

Discounting Rules for Risky Assets

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ABSTRACT

This paper develops a new rule for calculating the discount rate to value risky projects. The rule works under any linear asset pricing model and any equilibrium theory of debt and taxes. If securities are priced by the standard capital asset pricing model, the discount rate is a weighted average of the after-tax Treasury rate and the expected rate of return on the market portfolio, where the weight on the market portfolio is the project beta. We prove that this discount rate gives the correct project value and explain why it works. We also recast the rule in certainty equivalent form, restate it for multifactor capital asset pricing or arbitrage pricing models, and derive implications for the valuation of real options.

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DISCOUNTING RULES FOR RISKY ASSETS

Stewart C. Myers and Richard S. Ruback*

I. INTRODUCTION

We still do not understand the role of taxes in determining optimal capital structure, if there is an optimal capital structure. Therefore, we have so far no general rule for calculating discount rates for capital investments which are partly debt-financed. The only bulletproof existing rules apply to two special cases. Risk-free, after-tax nominal cash flows should be discounted the after-tax risk-free interest rate. And projects that exactly duplicate the firm's existing assets, both in risk and financing, are correctly valued by discounting at the firm's weighted average cost of capital.

The discounting rules for these two special cases work regardless of "right" theory of debt and taxes. For example, Ruback (1986) shows that discounting safe, nominal cash flows at the after-tax Treasury rate works when the benefits to leverage equal the corporate tax shield, as in Modigliani and Miller (1963), when personal taxes fully offset the corporate tax shield, as in Miller (1977), and in intermediate cases. Ruback observes that any stream of riskless future cash inflows can be offset by a borrowing plan that exactly matches after-tax debt service to the cash inflows. Since debt service is covered exactly, the initial amount borrowed under the plan is the value of the stream. ¹

We present a discounting rule which can be used to value any risky cash flow stream under any linear asset pricing model and any equilibrium theory of debt and taxes. The discounting rule works if the corporation adheres to a specific financing policy for the project or if the corporation can hold common stocks for its own account without incurring significant taxes or transactions costs.

Our rule does not require exotic ingredients -- only the risk-free interest rate, the marginal corporate tax rate, a risk measure or measures for the cash flow stream, and the expected rate of return on a reference portfolio of common stocks. If a one-factor capital asset pricing model is assumed, as we do for convenience in most of this paper, the risk measure is the asset beta and the reference portfolio is the market portfolio. In that case, our rule is:

$$r^* = r_f(1 - T_c)(1 - \beta) + \beta r_m \quad (1)$$

where r^* is the discount rate;² $r_f(1 - T_c)$ is the nominal Treasury rate, after taxes at the marginal corporate rate T_c ; r_m is the expected rate of return on the market, and β is the project's asset beta, the beta of a direct equity claim on the asset's cash flows.

The intuition for this rule is straightforward. The right discount rate for a risky project is the expected rate of return on an equivalent-risk capital market investment. A firm could invest in Treasury (T) bills and the market portfolio to form a replicating portfolio with the same risk as the project. The replicating portfolio includes the fraction $(1-\beta)$ invested in T-bills at an after-corporate-tax return of $r_f(1 - T_c)$, and the fraction β invested in the market at an expected return of r_m . (We assume for now that corporate investment in equities is untaxed.) The replicating portfolio has the same beta as the risky project and provides an after-corporate-tax return of r^* as given by (1). That rate is the firm's opportunity cost of capital for the project.

Our rule can also be interpreted as a project-specific, after-tax, weighted average cost of capital (WACC):

$$\text{WACC} = r_D(1-T_c) (D/V) + r_E (E/V) \quad (2)$$

Here r_D and r_E are expected returns on debt and equity, D and E are market values of debt and equity, with $V = D + E$. Our rule sets the debt ratio, D/V , equal to $1 - \beta$ and therefore $E/V = \beta$. With these weights, if the debt is riskless (so that $r_D = r_f$), the equity has a beta of one and $r_E = r_m$. Thus, our rule simplifies the weighted average cost of capital approach in two ways. First, the debt and equity weights are determined by the project's asset beta, and do not require a judgment about debt capacity or optimal leverage. Second, there is no need to estimate the costs of debt and equity for the firm's own securities.

The information requirements for capital budgeting are reduced in our rule to the practical minimum. We do not claim this minimum is easy. The decision-maker cannot avoid judging or estimating project risk and the expected return on some equity security or portfolio.

Existing Theory and Practice

Taggart (1991) reviews and tabulates existing discount rate formulas. These fall into two classes.

WACC. The tax-adjusted weighted average cost of capital shown in Eq. (2) appears in Modigliani and Miller (1963) and has a long textbook tradition -- see, for example, Guthman and Dougall (1962), Chapter 7. WACC is robust and general, but it is almost always estimated for the firm as a whole, and thus applies only to projects which match the firm in risk and financing. Our rule is a project-specific WACC, where the weights, but not the "costs" of debt and equity, depend on project risk.

Tax-Adjusted discount rates. These formulas start with the project's opportunity cost of capital, that is the expected rate of return that would be demanded on an unlevered equity claim on project cash flows. This rate is then adjusted downwards to account for the net value of interest tax shields on debt supported by the project. The hard part is determining the net value of interest tax shields and the slope of the security market line when the relevant marginal personal tax rates are not observable.³ This may explain why tax-adjusted discount rates are rarely used in practice. Our rule avoids this problem because it uses rates of return defined before personal income taxes.

Taggart's and our reviews of the literature uncover no direct ancestors of our discounting rule. However, our idea of varying debt ratios project-by-project to keep the sum of business and financial risk constant is not new. Perhaps its earliest expression is in Solomon (1963, pp. 76-78).

Outline

The next section of the paper offers two proofs of our discounting rule. Section III explains more specifically why it works. It turns out that the rule values projects as if firms face a Miller "Debt and Taxes" equilibrium in which there is no net tax advantage to borrowing -- yet the rule also works under MM's "corrected" theory in which there is a strong tax advantage. This is not as paradoxical as it sounds. Firms that borrow in an MM world of course gain valuable interest tax shields. However the slope of the security market line is less than in a Miller world. These two effects cancel.

Each of our initial proofs relies on special assumptions. However, the rule is not tricky or fragile. The special assumptions of Section II are mostly relaxed by the end of Section IV. Section IV also recasts the rule in certainty equivalent form, restates it for

multifactor capital asset pricing or arbitrage pricing models, and derives implications for the valuation of real options.

Section V sums up and notes certain implications of our analysis for capital structure theory and the takeover business.

II. A DISCOUNTING RULE FOR RISKY ASSETS

Consider an asset generating a single cash flow $X \equiv E(\tilde{X})$ to be received next period. \tilde{X} is net of corporate taxes. However, these taxes do not reflect any interest tax shields on debt associated with, or supported by, \tilde{X} . In other words, the corporate tax to be paid on the asset is calculated assuming all-equity financing. This follows standard capital budgeting practice. We assume that the firm has enough taxable income, from either the asset being valued or from other assets, to realize interest tax shields immediately upon payment of interest. We also ignore transaction costs and other possible market imperfections.

We assume the standard capital asset pricing model. The market portfolio is a convenient reference because it is actively traded and probably fairly priced. Its expected return should be easier to estimate than expected returns on other equity portfolios or specific common stocks.

Our risk measure is β . However, we make no specific assumptions about the intercept and slope of the security market line.

We now present two proofs. The proofs differ in assumptions and approach. They are not just two different versions of the same story. In the *own-equity financing* proof, the firm finances the project through riskless borrowing and its own equity. The firm sets the project's capital structure to equate the expected rates of return on the project's equity and the market portfolio. This gives

the discount rate for cash flows to the project's equity. The discount rate for overall project cash flows is then easily calculated.

In the *corporate opportunity cost* proof, investment in the project is traded off against investment in a mixture of the riskless asset and the market portfolio. This mixture replicates the project's expected cash flow and risk. The cost of this replicating portfolio determines the value of the project to the firm.

Own-equity financing

Suppose the firm "finances" the project with $D = (1-\beta)V$ dollars of debt, where V is the yet-to-be-determined project value, and β is its asset beta. In all cases the firm's equity claim on the project is worth βV . If $\beta > 1$, $(1-\beta)V$ is negative, that is, the firm borrows nothing and *lends* $(\beta - 1)V$ by investing in safe, short-term marketable securities.

The initial market value balance sheet for the project is:

$V = V(\tilde{X}, D)$ <hr style="width: 80%; margin: 0 auto;"/> V	$D = (1-\beta)V$ $E = \beta V$ <hr style="width: 80%; margin: 0 auto;"/> V
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Note that V may depend on debt policy. We do not assume that borrowing $(1-\beta)V$ is the best policy, only that it is a feasible policy. We do assume, provisionally, that the beta of $V(\tilde{X}, D)$ does not depend on D .⁴

The next step pins down the "cost of debt." We assume that debt is risk-free, so that the firm can, or could, borrow against this project at the rate r_f . This is an unusual assumption which we relax

later. However, in the absence of transaction or information costs, risk-free borrowing is a close-to-feasible strategy. Remember that \tilde{X} is realized after a short wait. If information about \tilde{X} arrives continuously (no jumps in V) then debt secured by \tilde{X} is risk-free in the limit as the wait shrinks to zero and debt levels are continuously rebalanced.⁵

This is consistent with the underlying assumptions of the conventional weighted average cost of capital. As Miles and Ezzell (1980) show, that approach implicitly assumes that debt and equity claims are rebalanced each period to maintain a constant market-value debt ratio.

The beta of the value-weighted portfolio of debt and equity equals the asset beta. With risk-free debt ($\beta_D = 0$),

$$\beta = \beta_D(D/V) + \beta_E(E/V) = \beta_E(E/V) \quad . \quad (3)$$

Rearranging (3), and substituting $D = (1-\beta)V$ and $E = \beta V$,

$$\beta_E = \beta \left(1 + \frac{D}{E}\right) = \beta \left(1 + \frac{1-\beta}{\beta}\right) = 1 \quad .$$

Since the equity beta equals one, $r_E = r_m$, the expected rate of return on the market portfolio.

At the end of the period, the expected inflow equals the expected after-tax cash flow from the project, X , plus the interest tax shield, $r_f T_c D$. The expected outflow to equity is $\beta V(1 + r_m)$ and the outflow to debt is $(1 - \beta)V(1 + r_f)$. We value the project by equating the end-of-period expected inflows and outflows.

$$X + r_f T_c D = D(1 + r_f) + E(1 + r_m). \quad (4)$$

Substituting the financing policy, $D = (1 - \beta) V$ and $E = \beta V$, and rearranging provides:

$$X = V(1 + \cancel{(1 - \beta) r_f(1 - \tau_c)} + \beta r_m) = V(1 + r^*).$$

The value of the after-tax cash flow, V , is calculated as:

$$V = X/(1 + r^*) \quad (5)$$

In application, (5) is the starting point, not the end result. The firm forecasts X , and discounts at r^* to obtain V . Then it can issue debt of $(1 - \beta)V$. Our proof shows that the actual market value of \tilde{X} (or of the debt plus the residual equity claim) is in fact V under the assumed financing policy.⁶

If the firm chooses not to follow this strategy, our discounting rule still works if such a financing strategy is feasible, *or if there is a feasible alternative financing strategy which yields the same project value*. In particular, suppose the firm chooses not to rebalance continuously, so that debt issued to finance the project is not riskless. This does not invalidate our proposition, because the net present value (NPV) of issuing risky debt is the same as the NPV of safe debt. Substituting the risky debt for the safe debt has NPV of zero in perfect and efficient capital markets. Thus the value obtained from using our rule is still correct. We discuss this further below.

Corporate Opportunity Cost

The opportunity cost of capital for a risky project can be determined by constructing a portfolio that replicates the expectation and risk of the project's after-tax cash flow. Since the firm can invest in either the project or the replicating portfolio, the expected return on the replicating portfolio is the appropriate project hurdle rate.

Let r_p be the expected after-tax rate of return on a portfolio of securities with the same risk and expected after-tax cash return as the project. The replicating portfolio is constructed by investing $(1-\beta)V$ in the riskless asset and βV in the market portfolio, where V is the yet-to-be-determined project value. The expected payoff of this replicating portfolio is:

$$V(1 + r_p) = (1 - \beta) V (1 + r_f(1-T_c)) + \beta V(1 + r_m). \quad (6)$$

The first right-hand-side term of (6) assumes that the firm pays taxes on riskless investments at the marginal tax rate, T_c ; the second right-hand-side term assumes the firm does not pay taxes on investments in the market portfolio.

The risk of the replicating portfolio is also determined by its covariance with the market portfolio.

$$\text{Cov}(V(1 + \tilde{r}_p), \tilde{r}_m) = V\text{Cov}(\beta\tilde{r}_m, \tilde{r}_m) = V\beta\sigma_m^2. \quad (7)$$

Substituting the definition of the asset beta, $\beta \equiv \text{Cov}(\tilde{X}, r_m) / V\sigma_m^2$, into (7) shows that the project and the replicating portfolio have the same covariance.

The after-tax cash flow has an expected payoff of X . We find the investment V in the replicating portfolio which likewise offers X

$$V(1 + \underbrace{(1 - \beta) r_f(1 - T_c) + \beta r_m}_r) = X \quad (8)$$

Rearranging (8),

$$V = X / (1 + (1 - \beta) r_f(1 - T_c) + \beta r_m)$$

$$V = X / (1 + r^*).$$

This completes the proof.

The corporate opportunity cost proof could also be run in reverse. The firm could sell $(1 - \beta)V$ of the riskless asset and short-sell βV of the market portfolio. This effectively sells a future cash flow X for immediate proceeds V .

If the firm can buy or sell X for the price V , then the value of an equivalent-risk investment project generating X must also be V . If the project can be taken on for less than V , it must have a positive NPV.

The reversed corporate opportunity cost proof connects to the own-equity financing proof. The firm could work through that analysis and then actually issue debt secured by the project and a residual equity claim. That claim would have $\beta=1$ and offer the same return as the market portfolio. This financing package would also be the replicating portfolio for the project.

Application to Long-Lived Assets

Moving from one to t -period discounting is easy once the t -period financing policy is specified. Our discounting rule can be applied period by period if debt is adjusted to the specified fraction of market value at the start of each period.

We assume that an unlevered equity claim \tilde{X}_t can be properly valued by discounting at a constant risk adjusted rate. That in turn means that the ingredients of our discount rate (i.e., β , r_f , and r_m) are also constant,⁷ and that equation (1) generates the same r^* for each future period.

Think of how V^u , the value of an unlevered claim on \tilde{X}_t , is determined at $t-2$. It is:

$$V_{t-2}^u = \frac{E_{t-2}(\tilde{V}_{t-1}^u)}{1+r} = \frac{E_{t-2}(\tilde{X}_t/(1+r))}{1+r} = \frac{E_{t-2}(\tilde{X}_t)}{(1+r)^2}$$

In other words, the unlevered value of \tilde{X}_t at $t-2$ is the expectation of its uncertain value at $t-1$, which in turn is linked to the expectation of \tilde{X}_t given information available at $t-2$.

The value \tilde{X}_t at $t-1$ under our assumed financing policy is proportional to V_{t-1}^u :

$$\tilde{V}_{t-1} = \frac{\tilde{E}_{t-1}(\tilde{X}_t)}{(1+r^*)} = \tilde{V}_{t-1}^u \left(\frac{1+r}{1+r^*}\right)$$

Given this proportional link, the "asset beta" of \tilde{V}_{t-1} as viewed from $t-2$ is identical to the beta of \tilde{V}_{t-1}^u viewed from the same point.

We can therefore treat \tilde{V}_{t-1} as if it were a cash payoff to investment at $t-2$. The cash payoff is discounted at r^* .

This argument obviously repeats for $t-3$, $t-4$, etc. In general:

$$V_0 = \frac{E_0(\tilde{X}_t)}{(1+r^*)^t} = \frac{X_t}{(1+r^*)^t}$$

Tax shields and discount rates. So far we have assumed that the risk of the total cash payout to debt and equity does not depend on the debt amount. This is not always right, because the corporate interest tax shield $T_c r_f D$ is a safe nominal flow, received when

interest is paid next period. The overall beta of debt and equity is thus reduced by borrowing whenever interest tax shields contribute to firm value.⁸ If they do contribute, our rule could be modified by putting more weight on $r_f(1-T_c)$, the after-tax risk-free rate, and less on the market return r_m . However, the modification makes hardly any difference. Exact and approximate rates rarely differ by more than half a percentage point.⁹ This difference easily fits in the typical confidence band for a cost of capital estimate.

Tax shields and unlevered betas. Asset betas are often calculated by unlevering observed equity betas. Several unlevering formulas have been developed. We suggest¹⁰

$$\beta_u = \beta_D D/V + \beta_E E/V \quad (9)$$

Where β_D and β_E are the betas of the firm's outstanding debt and equity. Of course $\beta_D \equiv 0$ if the firm follows our financing policy.

Equation (9) is almost exact when the firm rebalances its capital structure to maintain a constant D/V at the start of each period. Consider an MM world where firm value includes the full value of interest tax shields. The dollar payoff to debt and equity investors at $t = 1$ is $\tilde{X}_1 + \tilde{V}_1 + r_f T_c D_0$. The market value balance sheet is:

$V(\tilde{X}_1 + \tilde{V}_1)$	D
$V(r_f T_c D_0)$	E
V	V

Interest tax shields received in $t - 2$ may increase \tilde{V}_1 , but the value of these shields, viewed from $t = 0$, is strictly proportional to the unlevered value of the firm. The only safe (zero-beta) cash flow is next year's interest tax shield; its beta is zero.

We want the asset beta, that is the beta of $V(\tilde{X}_1 + \tilde{V}_1)$. Equation (9) gives us a weighted average of the asset beta and the beta of $V(r_f T_c D_0)$, i.e. zero. However, the present value of next year's interest tax shield is a very small fraction of overall firm value, and for practical purposes we can take β_u as an estimate of β .

Review of Assumptions

We now step back to review the assumptions supporting the two proofs of our discounting rule.

Taxation of intercorporate investment. Managers who calculate and use the weighted average cost of capital in practice routinely make unrealistic assumptions. For example, they assume that their firm will rebalance its capital structure to maintain a constant market-value debt ratio. The only unusual unrealistic assumption in the corporate opportunity cost proof is that the firm pays no tax on investments in the market portfolio. The effective tax rate on such investments -- call it T_{CM} -- is less than T_c because only 20 percent of intercorporate dividends are taxed. However, T_{CM} is clearly positive.

Of course a firm actually investing in equities would do what it could to minimize T_{CM} . For example, it could invest in a high-dividend portfolio with $\beta = 1$. Even better, it could buy the market via an overfunded pension plan, or take over another company, adjusting leverage so that the target's $\beta = 1$. It could buy its own shares -- our rule follows immediately if the firm's capital structure is chosen to give $\beta_E = 1$. Finally the corporate opportunity cost proof could be run in reverse, with the firm issuing an equity claim with $\beta = 1$.

Nevertheless, just assuming $T_{CM} = 0$ is not entirely plausible. Fortunately, that assumption is not required in the own-equity

financing proof. Each proof's unusual assumption is not required by the other.

Risk-free borrowing. The own-equity financing proof assumes the firm could finance with default-risk-free debt. Corporate debt would be (close to) risk-free if the firm committed to frequent rebalancing to maintain a market-value debt ratio less than one for the firm as a whole. This kind of rebalancing would raise no eyebrows in other financial contexts, for example, hedging strategies in options trading, or margin loans. Margin loans are virtually risk-free so long as the shares serving as collateral are actively traded and margin calls are promptly executed if the shares fall. It does not matter how risky the shares are so long as price discontinuities are ruled out.

However, corporations do not rebalance frequently and consistently. Therefore corporate lenders are exposed to default.

We think of the firm as implementing a two-step strategy:

1. Buy an asset, issuing new debt of $(1-\beta)V$. The debt is rebalanced at short intervals and therefore is close to risk-free.
2. Buy back the risk-free debt with a longer-term debt issue which is not rebalanced and therefore may default.

Step 2 is clearly zero-NPV in a taxless, frictionless world. If it is actually close to zero-NPV, then we can analyze capital investments as if financing were as assumed in the own-equity financing proof.

The real issue, then, is whether taxes or market imperfections make step 2's NPV positive or negative. We return to this point in Section IV.

Possible tax gains from leveraging up. The own-equity financing proof values projects correctly under any equilibrium theory of debt and taxes, so long as the corporation adheres to a specific financing policy for the project. We do not claim that this financing policy is optimal, only that it is feasible. If there is a different optimal policy, *and if the manager knows what that policy is*, extra value can be generated.

In general, using more debt than $(1-\beta)V$ increases the value of the firm if there are significant tax advantages to corporate debt. We emphasize that additional leverage may increase the value of the *firm* -- not the value of the *project* -- because the firm can obtain these advantages without the project. The firm could simply buy the replicating portfolio (or some other company) instead of the project and lever *it* up. Possible benefits from using more debt than in our financing strategy accrue to the tax status of the firm, not to the project, to the extent that the firm can buy common stocks tax-free.

Suppose a proposed investment offers a negative NPV at our discount rate r^* . Could investment ever be justified by leveraging up the project? Only if two conditions hold. First, the firm must already have reached the "debt capacity" of its other assets -- otherwise the firm could borrow more without taking the immediate project. Second, investment by the firm in the market portfolio or other equities must be closed off, taxed, or more costly than direct investment by the firm's shareholders. If the firm can earn r^* on a replicating portfolio, there is no reason to take less from the project.

Asset pricing. Our two proofs are not based on arbitrage. Both the replication and self-financing approaches require a capital asset pricing model to identify the priced characteristics of the project. The proofs match parameters -- expected return and covariance -- not realized cash flow. There is always residual risk to be absorbed or hedged by the firm and its shareholders. However, this residual

risk is unpriced, because it is diversifiable and investors do not demand a premium for bearing it.

III. Net Corporate Tax Advantages to Debt Financing

Our proposition and its two proofs focus on corporate taxes and rely on the market prices of securities to fully incorporate personal taxes. We now show why our valuation approach works under various theories of debt and taxes as long as the firm adheres to the debt policy that underlies our discounting rule.

Define T_{pe} and T_{pd} as the marginal investor's personal tax rates on equity and interest income, respectively. Also, define r_{fe} as the expected rate of return on zero-beta equity. If the personal tax rates on equity and interest income are equal, as in Modigliani and Miller (1963), then r_{fe} equals the riskless rate, r_f . In general, the after-personal-tax rates on safe debt and zero beta equity must be the same in equilibrium:

$$r_{fe}(1-T_{pe}) = r_f(1 - T_{pd}). \quad (10)$$

Thus in Miller's (1977) model, where the personal tax rate on equity income is zero and the marginal investor's personal tax rate on debt equals the corporate rate, the expected return on zero beta equity equals the after-corporate-tax riskless rate: $r_{fe} = r_f(1 - T_c)$.

We do not know the expected return on zero-beta equity or the personal tax rates of the relevant marginal investor. We assume the firm knows the market-wide rates of return, r_f and r_m , but not r_{fe} . The security market line for equities is $r = r_{fe} + \beta(r_m - r_{fe})$, but the intercept and slope of the security market line are unknown.

Figure 1 shows three possible security market lines: first, the "MM" line with $r_{fe} = r_f$, which matches the original capital asset pricing model; second, the "Miller line" with

$T_{pe} = 0$ and $r_{fe} = r_f(1 - T_c)$, and finally an intermediate case. The MM line implies a strong tax advantage to corporate borrowing, the intermediate line a weaker advantage, and the Miller line no advantage at all. We do not know which line is right. But the value of a future cash flow does not depend on the model so long as the firm adheres to the debt policy underlying our discounting rule.

Define T^* as the net corporate tax gain from one dollar of interest payments, including the effect of personal taxes. Suppose the firm issues debt paying one dollar of interest per year, using the debt proceeds to retire equity. The corporate tax shield is T_c , or $T_c(1 - T_{pe})$ after equity investors' taxes. The switch to debt also subjects one dollar of interest income to personal tax at T_{pd} rather than T_{pe} , at a cost to investors of $T_{pd} - T_{pe}$. The net gain after all taxes is $T_c(1 - T_{pe}) - T_{pe} + T_{pd}$. Divide by $1 - T_{pe}$ to express this as a before-personal-tax amount.¹¹

$$T^* = T_c - \frac{(T_{pd} - T_{pe})}{1 - T_{pe}} \quad (11)$$

The obvious cases are "MM", where $T_{pd} = T_{pe} = 0$ so that $T^* = T_c$, and "Miller" with $T_{pe} = 0$, and $T_{pd} = T_c$, so that $T^* = 0$.

Given some security market line, and thus some discount rate r for an unlevered equity claim on X , market value can be calculated using the adjusted present value (APV) approach. V is the sum of a pure equity claim on X plus the value of the interest tax shields:¹²

$$V = \frac{X}{1 + r} + \frac{T^* r_f (1 - \beta) V}{1 + r} \quad (12)$$

where $(1 - \beta)V$ is the debt issued against X ; r is the discount rate for an all-equity claim to the cash flow; and $T^* r_f (1 - \beta) V$ is the net interest tax shield after personal as well as corporate taxes.

Rearranging,

$$V = \frac{X}{1 + r - T^* r_f (1 - \beta)} \quad (13)$$

Thus the APV formulation implicitly discounts the risky after-tax cash flow at $r - T^* r_f (1 - \beta)$. We can now check whether this implicit discount rate always equals our r^* .

Compare our rule with the CAPM expression for r , the all-equity opportunity cost of capital.

$$r^* = r_f (1 - T_c) (1 - \beta) + \beta r_m \quad (\text{Discounting rule})$$

$$r = r_{fe} + \beta (r_m - r_{fe}) = r_{fe} (1 - \beta) + \beta r_m \quad (\text{CAPM})$$

Subtract the second from the first equation and express r^* as:

$$r^* = r - (1 - \beta) (r_{fe} - r_f (1 - T_c)) \quad (14)$$

From (10), $r_{fe} = r_f (1 - T_{pd}) / (1 - T_{pe})$. Substitute for r_{fe} in (14) and simplify:

$$\begin{aligned} r^* &= r - (1 - \beta) r_f \left(\frac{1 - T_{pd}}{1 - T_{pe}} - 1 + T_c \right) \\ &= r - (1 - \beta) r_f \left(T_c - \frac{T_{pd} - T_{pe}}{1 - T_{pe}} \right) \\ &= r - (1 - \beta) r_f T^*. \end{aligned}$$

This proves that our discounting rule will work without specifying personal tax rates as long as the firm adheres to a financing policy of $D = (1 - \beta)V$.

Example

Suppose we observe $r_f = .10$ and $r_m = .20$. The corporate tax rate is $T_c = .5$. The cash flow's expected value is $X = 100$ and its beta is $.5$. Our discounting rule gives $r^* = (1 - .5) (.10) (1 - .5) + (.5) (.20) = .125$ and a value $V = 100/1.125 = 88.89$.

Table 1 shows that *exactly* the same value is obtained under three different assumptions about debt, taxes and the security market line. The calculations in Table 1 clarify why our discounting rule works under any equilibrium model of debt and taxes. If we move from Case 1 (MM) to Case 2 (Miller), the expected cash flow X loses value because T^* drops from $.50$ to zero. But it also gains value because the all-equity opportunity cost of capital, r , falls from 15 to 12.5 percent. The loss and gain exactly offset. Given r_f , r_m and T_c , and given our proposed financing policy, calculated value can never be increased by assuming a higher value for T^* because a consistent assumption about the security market line requires increasing r to offset the tax gain.

IV. ExtensionsCertainty Equivalents

We know that certain cash flows should be discounted at the after-tax risk-free rate, $r_f(1-T_c)$. Our discounting rule gives the same rate for uncertain cash flows in the special case of $\beta = 0$. We now show that the certainty equivalent of a positive-beta cash flow should also be discounted at $r_f(1-T_c)$.

Note first that the conventional discounting formula $V = X/(1+r^*)$, where $r^* = (1-\beta)r_f(1-T_c) + \beta r_m$, is algebraically identical to the certainty equivalent valuation model

$$V = \frac{X - \lambda \text{cov}(X, r_m)}{1 + r_f(1 - T_c)},$$

where the certainty equivalent cash flow is $CEQ = X - \lambda \text{cov}(X, r_m)$,

$$\text{and } \lambda = \frac{r_m - r_f(1 - T_c)}{\sigma_m^2} .$$

This result can also be obtained from Black's (1988) discounting rule. Black assumes the cash flow \tilde{X} can be expressed as a linear function of the return on the market, $\tilde{X} = a + b\tilde{r}_m + \tilde{\epsilon}$. The error term $\tilde{\epsilon}$ is diversifiable noise, and so we ignore it. Note that $a + b\tilde{r}_m = a - b + b(1 + \tilde{r}_m)$. The present value of $b(1 + \tilde{r}_m)$, the payoff to b dollars invested in the market, is simply b ; $(a - b)$ is a fixed cash flow which must be discounted at the after-tax risk-free rate. Therefore

$$V(\tilde{X}) = \frac{a-b}{1+r_f(1-T_c)} + b = \frac{a + br_f(1-T_c)}{1+r_f(1-T_c)} \quad (17)$$

The certainty equivalent, $CEQ = a + br_f(1-T_c)$, is the expectation of \tilde{X} conditional on $r_m = r_f(1-T_c)$.

This result does not assume or imply that r_{fe} , the unconditional expected return on a zero-beta stock, equals $r_f(1-T_c)$; r_{fe} , whatever it is, never enters the certainty equivalent valuation. ¹³

Consider the following certainty equivalent version of our own-equity financing proof. Note that the beta of $(a + b)$ is zero; the beta of $b(1+r_m)$ is by definition one. Then

$$\beta V = V(a+b) \times 0 + V(b(1 + r_m)) \times 1 = b$$

The implied balance sheet is:

$V = V(\tilde{X}, D)$	$D = (1-\beta)V = V - b$
V	$E = \beta V = b$
V	V

The cash returns on each side are equal regardless of the outcome \tilde{r}_m :

$$a + b\tilde{r}_m + T_c r_f (V - b) = (1 + r_f)(V - b) + b(1 + \tilde{r}_m)$$

The final term $b(1 + \tilde{r}_m)$ is the cash return to equity. Thus the cash return to debt is

$$a + b\tilde{r}_m + T_c r_f (V - b) - b(1 + r_m) = (1 + r_f)(V - b). \quad (18)$$

Solving for V ,

$$V = \frac{a + b r_f (1 - T_c)}{1 + r_f (1 - T_c)}$$

The certainty equivalent version of the corporate opportunity cost proof is also interesting. If equity investments by corporations are untaxed, the firm can obtain $X = a + b\tilde{r}_m$ by the following strategy.

	Cost	Payoff
<i>Lend:</i>	$\frac{a - b}{1 + r_f(1 - T_c)}$	$a - b$
<i>Invest:</i>	b	$b(1 + \tilde{r}_m)$
<i>Totals:</i>	$\frac{a + br_f(1 - T_c)}{1 + r_f(1 - T_c)}$	$a + b\tilde{r}_m$

The value of $a + b\tilde{r}_m$ must equal the cost of replicating it. This cost is found by discounting at the after-tax risk-free rate.

Valuing corporate options

Standard option valuation techniques "discount" at the risk-free interest rate. For corporations, the appropriate risk-free rate is after-tax. Ruback (1986) showed that the time value of money for a tax-paying firm is $r_f(1 - T_c)$. This rate should be used to value real options held by corporations.

Suppose we discount X at our r^* to obtain a value $V(X)$. The implicit equity claim is $E = \beta V(X)$ with $\beta_E = 1$. We can value a call on \tilde{X} by considering a portfolio of the call option plus lending which locally replicates the returns to the equity position in the underlying asset. This determines the value of the option to the corporation. This value is the same as obtained by valuing the option directly, using $r_f(1 - T_c)$ as the risk-free interest rate. Of course the cash inflows and outflows associated with the real option all have to be defined after corporate tax.

Example. Consider an asset with $\beta = .5$ and $r^* = .125$ as in Table 1. The payoff is binomial, either 150 or 75, with $X = 112.5$ and $V = 100$. The implicit debt and equity claims are $D = E = 50$. The after-tax interest rate is .05. Now consider a call on X at an exercise price of $S = 100$. The payoffs are:

Asset	Equity	Call (Max $\tilde{X} - S, 0$)
$\tilde{X} = 150$	$150 - 52.50 = 97.50$	$150 - 100 = 50$
$\tilde{X} = 75$	$75 - 52.50 = 22.50$	0

Replicate the equity by purchasing N_c calls and borrowing. The sum of principal and after-tax interest is B . Replication requires $50N_c + B = 97.50$ and $0xN_c + B = 22.50$. Thus $N_c = 1.5$.

The implied balance sheet is:

1.5 calls = 28.57	
Debt = 21.43	E = 50
50	50

Debt is $B/(1+r_f(1-T_c)) = 22.50/1.05 = 21.43$. Since 1.5 calls plus 21.43 must equal 50 (the value of the replicated equity claim), the value of one call is 19.05.

We can also value the call by first assuming the expected payoff to the assets is the risk-free rate, and then discounting the resulting expected payoff to the call. This requires "up" and "down" probabilities of .4 and .6, so that $.4(150) + .6(75) = 105$. The present value of the call is

$$\text{Call Value} = \frac{E[\text{Max}(\tilde{X} - S, 0)]}{1+r_f(1-T_c)} = \frac{.4(50)}{1.05} = 19.05.$$

Default risk and taxes

We now return to the own-equity financing proof's assumption of risk-free borrowing. This requires frequent rebalancing: in particular, the firm has to invest additional equity, and pay down debt, every time project value falls.

Suppose instead that non-recourse debt is issued against a project. The firm does *not* rescue bad outcomes with additional equity, and so acquires a default put. The *cost* of this put shows up as a reduction in the amount that can be borrowed against a given *promised* payment to lenders. If the default put is worth P , the implied project balance sheet is:

$V(X)$	$D = (1-\beta)V - P$
<hr style="width: 80%; margin: 0 auto;"/> V	$E = \beta V + P$
<hr style="width: 80%; margin: 0 auto;"/> V	<hr style="width: 80%; margin: 0 auto;"/> V

The firm "purchases" the default put worth P by investing that much more in the project. Lenders give up the put but invest P dollars less. In a perfect and efficient capital market, this transaction is clearly zero-NPV. Therefore we can ignore default and analyze project value as if only risk-free debt were issued.

Straightforward taxes do not disturb this argument, even if tax rates differ on the two sides of the put transaction. The put can be valued by (1) assuming that the underlying asset $V(X)$ offers an expected rate of return of $r_f(1-T_c)$, (2) calculating the expected

payoff to the put in the world assumed in step (1), and (3) discounting at $r_f(1-T_c)$. Let the expected pre-tax payoff to the put in step (2) be Y . If the cost of the put (P) is tax-deductible, the after tax payoff is $Y(1-T_c) + T_cP$. The put's value is

$$P = \frac{Y(1-T_c) + T_cP}{1 + r_f(1-T_c)} = \frac{Y}{1+r_f} \quad (19)$$

In other words, taxes wash out of the option valuation. If purchase of the put is zero-NPV after-tax, it is zero-NPV pretax, and conversely.

It would be rash to claim that taxes always wash out of the valuation of default puts. The intricacy of actual tax codes may hide features which in effect subsidize firms which issue risky rather than safe debt. However, we find no such subsidies in the obvious cases. A further discussion of these cases is given in the Appendix.

Multifactor Arbitrage Pricing Models

Our analysis can be extended to an environment in which asset prices are a linear function of more than one factor. Take Arbitrage Pricing Theory (APT) as an example. Under APT, the expected return on an equity security, r , equals:

$$r = r_{fe} + \beta_1 (r_1 - r_{fe}) + \beta_2 (r_2 - r_{fe}) + \dots + \beta_k (r_k - r_{fe}), \quad (20)$$

where r_{fe} is the return on riskless equities, r_1 through r_k are the expected returns to factors 1 thorough k , and β_1 through β_k are the asset betas for the respective factors. Imagine forming a composite portfolio in which the factors are weighted in proportion to the asset betas. The expected return of this portfolio, r_p , is

$$r_p = (\beta_1/B) r_1 + (\beta_2/B) r_2 + (\beta_k/B) r_k \quad (21)$$

where $B \equiv \beta_1 + \beta_2 + \dots + \beta_k$.

All our analysis can simply be repeated substituting r_p for r_m . In an APT environment, the discounting rule becomes:

$$r^* = (1-B) r_f (1-T_c) + B r_p. \quad (22)$$

The APT version of our discounting rule is more difficult to apply than the CAPM version. The APT requires the identification of multiple factors and estimation of expected returns and betas for each identified factor. Furthermore, the composite portfolio will change from project to project as the underlying asset betas change. In principle, however, our arguments follow through exactly for the APT, as they do for any linear capital asset pricing model.

V. CONCLUSION

This paper presents a rule for calculating discount rates for risky projects. The rule works for any linear capital asset pricing model. It works for any equilibrium theory of debt and taxes. It works because it treats all projects as combinations of two assets: Treasury bills and the market portfolio. We know how to value each of these assets under any assumption about the slope and intercept of the market line for equities.

Generality on these dimensions requires one of two special assumptions. The analyst can assume either (1) that project financing is arranged to keep the sum of business and financial risk constant, so that equity claims on projects match the risk of some reference portfolio of equities; or (2) that the firm can buy or sell that reference portfolio without incurring corporate taxes. The obvious (but not the only) reference portfolio is the market, which requires that the implicit risk of project equity be $\beta_E = 1$.

Each of these assumptions could be questioned. For example, if there are significant net tax gains to corporate borrowing, *and* if firms cannot buy (sell) equities without paying (saving) significant taxes, then firms could generate additional value by leveraging up each project so that $\beta_E > 1$. If such opportunities exist, and firms systematically pursue them, then correct project discount rates are less than ours. Even in this case, however, our rule would be helpful in establishing an upper bound for the correct discount rate and a lower bound for project value.

Our rule values projects as if security markets conformed to a Miller equilibrium in which there is no net tax advantage to borrowing and the intercept of the security market line is the after-tax Treasury rate. However, our discount rate does not change in other possible equilibria, because the security market line shifts to offset any net tax advantage to corporate debt.

Thus managers confronting capital investment decisions do not have to profess their faith in any theory of taxes and capital structure; we and they can stay agnostic.

However, we can not help observing that if value-maximizing corporations are given tax-free access to equity markets, then it's difficult to see how interest tax shields could have significant value in the long run. Suppose equities were priced by the traditional capital asset pricing model (the CAPM which says $r = r_f + \beta(r_m - r_f)$), which gives a strong tax value to corporate debt. This would allow corporations to make easy money. That CAPM says that all stocks with $\beta < 1$ offer expected rates of return higher than the hurdle rate established by our rule. A firm could buy any one of those stocks, financing the purchase with $100 \times (1 - \beta)$ percent debt, and earn the spread between the CAPM's r and our r^* . (Buying up all the shares of companies with low-beta stocks -- i.e., taking them over, would be even better.) This would continue until low-beta stock prices were bid up enough to drive r and r^* together. The end result would be a

Miller equilibrium, in which the security market line and the equation for our discounting rule exactly correspond.

Table 1

Calculating the Adjusted Present Value under different theories about debt and taxes

The example assumes an expected cash flow after one period, X, of \$100; an asset beta, β , of .5; a Treasury Bill rate of 10%; and an expected market return of 20%. The value of the project using the general discounting rule with $r^* = (1 - \beta)r_f + \beta r_m$ is:

$$V = \frac{X}{1 + r^*} = \frac{100}{1.125} = 88.89$$

PARAMETER VALUES	CAPITAL STRUCTURE THEORY		
	MM (Modigliani and Miller, 1963)	Miller, 1977	Intermediate
$T_{pd} - T_{pe}$ $T_C = 0$	$T_{pd} - T_{pe}$ $T_C = 0$	$T_{pd} - T_C = 50\%$ $T_{pe} = 0$	$T_{pe} = 10\%$ $T_{pd} = 30\%$ $T_C = 50\%$
NET CORPORATE GAIN PER DOLLAR INTEREST: $T^* = T_C - \{(T_{pd} - T_{pe}) / (1 - T_{pe})\}$	$T^* = 50\%$	$T^* = 0$	$T^* = 27.78\%$
EXPECTED RETURN ON ZERO-BETA EQUITY: $r_{fe} = r_f \{(1 - T_{pd}) / (1 - T_{pe})\}$	$r_{fe} = 10\%$	$r_{fe} = 5\%$	$r_{fe} = 7.78\%$
ALL-EQUITY COST OF CAPITAL: $r = r_{fe} + \beta(r_m - r_{fe})$	$r = 15\%$	$r = 12.5\%$	$r = 13.89\%$
ADJUSTED PRESENT VALUE OF THE PROJECT: $APV = \frac{X + T_C(1 - \beta)APV}{1 + r}$	$APV = \frac{100 + .2778(.1)(1 - .5)APV}{1.15}$ - 88.89	$APV = \frac{100 + 0(.10)(1 - .5)APV}{1.125}$ - 88.89	$APV = \frac{100 + .2778(.1)(1 - .5)APV}{1.1389}$ - 88.89

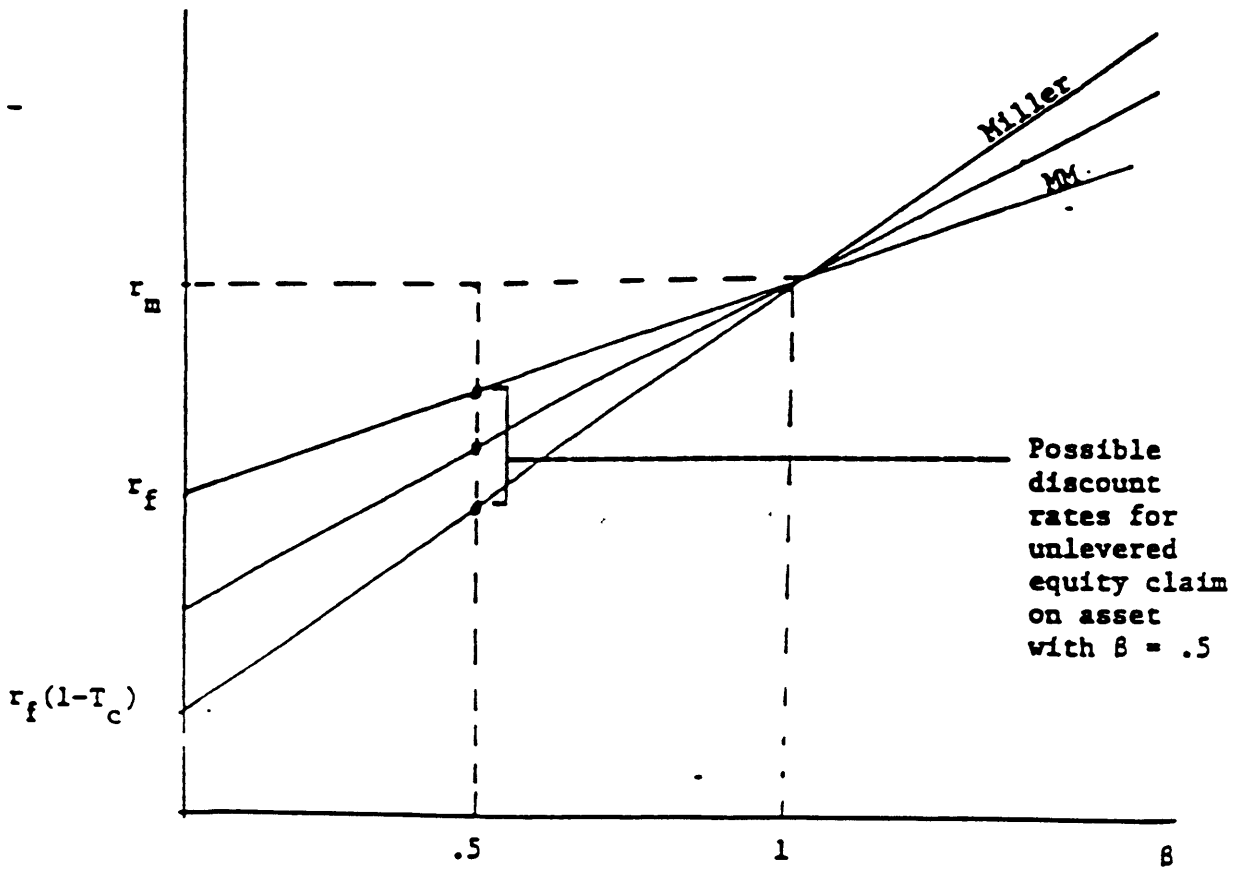


Figure 1

Security market lines implied by three theories of debt and taxes. For each case the intercept, r_{fe} , is given by $r_{fe} (1-T_{pe}) = r_f (1-T_{pd})$.

APPENDIX: DEFAULT RISK AND TAXES

One proof of our discounting rule assumes that default-risk free debt can be issued against the assets being valued. We argue that substituting risky for safe debt is a zero-NPV transaction, so that our rule works when there is default risk. The following examples illustrate the argument.

Think of the project as a "sub" of a corporate parent that will pay tax. The parent may choose whether to stand behind the sub's debt. The sub may be taxed stand-alone, or its profits and losses may be passed up to its parent.

The following numerical example is set up so that $NPV = 0$ in case 1, when the sub is taxed as part of the firm and does not borrow separately (the firm could borrow according to our rule). Then we show that *NPV is unchanged* if the sub issues debt which may default.

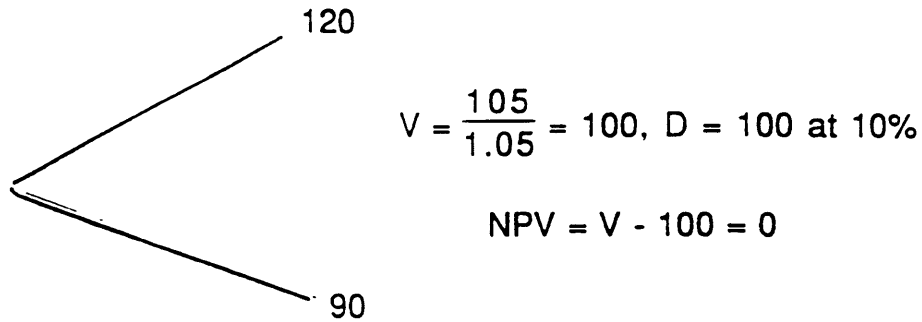
Example

The sub lasts only one period. It faces 50-50 probabilities of good and bad states. Suppose it invests 100 now. At period 1,

	<u>Good state</u>	<u>Bad state.</u> <u>consolidated</u>	<u>Bad state.</u> <u>stand-alone tax</u>
Pre-tax cash flow	140	80	80
<i>less</i> depreciation	<u>-100</u>	<u>-100</u>	
EBIT	40	-20	
Tax	<u>-20</u>	<u>+10</u>	
Net income	20	-10	
<i>plus</i> depreciation	<u>100</u>	<u>100</u>	
Cash flow	120	90	<u>80</u>

Note that with stand-alone tax the government has a call option on the assets of the firm.

The following examples assume $\beta = 0$, $D/V = 1.0$, $T_c = .50$, $r_f = .10$, and $r^* = r_f(1-T_c) = .05$. By our valuation rule $V = 100$ with consolidated tax:



Suppose the sub's debt is issued with recourse to the parent. The equity returns are

$$120 - 110 + 5 = +15$$

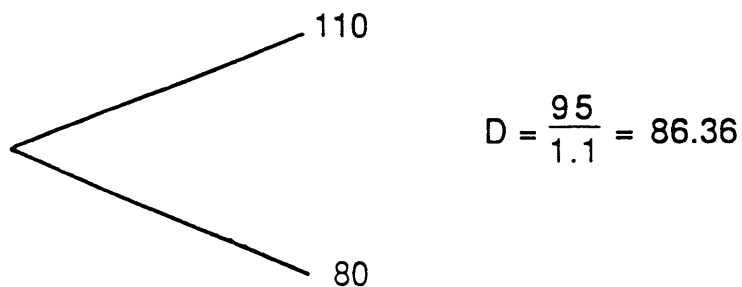
$$90 - 110 + 5 = -15$$

} $E = 0$
 Interest tax shield of 5
 Put in 20 to pay off debt

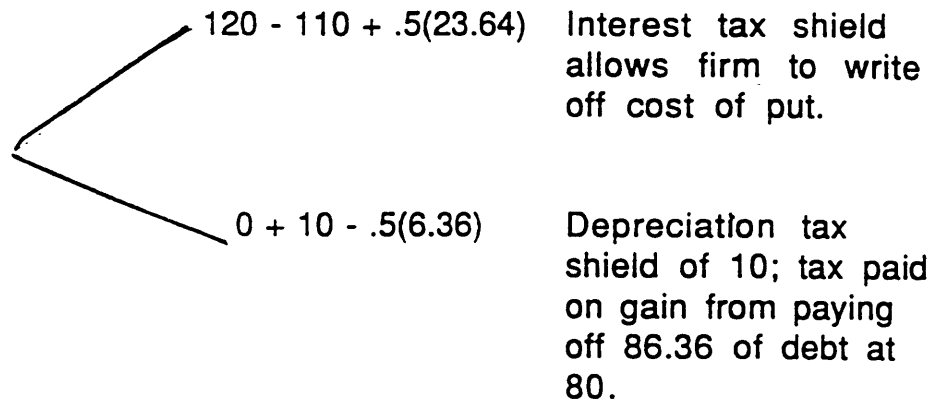
Now assume the sub issues non-recourse debt with a promised payoff of 110. The sub will default in the bad state. We consider this case for two tax regimes: in the first, the sub and parent are consolidated for tax purposes, even in default; in the second, the sub is taxed as a separate corporation.

Non-recourse debt, consolidated tax. Hold the *promised* payment to debt at 110. Assume that debt gets only 80 in the bad state because the parent keeps the depreciation tax shield. (The end result of $NPV = 0$ for equity is actually independent of the payoff to debt in the bad state. Any default put that's fair to debt investors has zero NPV to equity.)

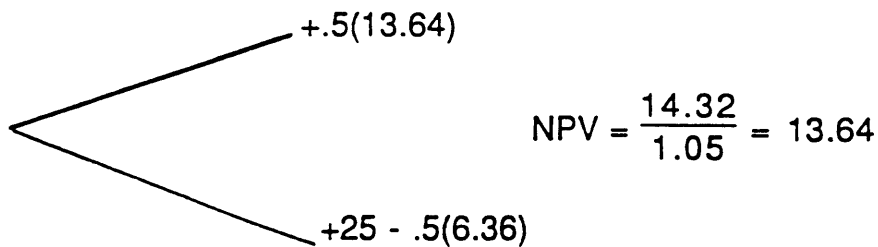
The debt will be sold for:



The equity returns will be:

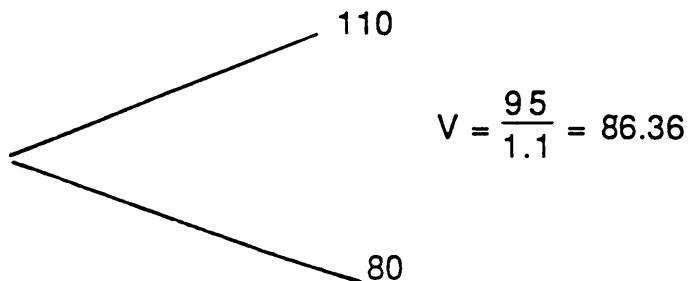


Compare these payoffs to those in case 1. The default put gives:

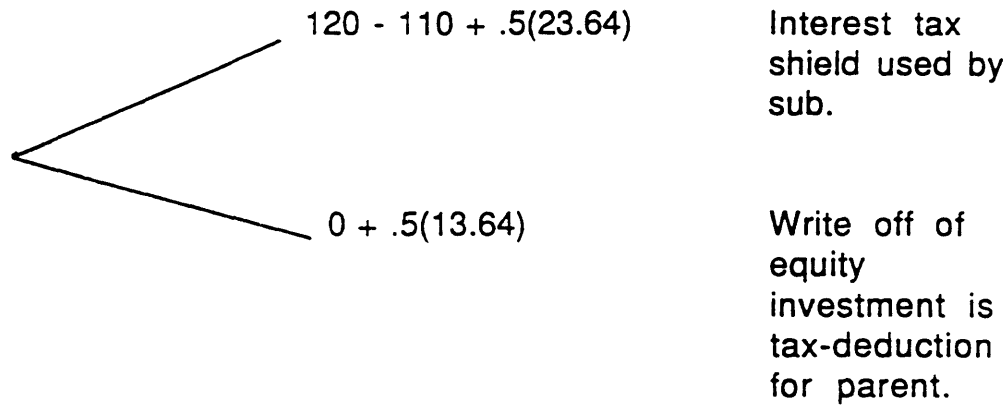


But the put cost 13.64 because the parent had to put up that much more equity to start with. Thus the put has zero NPV. We can value the sub as *if* the parent collateralized its debt.

Non-recourse debt, stand-alone tax. Here the sub is set up entirely on its own bottom. It issues debt of 86.36 against the promised payment of 110. The debt payoffs are



Thus the parent again has to put in 13.64 of equity. The payoffs to equity are:



Equity value is

$$E = \frac{.5(10 + .5(23.64)) + .5(5(13.64))}{1.05} = 13.64,$$

and the NPV of equity investment is again zero.

Conclusion

The own-equity financing proof does not require default-risk free debt. Substituting default-exposed debt for default-protected debt is a zero-NPV transaction. Thus projects or subs can be valued as if they were financed with default-risk free debt issued at the Treasury rate r_f .

FOOTNOTES

*This paper is a substantially expanded and generalized treatment of ideas first expressed in Myers and Ruback (1988). We thank Carliss Baldwin, Dick Brealey, Mervyn King, Lawrence Kolbe, James Miles, Bruno Solnik, and Jim Wiggins for helpful comments. We also thank MIT's Center for Energy Policy Research, the London Business School and the Harvard Business School for research support.

1. Franks and Hodges (1978) first used this argument to value financial leases.
2. The corresponding real r^* is calculated as $(1 + r^*)/(1 + i) - 1$, where i is the expected inflation rate.
3. The adjusted present value (APV) rules developed by Myers (1974) also require knowledge of (or an assumption about) marginal personal tax rates on debt and equity income. See Taggart (1991), Exhibit III.
4. The beta actually declines with D when interest tax shields add to firm value. See fn. 8 below. However, the error introduced by assuming $d\beta/dV = 0$ is trivial for long-lived assets when capital structure is rebalanced period by period. We show this below.

When D is continuously rebalanced, the β of $V(\tilde{X}, D)$ is totally disconnected from D . Debt tax shields may make $dV/(dD) > 0$, but the value of those tax shields will be strictly proportional to $V(\tilde{X}, 0)$ and therefore have the same beta as a pure equity claim on X .

5. This does not follow when $\beta = 0$ and $D = V$ unless \tilde{X} is known with absolute certainty. However, our assumed financing policy is still feasible if debt capacity can be poached from other positive-beta assets. Poaching is costless, since with continuously adjusted financing more debt can be issued against other assets without risking default.
6. Our rule implies $D = V$ and $r^* = (1 - T_c)r_f$ when $\beta = 0$. Perhaps this case requires further discussion. Note $\tilde{X} = V(1 + r^* + \tilde{e})$ where \tilde{e} is a

diversifiable "error" with zero mean. If $D = V$, then the equity claim receives only $\tilde{\epsilon}$, which must have $NPV = 0$. Thus $V = D = X/(1 + r^*)$.

Here is a consistency check. Suppose we are right that $V_0 = X_0/(1+r_f(1-T_c))$ -- here the subscript refers to "zero beta." Take another asset with value V_1 (for simplicity assume $V_1 = V_0$) and $\beta=1$. Now value the portfolio of V_0 and V_1 , which has $\beta = 1/2$.

$$V(X_0 + X_1) = \frac{X_0 + X_1}{1 + r_f(1 - T_c) (1 - \beta/2) + (\beta/2)r_m}$$

Now check that $V_0 = V(X_0 + X_1) - V_1 = X_0/(1 + r_f(1 - T_c))$. It does.

This is also a useful way of thinking about negative-beta assets. Suppose $\beta = -.5$ and $V = X/(1+r^*)=100$, and that the firm holds the asset plus 50 in the market (M) or some other asset with $\beta = 1$. The project balance sheet in this case is

$V = 100$	$D = 150$
$M = 50$	$E = 0$
<hr/> 150	<hr/> 150

The firm borrows 150 and holds assets with an overall beta of zero. The return $r_m M$ is used to pay part of the debt interest, so the net "cost of financing" is

$$100 (1 + r_f(1 - T_c) 1.5 - .5r_m),$$

which just offsets the expected payoff $X = V(1+r^*)$. In this case equity investors hold no net investment, and receive a payoff with zero mean, zero beta, and thus zero value. Of course if the asset can be purchased for less than 100, that NPV is cash in the bank.

7. Three conditions are usually considered necessary for discounting a cash flow at a constant risk-adjusted rate: (1) a known, constant

beta for an all-equity claim on the cash flow, (2) a known, constant market risk premium, (3) a known, constant Treasury bill rate.

Condition (1) implies that uncertainty is resolved at a constant rate over time. It also implies that the detrended stream of project cash flows would follow a multiplicative random walk. ("Detrended" cash flows are expressed as percentages of their ex ante expectations.) see Myers and Turnbull (1977) and Fama (1977).

8. If they do not contribute, the overall beta is unchanged by borrowing despite the addition of the safe interest tax shields. Consider the beta of investing in the total cash payout to debt and equity investors. It depends on the covariance of the return on this investment with the market return \bar{r}_m , that is:

$$\text{COV}[(\tilde{X} + r_f T_c D)/V, r_m] = \text{COV}(\tilde{X}, \bar{r}_m)/V.$$

The safe tax shield $r_f T_c D$ affects this covariance only as it affects V .

9. Suppose the interest tax shield adds yD to the value of the firm, and that the firm increases borrowing by this amount. The market value balance sheet is:

$V - yD$	$D = (1 - \beta)(V - yD) + yD$
yD	E
<hr/> $V = V(\tilde{X}, D)$	<hr/> V

The debt and equity weights work out to $(1 - \beta)/(1 - \beta y)$ and $\beta(1 - y)/(1 - \beta y)$ respectively. The revised discount rate is:

$$r^* = \left(\frac{1 - \beta}{1 - \beta y}\right) r_f (1 - T_c) + \frac{\beta(1 - y)}{1 - \beta y} r_m \quad (10)$$

β_E , the beta of the equity claim, is again one despite the addition of the safe asset yD to the left-hand side of the balance sheet. The

rest of the proofs follow as before. For example, under the assumptions of Table 1 below,

$$y = \frac{T_c r_f}{1 + r_f(1 - T_c)} = \frac{.5(.10)}{1 + .10(1 - .5)} = .0476$$

The weight on the after-tax risk free rate would change from $1 - \beta = .5$ to:

$$\frac{(1 - \beta)}{1 - \beta y} = \frac{.5}{1 - .5(.0476)} = .512.$$

The discount rate changes by only 20 basis points, from $r^* = .125$ in Table 1 to:

$$r^* = .512 (.10)(1 - .5) + .488 (.20) = .123.$$

10. The "MM" unlevering formula is more common:

$$\beta_u = (1 - T_c) \beta_D D/V + \beta_E E/V$$

However, this assumes perpetual debt, no rebalancing, and the maximum possible tax gain to leverage. We believe the assumption of period-by-period rebalancing implicit in equations (1) and (2) is more reasonable.

11. Taggart (1991) expresses the value of interest tax shields as

$$G_L = 1 - \frac{(1 - T_c)(1 - T_{pe})}{1 - T_p},$$

where G_L is the tax gain to leverage. Obviously this does not match T^* in (11). However, his APV formula multiplies G_L by r_{fe} , the expected return on risk-free equity; in (12) below we multiply T^* by the interest rate on risk-free debt. His and our formulas give exactly the same final values.

12. The interest tax shield is a safe, nominal cash inflow. Strictly speaking, it should be discounted at a risk-free interest rate, not r , the opportunity cost of capital for an all-equity financed project.

Discounting both terms of (12) at r eliminates algebraic clutter that's unrelated to the main point of this section. Moreover, in practice discounting both terms at r is an excellent approximation, since tax shields are known only for the first period. (See Taggart (1991), p. 17) After that, capital structure is rebalanced, and tax shields become just as risky as the project itself. With continuous discounting and rebalancing, (12) is exact for cash flows received at any future time.

13. There is a certainty-equivalent version of our discounting rule in which r_{fe} appears. Rewrite (18) below with $r_m = r_{fe}$:

$$a + br_{fe} + T_c r_f (V - b) = (1 + r_f)(V - b) + b(1 + r_{fe})$$

This equates cash returns conditional on $r_m = r_{fe}$. Since $V - b = (1 - \beta)V$ and $b = \beta V$,

$$V = \frac{a + br_{fe}}{1 + r_f^*}$$

$$r_f^* = r_f(1 - T_c)(1 - \beta) + \beta r_{fe}$$

Thus r_{fe} , if known, could replace r_m in a discounting rule for CEQs. In this case the expectation of X would have to be conditional on $\bar{r}_m = r_{fe}$.

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