

Magnetic fluxes and moduli stabilization

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Abstract

Stabilization of closed string moduli in toroidal orientifold compactifications of type IIB string theory are studied using constant internal magnetic fields on D -branes and 3-form fluxes that preserve $\mathcal{N} = 1$ supersymmetry in four dimensions. Our analysis corrects and extends previous work by us, and indicates that charged scalar VEV's need to be turned on, in addition to the fluxes, in order to construct a consistent supersymmetric model. As an explicit example, we first show the stabilization of all Kähler class and complex structure moduli by turning on magnetic fluxes on different sets of $D9$ -branes that wrap the internal space T^6 in a compactified type I string theory, when a charged scalar on one of these branes acquires a non-zero VEV. The latter can also be determined by adding extra magnetized branes, as we demonstrate in a subsequent example. In a different model with magnetized $D7$ -branes, in a IIB orientifold on T^6/\mathbb{Z}_2 , we show the stabilization of all the closed string moduli, including the axion-dilaton at weak string coupling g_s , by turning on appropriate closed string 3-form fluxes.

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1 Introduction

String theory is known to possess a large number of vacua which contain the basic structure of grand unified theories, and in particular of the Standard Model. However, one of the major stumbling blocks in making further progress along these lines has been the lack of a guiding principle for choosing the true ground state of the theory, thus implying the loss of predictivity. In particular, string vacua depend in general on continuous parameters, characterizing for instance the size and shape of the compactification manifold, that correspond to vacuum expectation values (VEV's) of the so-called moduli fields. These are perturbative flat directions of the scalar potential, at least as long as supersymmetry remains unbroken. It is therefore of great interest that during the last few years there has been a considerable success in fixing the string ground states, by invoking principles similar to the spontaneous symmetry breaking mechanism, now in the context of string theory. In particular, it has been realized that closed, as well as open, string background fluxes can be turned on, fixing the VEV's of the moduli fields and therefore providing the possibility for choosing a ground state as a local isolated minimum of the scalar potential of the theory. This line of approach allows string theory to play directly a role in particle unification, predicting the strength of interactions and the mass spectrum. In particular, the string coupling becomes a calculable dynamical parameter that fixes the value of the fine structure constant and determines the Newtonian coupling in terms of the string length.

On one hand, moduli stabilization using closed string 3-form fluxes has been discussed in a great detail in the literature [1, 2]. $\mathcal{N} = 1$ space-time supersymmetry and various consistency requirements imply that the 3-form fluxes must satisfy the following conditions formulated on the complexified flux defined as $G = F - \phi H$, where F and H are the R-R (Ramond) and NS-NS (Neveu-Schwarz) 3-forms, respectively, and ϕ is the axion-dilaton modulus: (1) The only non-vanishing components of G are of the type $(2, 1)$, pointing along two holomorphic and one anti-holomorphic directions, implying that its $(1, 2)$, $(3, 0)$ and $(0, 3)$ components are zero; (2) G is primitive, requiring $J \wedge G = 0$ with J being the Kähler form. This approach has been applied to orientifolds of both toroidal models as well as of Calabi-Yau compactifications. However, a drawback of the method is that the Kähler class moduli remain undetermined due to the absence of a harmonic $(1, 0)$ form on Calabi-Yau spaces, implying that the constraint $J \wedge G = 0$ is trivially satisfied. In the toroidal orientifold case, it turns out that one is able to stabilize the Kähler class moduli only partially, but in particular the overall volume remains always unfixed.

On the other hand, in [3] two of the present authors have shown that both complex structure

and Kähler class moduli may be stabilized in the type I string theory compactified down to four dimensions.¹ This can be achieved by turning on magnetic fluxes which couple to various $D9$ -branes, that wrap on T^6 , through a boundary term in the open string world-sheet action. The latter modifies the open string Hamiltonian and its spectrum, and puts constraints on the closed string background fields due to their couplings to the open string action. More precisely, supersymmetry conditions in the presence of branes with magnetic fluxes, together with conditions which define a meaningful (string) theory, put restrictions on the values of the moduli and fix them to specific constant values. This also breaks the original $\mathcal{N} = 4$ supersymmetry of the compactified type I theory to an $\mathcal{N} = 1$ supersymmetric gauge theory with a number of chiral multiplets. A detailed analysis of the final spectrum, as well as other related issues have been discussed in [6].

In the simplest case, the above model has only $O9$ orientifold planes and several stacks of magnetized $D9$ -branes. The main ingredients for moduli stabilization are then: (1) the introduction of “oblique” magnetic fields, needed to fix the off-diagonal components of the metric, that correspond to mutually non-commuting matrices similar to non-abelian orbifolds; (2) the property that magnetized $D9$ -branes can lead to negative 5-brane tensions; and (3) the non-linear part of Dirac-Born-Infeld (DBI) action which is needed to fix the overall volume. Actually, the first two ingredients are also necessary for satisfying the 5-brane tadpole cancellation without adding $D5$ -branes or $O5$ planes, while the last two properties are only valid in four-dimensional compactifications (and not in higher dimensions). It turns out however that the conditions of supersymmetry and tadpole cancellation cannot be satisfied simultaneously in such simple setups, contrary to our previous claim.²

In this work, we point out that an additional ingredient, namely non-vanishing VEV’s for the scalars fields on some of the branes, is needed for consistent model building. Indeed, it is known that the brane fluxes generate a D -term potential for the Kähler moduli in the form of a Fayet-Iliopoulos (FI)-term. The supersymmetry conditions used in [3] are then precisely the D -flatness conditions, implying the vanishing of the FI-parameters. But this is valid only in the case where the VEV’s of the charged scalars vanish. In their presence, the D -flatness condition for the closed string moduli is modified and a more general supersymmetric minimum can be obtained.

¹For partial Kähler moduli stabilization, see also [4, 5].

²This is due to an overlooked sign in [3], as explained in Section 2.

In this paper, we first implement this new ingredient on the models of [3]. We show that by a minimal modification of those models, namely a modification in the supersymmetry condition on one of the branes by switching a non-zero VEV v , the results of [3] still hold. In other words, one is able to stabilize the complex structure and Kähler class moduli, while cancelling 5-brane tadpoles among the magnetized branes, and $D9$ -brane tadpoles by the $O9$ planes of type I string theory. Furthermore, this method of moduli stabilization can be extended for the T^6/\mathbb{Z}_2 orientifold compactification of IIB string theory. One can then try to stabilize also the dilaton-axion modulus by turning on simultaneously 3-form fluxes. Since in their presence, the gauge groups of the $D9$ -branes are in general anomalous [7], we construct models with magnetized $D7$ -branes. In this case, we explicitly show that all closed string moduli are now fixed using brane fluxes, 3-form closed string fluxes and charged scalar VEV's on some of the branes.

Stabilization of all closed string moduli, using magnetized $D9$ -branes, were attempted previously by us in [8]. However due to the use of an inappropriate definition of wrapping numbers for oblique fluxes, erroneous tadpole contributions were obtained. In this paper, we have corrected the error and obtained a model with tadpole cancellations, by turning on charged scalar VEV's, where all the closed string moduli are fixed.

Our moduli stabilization scheme, where some of the open string scalars have acquired VEV's to generate a non-zero FI parameter, can be thought of as a mixing of Kähler moduli with open string fields through the D-term, in such a way that only one linear combination is fixed by the presence of the corresponding magnetic field [9], while its orthogonal combination remains a flat direction. However, we show that the latter can also be fixed by the same principle, implying a particularly interesting algorithm for moduli stabilization in toroidal type I compactifications: (1) All geometric moduli are first fixed using a minimal set of (at least nine) magnetized branes, in the absence of charged scalar VEV's, in the spirit of [3]. This has the advantage of being exact in α' (world-sheet) perturbation theory, but does not satisfy tadpole cancellation. (2) The latter is achieved by adding extra magnetized branes on which some charged scalars are forced to acquire non-vanishing VEV's. Since the inclusion of charged fields in the D-terms is not known exactly, their VEV's can be determined only perturbatively in α' , when their values are small compared to the string scale.

The supersymmetry conditions, as well as the tadpole equations, possess some scaling symmetries observed in [3], where fluxes and Kähler moduli are rescaled appropriately for fixed charged scalar VEV's. These symmetries lead to an infinite discrete class of vacua with stabilized moduli,

differing by their spectra and values of gauge couplings and Planck mass in string units.

The rest of the paper is organized as follows. In Section 2, we write down the consistency conditions for magnetic fluxes on $D9$ -branes in the context of type I toroidal compactifications. Explicit models are then presented in Section 3. The first model is a minimal modification of the main example of [3], illustrating our method of moduli stabilization in the presence of a single VEV for one of the brane stacks. It also serves as an example to demonstrate the existence of the discretum of vacua and to discuss models with large dimensions. In a second model, we show how the open string scalar VEV's are fixed consistently by adding extra stacks of magnetized branes. Consistency conditions, as well as a model with stabilized moduli, for magnetized $D7$ -branes in a IIB compactification on T^6/\mathbb{Z}_2 is given in Section 4. In Section 5, we deal with stabilization of axion-dilaton modulus, using close string 3-form fluxes and show that we are able to stabilize all the close string moduli at a weak string coupling $g_s = \frac{1}{4}$. Some concluding remarks are presented in Section 6, while Appendix A contains technical details on T^6 parametrization that we mainly use in Section 5.

2 Magnetized $D9$ -branes

2.1 Setup

The stacks of $D9$ -branes are characterized by three independent sets of data ; their multiplicities N_a , winding matrices $W_{a,I}^{\hat{I}}$ and first Chern numbers $m_{I\hat{J}}^a$ of the $U(1)$ background on their world-volume Σ_9^a , $a = 1, \dots, K$. The first describes the rank of the the unitary gauge group $U(N_a)$ on each $D9$ stack. The second is the covering of the world-volume of each stack of $D9$ -branes on the ambient space. In other words, it gives the winding of the branes around the different cycles of the internal space [6]. Their entries are therefore integrally quantized. The last set of parameters is the first Chern numbers of the $U(1) \subset U(N_a)$ background on the world-volume of the $D9$ -branes. For each stack, a linear combination of the $U(N_a)$ generators lying in the Cartan subalgebra is chosen. It forms a $U(1)$ subalgebra whose constant field strength is introduced on the covering of the internal space. These are subject to the Dirac quantization condition. On the world-volume of each stack of $D9$ -branes, they are therefore integrally quantized .

In type I string theory, the number of magnetized $D9$ -branes must be doubled. Indeed the orientifold projection $\mathcal{O} = \Omega_p$ is defined by the world-sheet parity, it maps the field strength $F_a = dA_a$ of the $U(1)_a$ gauge potential A_a to its inverse $\mathcal{O} : F_a \rightarrow -F_a$. The magnetized

$D9$ -branes are therefore not an invariant configuration as it stands. For each stack, a mirror stack must be added with an inverse flux on their world-volume. The complete gauge group of this construction remains a product of unitary groups $\otimes_a U(n_a)$, since the associated open strings attached on a given stack are identified with the ones attached on the mirror stack. In addition to these vectors, the massless spectrum contains adjoint scalars and fermions which form a $\mathcal{N} = 4$, $d = 4$ supermultiplet. On the other hand, open strings stretched between the a -th and b -th stack give rise to chiral spinors in the bifundamental representation (N_a, \bar{N}_b) . Their multiplicity I_{ab} is given by the index theorem of the product bundle $\mathcal{E}_a \otimes \mathcal{E}_b$ associated to the $U(1)_a \times U(1)_b$ flux [6].

$$I_{ab} = C_3(\mathcal{E}_a \otimes \mathcal{E}_b) = \frac{\det W_a \det W_b}{(2\pi)^3} \int_{T^6} (q_a F_a + q_b F_b)^3, \quad (2.1)$$

where F_a is the pullback of the integrally quantized world-volume flux m_{ij}^a on the target torus, and q_a the corresponding $U(1)_a$ charge; in our case $q_a = +1$ (-1) for the fundamental (anti-fundamental representation). The transformation under the gauge group and their multiplicities are thus determined in terms of the data $(N_a, W_{a,I}^{\hat{I}}, m_{ij}^a)$.

Open strings stretched between the a -th brane and its mirror a^* give rise to massless modes associated to I_{aa^*} chiral fermions. These transform either in the antisymmetric or symmetric representation of $U(N_a)$. In addition to the massless chiral spinors, there exist in all twisted open string sectors a set of massive (or tachyonic) scalars in the same representation as the associated spinor corresponding to their superpartners under the supersymmetries generally broken by the magnetic fluxes.

Let us be more specific and assume the presence of K stacks of N_a magnetized $D9$ -branes, $a = 1, \dots, K$. Each stack is associated with a corresponding $U(N_a)$ gauge symmetry. We choose K linear combinations of the generators of $U(N_a)$ which lie in the Cartan subalgebra and denote their abelian gauge potentials by A^a ; for simplicity, we identify them with $U(1)_a$. Their field strengths is assumed to take constant values on the torus T^6 . Thus there is a set of K $U(1)$ gauge potentials A^a with constant background field strengths

$$A_\alpha^a = \frac{1}{2} F_{\alpha\beta}^a X^\beta \quad \text{where } a = 1, \dots, K. \quad (2.2)$$

Moreover the magnetized $D9$ -branes couple only to the $U(1)$ flux associated with the gauge fields located on their own world-volume. In other words, the charges of the endpoints q_R and q_L of the open strings stretched between the i -th and the j -th $D9$ brane can be written as $q_L \equiv q_i$ and $q_R \equiv -q_j$ and the Cartan generator h is given by $h = \text{diag}(h_1 \mathbb{1}_{N_1}, \dots, h_N \mathbb{1}_{N_K})$, with $\mathbb{1}_{N_a}$ being

the $N_a \times N_a$ identity matrix.

The magnetized $D9$ -branes are also characterized by their wrapping numbers around the different 1-cycles of the torus which are encoded in the covering matrices $W_{\alpha}^{\hat{\alpha}, a}$ defined as

$$W_{\hat{J}}^{\hat{I}} = \frac{\partial \xi^{\hat{I}}}{\partial X^{\hat{J}}} \quad \text{for } \hat{I}, \hat{J} = 0, \dots, 9, \quad (2.3)$$

where the coordinates on the world-volume are denoted by $\xi^{\hat{I}}$ while the coordinates on the spacetime \mathcal{M}_{10} are X^I . Similarly to the space-time which is assumed to be the direct product of a four-dimensional Minkowski manifold with a six-dimensional torus, the form of the covering matrix is assumed to be as

$$W_{a, J}^{\hat{I}} = \begin{pmatrix} \delta_{\mu}^{\hat{\mu}} & 0 \\ 0 & W_{\alpha}^{\hat{\alpha}, a} \end{pmatrix} \quad \text{for } \mu, \hat{\mu} = 0, \dots, 3 \text{ and } \alpha, \hat{\alpha} = 1, \dots, 6, \quad (2.4)$$

where the upper block corresponds to the covering of the Σ_4 on the four-dimensional spacetime \mathcal{M}_4 . Since it is assumed that both are identical, the associated covering map $W_{\mu}^{\hat{\mu}}$ is the identity, $W_{\mu}^{\hat{\mu}} = \delta_{\mu}^{\hat{\mu}}$. The entries of the lower block, on the other hand, describe the wrapping numbers of the $D9$ around the different 1-cycles of the torus. These are therefore restricted to be integers $W_{\alpha}^{\hat{\alpha}} \in \mathbb{Z}, \forall \alpha, \hat{\alpha} = 1, \dots, 6$.

The K $D9$ stacks are then ten-dimensional objects which fill the four-dimensional spacetime and cover the internal torus T^6 . Thus there are K different coverings \mathcal{T}_6^a of the torus T^6 described by the K covering maps $W_{\alpha}^{\hat{\alpha}, a}$, for $a = 1, \dots, K$. The fields $F_{\alpha\beta}^a$ then correspond to a non-trivial $U(1)$ gauge bundle on the torus T^6 . Equivalently, their world-volume field strengths $F_{\hat{\alpha}\hat{\beta}}^a$ correspond to a non-trivial $U(1)$ gauge bundle on the covering \mathcal{T}_6^a of the torus T^6 . The Dirac quantization condition applies independently to the K fluxes $F_{\hat{\alpha}\hat{\beta}}^a$

$$\left\{ \begin{array}{l} F_{\hat{\alpha}\hat{\beta}}^a = m_{\hat{\alpha}\hat{\beta}}^a \in \mathbb{Z} \quad , \quad \forall \hat{\alpha}, \hat{\beta} = 1, \dots, 6 \\ p_{\alpha\beta}^a = (W^{-1})_{\alpha}^{\hat{\alpha}, a} (W^{-1})_{\beta}^{\hat{\beta}, a} m_{\hat{\alpha}\hat{\beta}}^a \in \mathbb{Q} \quad , \quad \forall \alpha, \beta = 1, \dots, 6 \end{array} \right. \quad \forall a = 1, \dots, K \quad (2.5)$$

Note that these rationally quantized fluxes are equivalent to the one introduced in [3]. Here, the entries of the winding matrix describe the 1-cycle winding numbers, whereas in [3], the spacetime fluxes are defined via the 2-cycle winding numbers $n_{\alpha\beta}$. For the simplest flux configurations, the winding numbers $n_{\alpha\beta}$ are the denominators of the entries of the matrix $p_{\alpha\beta}^a$.

2.2 Consistency conditions

Necessary conditions for a consistent construction involving K stacks of N_a magnetic $D9$ -branes on a compact orientifold compactification follow from the Ramond-Ramond (R-R) tadpole cancellations. These account for the absence of UV divergencies in the one loop amplitude and ensure, via a generalized Green-Schwarz mechanism, the cancellation of gauge anomalies in the associated four dimensional field theories. In the toroidal compactification of type I string theory, the magnetized $D9$ -branes induce 9-brane and 5-brane charges, while the 3-brane and 7-brane charges automatically vanish due to the presence of mirror branes with opposite flux. For general magnetic fluxes, they can be written in terms of the Chern numbers and winding matrix [6]. The tadpole conditions read in this case

$$16 = \sum_{a=1}^K N_a \det W_a \quad (2.6)$$

$$0 = \sum_{a=1}^K N_a \det W_a \mathcal{Q}_a^{\alpha\beta} \quad \forall \alpha, \beta = 1, \dots, 6 \quad (2.7)$$

where

$$\mathcal{Q}_a^{\alpha\beta} = \epsilon^{\alpha\beta\delta\gamma\sigma\tau} p_{\delta\gamma}^a p_{\sigma\tau}^a. \quad (2.8)$$

The l.h.s of eq. (2.6) arises from the charge contribution of the $O9$ plane. Moreover, the toroidal compactification implies the absence of any $O5$ -planes and thus the l.h.s of eq. (2.7) vanishes. Note that equations (2.7) are in agreement with the 5-brane tadpole condition given in [3], where the factor \mathcal{K} accounts for the ϵ tensor in eq. (2.8), while the 9-brane tadpole condition (2.6) disagrees with [3] because the factor \mathcal{K} should be absent.³

The above tadpole conditions restrict the allowed choices for the ranks of the gauge groups, winding matrices, Chern numbers and consequently the allowed spectra. They ensure in particular that the spectrum is anomaly-free. Note also that they are invariant under a discrete rescaling of the Chern numbers in some given direction for all stacks, keeping the winding matrix invariant

$$\{m_{\hat{\alpha}\hat{\beta}}^a, W_{a,\alpha}^{\hat{\beta}}\} \rightarrow \{\Lambda m_{\hat{\alpha}\hat{\beta}}^a, W_{a,\alpha}^{\hat{\beta}}\} \quad \forall a = 1, \dots, K \quad \text{and} \quad \text{for a given } \hat{\alpha} \quad ; \quad \Lambda \in \mathbb{Z}. \quad (2.9)$$

This rescaling affects the spectrum at the intersection of all pairs of stacks. Form eq. (2.1), we see that the number of chiral fermions in all intersections is also rescaled. Note that this invariance of the tadpole conditions is not true anymore in the case of orbifold compactifications

³We thank F. Denef and F. Marchesano for correspondence and enlightening discussions on this point.

with $O5$ -planes. In this case, the $O5$ -planes carry a R-R charge which is not insensitive to the rescaling (2.9). Since these contribute to the 5-brane tadpole (2.7), the consistency conditions would not be invariant.

2.3 Supersymmetry conditions

For a given configuration of magnetized branes, one may ask whether the different stacks forming the brane configuration preserve some common supersymmetries. Via a Zeeman-like effect, the scalars and fermions of the twisted sectors, corresponding to the intersection of two brane stacks acquire different masses. Supersymmetry is thus broken [10]. Similarly, a single magnetized $D9$ -brane in type I string theory is not generically supersymmetric. Indeed, the orientifold projection implies the presence of mirror branes. Twisted scalars from the Neveu-Schwarz (NS) sector of open string stretched between a brane and its image are generically massive, while some chiral spinors from the Ramond (R) sector remain massless. In other words, the $D9$ -brane does not preserve the same supersymmetry as the orientifold projection. Part of supersymmetry may however be restored. The supersymmetry conditions involve the flux quanta and winding matrix, but also the metric moduli.

In the case of toroidal compactification of type I string theory, starting from the real orthonormal basis of T^6 : $x_i = x_i + 1$ and $y^i = y^i + 1$, $i = 1, 2, 3$ with unit periodicity, the moduli decompose in a complex structure variation which is parametrized by the matrix τ_{ij} entering in the definition of the complex coordinates $z_i = x_i + \tau_{ij}y^j$ and in the variation of the mixed part of the metric described by the real (1, 1)-form $J = i\delta g_{i\bar{j}}dz^i \wedge d\bar{z}^j$. The supersymmetry conditions then read [3]

$$F_{(2,0)}^a = 0 ; \quad \mathcal{F}_a \wedge \mathcal{F}_a \wedge \mathcal{F}_a = \mathcal{F}_a \wedge J \wedge J ; \quad \det W_a (J \wedge J \wedge J - \mathcal{F}_a \wedge \mathcal{F}_a \wedge J) > 0 \quad \forall a = 1, \dots, K. \quad (2.10)$$

The complexified fluxes can be written as

$$F_{(2,0)}^a = (\tau - \bar{\tau})^{-1T} [\tau^T p_{xx}^a \tau - \tau^T p_{xy}^a - p_{yx}^a \tau + p_{yy}^a] (\tau - \bar{\tau})^{-1} \quad (2.11)$$

$$F_{(1,1)}^a = (\tau - \bar{\tau})^{-1T} [-\tau^T p_{xx}^a \bar{\tau} + \tau^T p_{xy}^a + p_{yx}^a \bar{\tau} - p_{yy}^a] (\tau - \bar{\tau})^{-1} \quad (2.12)$$

where the matrices $(p_{xx}^a)_{mn}$, $(p_{xy}^a)_{mn}$ and $(p_{yy}^a)_{mn}$ enter in the quantized field strength (2.5) in the directions (x^m, x^n) , (x^m, y^n) and (y^m, y^n) , respectively. The field strengths $F_{(2,0)}^a$ and $F_{(1,1)}^a$

are 3×3 matrices that correspond to the upper half of the matrix \mathcal{F}^a :

$$\mathcal{F}^a = -(2\pi)^2 i\alpha' \begin{pmatrix} F_{(2,0)}^a & F_{(1,1)}^a \\ -F_{(1,1)}^{a\dagger} & F_{(2,0)}^{a*} \end{pmatrix}, \quad (2.13)$$

which is the total field strength in the cohomology basis $e_{i\bar{j}} = idz^i \wedge d\bar{z}^j$.

The first set of conditions of eq. (2.10) states that the purely holomorphic flux vanishes. For given flux quanta and winding numbers, this matrix equation restricts the complex structure τ . Using eq. (2.11), the supersymmetry conditions for each stack can first be seen as a restriction on the parameters of the complex structure matrix elements τ :

$$F_{(2,0)}^a = 0 \quad \rightarrow \quad \tau^T p_{xx}^a \tau - \tau^T p_{xy}^a - p_{yx}^a \tau + p_{yy}^a = 0. \quad (2.14)$$

Similarly with the tadpole conditions, the ‘‘holomorphicity’’ equation (2.14) is invariant under a general rescaling of all fluxes

$$m_{\hat{\alpha}\hat{\beta}}^a \rightarrow \Lambda^a m_{\hat{\alpha}\hat{\beta}}^a \quad ; \quad \forall \hat{\alpha}, \hat{\beta} = 1, \dots, 6 \quad \text{and } a = 1, \dots, K. \quad (2.15)$$

This may be compared to the tadpole conditions which have a wider invariance under rescaling of Chern numbers in given direction, while the conditions (2.14) are only invariant under the same rescaling in all directions.

The second set of conditions of eq. (2.10) gives rise to a real equation and restricts the Kähler moduli. This can be understood as a D -flatness condition. In the four-dimensional effective action, the magnetic fluxes give rise to topological couplings for the different axions of the compactified field theory. These arise from the dimensional reduction of the Wess Zumino (WZ) action. In addition to the topological coupling, the $\mathcal{N} = 1$ supersymmetric action yields a Fayet-Iliopoulos (FI) term of the form

$$\frac{\xi_a}{g_a^2} = \frac{1}{(4\pi^2\alpha')^3} \int_{T^6} (\mathcal{F}_a \wedge \mathcal{F}_a \wedge \mathcal{F}_a - \mathcal{F}_a \wedge J \wedge J). \quad (2.16)$$

The D -flatness condition in the absence of charged scalars requires then that $\langle D_a \rangle = \langle \xi_a \rangle = 0$, which is equivalent to the second equation of eq. (2.10). Finally, the last inequality in eq. (2.10) may also be understood from a four-dimensional viewpoint as the positivity of the $U(1)_a$ gauge coupling g_a^2 , since its expression in terms of the fluxes and moduli reads

$$\frac{1}{g_a^2} = \frac{1}{(4\pi^2\alpha')^3} \int_{T^6} (J \wedge J \wedge J - \mathcal{F}_a \wedge \mathcal{F}_a \wedge J). \quad (2.17)$$

The above supersymmetry conditions are only valid in the absence of charged scalars. The situation in the presence of scalars charged under the $U(1)$ gauge groups is different. The D -flatness condition is modified. In the low energy field theoretical approximation, the D-term reads

$$D_a = - \left(\sum q_a^\phi |\phi_a|^2 + M_s^2 \xi_a \right), \quad (2.18)$$

where $M_s = \alpha'^{-1/2}$ is the string scale⁴, and the sum is extended over all scalars ϕ_a charged under the a -th $U(1)_a$ with charge q_a^ϕ . Such scalars arise in the compactification of magnetized $D9$ -branes in type I string theory, for instance from the NS sector of open strings stretched between the a -th brane and its image a^* . When one of these scalars acquire a non-vanishing VEV $\langle |\phi_a|^2 \rangle = v_a^2$, the calibration condition of eq. (2.10) is modified to

$$q_a v_a^2 \int_{T^6} (J \wedge J \wedge J - \mathcal{F}_a \wedge \mathcal{F}_a \wedge J) = -M_s^2 \int_{T^6} (\mathcal{F}_a \wedge \mathcal{F}_a \wedge \mathcal{F}_a - \mathcal{F}_a \wedge J \wedge J) \quad (2.19)$$

$$\det W_a (J \wedge J \wedge J - \mathcal{F}_a \wedge \mathcal{F}_a \wedge J) > 0, \quad \forall a = 1, \dots, K. \quad (2.20)$$

In contrast with the "holomorphicity" equation (2.14), the conditions (2.19) and (2.20) are not invariant under the rescaling (2.15). This now leads to a family of solutions which differ by their overall volume. Indeed, the rescaling (2.15) at fixed winding numbers corresponds to the rescaling of all fluxes \mathcal{F}_a . If all stacks scale in the same way $\Lambda_a \equiv \Lambda, \forall a = 1, \dots, K$, one obtains an infinite family of solutions

$$\{\mathcal{F}_a, J\} \rightarrow \{\Lambda \mathcal{F}_a, \Lambda J\} \quad \text{for} \quad \Lambda \in \mathbb{N} \quad \text{and} \quad \forall a = 1, \dots, K. \quad (2.21)$$

Since the tadpole and holomorphicity conditions are invariant under this rescaling, one obtains an infinite discretum of vacua which differ by their spectra and the overall volume. All of them have the same gauge symmetry but with different gauge couplings (2.17). Similarly, their total internal volume and consequently their four-dimensional Planck mass differs by a factor of Λ^3 . However, not all of these vacua are phenomenologically viable. Indeed, the experimental bounds on the string scale, on the numbers of chiral families and on the value of the longitudinal volumes strongly restricts the permissible vacua. Nevertheless, all of them are fully consistent from the viewpoint of string theory.

It turns out that there exist no toroidal supersymmetric models of magnetized $D9$ -branes with chiral matter in the literature. Despite the absence of a full no-go theorem, it is widely believed that the tadpole conditions (2.6) and (2.7) are not compatible with the supersymmetry

⁴When mass scales are absent, string units are implicit throughout the paper.

conditions (2.10) at zero open string VEV's.⁵ In the following section, we show that this is not true when one turns on a non-vanishing VEV's for charged scalars on the branes. Actually, one such VEV, which does not even break gauge symmetries, is sufficient to render compatible the supersymmetry with the tadpole conditions.

3 Supersymmetric toroidal model

Our aim is first to show that the tadpole equations are compatible with the deformed supersymmetry condition (2.19). An explicit example of a consistent configuration of supersymmetric $D9$ -branes is presented where the tadpole conditions (2.6) and (2.7) are satisfied. We will give an example of nine magnetized branes which are supersymmetric for fixed values of the metric moduli of the torus. The only remaining closed string modulus is then the dilaton and the associated axion field. One can then analyze the case of usual toroidal models in the limit of vanishing VEV's for the charged scalars. This will be shown to approach points at the boundary of moduli space corresponding to the decompactification limit where the volume of the internal torus becomes infinite. We will finally show the existence of many infinite discreta of vacua which differ by their spectra, gauge couplings and four-dimensional Planck mass in string units.

To this end, one slightly modifies the configuration of branes presented in [3]. Inspection of eqs. (2.14) and (2.19) shows that for each stack of magnetized $D9$ -branes, we have up to three complex conditions for the moduli of the complex structure, depending on the directions in which the fluxes are switched on, whereas only one real condition can be set on the Kähler moduli. Therefore, to fix all Kähler moduli in a toroidal compactification, at least nine stacks of branes must be added. The first six branes have *oblique* fluxes on their world-volume. They do not have any chiral fermions on their intersections and preserve (each one) an $\mathcal{N} = 2$, $d = 4$ supersymmetry for restricted complex structure and Kähler moduli. Moreover in our example, since the number of intersections of any pair vanishes, the intersections preserve also extended $\mathcal{N} = 2$ supersymmetry, although not the same for each pair. All complex structure moduli are fixed, while three Kähler moduli remain undetermined. These are stabilized, in terms of a single charged scalar VEV, by the three last stacks which have usual parallel fluxes. Their intersections are generically not trivial and the massless spectrum contains chiral $\mathcal{N} = 1$ supermultiplets.

⁵The examples found in [3] were due to the presence of the sign factor \mathcal{K} in the 9-brane tadpole condition (2.6).

Stack#	Multiplicity	Fluxes	Fixed moduli	5 – brane localization
#1	$N_1 = 1$	$(F_{x_1 y_2}^1, F_{x_2 y_1}^1) = (1, 1)$	$\tau_{31} = \tau_{32} = 0$ $\tau_{11} = \tau_{22}$ $\text{Re}J_{1\bar{2}} = 0$	$[x_3, y_3]$
#2	$N_2 = 1$	$(F_{x_1 y_3}^2, F_{x_3 y_1}^2) = (1, 1)$	$\tau_{21} = \tau_{23} = 0$ $\tau_{11} = \tau_{33}$ $\text{Re}J_{1\bar{3}} = 0$	$[x_2, y_2]$
#3	$N_3 = 1$	$(F_{x_1 x_2}^3, F_{y_1 y_2}^3) = (1, 1)$	$\tau_{13} = 0, \text{Im}J_{1\bar{2}} = 0$ $\tau_{11}\tau_{22} = -1$	$[x_3, y_3]$
#4	$N_4 = 1$	$(F_{x_2 x_3}^4, F_{y_2 y_3}^4) = (1, 1)$	$\tau_{12} = 0, \text{Im}J_{2\bar{3}} = 0$	$[x_1, y_1]$
#5	$N_5 = 1$	$(F_{x_1 x_3}^5, F_{y_1 y_3}^5) = (1, 1)$	$\text{Im}J_{1\bar{3}} = 0$	$[x_2, y_2]$
#6	$N_6 = 1$	$(F_{x_2 y_3}^6, F_{x_3 y_2}^6) = (1, 1)$	$\text{Re}J_{2\bar{3}} = 0$	$[x_1, y_1]$

Table 1: Fixed complex structure moduli for each magnetized stack # of $D9$ -branes depending on the quantized fluxes. The last column gives the localization on the 2-cycles $[x_i, y_i]$, of the induced 5-brane charges.

Stack#	Multiplicity	Fluxes	D5branes localization
#7	$N_7 = 1$	$(F_{x_1 y_1}^7, F_{x_2 y_2}^7, 0) = (2, -3, 0)$	$[x_3, y_3]$
#8	$N_8 = 3$	$(F_{x_1 y_1}^8, 0, F_{x_3 y_3}^8) = (-2, 0, 1)$	$[x_2, y_2]$
#9	$N_9 = 2$	$(F_{x_1 y_1}^9, F_{x_2 y_2}^9, F_{x_3 y_3}^9) = (4, 1, 1)$	$[x_1, y_1]$ $[x_2, y_2]$ $[x_3, y_3]$

Table 2: Additional stacks of magnetized $D9$ -branes allowing the stabilization of the diagonal part of the Kähler form. The last column gives the localization on the 2-cycles $[x_i, y_i]$, of the induced 5-brane charges.

3.1 Explicit Model

Here, we prove the existence of a family of supersymmetric toroidal compactifications with magnetized $D9$ -branes. The presence of VEV's for the charged fields cures the apparent incompatibility between the supersymmetry condition (2.10) and the tadpole conditions (2.6) and (2.7).

The model is then constructed out of the nine stacks presented in Tables 1 and 2, with all winding matrices W_a equal to the identity. Following the setup of [3], the first six branes have purely oblique fluxes and each preserves separately $\mathcal{N} = 2$ supersymmetry. They fix the complex structure moduli to be of the form

$$\tau_{ij} = i\delta_{ij}, \quad (3.1)$$

and all off-diagonal components of the Kähler form to be vanishing,

$$J_{i\bar{j}} = 0. \quad (3.2)$$

This geometry corresponds to factorizable tori $T^6 = T^2 \times T^2 \times T^2$, where each T^2 is a squared lattice. Furthermore, the contribution of these off-diagonal fluxes to the 5-brane tadpoles is diagonal and negative. It sums up to

$$(Q_5^{x_1 y_1}, Q_5^{x_2 y_2}, Q_5^{x_3 y_3}) = (-2, -2, -2). \quad (3.3)$$

The last three brane stacks stabilize the remaining three Kähler moduli $J_{i\bar{i}}$. The minimal modification of the setup of [3] is the addition of a non-vanishing VEV for a single scalar field charged under one of the last three $U(1)$'s, for instance the ninth, $v_9^2 \neq 0$. In fact, the choice of quanta of Table 2 cancels the tadpole (3.3) induced by the first six stacks. Furthermore, these are supersymmetric for restricted values of the Kähler moduli. Indeed, from the conditions (2.19), these are fixed to be

$$2J_{3\bar{3}} = \frac{2}{3}J_{2\bar{2}} = J_{1\bar{1}} := J, \quad (3.4)$$

where

$$q \frac{v_9^2}{M_s^2} = -\xi_9 = -\frac{16 - 20J^2}{3J^3 - 36J^2} \quad \text{with} \quad q = 2. \quad (3.5)$$

The VEV v_9 corresponds to a charged scalar in the antisymmetric representation of $U(N_9)$ with $U(1)_9$ charge $q = 2$; there are no states in the symmetric representation since the winding number is 1. The relative sign between the value of q and the FI parameter ξ_9 , appearing in the D -term (2.18), can be easily verified by the presence of a tachyonic state in the spectrum in the large volume limit, according to the formula (3.5). For the above particular points of the Kähler moduli space, the D -flatness condition (2.19) is satisfied, while keeping the gauge coupling positive, or equivalently satisfying the condition (2.20). Finally, since each brane contributes one unit of 9-brane R-R charge in the r.h.s. of eq. (2.6), the total contribution is

$$Q_9 = 12. \quad (3.6)$$

One can then add for instance four extra non-magnetized $D9$ -branes to account for the left-over charge.

Note that our computation is valid for small values of v_9 (in string units), since the inclusion of the charged scalars in the D -term is in principle valid perturbatively. From eq. (3.5), this corresponds to large values of the Kähler parameter J . Actually, this equation is valid for $J > 12$ in order to satisfy the positivity condition (2.20). It follows that in this region there is always a solution of the supersymmetry equation (3.5). Consequently, the above configuration of nine magnetized $D9$ -branes forms a consistent supersymmetric model where all metric moduli are fixed. Moreover from the usual Stückelberg couplings, the R-R moduli are absorbed in the longitudinal polarization of the $U(1)$ gauge bosons. All nine $U(1)$ gauge fields become then massive. The gauge symmetry of this vacuum is therefore

$$SU(3) \times SU(2) \times SO(8) \quad (3.7)$$

where the last factor corresponds to the four additional non-magnetized $D9$ -branes needed to satisfy the 9-brane tadpole condition (2.6). To obtain the standard model gauge group, an extra $U(1)$ factor can easily be added in a supersymmetric configuration, reducing the $SO(8)$ symmetry to $SO(6)$. The Stückelberg couplings give mass to nine $U(1)$ gauge bosons, while in general a linear combination of the ten $U(1)$'s remains massless. One can therefore obtain an additional abelian factor with chiral spectrum.

Finally, one may expect that the presence of non-trivial VEV's for charged scalars break the gauge symmetry. This is not in general true. For instance in our example, as we saw above, the charged scalars from the ninth stack transform in the antisymmetric representation of $U(2)$ which is $SU(2)$ singlet. Moreover, due to the Stückelberg coupling, the $U(1)$ gauge boson is massive and the abelian gauge symmetry is already broken. Nevertheless, the left-over global $U(1)$ symmetry is spontaneously broken by the presence of v_9 , signaled by the presence of a Goldstone boson.

3.2 Extensions to other models

The non-vanishing VEV v_9 appearing in eq. (3.5) corresponds to charged scalars from the NS sector of open strings stretched between the ninth brane stack and its mirror. At the supersymmetric points $\langle D \rangle = 0$, a linear combination of the Kähler form and the open string scalar remains massless. Thus, in this example, the direction of the charged field is flat and there exist no preferred values for its VEV v_9 . In Section 3.4 below, we show how this VEV can be fixed by adding extra stacks of magnetized branes. Indeed, we present an explicit example, where the last three stacks #7, 8, 9 of the model described in Section 3.1 are replaced by five others with the following properties: (1) Three of them are used to fix the three diagonal Kähler moduli for vanishing scalar VEV's, in terms of the magnetic fluxes. (2) The remaining two are used to satisfy tadpole cancellation conditions, while supersymmetry requirement fix two charged scalar VEV's to be non-vanishing, at values smaller than the string scale consistently with the α' -expansion. Actually, even in the example presented above as we mentioned already, the field theoretical approximation which led to the D -flatness condition (2.19) is only valid for small VEV $v_9^2 \ll M_s^2$, since higher powers in $|\phi|^2$ have been neglected. This yields minima at large values for the Kähler moduli $J \gg \alpha'$.

Note also that the usual compactification in the absence of VEV's for the charged scalars can be obtained in the limit where $v_9 \rightarrow 0$. From eq. (3.5), one observes that it corresponds to the

decompactification limit where the overall volume of the torus becomes infinite, $J \wedge J \wedge J \rightarrow \infty$. In other words traditional toroidal compactification of magnetic branes may lead to supersymmetric vacua only at the boundary of the moduli space.

Further models can finally be constructed apart from the one presented above. Moreover, aside from the discretum of vacua arising by the general rescaling of the fluxes and volume, there exists a second one. It is characterized by a discrete set of volumes of one of the three T^2 , while keeping the other volumes, as well as the shape moduli, invariant. In other words, this family of vacua is given by the set of volumes

$$J^\Lambda = (\Lambda^2 J, \frac{3}{2} J, \frac{1}{2} J) \tag{3.8}$$

where J is always a solution to the equation (3.5). For this, the Chern numbers $m_{x_1 y_1}^b$ of the last three stacks $b = 7, 8, 9$ are rescaled by Λ^2 , $m_{x_1 y_1}^b \rightarrow \Lambda^2 m_{x_1 y_1}^b$. This modifies also the 5-brane tadpole charges induced by the last three stacks:

$$Q_5 = (2, 2\Lambda^2, 2\Lambda^2). \tag{3.9}$$

In order to compensate them in such a way that the complex structure remains of the diagonal form (3.1), the fluxes of the stacks #1, #2, #3 and #5 must be rescaled such that

$$m_{x_1 y_2}^{1,\Lambda} = m_{x_1 y_3}^{2,\Lambda} = m_{x_1 x_2}^{3,\Lambda} = m_{x_1 x_3}^{5,\Lambda} = \Lambda. \tag{3.10}$$

The tadpole conditions are then satisfied and the rescaled model is consistent. The metric remains diagonal describing the product of three orthogonal T^2 's, while the volume of the first T^2 is rescaled by Λ^2 and the other two remain the same. The replication of chiral fermions, the gauge couplings and the Planck mass are also affected since the total internal volume is rescaled.

By a similar argument one can show that it is possible to rescale the areas of two of the tori, while keeping the third one fixed, leading yet to another discretum of vacua.

3.3 Hierarchy Problem

From the analysis of Sections 2.3 and 3.2, one observes the possibility to obtain large volume compactifications [11] having two, four or six large dimensions. Here, we study the consequence on the hierarchy of the string scale with respect to the Planck mass.

If one assumes that the standard model lives on some of the magnetized $D9$ -branes, the dimensions longitudinal to these branes are constrained by accelerator experiments. They can

not be hierarchically larger than the string scale. On other hand, since the $D9$ -branes are space-time filling, there exist no dimensions transverse to their world-volume. The moduli stabilization at very large volume can not therefore be implemented to decouple the string scale from the Planck mass.

However, the model presented in Section 3.1 can be easily modified to allow the presence of unmagnetized $D5$ -branes, transverse for instance to the first torus with large value. In order for them to preserve the same supersymmetry as the magnetic fluxes, their GSO projection must be chosen in a way that their tension and R-R charge have opposite sign. Moreover, in the sectors of open strings stretched between magnetized $D9$ -branes and the unmagnetized $D5$ -branes, there exist chiral fermions.

3.4 Charged scalar VEV's determination

In the construction presented in section 3.1, the VEV v_9 of the charged field is undetermined, corresponding to an open string modulus with flat potential. Here, we present an extension based on the same principle of magnetized branes, where such open string scalar VEV's are fixed. To this end, one introduces more than nine stacks of magnetized $D9$ -branes. The vanishing of the extra D -terms induces further conditions. Once the nine metric moduli are stabilized, the new conditions may then stabilize the above open string moduli that enter the D -terms.

Let us present an explicit example, where besides the metric moduli we turn on VEV's for two massless charged fields. The latter are charged under two additional $U(1)$'s. These are embedded in a model defined by eleven stacks of magnetized branes. The first six are those with oblique flexes given in Table 1. The next five branes are new and given in Table 3. The D -flatness conditions for the first nine stacks restrict the metric moduli of the torus to be of the diagonal form

$$\tau_{ij} = i\delta_{ij} \quad ; \quad (J_{x_1y_1}, J_{x_2y_2}, J_{x_3y_3}) = 4\pi^2\alpha' \sqrt{\frac{3}{22}}(44, 66, 19), \quad (3.11)$$

in the absence of VEV's for the fields charged under these nine branes. The tadpole conditions (2.6) and (2.7) ask however for additional branes. These are the stacks #10 and #11, as well as four unmagnetized $D9$ -branes. Due to the usual Stückelberg couplings, this model defines a consistent brane configuration with gauge symmetry

$$SU(3) \times SU(2)^3 \times U(1)^2. \quad (3.12)$$

However, it can be supersymmetric only in the presence of non-trivial VEV's for open string states charged under the $U(1)$ gauge bosons of the last two magnetized $D9$ -brane stacks.

Let us then switch on VEV's for the fields ϕ_{10} and ϕ_{11} , transforming in the antisymmetric representations of the corresponding $SU(2)$ gauge groups and charged under the $U(1)$'s of the last two stacks. Their respective VEVs v_{10} and v_{11} , are fixed by the supersymmetry conditions. Indeed, from the quanta given in Table 3 and the values for the Kähler moduli (3.11), the positivity conditions (2.20) for these branes are satisfied. Moreover, since the Kähler form is already fixed, the supersymmetry conditions (2.19) determine the values of v_{10} and v_{11} as:

$$v_{10}^2 l_s^2 \simeq \frac{0.71}{q} \simeq 0.35 \quad ; \quad v_{11}^2 l_s^2 \simeq \frac{0.31}{q} \simeq 0.15, \quad (3.13)$$

where we used that the $U(1)$ charge of the fields in the antisymmetric representation is $q = 2$ in our conventions, as mentioned earlier. These VEV's break the two $U(1)$ factors and the gauge group becomes $SU(3) \times SU(2)^3$. Note that the above values of the VEV's are reasonably small in string units, consistently with our perturbative approach of including the charged scalar fields in the D -terms. We have thus presented a model where the open string moduli corresponding to charged scalar VEV's are also fixed by the magnetic fluxes. In principle, the same method can be applied for stabilizing other open string moduli, as well.

4 Magnetized $D7$ -branes

4.1 Generalities

In previous sections, we have shown that all the complex structure and Kähler class moduli are stabilized using magnetic fluxes on $D9$ -brane world-volume, when charged scalars acquire VEV's. However, one is still left with an unstabilized axion-dilaton modulus. A mechanism to implement this stabilization is to use closed string 3-form fluxes [12]. In this section, we show how the two kinds of fluxes, namely the magnetic and 3-form fluxes, can be simultaneously turned on in a consistent way in order to stabilize all closed string moduli. To this end, it has to be imposed that the supersymmetry preserved by the 3-form fluxes is the same as the one preserved by an $O3$ plane, implying that these fluxes $G_{(3)} \equiv H_{(3)} - \tau F_{(3)}$ are primitive and of the type $(2, 1)$:

$$G_{(3)} \wedge J = 0 \quad ; \quad G_{(3)} \in H^{2,1}, \quad (4.1)$$

where $H_{(3)}$ and $F_{(3)}$ are the field strengths of the NS-NS and R-R 2-forms. In this case, the compactification is therefore chosen to be the torus orientifold T^6/\mathbb{Z}_2 , where the \mathbb{Z}_2 orientifold

Stack#	Multiplicity	Fluxes
#7	$N_7 = 1$	$(F_{x_1 y_1}^7, F_{x_2 y_2}^7, 0) = (-4, -4, 3)$
#8	$N_8 = 2$	$(F_{x_1 y_1}^8, 0, F_{x_3 y_3}^8) = (-3, 1, 1)$
#9	$N_9 = 3$	$(F_{x_1 y_1}^9, F_{x_2 y_2}^9, F_{x_3 y_3}^9) = (-2, 3, 0)$
#10	$N_{10} = 2$	$(F_{x_1 y_1}^{10}, F_{x_2 y_2}^{10}, F_{x_3 y_3}^{10}) = (5, 1, 2)$
#11	$N_{11} = 2$	$(F_{x_1 y_1}^{11}, F_{x_2 y_2}^{11}, F_{x_3 y_3}^{11}) = (0, 4, 1)$

Table 3: Additional stacks of magnetized $D9$ -branes allowing the stabilization of the diagonal part of the Kähler form and some charged moduli.

action $\mathcal{O}_2 = \Omega_p G_2 (-1)^{F_L}$ is a composition of the world-sheet parity Ω_p with the left fermion number $(-1)^{F_L}$ and the orbifold G_2 acting on the torus coordinates $G_2 : z_i \rightarrow -z_i, \forall i = 1, 2, 3$. This induces $O3$ planes that carry an overall charge $Q_{O3} = -16$.

In this closed string background, one may introduce Dp -branes. The presence of Dp -branes is however constrained by the Freed-Witten (FW) anomaly [7, 13]. NS-NS 3-form flux $H_{(3)}$ implies that the Bianchi identity for the gauge-invariant world-volume gauge field strengths $\mathcal{F}_{\hat{\alpha}\hat{\beta}}$ is not satisfied along the brane world-volume:

$$\partial_{[\hat{\alpha}} F_{\hat{\beta}\hat{\gamma}]} = H_{\hat{\alpha}\hat{\beta}\hat{\gamma}}. \quad (4.2)$$

The anomaly is therefore absent in the presence of $H_{(3)}$ fluxes, only if the flux on the world-volume of each separate brane vanishes. Since a $D9$ -brane covers the whole ten-dimensional space, its use for stabilizing the axion-dilaton modulus is therefore ruled out. For $D7$ -branes on the other hand, one can obtain models free of any FW-anomaly by choosing at least one index $(\hat{\alpha}, \hat{\beta}, \hat{\gamma})$ of $H_{\hat{\alpha}\hat{\beta}\hat{\gamma}}$ along the directions transverse to the 7-brane world-volume.

Furthermore, similar to the toroidal compactification of Section 2.1, the number of magnetized $D7$ -branes must be doubled. Indeed, the orientifold action \mathcal{O}_2 maps the fluxes F on a stack of $D7$ -branes to its inverse $\mathcal{O}_2 : F \rightarrow -F$. To obtain an invariant configuration, a mirror stack must be added with the opposite flux on their world-volume but the same multiplicity and winding numbers.

4.2 Supersymmetry, tadpoles and charged scalar VEV's

Let us assume there exist K stacks of N_a magnetized $D7$ -branes, $a = 1, \dots, K$. For the sake of simplicity, it is assumed that the covering matrices W_a are diagonal. Their entries are thus the wrapping numbers around four of the 1-cycles of T^6 .

$$W_{\hat{\alpha}}^{\alpha, a} = n_{\hat{\alpha}}^a \delta_{\hat{\alpha}}^{\alpha}. \quad (4.3)$$

Each of the $D7$ -branes covers a 4-cycle of the torus T^6 . The winding matrix has therefore rank four and two out of the six entries must vanish. Finally, the quantized first Chern numbers $m_{\hat{\alpha}\hat{\beta}}^a$ of the magnetic fluxes are given in eq. (2.5).

This configuration of magnetized $D7$ -branes with 3-form fluxes must satisfy the tadpole conditions. Generally, magnetized $D7$ -branes generate 5-brane and 3-brane tadpoles in addition to the 7-brane charges. Similarly as in Section 2.2, the presence of mirror branes cancels the

5-brane charges. One is then left with 7-brane and 3-brane tadpole contributions. Together with the contribution of the $O3$ planes and of the 3-form fluxes, the tadpole conditions read

$$16 = \sum_a N_a \mathcal{Q}_3^a + N_3 \quad (4.4)$$

$$0 = \sum_a N_a n_\alpha^a n_\beta^a n_\gamma^a n_\delta^a \quad , \quad \forall \alpha, \dots, \delta = 1, \dots, 6 \quad (4.5)$$

where

$$\mathcal{Q}_3^a = \epsilon^{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\delta}} m_{\hat{\alpha}\hat{\beta}}^a m_{\hat{\gamma}\hat{\delta}}^a \quad ; \quad N_3 = -\frac{1}{2} \frac{1}{(2\pi)^4 \alpha'^2} \int_{T^6} H_{(3)} \wedge F_{(3)} \quad (4.6)$$

These conditions restrict the rank of the gauge symmetry as well as the chiral spectrum. It ensures in particular via the Green-Schwarz mechanism that the spectrum is anomaly-free.

One may ask whether the different stacks of magnetized $D7$ -branes preserve the same supersymmetry. The supersymmetry conditions, for a magnetized $D7$ -brane on T^4 , can be read from general expressions of the central charge or the Born-Infeld action. The conditions depend on the wrapping matrices W_a , the flux \mathcal{F}_a and the metric moduli of T^4 :

$$F_{(2,0)}^a = 0 \quad ; \quad J \wedge \mathcal{F}_a = 0 \quad ; \quad \left(\prod_\alpha n_\alpha^a \right) (J \wedge J - \mathcal{F}_a \wedge \mathcal{F}_a) < 0 \quad , \quad \forall a = 1, \dots, K. \quad (4.7)$$

From the 7-brane tadpole condition (4.5), it is obvious that one needs stacks of branes with both positive and negative values for the overall winding to cancel $D7$ tadpoles. For negative overall winding number $\prod_\alpha n_\alpha^a < 0$, the positivity condition of eq. (4.7) can be easily satisfied, consistently with the supersymmetry requirement $J \wedge \mathcal{F}_a = 0$. However, for positive winding $\prod_\alpha n_\alpha^a > 0$, the second set of conditions are impossible to satisfy simultaneously with the inequality of (4.7). It is therefore not possible to obtain supersymmetric models with cancelled tadpoles, purely from magnetized $D7$ -branes.

To construct consistent supersymmetric models, one may turn on charged scalar VEV's, as was done for the magnetized $D9$ -branes in Section 2.3. The supersymmetry conditions then read:

$$F_{(2,0)}^a = 0 \quad ; \quad \int_{T^4} J \wedge \mathcal{F}_a = -v_a \int_{T^4} (J \wedge J - \mathcal{F}_a \wedge \mathcal{F}_a) \quad ; \quad \left(\prod_\alpha n_\alpha^a \right) (\mathcal{F}_a \wedge \mathcal{F}_a - J \wedge J) > 0 \quad , \quad \forall a \quad (4.8)$$

where the integration space T^4 is the appropriate 4-cycle, $v_a = \sum_k q_k^a |V_{a,k}|^2$, with q_k^a being the charge of the k -th scalar field acquiring a VEV $V_{a,k}$. Also, in general, v_a can take different values in different stacks. The last inequality is a consequence of the positivity of gauge couplings, as in the case of magnetized $D9$ -branes (2.17):

$$\frac{1}{g_a^2} = \frac{1}{(4\pi^2 \alpha')^2} \left(\prod_\alpha n_\alpha^a \right) \int_{T^4} (\mathcal{F}_a \wedge \mathcal{F}_a - J \wedge J) \quad . \quad (4.9)$$

Stack No.	Multiplicity	Flux	Fixed moduli	Windings
Stack-1	$N_1 = 1$	$p_{x^1 y^2} = p_{x^2 y^1} = 1$	$\tau_{12} = \tau_{21} = 0$	$(1, 1, 1, -1, 0, 0)$
Stack-2	$N_2 = 1$	$p_{x^1 x^2} = p_{y^1 y^2} = 1$	$\tau_{11} \tau_{22} = -1$	$(1, 1, 1, -1, 0, 0)$
Stack-3	$N_3 = 1$	$p_{x^2 y^3} = p_{x^3 y^2} = 1$	$\tau_{23} = \tau_{32} = 0$	$(0, 0, -1, 1, 1, 1)$
Stack-4	$N_4 = 1$	$p_{x^2 x^3} = p_{y^2 y^3} = 1$	$\tau_{22} \tau_{33} = -1$	$(0, 0, -1, 1, 1, 1)$
Stack-5	$N_5 = 1$	$p_{x^3 y^1} = p_{x^1 y^3} = 1$	$\tau_{31} = \tau_{13} = 0$	$(1, 1, 0, 0, 1, -1)$
Stack-6	$N_6 = 1$	$p_{x^3 x^1} = p_{y^3 y^1} = 1$	$\tau_{33} \tau_{11} = -1$	$(1, 1, 0, 0, 1, -1)$

Table 4: Brane configuration for the first six stacks of $D7$ -branes with oblique magnetic fluxes.

We now introduce two sets of brane stacks: the first set has negative overall winding numbers $\prod_{\alpha} n_{\alpha}^b < 0$, $b = 1, \dots, 6$ and no VEV for the charged scalars, while the second has positive windings and a non-vanishing VEV for some charged scalars. We use supersymmetry conditions (4.8) and explicitly construct a model where all the Kähler and complex structure moduli are stabilized. Specifically, the model we construct has nine stacks. The first six stacks contain purely off-diagonal (oblique) fluxes, i.e. components mixing the two T^2 's of T^4 . Among these six stacks, the fluxes along the first three are purely symmetric in complex coordinates $z_i = x^i + \tau y^i$, $i = 1, 2, 3$ describing the three tori, whereas the remaining three are antisymmetric. The last three stacks have fluxes purely along the diagonal components, $F_{i\bar{i}}$. Such fluxes are consistent supersymmetric solutions of the tadpole equations provided some scalar VEV's V_i are also simultaneously turned on. We are able to show that one can consistently stabilize the Kähler moduli as a function of V_i 's, for $V_i \ll 1$. We now go on to present the model explicitly.

4.3 A model

Below, we give an explicit model of magnetized $D7$ -branes, where all metric moduli are stabilized.

The first six stacks have multiplicity $N_b = 1$, $b = 1, \dots, 6$ and a negative 7-brane charge. Their fluxes are all oblique (given in Table 4). These then fix all metric moduli to be at a point of the moduli space where the six-dimensional torus is a direct product of three squared tori $T^6 = T^2 \times T^2 \times T^2$, or

$$\tau_{ij} = i\delta_{ij} \quad ; \quad J = J_{i\bar{i}} dz^i \wedge d\bar{z}^{\bar{i}} \quad , \quad J_{i\bar{j}} = 0 \quad (4.10)$$

while the Kähler moduli of the three T^2 remain unfixed. In addition to the above six branes,

Stack No.	Multiplicity	Flux	Windings
Stack-7	$N_7 = 2$	$p_{x^1 y^1} = p_{x^2 y^2} = 1$	$(1, 1, 1, 1, 0, 0)$
Stack-8	$N_8 = 2$	$p_{x^2 y^2} = p_{x^3 y^3} = 1$	$(0, 0, 1, 1, 1, 1)$
Stack-9	$N_9 = 2$	$p_{x^1 y^1} = p_{x^3 y^3} = 1$	$(1, 1, 0, 0, 1, 1)$

Table 5: Brane configuration for the last three stacks of $D7$ -branes with diagonal magnetic fluxes and charged scalar VEV's.

three more stacks are added where charged scalar VEV's are turned on. For these stacks fluxes and windings are given in Table 5. Let us assume for the sake of simplicity that all charged fields acquire the same VEV $v_a \equiv v$, $a = 7, 8, 9$. The fluxes are diagonal and fix the remaining three Kähler moduli in terms of the magnetic fluxes and the VEV of the charged fields. These are constrained by the second set of conditions of eq. (4.8) to

$$(J_{1\bar{1}} + J_{2\bar{2}}) = -v(J_{1\bar{1}}J_{2\bar{2}} - 1) \quad (4.11)$$

$$(J_{2\bar{2}} + J_{3\bar{3}}) = -v(J_{2\bar{2}}J_{3\bar{3}} - 1) \quad (4.12)$$

$$(J_{3\bar{3}} + J_{2\bar{2}}) = -v(J_{3\bar{3}}J_{1\bar{1}} - 1). \quad (4.13)$$

These equations have a unique solution:

$$J_{1\bar{1}} = J_{2\bar{2}} = J_{3\bar{3}} = J, \quad (4.14)$$

with

$$J = -\frac{1}{v} \left(1 - (1 + v^2)^{\frac{1}{2}} \right). \quad (4.15)$$

For small v , the positivity conditions of eq. (4.8) are satisfied and one has

$$J \simeq \frac{v}{2} \quad (4.16)$$

for v positive. The sign of v and consequently the positivity of the volume J can be easily verified by the presence of a tachyonic state in the spectrum in the limit where the VEVs vanish. We have therefore shown that the above 9 stacks stabilize all Kähler moduli.

4.4 Tadpole cancellation

The non-zero 7-brane tadpole contributions for various branes may be computed with the flux quanta given in Tables 4 and 5. One can easily check that all 7-brane tadpoles vanish, while the 3-brane tadpoles add up to

$$\mathcal{Q}_3^{tot} = \sum_{a=1}^9 N_a \mathcal{Q}_3^a = 12, \quad (4.17)$$

with $\mathcal{Q}_3^a = 1$ for each brane.

5 Dilaton Stabilization

5.1 3-form fluxes

A possible stabilization mechanism for the dilaton is by turning on R-R and NS-NS 3-form closed string fluxes, that for generic Calabi-Yau compactifications can fix also the complex structure [1]. As we are going to combine the two mechanisms, namely magnetic and 3-form fluxes, to stabilize the axion-dilaton modulus for our model in Section 4, we first review briefly the main properties of 3-form fluxes.

Let $H_{(3)}$ and $F_{(3)}$ be the field strengths of the NS-NS 2-form $B_{(2)}$ and of the R-R 2-form $C_{(2)}$, respectively, $H_{(3)} = dB_{(2)}$ and $F_{(3)} = dC_{(2)}$, subject to the usual Dirac quantization condition in the compact space. In the basis (α_a, β_b) given in eq. (A.2) of Appendix A, $H_{(3)}$ and $F_{(3)}$ can be written as

$$\begin{aligned} \frac{1}{(2\pi)^2 \alpha'} H_{(3)} &= \sum_{a=0}^{h_{2,1}} (h_1^a \alpha_a + h_2^a \beta_a) \\ \frac{1}{(2\pi)^2 \alpha'} F_{(3)} &= \sum_{a=0}^{h_{2,1}} (f_1^a \alpha_a + f_2^a \beta_a), \end{aligned} \quad (5.1)$$

where h_1^a, h_2^a, f_1^a and f_2^a are integers. Using the complex dilaton modulus, one can then form the 3-form $G_{(3)}$

$$G_{(3)} = F_{(3)} - \phi H_{(3)} \quad , \quad \phi = C_{(0)} + i g_s^{-1}, \quad (5.2)$$

where g_s is the string coupling. The 3-form background fields preserve then a common supersymmetry with the \mathbb{Z}_2 -orientifold projection of T^6/\mathbb{Z}_2 if the following conditions are fulfilled: $G_{(3)}$ has to be a primitive $(2, 1)$ form [14]:

$$G_{(3)} \wedge J = 0 \quad , \quad G_{(3)} \in H^{2,1}. \quad (5.3)$$

Actually, the second of the conditions above corresponds to finding a minimum of the GVW superpotential [15]

$$W = \int_{T^6} G_{(3)} \wedge \Omega, \quad (5.4)$$

with Ω the holomorphic 3-form (A.6). W has then to be covariantly constant with respect to all moduli, $D_I W = 0$, or equivalently:

$$W = 0 \quad , \quad \partial_\phi W = 0 \quad , \quad \partial_{\tau_{ij}} W = 0, \quad (5.5)$$

where ϕ is defined in (5.2). Note that all primitive (2,1)-forms are imaginary self-dual (ISD), $\star_6 G_{2,1} = iG_{2,1}$, where the star map \star_6 is the usual Hodge map on the torus.

Let us analyze further the supersymmetry conditions (5.5). For given flux quanta (5.1), they can be understood as conditions on the dilaton and complex structure moduli. More precisely, using the symplectic structure (A.3), the superpotential (5.4) reads

$$W = \frac{1}{(2\pi)^2 \alpha'} \int_{T^6} G_{(3)} \wedge \Omega = -(f_1^0 - \phi h_1^0) \det \tau + (f_2^0 - \phi h_2^0) + (f_1^{ij} - \phi h_1^{ij})(\text{cof} \tau)_{ij} + (f_2^{ij} - \phi h_2^{ij}) \tau_{ij}. \quad (5.6)$$

We can now express the three supersymmetry conditions (5.5) explicitly in the form :

$$0 = -(f_1^0 - \phi h_1^0) \det \tau + (f_2^0 - \phi h_2^0) + (f_1^{ij} - \phi h_1^{ij})(\text{cof} \tau)_{ij} + (f_2^{ij} - \phi h_2^{ij}) \tau_{ij} \quad (5.7)$$

$$0 = h_1^0 \det \tau - h_2^0 - h_1^{ij}(\text{cof} \tau)_{ij} - h_2^{ij} \tau_{ij} \quad (5.8)$$

$$0 = -(f_1^0 - \phi h_1^0)(\text{cof} \tau)_{kl} + (f_2^{kl} - \phi h_2^{kl}) + (f_1^{ij} - \phi h_1^{ij}) \epsilon_{ikm} \epsilon_{jln} \tau_{mn}, \quad (5.9)$$

where $\text{cof} \tau = (\det \tau) \tau^{-1, T}$. These are eleven conditions on the complex structure, parametrized by the nine elements τ_{ij} and the (complex) dilaton field ϕ . It is then in principle possible to fix all complex structure and dilaton moduli in terms of adequate quanta [1]. Let us now examine the primitivity condition $G_{(2,1)} \wedge J = 0$. We could naively think that this can be interpreted, for given fluxes, as conditions on the Kähler moduli. However, this condition is trivially satisfied in the case of generic Calabi-Yau compactifications, because there are no harmonic (3,2) forms on these manifolds. Therefore, this condition can only become partially non-trivial on Kähler moduli for compactification manifolds with more symmetries, such as the torus.

There exist however alternative possibilities to fix the metric moduli. As shown in previous sections, the presence of internal magnetic fluxes leads to conditions on both the Kähler class and complex structure moduli. For generic Calabi-Yau spaces one can fix only the former⁶, while for

⁶Note however that one can also fix complex structure moduli when non-trivial “fluxes” are turned on for scalar fields or for gauge field components with no physical zero modes [3].

toroidal compactifications it is possible to fix all metric moduli by a suitable choice of stacks of magnetized $D9$ or $D7$ -branes. On the other hand, the dilaton modulus remains unfixed, but can be stabilized using 3-form closed string fluxes. In fact, for fixed complex structure, the conditions (5.7) and (5.9) constrain exclusively the dilaton. Moreover, as the Kähler form is fixed by the presence of magnetic fields, the primitivity condition $G_{(2,1)} \wedge J = 0$ restricts the possible fluxes $G_{(2,1)}$ we can switch on. Finally, the value of the string coupling we can obtain in this way is strongly constrained by the tadpole conditions. The latter can be read off from the topological coupling of the 3-form fluxes with the R-R 4-form $C_{(4)}$ potential in the effective action of the ten-dimensional type IIB supergravity:

$$S_{CS} = \frac{1}{4i(2\pi)^7 \alpha'^4} \int_{\mathcal{M}_{10}} \frac{C_{(4)} \wedge G_{(3)} \wedge \bar{G}_{(3)}}{\text{Im}\phi} = -\mu_3 \frac{1}{2} \frac{1}{(2\pi)^4 \alpha'^2} \int_{\mathcal{M}_{10}} C_{(4)} \wedge H_{(3)} \wedge F_{(3)}, \quad (5.10)$$

where we defined the R-R charge μ_3 in terms of α' as $\mu_3 = (2\pi)^{-3} \alpha'^{-2}$. The coupling of $C_{(4)}$ to the magnetized $D9$ -branes gives its effective $R-R$ -charge, while the coupling of the $O3$ orientifold plane reads

$$S_{O3} = \mu_3 Q_{O3} \int_{\mathcal{M}_4} C_{(4)}, \quad (5.11)$$

where Q_{O3} is the R-R-charge of the $O3$ plane. Therefore, the integrated Bianchi identity for the modified R-R 5-form field strength $F_{(5)}$ reads

$$-\frac{1}{2} \frac{1}{(2\pi)^4 \alpha'^2} \int_{T^6} H_{(3)} \wedge F_{(3)} + \mathcal{Q}_3^{tot} + Q_{O3} = 0, \quad (5.12)$$

where the factor $\frac{1}{2}$ comes from the fact that the volume of the orientifold T^6/\mathbb{Z}_2 is half the volume of the torus T^6 .⁷

It follows from the ISD condition, that the contribution to (5.12) coming from the 3-form flux is always positive :

$$N_3 =: -\frac{1}{2} \frac{1}{(2\pi)^4 \alpha'^2} \int_{T^6} H_{(3)} \wedge F_{(3)} = \frac{1}{2g_s} \int_{T^6} H_{(3)} \wedge \star_6 H_{(3)} > 0. \quad (5.13)$$

Finally, the 3-brane tadpole could also receive contributions from ordinary $D3$ -branes. All together, the tadpole condition is now modified as

$$N_3 + \mathcal{Q}_3^{tot} + N_{D3} + Q_{O3} = 0, \quad (5.14)$$

where $Q_{O3} = -16$. As the first three terms in the l.h.s. of equation (5.14) contribute positively, the possible values of N_3 as well of \mathcal{Q}_3^{tot} are bounded. This restricts strongly the possible values

⁷Note that it does not come from the factor $\frac{1}{2}$ in (5.10) which is compensated by the magnetic coupling to $C_{(4)}$; see [16] for more details.

of the string coupling g_s . In order to obtain a small value for the string coupling, N_3 should be as large as possible and a small contribution from the integral $\int_{T^6} H_{(3)} \wedge \star_6 H_{(3)}$. This depends on the quanta h_1^a , h_2^a of (5.1) and on the Hodge star operator. The latter only depends on the complex structure [17]. It is therefore in principle possible to fix the string coupling g_s at small value with the help of either internal magnetic fields or 3-form fluxes. This is discussed in the next sub-section.

5.2 Application to the model of Section 4

To stabilize the dilaton, consistently with the geometric moduli stabilization achieved by magnetized branes in Section 4, we now introduce 3-form NS-NS and R-R fluxes to saturate the total 3-brane tadpole to 16. As mentioned in Section 4.4, each magnetized $D7$ -brane in our model of Section 4.3 contributes one unit in the effective 3-brane charge, so that that the nine magnetized 7-brane stacks, stabilizing all other close string moduli, already contribute $Q_3^{tot} = 12$. They also fix the complex structure to $\tau_{ij} = i\delta_{ij}$. We therefore look for a 3-form flux solution with $N_3 = 4$.

We thus introduce the minimal 3-form flux which is free of FW-anomalies by taking:

$$h_1^0 \quad , \quad f_2^0 \quad , \quad h_2^{11} \quad , \quad f_1^{11} \quad (5.15)$$

for H_3 and F_3 given in eqs. (5.1). The supersymmetry conditions eqs. (5.7) - (5.9) then imply:

$$h_1^0 = -h_2^{11} \quad ; \quad f_2^0 = f_1^{11} \quad ; \quad g_s = \frac{h_1^0}{f_2^0} = -\frac{h_2^{11}}{f_1^{11}} \quad (5.16)$$

and the induced 3-brane charge is

$$N_3 = \frac{1}{2} \int_{T^6} H_3 \wedge F_3 = \frac{1}{2} (h_1^0 f_2^0 - f_1^{11} h_2^{11}) = h_1^0 f_2^0. \quad (5.17)$$

It follows that the minimal value for the string coupling is obtained by $h_1^0 = 1$, $f_2^0 = 4$, such that the 3-brane tadpole condition (5.14) is saturated and $g_s = \frac{1}{4}$. Also, for the above choice of flux quanta, $G_{(3)}$ as defined in eq. (5.2) has the form:

$$G_{(3)} = -2idz_1 \wedge (dz_2 \wedge d\bar{z}_3 - dz_3 \wedge d\bar{z}_2), \quad (5.18)$$

which is a $(2,1)$ -form as expected and satisfies the primitivity condition $J \wedge G = 0$ for $J = \sum_i J_{i\bar{i}} dz_i \wedge d\bar{z}_i$.

6 Conclusions

We have therefore shown the stabilization of all geometric closed string moduli by using gauge field fluxes and charged scalar VEV's. Moreover, we have shown that the above stabilization method can be consistently implemented together with the ones using 3-form fluxes, in order to stabilize the dilaton-axion modulus, as well. A drawback of the construction is that the charged scalar VEV's, spoil the exact nature of the string construction, although they do not

influence in general the geometric moduli stabilization per se, which can be done in the absence of such VEV's as it was illustrated in explicit examples. Moreover, the string effective action description remains valid, as long as the inclusion of charged scalar effects are kept smaller than the string corrections whose strength is given by the inverse string tension. For this reason, we have required that the scalar VEV's, V_i 's, (with $v = \sum_i q_i |V_i|^2$), are small. Also, one has to ensure that the string coupling g_s is stabilized at a small value. Our solution satisfies this condition as well.

In the case of a generic Calabi-Yau compactification, the effective potential of the model has two contributions: 3-form closed string fluxes generate F -term potential for the complex structure moduli and dilaton-axion, whereas magnetic open string fluxes along branes (including the charged scalar VEV's) generate a D -term potential for the Kähler moduli. Both these terms are separately stabilized to zero value, giving a Minkowski vacuum.

An interesting exercise would be to find out if the fluxes along the branes can give rise to AdS_4 minima as well. Since the fluxes contribute to the space-time energy-momentum tensor, one can hope to obtain such a vacuum even when the charged scalar VEV's are tuned to zero value. AdS branes have in fact been studied in different contexts [18]. It remains to be seen though, whether one is able to obtain such backgrounds in the presence of magnetic fluxes for toroidal compactifications.

Another interesting question is to combine this method with D -brane model building and study properties of the corresponding effective field theory.

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A Appendix: Parametrization of T^6

Consider a six-dimensional torus T^6 having six coordinates with periodicity normalized to unity $x^i = x^i + 1$, $y_i = y_i + 1$ [19]. Writing the coordinates as x^i , y_i , $i = 1, 2, 3$, we choose then the orientation⁸

$$\int_{T^6} dx^1 \wedge dy_1 \wedge dx^2 \wedge dy_2 \wedge dx^3 \wedge dy_3 = 1 \quad (\text{A.1})$$

and define the basis of the cohomology $H^3(T^6, \mathbb{Z})$

$$\begin{aligned} \alpha_0 &= dx^1 \wedge dx^2 \wedge dx^3 \\ \alpha_{ij} &= \frac{1}{2} \epsilon_{ilm} dx^l \wedge dx^m \wedge dy_j \\ \beta^{ij} &= -\frac{1}{2} \epsilon^{jlm} dy_l \wedge dy_m \wedge dx^i \\ \beta^0 &= dy_1 \wedge dy_2 \wedge dy_3, \end{aligned} \quad (\text{A.2})$$

forming a symplectic structure on T^6 :

$$\int_{T^6} \alpha_a \wedge \beta^b = -\delta_a^b, \quad \text{for } a, b = 1, \dots, h_3/2, \quad (\text{A.3})$$

with $h_3 = 20$, the dimension of the cohomology $H^3(T^6, \mathbb{Z})$.

We can also choose complex coordinates

$$z^i = x^i + \tau^{ij} y_j, \quad (\text{A.4})$$

where τ^{ij} is a complex 3×3 matrix parametrizing the complex structure. In this basis, the cohomology $H^3(T^6, \mathbb{Z})$ decomposes in four different cohomologies corresponding to the purely holomorphic parts and those with mixed indices:

$$H^3(T^6) = H^{3,0}(T^6) \oplus H^{2,1}(T^6) \oplus H^{1,2}(T^6) \oplus H^{0,3}(T^6). \quad (\text{A.5})$$

The purely holomorphic cohomology $H^{3,0}$ is one-dimensional and is formed by the holomorphic three-form Ω for which we choose the normalization

$$\Omega = dz^1 \wedge dz^2 \wedge dz^3. \quad (\text{A.6})$$

⁸This is the orientation of [19], which is different from the one of [1].

In terms of the real basis (A.2), this can be written as

$$\Omega = \alpha_0 + \tau^{ij} \alpha_{ij} - \text{cof} \tau_{ij} \beta^{ij} + \det \tau \beta_0, \quad (\text{A.7})$$

where $\text{cof} \tau_{ij}$ is given by $\text{cof} \tau = (\det \tau) \tau^{-1, T}$. We can then define the periods of the holomorphic 3-form to be

$$\tau^a = \int_{A_a} \Omega \quad , \quad F_b = \int_{B^b} \Omega. \quad (\text{A.8})$$

Note that the period F_b can be written as the derivative of a prepotential F : $F_b = \partial_{\tau^b} F$.

Similarly, the cohomology $H^2(T^6, \mathbb{Z})$ decomposes also in three cohomologies

$$H^2(T^6) = H^{2,0}(T^6) \oplus H^{1,1}(T^6) \oplus H^{0,2}(T^6). \quad (\text{A.9})$$

We choose the basis $e^{i\bar{j}}$ of $H^{1,1}$ to be of the form

$$e^{i\bar{j}} = i dz^i \wedge d\bar{z}^{\bar{j}}. \quad (\text{A.10})$$

The Kähler form can therefore be parametrized as

$$J = J_{i\bar{j}} e^{i\bar{j}}. \quad (\text{A.11})$$

As the Kähler form is a real form, its elements satisfy the reality condition $J_{i\bar{j}}^\dagger = J_{j\bar{i}}$. Therefore J depends only on nine real parameters.

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