that project under that price model.[13]

The value functions for each of the two projects differ under the two price models. First, for each project, the dependence of the value function on the starting price is weaker with reversion. This difference is greater for the long-term project. Second, the starting price at which each project has a "now-or-never" value of zero (which we call the starting-price intercept of the value function) is lower with reversion. Again, the difference is greater for the long-term project.

The differences in the sensitivity of the value to the starting price are the result of counterbalancing forces, of which there are three.

The first is a future reversion effect. Without reversion, shocks to the price have permanent effects, and higher or lower prices are likely to be maintained. With reversion, the term structure of price distributions will tend in the long-term toward the original distribution. This decreases the dependence of statistics of the conditional price measure (such as the value function) on the conditioning price. The longer the term and the larger the degree of reversion, the greater this future-reversion effect tends to be. It tends to decrease the dependence of the value function on the starting price for a project under reversion, and the effect is stronger for longer-term projects.

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[13] For these calculations, the cash-flows are modelled, for the sake of this valuation, to occur at annual intervals.

The second and third forces are due to risk discounting effects. Greater risk discounting of short-term revenues under the reversion model also tends to decrease the dependence of the value function on the starting price, simply by decreasing the value at all starting prices. However, unlike the first force, it does so more for shorter-term projects. Decreased risk discounting for longer-term revenues increases the starting-price dependence of the value functions under the reversion model by increasing the value for all starting prices. It does so more for the longer project than the shorter.

The dominant force of the three is the first in this example.

The differences in the starting-price intercept of the value function also result from a combination of effects. At prices below the current $20 price, the revenues are more valuable if there is reversion, both because reversion decreases risk discounting in the valuation of the revenues and because, in states defined by price levels below the current price, reversion increases the conditional price median term structure. The risk-discounting effect occurs also in future states at the current (and long-term median) price, and it is sufficiently large so that, if there is reversion, both projects are in the money, more so for the long-term project. Recall that, without reversion, the short-term project is at the money and the long-term project is out of the money.

In the second step of the procedure, the value function of each project is used as an input to a Black-Scholes-Merton formulation of the American option for starting that project.[14] The result of this calculation is the value and the critical starting-price exercise boundary, P^*_S, at each possible starting time s for this timing option.[15] Figure 8 shows this boundary for each project under each price model, given an option of length T = 5 years. At year 5, the option offers only a now-or-never choice, and the project is started at prices where its "now" value is positive. For each project, the critical starting price, P^*_S, at

[14] In these calculations, it is presumed that an option may be exercised only at times occurring at annual intervals. This corresponds to the annual binning of the project cash-flows used in the calculation of the project value functions. If the possible exercise times were modelled to be continuously distributed, and if cash-flows were modelled to flow continuously, the qualitative results would not differ.

[15] For study of the details of the optimal solution, and for representation of the results in a simulation context, the values may then be recomputed conditional on P^* (Laughton and Jacoby 1991 a, b).

s = T = 5 years is the starting-price intercept of the value function in Figure 7. For the years before s = T = 5 years, the price must be higher than the now-or-never break-even price to justify starting instead of waiting, because the option to wait has value. The value of the option to wait is greater, and the critical starting price is higher, the longer the option has to run. Note that, because there is greater chance in the model without reversion of still higher prices when the contemporaneous price is above its original median, the wait option is worth more in any future high price state and a higher price is required to justify starting either project. This effect is larger for the longer-term project.

Figure 9 shows the value, $V_0(L,T)$, of options on each projects under both price models for option lengths ranging from T = 0 (now-or-never) to a maximum of T = 10 years. (The results for T = 0 are the same as those shown for "now" projects in Table 1, with negative values set to zero.) As one would expect, the option value is a non-decreasing function of the option length, T, whatever the project or price model.

If the proper specification of price reversion is H = 3 years, note first that, for all option lengths from T = 0 to T = 10 years, the 20-year project would be begun now and the option to undertake it is more valuable than the option on the 10-year project, although by decreasing amounts for longer options as the timing option on the 10-year project increases in value. However, without reversion, the option on the 10-year project would be more valuable. Therefore, the neglect of reversion in an otherwise correct valuation would result in a bias against the project with a long operating duration if there is management flexibility about when to commit to the project, just as it would in a setting in which management is faced with a now-or-never decision. Although the option to wait somewhat reduces the effect of the bias introduced by a mis-specification of the degree of reversion, the effect of the option value does not overcome the much greater difference in now-or-never values for the longer project.

For either project, neglecting reversion would provide a relative bias in favour of longer-term timing options. This is the opposite of the result for the projects with one time of production and sales, shown in Section 3. The difference is that the value function for the 10-year and 20-year projects depends on the amount of reversion, while the value function for the instantaneous project does not. For the 10-year project, the decreased slope of the value function under reversion makes the major difference. For the 20-year project, the options are out of the money without reversion, while, under reversion, they are so far in the money that the option to wait has no incremental value for any option length.

5. CONCLUSIONS AND EXTENSIONS

Reversion in the cash-flows of different decision alternatives can have a big impact on their relative evaluation, particularly if some of the alternatives have only short-term effects, while others have long-term implications. We have classified the effects of such reversion for situations in which the decision cash-flows increase with a variable that reverts. We have shown examples of these effects in the analysis of production opportunities where the reverting variable is the output price.

To increase the potential for use by non-financial corporations, we have restricted ourselves to a case where the output price bonds may be valued within a nonstochastic discounting framework, and to a pattern of reversion that allows the economic state at any time to be parameterised simply by the contemporaneous output price. The resulting model exhibits a decaying term structure of price expectation volatilities, thus linking the notion of the decay over time of the effect of economic shocks with reversion to a long-term equilibrium measure of economic states.

Reversion can affect value through three channels. First, by decreasing long-term uncertainty, it decreases the amount of risk discounting that is appropriate for those long-term cash-flows that are discounted for risk, and increases their value (the risk discounting or value effect). Second, by decreasing long-term uncertainty, it also will tend directly to decrease the value of any optional element in uncertain cash-flows (the variance effect). Finally, the future reversion of the term structure of the cash-flow determinants can have direct effects on value (future reversion effects).

The analysis of a choice between projects with differing operating lives shows that reversion may be a key element in justifying the use of lower effective discount rates in the valuation, on a now-or-never basis, of long-term as compared to short-term projects. If output prices do exhibit reversion, there may be a bias against projects with long operating durations if all alternatives are evaluated using the same single discount rate.

In otherwise correct valuations that neglect reversion by modelling the price as following a random walk, there may also be a bias for or against long-term decision alternatives depending on the details of the situation. A bias against the long-term is typically caused by overdiscounting of long-term cash-flows. A bias in *favour* of the long-term occurs when the overdiscounting is dominated by the overestimate of the value of the options imbedded in the decision alternative being considered, which is typically caused by the overestimation of the variance of the long-term cash-flows. The effects of reversion in the future term structure of the cash-flows can result in different biases depending on the situation.

In our timing option examples, the neglect of reversion results in a bias against options for projects with long operating duration. However, for the same project with at least a moderate operating duration, neglecting price reversion may introduce a relative bias in favour of longer options to undertake the project.

This work can be extended in several directions. First, random-walk models, which are the mainstay of the "real options" literature, do not exhibit reversion. Our results suggest that it may be useful to revisit work based on random walk models, such as Brennan and Schwartz (1985), Pindyck (1991) and Laughton and Jacoby (1991a, b), to extend the analysis to situations where reversion may be a consideration.

Second, a more complete study should be done of the effects on project values of the interaction of cash-flow reversion with different measures of the asset duration, as well as with other aspects of asset structure such as the degree of operating leverage, the relative size of early sunk costs, and different types of operating flexibility. In addition to the initial timing option considered in this paper, the types of flexibility to be addressed include initial capacity and technology choices, and post-startup options such as capacity or technology changes, temporary shut-downs, and abandonment. More difficult tasks would include finding the effects of other types of term-dependent uncertainty (such as cyclic behaviour) in the project setting of this paper, or of categorising the effects of this or other types of term-dependent behaviour in more complex settings where the project being considered is not stationary with respect to the time of its initiation.

APPENDIX

For $H = \infty$, the current associated variance of the price at time t_{ref} has the form $\sigma^2 t_{ref}$. We calculate the magnitude of the appropriate adjusted volatility for a finite half-life relative to the volatility for $H = \infty$, which in these examples is maintained at 0.1 in annual terms. Therefore, the σ appropriate for a finite half-life is a function of this short-term volatility for $H = \infty$ as well as the reference time and the half-life. It is denoted $\sigma(t_{ref}, H, \sigma)$. The current associated variance of the price at t_{ref}, given a finite half-life H, is of the form

$$\frac{\sigma^2(t_{ref}, H, \sigma)}{2\gamma}(1 - \exp(-2\gamma t_{ref})), \tag{A1}$$

where $\gamma \equiv \ln(2)/H$. Equating this to the variance for $H = \infty$ gives the following expression for the adjusted volatility:

$$\sigma(t_{ref}, H, \sigma) = \sigma\left(\frac{2\gamma t_{ref}}{1 - \exp(-2\gamma t_{ref})}\right)^{1/2}. \tag{A2}$$

For example, the adjusted volatility for $t_{ref} = 5$ years, $H = 3$ years and $\sigma = 0.1$ is $\sigma(5, 3, 0.1) = 0.160$.

Similarly, we want to preserve the current risk discounting of the price bond maturing at t_{ref}, which can be accomplished by an adjustment in the price of risk ϕ. It also becomes a function of σ, H and t_{ref} as well as the price of risk for $H = \infty$, denoted ϕ, which is set at 0.4 in annual terms for all of these examples. The risk discount factor with $H = \infty$ has the form $\exp(-\phi\,\sigma\,t)$. If the adjusted price of risk is denoted $\phi(t_{ref}, H, \sigma, \phi)$, the risk discount factor for finite H is

$$\exp\left(-\frac{\phi(t_{ref}, H, \sigma, \phi)\,\sigma(t_{ref}, H, \sigma)}{\gamma}(1 - \exp(-\gamma t_{ref}))\right). \tag{A3}$$

Combining Equations A2 and A3 gives the following expression for the adjusted price of risk:

$$\phi(t_{ref}, H, \sigma, \phi) = \phi \frac{\gamma t_{ref}}{1 - \exp(-\gamma t_{ref})} \left(\frac{2 \gamma t_{ref}}{1 - \exp(-2 \gamma t_{ref})} \right)^{-1/2} . \tag{A4}$$

For $\phi = 0.4$, the adjusted price of risk is ϕ (5, 3, 0.1, 0.4) = 0.421.

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Table 1. Bias in the Now-or-Never Valuation

Project Length	H = 3 years		$H = \infty$	
	10 years	20 years	10 years	20 years
Value	3.62	17.02	0.00	-19.67
Discount rate, Project	0.093	0.075	0.101	0.110
Discount rate, Revenue	0.063	0.053	0.067	0.067
Value (10yr rate)	3.62	-3.76	0.00	-11.83

Fig. 1 Current Output Price Distributions
0.5 Fractile or Median (Dotted)
0.9 and 0.1 Fractiles
Half-life = ∞ (Solid), 3yr (Dashed)

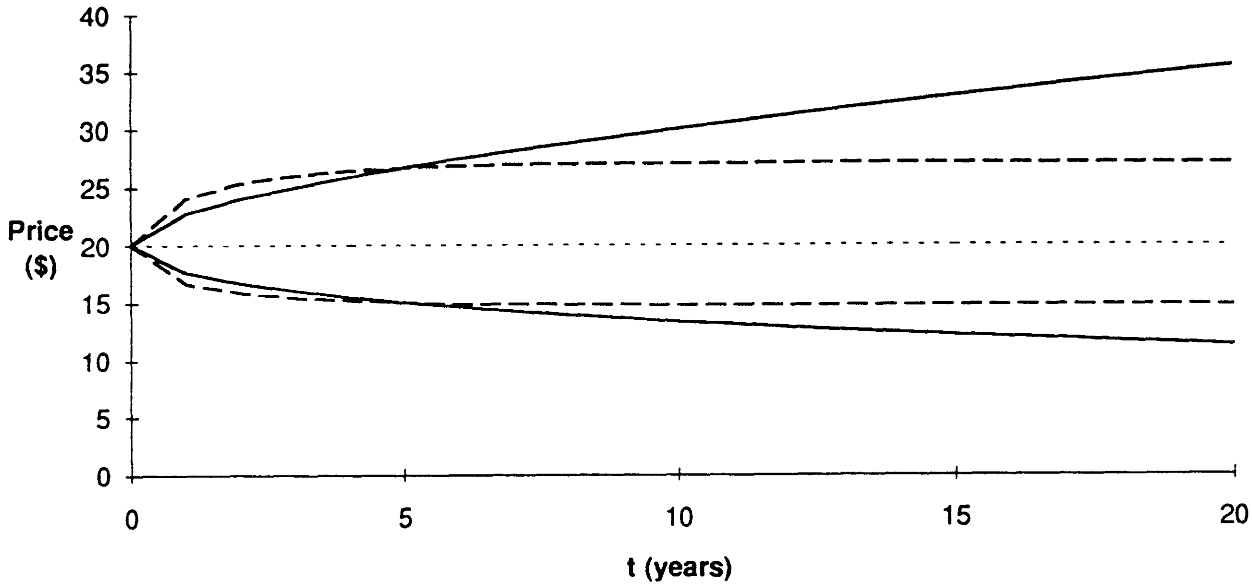

Fig. 2 Call Obligation Value vs. Maturity Time
Half-life = 1yr, 3yr, 5yr, ∞

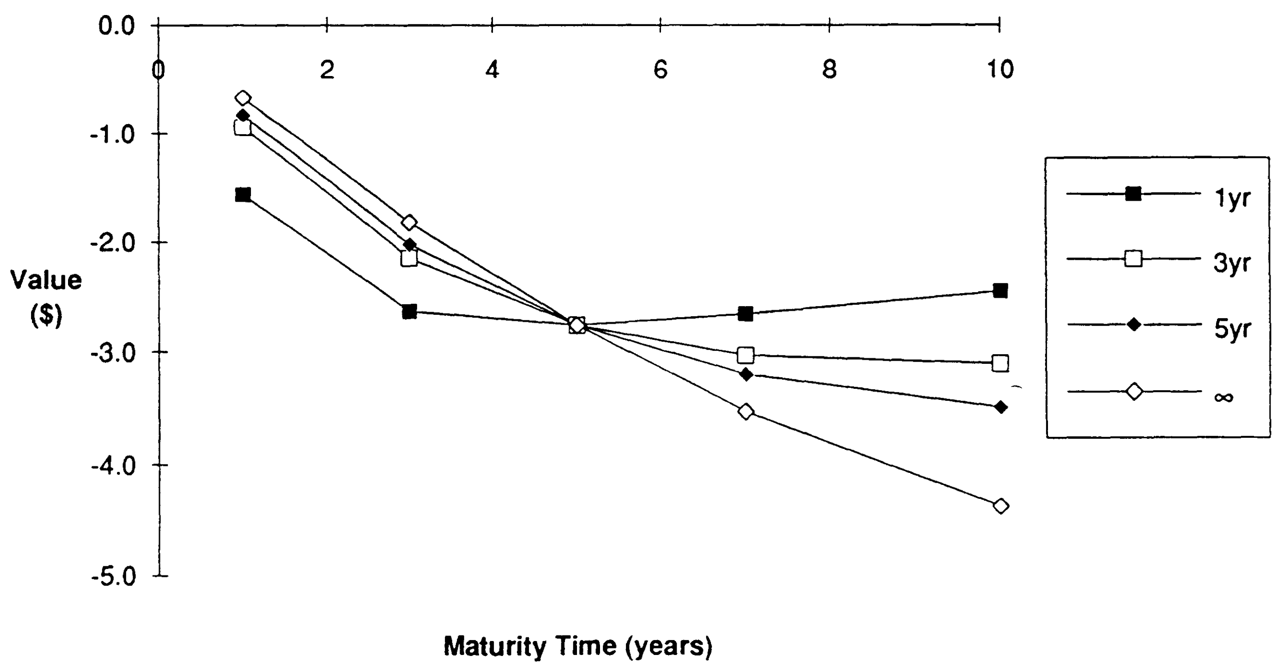

Fig. 3 European Put Value vs. Maturity Time
Half-life = 1yr, 3yr, 5yr, ∞

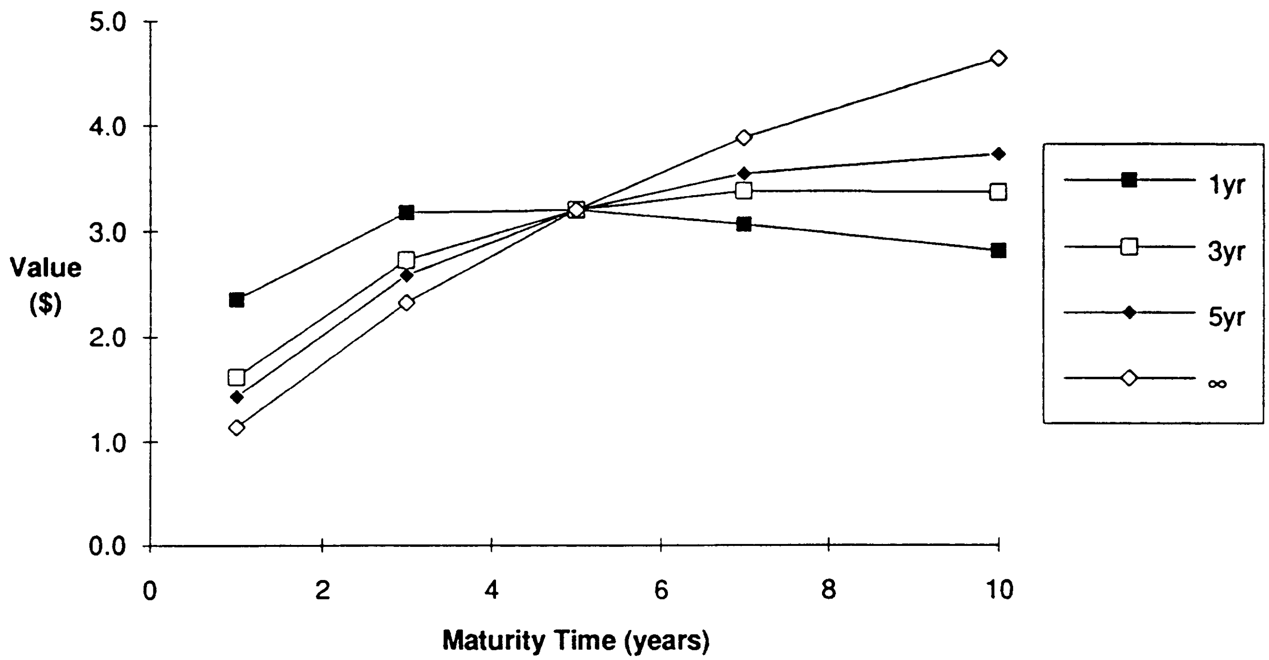

Fig. 4 European Call Value vs. Maturity Time
Half-life = 1yr, 3yr, 5yr, ∞

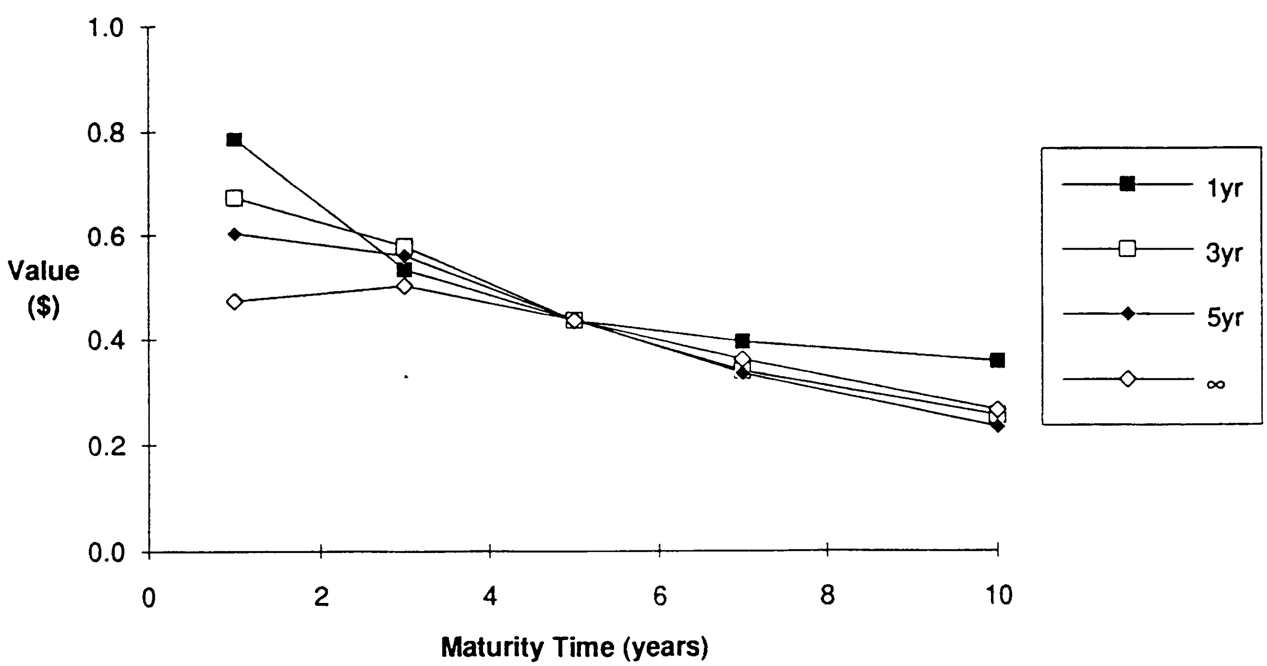

Fig. 5 Call Early Exercise Premium vs. Maturity Time
Half-life = 1yr, 3yr, 5yr, ∞

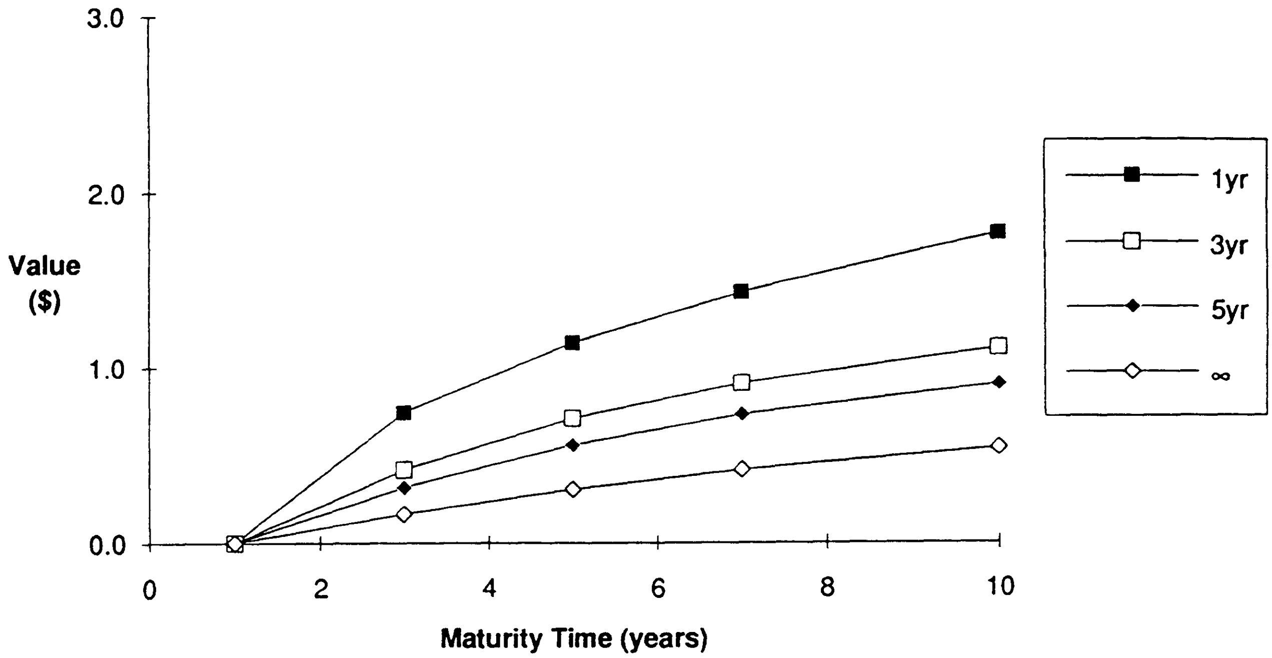

Fig. 6 American Call Value vs. Maturity Time
Half-life = 1yr, 3yr, 5yr, ∞

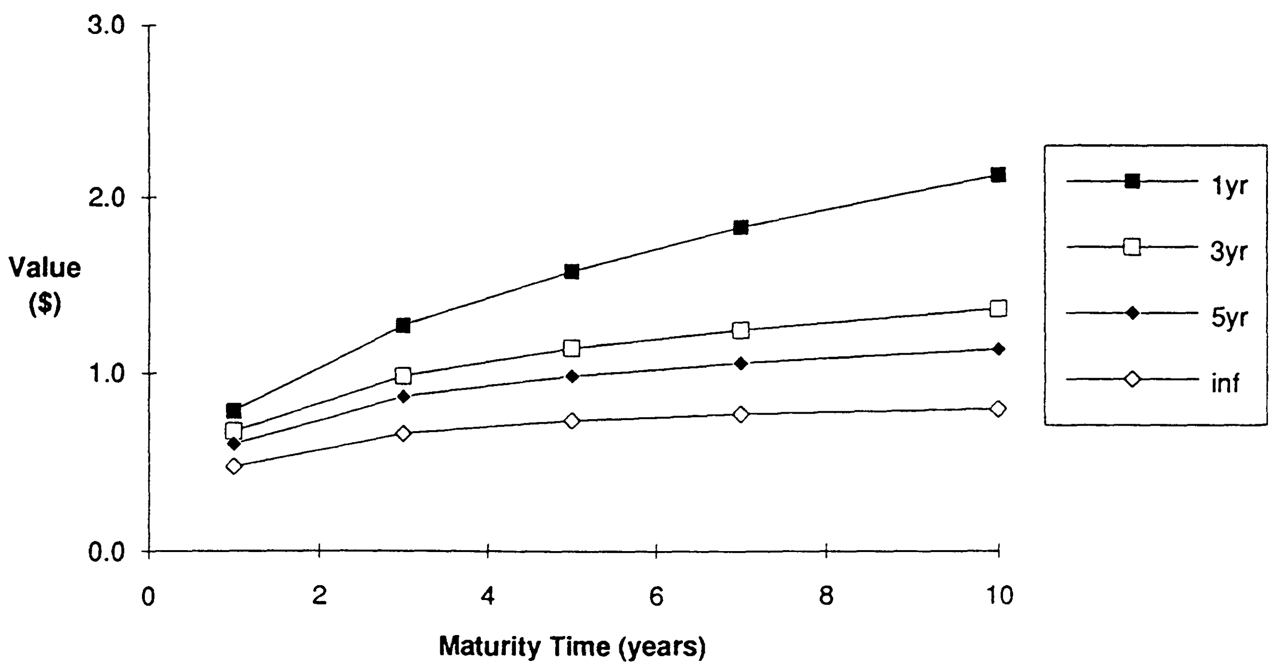

Fig. 7 Now or Never Project Value vs. Starting Price
Project Length = 10yr, 20yr
Half-life = 3yr, ∞

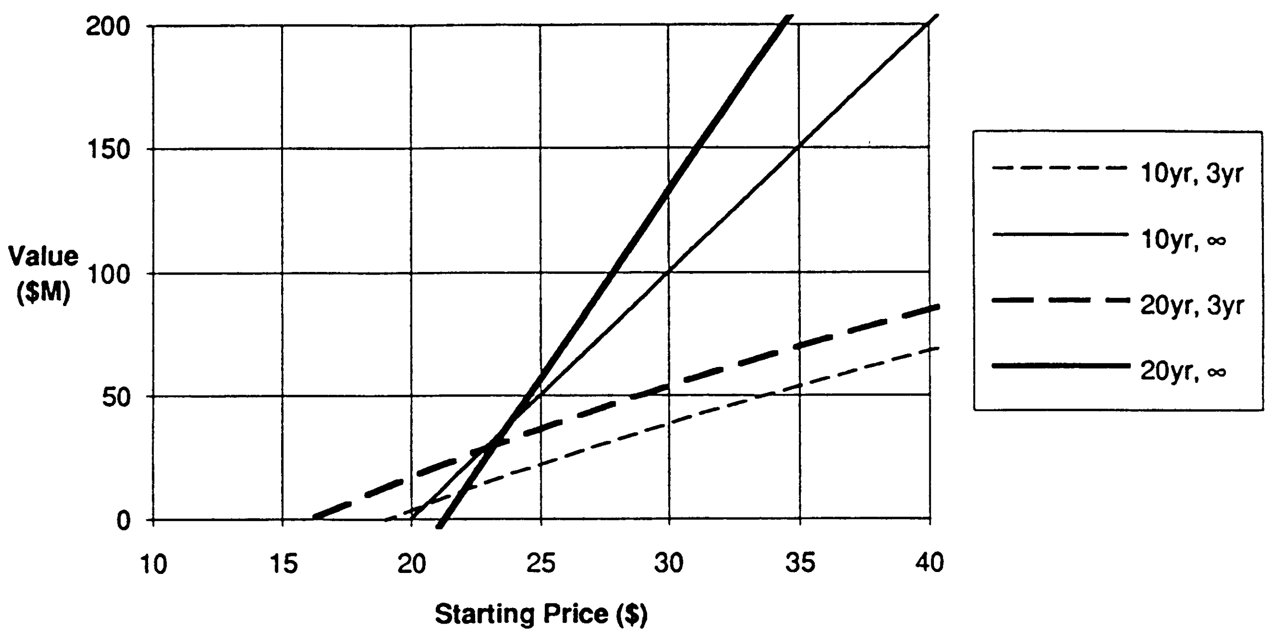

Fig. 8 5-Year Timing Options
Critical Starting Price vs. Starting Time
Project Length = 10yr, 20yr
Half-life = 3yr, ∞

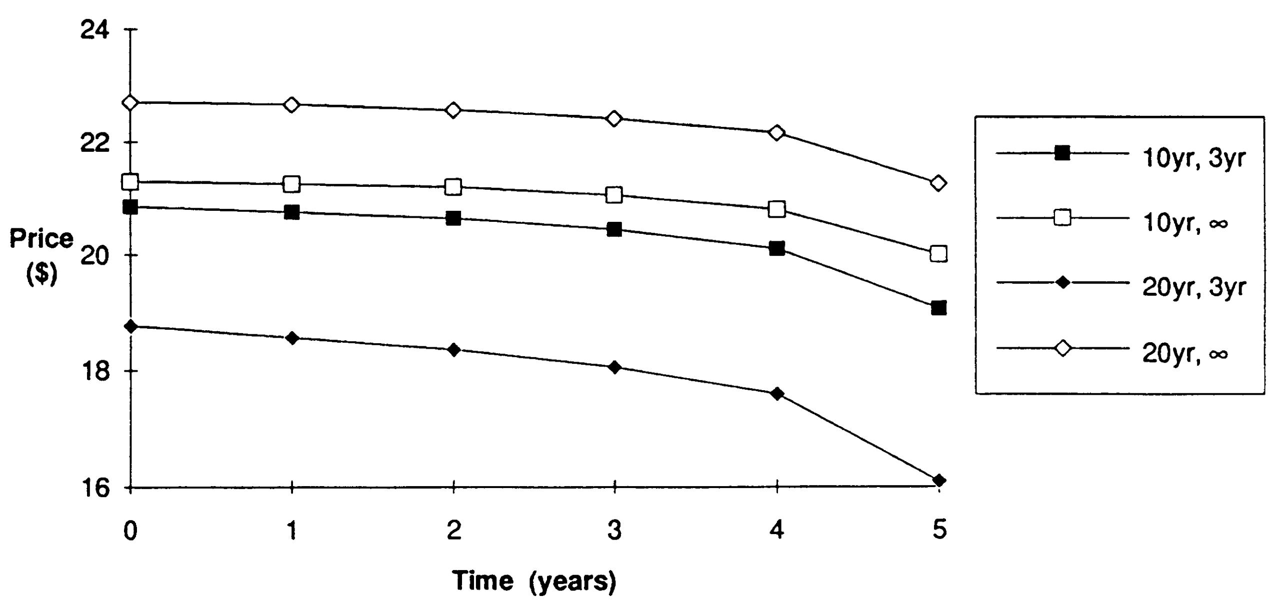

Fig. 9 Project Timing Option Value vs. Maturity Time
Project Length = 10yr, 20yr
Half-life = 3yr, ∞