The Economics of Rotating Savings and Credit Associations

by

Timothy Besley, Stephen Coate,
and Glenn Loury

MIT-CEPR 90-014WP

May 1990

The Economics of Rotating Savings and Credit Associations*

by

Timothy Besley
Princeton University

Stephen Coate
Harvard University

and

Glenn Loury
Harvard University

May 1990

Abstract

This paper examines the role and performance of an institution for allocating savings which is observed world wide - rotating savings and credit associations. We develop a general equilibrium model of an economy with an indivisible durable consumption good and compare and contrast these informal institutions with credit markets and autarkic saving in terms of the properties of their allocations and the expected utility which they obtain. We also characterize Pareto efficient and expected utility maximizing allocations for our economy, which serve as useful benchmarks for the analysis. Among our results is the striking finding that rotating savings and credit associations which allocate funds randomly may sometimes yield a higher level of expected utility to prospective participants than would a perfect credit market.

*The authors are grateful to Anne Case, Richard Zeckhauser, and seminar participants at Boston, Cornell, Harvard, Oxford, Princeton and Queen's Universities for their helpful comments. Financial support from the Center for Energy Policy studies, M.I.T., and the Japanese Corporate Associates Program at the Kennedy School of Government, Harvard University is gratefully acknowledged.

I. Introduction

This paper examines the economic role and performance of Rotating Savings and Credit Associations (Roscas). These informal credit institutions are found all over the world, particularly in developing countries[1]. While their prevalence and, to some degree, robustness has fascinated anthropologists[2], there has been very little work devoted to understanding their economic performance. Here we shall subject them to scrutiny in a general equilibrium setting, contrasting and comparing them with other financial institutions.

Roscas come in two main forms. The first type allocates funds using a process of random allocation. In a *Random Rosca*, members commit to putting a fixed sum of money into a "pot" for each period of the life of the Rosca. Lots are drawn and the pot is randomly allocated to one of the members. In the next period, the process repeats itself, except that the previous winner is excluded from the draw for the pot. The process continues, with every past winner excluded, until each member of the Rosca has received the pot once. At this point, the Rosca is either disbanded or begins over again.

Roscas may also allocate the pot using a bidding procedure. We shall refer to this institution as a *Bidding Rosca*. The individual who receives the pot in the present period does so by bidding the most in the form of a pledge of higher future contributions to the Rosca or one time side payments to other Rosca members. Under a Bidding Rosca, individuals may still only receive the pot once – the bidding process merely establishes priority[3].

We take the view, supported in the literature, that these institutions are primarily a means of saving up to buy indivisible goods, such as bicycles, or to finance major events, such as weddings. Random Roscas are not particularly effective as institutions for buffering against risk since the probability of obtaining the pot need not be related to one's immediate circumstances. Even Bidding Roscas, which may allow an individual to obtain the pot immediately, only permit individuals to deal with situations which cannot recur, since the pot may be obtained no more than once. Furthermore, since many kinds of risks in LDC's are covariant, many individuals will have high valuations at the same instant. Roscas do play a greater role in transferring resources to meet life–cycle needs such as financing a wedding. However, even in this context, they seem more appropriate for dealing with significant, idiosyncratic events, rather than the hump saving required for old age.

Despite its manifest importance, there has been relatively little analysis in the savings literature on the notion of saving up to buy an indivisible good. Yet, the existence of indivisible goods does, by itself, provide an argument for developing institutions which intermediate funds. In a closed economy without access to external funds, individuals have to save from current income to finance lumpy expenditures, and can gain from trading with one another. The savings of some agents can be used to finance the purchases of others. By contrast, when all goods are divisible individuals can accumulate them gradually and there need be no gains from intertemporal trade. Clearly autarky with indivisible goods is inefficient. Savings lie idle during the accumulation process when they could be employed to permit some individuals to enjoy the services of the indivisible durable good.

As we explain in more detail in the next section, Roscas provide one method of making these joint savings work[4]. Given the worldwide prevalence of these institutions, it is of interest to understand precisely how they do mobilize savings and, in particular, the differences between Random and Bidding Roscas. It is also interesting to consider how the allocations resulting from Roscas differ from that which would emerge with a fully functioning credit market. Such markets are the economist's natural solution to the problem of intermediation. Yet there are many settings in which they are not observed. We hope, through our comparative analysis, to suggest some reasons why this is so.

Our study, in the context of a simple two–good model with indivisibilities, characterizes the path of consumption and of the accumulation of indivisible goods through time under these alternative institutional arrangements. We also investigate the welfare properties of Roscas and of a competitive credit market, employing the criteria of ex–post Pareto efficiency, and of *ex–ante* expected utility. Neither type of Rosca, as modeled here, is *ex–post* efficient. We find that, with homogeneous agents, randomization is preferred to bidding as a method of allocating funds within Roscas. Moreover, while Bidding Roscas are always dominated by a perfect credit market, we show that, in terms of *ex–ante* expected utility, this is not generally the case for Random Roscas.

We use a model which makes no claim to generality, but which we believe captures the essential features of the problem described above. We keep the model simple in the interests of making the insights obtained from it as sharp as possible. Some important issues concerning Roscas are not dealt with here. In particular, we do not address the question of why, once an individual has received the "pot", he may be presumed to keep his commitment to pay into the Rosca. The anthropological literature reveals that the incentive to defect from a Rosca is curbed quite effectively by social constraints[5]. Roscas are typically formed among individuals whose circumstances and characteristics are well known to each other. Defaulters are sanctioned socially as well as not being permitted to take part in any future Rosca. Thus, in contrast to the anonymity of markets, Roscas appear to be institutions of financial resource allocation which rely in an essential way upon the "social connectedness" of those among whom they operate. It would be interesting to try to formalize this idea in a model allowing default. In future work we will consider the sustainability of Roscas in greater detail.

The remainder of the paper is organized as follows. Section II provides an informal overview of the economic role of Roscas and outlines the main questions to be addressed in this paper. Section III describes the model which provides the framework for our analysis. Section IV then formalizes the workings of Roscas and a credit market in the context of this model. Section V develops properties of efficient and optimal allocations of credit. The role of Section VI is to compare the different institutions. We describe how the accumulation and consumption time paths differ under the alternative arrangements. We also establish the normative properties of the allocations. Section VII concludes.

II. The Economic Role of Roscas: An Informal Overview

Consider a community populated by 20 individuals, each of whom would like to own some durable, indivisible consumption good — say, a bicycle. Suppose that bicycles cost \$100. If individuals, left to their own devices, were to save \$5 per month, then each individual would acquire a bicycle after 20 months. That is, under autarky and given the savings behavior assumed, nobody would have the good before 20 months, at which time everyone would obtain it. Obviously this is inefficient since, with any preference for the early receipt of their bicycles, it would be possible, by using the accumulating savings to buy one bicycle per month, to make everyone except the final recipient strictly better off.

Roscas represent one response to this inefficiency. Let our community form a Random Rosca which meets once a month for 20 months, with contributions set at \$5 per month. Every month one individual in the Rosca would be randomly selected to receive the pot of \$100, which would allow him to buy a bicycle. It is clear that this results in a Pareto superior allocation. Each individual saves \$5 per month as under autarky but now gets a bicycle on average 10 months sooner. Note that risk aversion is not an issue since, viewed *ex ante*, the Random Rosca does as well as autarky in every state of the world, and strictly better in all but one.

Forming a Random Rosca which lasts for 20 months is only one possibility. The community could, for example, have a Rosca which lasts for 10 months with members making the \$5 contribution, and someone getting a bicycle every half–month. Or, the Rosca could last for 30 months with members contributing \$5, and a bicycle being acquired, every month and a half. Given the uniform spacing of meeting dates and the constant contribution rate, common features of Roscas as they are observed in practice, the length of the Rosca will be inversely proportional to the rate at which the community saves and accumulates bicycles. It seems reasonable to suppose that the Rosca will be designed with a length which maximizes the (ex–ante expected) utility of its "representative" member. Supposing that this is the case, one can then characterize the optimal length of the Rosca and the savings rate which it implies.

If instead of determining the order of receipt randomly individuals were to bid for the right to receive the pot early, the community would be operating a Bidding Rosca. Suppose that Rosca members, at their initial meeting, bid for receipt dates by promising to make future contributions at various (constant) rates, with a higher bid entitling one to an earlier receipt date. In our 20 month example, the individual receiving the pot first might agree to contribute (say) \$6 per meeting for the life of the Rosca, the individual who is to be last might only pay \$4 per meeting, and others might pay something in between. Whatever the exact numbers, two things should be true in equilibrium: overall contributions should total \$100 per meeting; and, individuals should not prefer the contribution/receipt date pair of any other Rosca member to their own.

While both Bidding and Random Roscas allow the community to utilize its savings more effectively than under autarky, they clearly will not result in identical allocations. Even when they have the same length, the consumption paths of members differ since, under a Bidding Rosca, those who get the good early must forego consumption to do so. Moreover, there is no reason to suppose that the optimal length will be the same for both institutions. Of particular interest are the welfare comparisons between the institutions. Understanding what determines their relative performance gives insights into the circumstances under which we might expect to see one or the other type of Rosca in practice.

Of course, Roscas are not the only institutions able to mobilize savings. Imagine the introduction of (competitive) banking into the community. Bankers would attract savings by offering to pay interest, while requiring that interest be paid on loans made to finance the early acquisition of the bicycle. These borrowers would enjoy the durable's services for a longer period of time at the cost of the interest payments on the borrowed funds. Those deferring their purchase of the bicycle would receive interest on their savings with which to finance greater non–durable good consumption over the course of their lives.

Under ideal competitive conditions a perfect credit market would establish a pattern of interest rates such that, when individuals optimally formulate their intertemporal consumption plans, the supply of savings would equal the demand for loans in each period. Such a market is evidently a much more complex mechanism than the Roscas described above. It is natural, however, to wonder how the intertemporal allocation of durable and non–durable consumption which it produces compares with those attained under Roscas. Of particular interest are the welfare comparisons: how do these simple institutions perform relative to a fully functioning credit market?

III. The Model

We consider an economy populated by a continuum of identical individuals. Time is continuous and consumers have finite lives of length T. These assumptions allow us to treat the time individuals decide to purchase the indivisible good, the length of Roscas, and the fraction of the population who have the indivisible good at any moment, as continuous variables. This avoids some technically awkward, but inessential, integer problems.

Each individual in the economy receives an exogenously given flow of income over his lifetime at a constant rate of $y>0$. At any moment, individuals can receive utility from the flow of a consumption good, denoted by c, and the services of a durable good. We imagine that the durable good is indivisible and can be purchased at a cost of B.[6] Once purchased, we assume that it does not depreciate, yielding a constant flow of services for the remainder of an individual's lifetime. We assume that the durable good is produced for a large (global) market whose price is unaffected by the actions of agents in our economy. Moreover, we assume that the services of the durable are not fungible across agents – one must own it to enjoy its services.[7]

We assume, for analytical convenience, that preferences are intertemporally separable, and quasi–linear in the flows of consumption and of the durable's services. At each instant an individual receives a flow of utility at the rate v(c) if he does not have the durable good, and v(c) + ξ if he does. Moreover, we assume that individuals do not discount the future. Thus there is no motive for saving except to acquire the indivisible good. These assumptions make our analysis much less cumbersome without substantially affecting the main insights of the paper. We make the following assumption concerning the utility function $v(\cdot)$:

Assumption 1: $v: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is three times continuously differentiable, and satisfies $v'(\cdot) > 0$, $v''(\cdot) < 0$ and $v'''(\cdot) > 0$.

As well as being increasing and strictly concave, we require the utility function to have a positive third derivative. This assumption, which is not essential for much of the analysis, is satisfied by most plausible utility functions. It is sufficient for individuals to have a demand for precautionary savings (see Leland (1968)) and also guarantees proper risk aversion in the sense of Pratt and Zeckhauser (1987).

A *consumption bundle* for an individual in this economy may be described by a pair $<s,c(\cdot)>$, where $s \in \mathcal{T} \equiv [0,T] \cup \{\nu\}$ denotes the date of receipt of the durable good, and $c:[0,T] \rightarrow \mathbb{R}_+$ gives the rate of consumption of the non–durable at each date. By "$s = \nu$" we mean the individual *never* receives the durable good. We suppose, without loss of generality, that the population is uniformly distributed over the unit interval and we index different consumers with numbers $\alpha \in [0.1]$. An *allocation* is then a set of consumption bundles, one for each consumer type, and may be represented by a pair of functions $<s(\cdot),c(\cdot,\cdot)>$, such that $s:[0,1] \rightarrow \mathcal{T}$ and $c:[0,1]\times[0,T] \rightarrow \mathbb{R}_+$. The function $s(\cdot)$, hereafter referred to as the *assignment function*, tells us the dates at which different individuals receive the durable. By relabelling individuals as required, we may assume with no loss of generality that individuals with lower index numbers receive the durable earlier. Thus, letting $\bar{\alpha}$ denote the fraction of the population *ever* to receive the durable, we may suppose that $s(\cdot)$ is non-decreasing on $[0,\bar{\alpha}]$, $0 \leq \bar{\alpha} \leq 1$, and that $s^{-1}(\nu) = (\bar{\alpha},1]$. If $\bar{\alpha} = 1$, then $s^{-1}(\nu) = \emptyset$ and everyone receives the good – this being the case on which most of the analysis will focus. The second component of an allocation gives us the consumption path $\{c(\alpha,\tau):\tau\epsilon[0,T]\}$ of each individual α.[8]

Under the allocation <s,c>, an individual of type α enjoys utility:

$$(3.1)\qquad u(\alpha;<s,c>) \equiv \begin{cases} \int_0^T v(c(\alpha,\tau))d\tau + \xi(T-s(\alpha)), & s(\alpha)\in[0,T] \\ \int_0^T v(c(\alpha,\tau))d\tau, & s(\alpha)=\nu. \end{cases}$$

To be feasible, the allocation must consume no more resources than are available over any time interval. To make this precise, for any assignment function $s(\cdot)$ and any date $\tau\epsilon[0,T]$, we define $N(\tau;s)$ to be the fraction of the population which has received the durable by time τ with assignment function $s(\cdot)$.[9] Then an allocation <s,c> is *feasible* if, for all $\tau\epsilon[0,T]$:

$$(3.2)\qquad \int_0^\tau (y - \int_0^1 c(\alpha,x)d\alpha)dx \geq N(\tau;s)B.$$

The left hand side of (3.2) denotes aggregate saving at τ and the right hand side denotes aggregate investment. Hence, an allocation is feasible if aggregate investment never exceeds aggregate saving. In the sequel we will let F denote the set of feasible allocations.

IV. Alternative Institutional Arrangements

The object of this paper is to compare different institutional arrangements for allocating savings and it is the task of this section to describe four possible ways of allocating resources in our model – Autarky, a Random Rosca, a Bidding Rosca, and a perfect credit market. Each institution that we consider will result in a particular assignment function and set of consumption paths, and thus a particular distribution of utility on [0,1]. We can evaluate the institutional alternatives by comparing these utility distributions.

IV.1 Autarky

The simplest institution is *Autarky* – i.e., no financial intermediation at all. In this case saving occurs in isolation. An individual desiring to acquire the durable, given strictly concave utility and no discounting, will save the required sum B at the constant rate $y-c$, over some period of accumulation $[0,t]$. Acquisition is desirable if the utility from such a program exceeds $T\cdot v(y)$, i.e. the life–time utility attainable without saving to buy the durable. Consider, therefore, an individual's life–time utility maximization problem of choosing c and t to:

$$\text{(4.1)} \qquad \begin{gathered} \text{Max } \{t\cdot v(c) + (T-t)\cdot(v(y)+\xi)\} \\ \text{subject to } t\cdot(y-c)=B,\ 0\leq c\leq y \text{ and } 0\leq t\leq T. \end{gathered}$$

Let (c_A,t_A) denote the solution to this problem and let W_A denote the maximal value.

To develop an insightful expression for W_A, it is convenient to define

$$\text{(4.2)} \qquad \mu(\xi) \equiv \underset{0\leq c\leq y}{\text{Min}} \left[\frac{v(y)-v(c)+\xi}{y-c}\right],\ \xi \geq 0.$$

Assuming that $t_A < T$ at the optimum in (4.1),[10] then substitution and rearrangement leads to the following:

$$\text{(4.3)} \qquad W_A = T\cdot[v(y)+\xi] - B\cdot\mu(\xi).$$

Expression (4.3) admits an appealing interpretation, anticipating a more general result obtained in section V. The value of optimal acquisition of the durable under Autarky is the difference between the two terms on the right hand side of (4.3). The first equals what lifetime utility would be if the durable were a free good. Hence, the second term may be understood as the utility cost of acquiring the durable imposed by the need to save up. From (4.2), it is easily seen that this cost is directly proportional to the price of the durable and to the loss of utility per unit time during the accumulation phase, and is inversely proportional to the length of that phase. The optimal autarkic consumption (and hence savings) rate, c_A, is chosen to minimize this cost.

It should be noted that the function $\mu(\cdot)$ possesses all the usual properties of a cost function – it is non–negative, increasing and strictly concave[11]. Moreover, from the Envelope Theorem the optimal acquisition date, t_A, may be obtained by differentiating the cost function, i.e. $t_A = B\cdot\mu'(\xi)$. These facts will be used repeatedly below.

It is apparent from (4.3) that acquisition of the durable under Autarky will be desirable [i.e. $W_A \geq T\cdot v(y)$] if and only if $B\cdot\mu(\xi) \leq \xi\cdot T$. We formalize this requirement in the assumption that follows:

Assumption 2: (Desirability) Let $\underline{\xi}(y,\frac{T}{B})$ be defined by $\mu(\underline{\xi})/\underline{\xi} = \frac{T}{B}$. Then $\xi \geq \underline{\xi}(y,\frac{T}{B})$.[12]

Desirability requires that some combination of the utility of durable services (ξ), the length of the lifetime over which these services may be enjoyed (T), and the income flow (y) be large enough relative to the cost of the durable (B) to make its acquisition worthwhile. Even if durable good acquisition is not desirable under Autarky, it may be so under other institutional arrangements. We shall return to this point below.

If the durable good is desirable, then lives under Autarky will be characterized by two phases. In the first of these individuals save at a constant rate until they have sufficient funds to buy the durable and, since individuals are the same, this phase and the consumption rate will be identical for all agents (see (4.1) and (4.3)). Hence, letting $<s_A,c_A>$ denote the allocation under Autarky, $s_A(\alpha) = t_A$ and $c_A(\alpha,\tau) = c_A$ when $\tau\epsilon[0,t_A]$ for all $\alpha \in [0,1]$. The second phase begins after the durable has been purchased. In this phase, individuals consume all of their income while enjoying durable services as well, i.e. $c_A(\alpha,\tau) = y$ when $\tau\epsilon[t_A,T]$ for all $\alpha \in [0,1]$. Hence the allocation achieved under Autarky is as illustrated in Figure 1. Each individual has an identical piecewise horizontal consumption path and the fraction of individuals who have the durable at time τ jumps from 0 to 1 at date t_A.[13]

The Autarkic allocation is not Pareto efficient since at each moment before the acquisition date, the accumulated savings of $\tau\cdot(y-c_A)$ lie idle, even though they could be used to finance durable good acquisition for some agents without lowering the utility of anybody else in the economy. This inefficiency arises solely because of the indivisibility of the durable – Autarkic accumulation of a divisible good would be Pareto efficient. Any effort to mobilize savings must simultaneously provide a mechanism to determine the order in which individuals will have access to them. The institutions of financial intermediation which we consider now are distinguished primarily by the different ways in which they treat this "rationing" requirement.

IV.2 Random Rosca

A Random Rosca allocates access to the savings which it generates by lot. Explaining how such a Rosca operates in our continuum economy requires some care. Consider a Random Rosca of length t, with contributions set so that the pot available to each winning member is equal to the cost of the indivisible good, B. To fix ideas, assume there are a finite number n members of the Rosca. Further suppose that it meets at the uniformly spaced dates $\{t/n, 2t/n, 3t/n, ...,t\}$, and that at each meeting every member contributes the sum B/n. A different individual is selected at each meeting to receive the pot of B, which allows him to buy the durable good.[14] Prior to its initiation, a representative member of this Rosca perceives his receipt date for the pot to be a random variable, $\tilde{\tau}$, with a uniform distribution on the set $\{t/n, 2t/n, \ldots, t\}$. Given Assumption 1, each member saves at the constant rate equal to B/t over the life of the Rosca. Thus each member's lifetime utility is the random variable:

$$(4.4) \qquad W(t,\tilde{\tau}) \equiv t \cdot v(y-B/t) + (T-t) \cdot (v(y)+\xi) + (t-\tilde{\tau})\xi.$$

The continuum case may be understood as the limit of this finite case as n approaches infinity.[15] As n grows, the Rosca meets more and more frequently.[16] In the limit it is meeting at each instant of time, and the receipt date of the pot, $\tilde{\tau}$, becomes a continuous random variable which is uniformly distributed on the interval [0,t]. Thus the *ex ante* expected utility of a representative individual who joins the Rosca of length t is given by the expectation of (4.4), taking $\tilde{\tau}$ to be uniform on [0,t]. Hence,

$$(4.5) \qquad W(t) = t \cdot v(y-B/t) + (T-t) \cdot (v(y)+\xi) + t\xi/2.$$

In the above discussion we took the length of the Random Rosca, t, as given. It seems natural, however, to assume that it will be chosen to maximize members' *ex-ante* expected utility – W(t). Thus consider than the problem of choosing t to maximize (4.5). We denote the solution by t_R and use W_R to represent the maximum expected utility. Notice that this problem is almost identical to that in (4.1) (after solving for c from the budget constraint in (4.1)). We may therefore proceed by analogy with (4.1) through (4.3) and, assuming $t_R < T$, write[17]

$$(4.6) \qquad W_R = T \cdot [v(y)+\xi] - B \cdot \mu(\xi/2).$$

As in the case of Autarky, we can use the Envelope Theorem to deduce that

$t_R = B \cdot \mu'(\xi/2)$.

Expression (4.6) reveals the elegant simplicity of our model of a Random Rosca. Comparing (4.6) with (4.3), it is immediately apparent how a Random Rosca improves upon Autarky. The financial intermediation provided by the Rosca lowers the utility cost of acquiring the durable, i.e. $W_R - W_A = B \cdot [\mu(\xi) - \mu(\frac{\xi}{2})] > 0$, since as we noted above, $\mu(\cdot)$ is increasing.

So far, we have taken the question of desirability under a Random Rosca for granted. This does, after all, seem reasonable. The savings in a Rosca are being put to work and each member of a Random Rosca expects to enjoy the services of the durable $t_R/2$ units of time more than he would by choosing the same saving rate under Autarky. So even if acquisition of the durable good is undesirable under Autarky, it seems reasonable to suppose that the agents would prefer to form a Random Rosca and acquire the durable good rather than go without it. This situation does indeed occur if $W_R > T \cdot v(y) > W_A$.

This belies, however, two more subtle issues related to desirability. First, for the model of a Random Rosca developed here to be consistent, it must be that each member would actually choose to invest in the durable good upon winning the pot, as is assumed in the foregoing derivations. This will be true only if the horizon remaining after the last member has received his winnings is long enough to justify the investment, which will depend upon the consumption value of the durable's services.

Second, the finite formulation whose limit is characterized in (4.6) assumes that the number of meeting dates of the Rosca is equal to the number of members. Relaxation of this assumption allows for the possibility that members might elect to face an *improper* probability distribution for the random receipt date $\tilde{\tau}$. Suppose, for example, that with an an even number of members, meetings occurred on dates $\{2t/n, 4t/n, 6t/n, \ldots, t\}$. Now, to make his contribution of B/n at each meeting, an individual would save at the constant rate of $B/2t$. In the continuum case, conditional on winning the pot, each agent would face a uniform distribution of receipt dates $\tilde{\tau} \epsilon [0,t]$, but the unconditional probability of ever receiving the pot would only be one–half. Could it be that such a "partially funded" Rosca would be preferred to the "fully funded" version whose version whose welfare is given in (4.5)?

To guarantee that Rosca members will always want to design a Rosca so that they all eventually receive the pot and that buying the durable is always an optimal response to winning, we require a stronger desirability assumption than we made under Autarky. It turns out that if ξ is twice as big as the minimal value needed to induce acquisition of the durable good under Autarky, then (4.6) does indeed characterize the performance of an optimal Random Rosca. Thus we replace Assumption 2 with:

Assumption 2′: (Strong Desirability) $\xi \geq 2\underline{\xi}(y,\frac{T}{B})$.

Then we have:

Lemma 1: Under Assumption 2′ an optimal Random Rosca involves every member receiving the pot during the life of the Rosca. Moreover, every participant of the optimal Random Rosca will desire to use the proceeds to acquire the durable good.

Proof: See the Appendix.

In fact, specifying the Random Rosca so that everyone receives the pot once is not essential for much of our comparative welfare discussion in section VI. Our analysis there compares Random Roscas constrained to provide probability one of eventual receipt of the pot to alternative arrangements. The finding that a Random Rosca is preferred to other institutions is only strengthened by allowing the possibility that not everyone wins. On the other hand, partially funded Roscas do not appear to be common in practice. We do need Assumption 2′, however, for its implication that all winners will prefer to invest in the durable good. Otherwise the analysis culminating in (4.6) is inconsistent.

The intuitive reason for the stronger desirability requirement comes from noting that randomization is tantamount to introducing divisibility of the durable good, since being offered a probability of receiving the durable good is like being offered a fraction of it *ex ante*. Assumption 2 assures that "all" is preferred to "nothing" – the only choices facing an Autarkic individual. Assumption 2′ must be stronger since it has to guarantee that "all" is preferred to "anything less", i.e. intermediate levels of durable consumption must also be dominated.

Let $<s_R,c_R>$ denote the allocation achieved with the optimal Random Rosca under Strong Desirability. Recalling that an individual's type, α, is identified with his order of receipt of the durable, the assignment function will be $s_R(\alpha)=\alpha t_R$. As under Autarky, all individuals have identical consumption paths which fall into two distinct phases. To be more precise, $c_R(\alpha,\tau)=c_R$, for $\tau\epsilon[0,t_R]$; and, $c_R(\alpha,\tau)=y$, for $\tau\epsilon(t_R,T]$, where $c_R=y-B/t_R$. The allocation with the optimal Random Rosca is exhibited in Figure 1. While consumption is still a piecewise horizontal function for each agent, the fraction of the population who have received the durable at time t is now increasing and linear (see Figure 1b). Moreover, while exhibiting the same general pattern for consumption, we shall show below that the accumulation phase will not last the same length of time as under Autarky.[18]

IV.3 Bidding Rosca

We turn next to the Bidding Rosca, where the order in which individuals receive the accumulated savings is determined by bidding. As discussed in section II, the simplest assumption to make is that the bidding takes place when the Rosca is formed at time zero, and involves individuals committing themselves to various contribution rates over the life of the Rosca. We do not model the auction explicitly, but instead simply characterize what we believe would be the most plausible outcome in this context. Specifically we will require that, for a Bidding Rosca of length t, an individual receiving the "pot" at date $\tau \epsilon [0,t]$ commits to contributing into the Rosca at the constant rate $b(\tau)$ over this interval. A set of bids $\{b(\tau):\tau\epsilon[0,t]\}$ constitutes an equilibrium if: (i) no individual could do better by out bidding another for his place in the queue; and (ii) contributions are sufficient to allow each participant to acquire the durable upon receiving the pot. The first condition is an obvious requirement of any bidding equilibrium. The second precludes the desirability of saving outside the Rosca. Any such saving would lie idle and thus be inefficient.

Now in the equilibrium of a Bidding Rosca of length t, the utility of an individual receiving the pot at time $\tau\epsilon[0,t]$ is given by: $u = tv(y-b(\tau))+(T-t)(v(y)+\xi)+(t-\tau)\xi$. Condition (i) then implies that, for all $\tau,\tau'\epsilon[0,t]$:

$$v(y-b(\tau)) + (1-\tfrac{\tau}{t})\xi = v(y-b(\tau')) + (1-\tfrac{\tau'}{t})\xi; \tag{4.7}$$

while condition (ii) implies that

$$\int_0^T b(\tau)d\tau = B. \tag{4.8}$$

These two equations uniquely determine the function $b(\cdot)$, given the Rosca's length t. An optimal Bidding Rosca will have its length chosen so that the utility level of a representative member is maximized subject to (4.7) and (4.8).

To characterize the optimal Bidding Rosca, consider the consumption rate of an individual who receives the pot at date τ, which we denote by $\bar{c}(\tau) \equiv y - b(\tau)$ for $\tau \in [0,t]$. We may describe the Bidding Rosca just as well with the function $\bar{c}(\cdot)$ as we could by considering the bids directly. Substituting this into (4.8) and making the change of variable $\alpha = \tau/t$, yields

$$(4.9) \qquad t\cdot[y-\int_0^{1}\tilde{c}(\alpha)d\alpha] = B.$$

Now consider the consumption level, $c = \tilde{c}(x)$, of an individual who receives the durable at xt, ($x\epsilon[0,1]$) and note that (4.7) implies that $\tilde{c}(\alpha) = v^{-1}(v(c)-(x-\alpha)\xi)$, for all $\alpha\epsilon[0,1]$. Hence all other individuals' consumption levels can be determined from the equal utilities condition once c is known. Moreover, defining $\lambda(c,x)\equiv\int_0^1[v^{-1}(v(c)-(x-\alpha)\xi)]d\alpha$ as the average consumption level and using (4.9), we may write $t = B/(y-\lambda(c,x))$. Hence the utility level of the member who receives the pot at date xt can be written as:

$$(4.10) \qquad W(c) = T\cdot[v(y)+\xi] - B\cdot[v(y)-v(c)+x\xi]/[y-\lambda(c,x)]$$

Since, all members have identical utility levels, by construction, maximizing the utility of a particular member is the same as maximizing the common utility level. Thus we can view the problem solved by the Bidding Rosca as choosing c to maximize (4.10). Let $c_B(x)$ denote the optimal consumption level for type x individuals and let W_B denote the welfare level achieved at the optimum. Using by now familiar arguments we may write:

$$(4.11) \qquad W_B = T\cdot[v(y)+\xi] - B\cdot\mu_B(\xi),$$

where

$$\mu_B(\xi) \equiv \underset{0\leq c\leq y}{\text{Min}} \left[\frac{v(y)-v(c)+x\xi}{y-\lambda(c,x)}\right],\ \xi \geq 0.$$

This admits the same interpretation that we noted for both Autarky and the Bidding Rosca, as the difference between utility were the durable free and the cost of saving up. Note that W_B and mean consumption at the optimum, denoted by λ_B

($\equiv\lambda(c_B(x),x)$), are both independent of x by construction.[19] Note also that the optimal length of the Bidding Rosca, which we denote by t_B, will be given by $t_B=B/(y-\lambda_B)$.

Once again, the above formulation assumes that individuals will always buy the durable when they receive the pot. Thus if it turns out to be the case that some individuals could actually do better by not purchasing the durable, the analysis will be inconsistent. Intuitively, it seems reasonable to suppose that the (weak) desirability condition given in Assumption 2 should rule this out. After all, the financial intermediation afforded by the Bidding Rosca mean that the costs of saving up are less than under Autarky and, since the allocation of the pot is not random, none of the issues raised in the previous sub–section arise. Unfortunately, however, we have been unable to establish the validity of this conjecture. While it is clear that ξ must be "sufficiently large" relative to y, B, and T, it is not obvious that Assumption 2 or even Assumption 2′ provide appropriate bounds. Hence the exact condition necessary to ensure desirability in the optimal Bidding Rosca awaits determination

Let $<s_B,c_B>$ denote the allocation generated by the optimal Bidding Rosca. As with the Random Rosca, the assignment function is linear, i.e. $s_B(\alpha) = \alpha t_B$. Below, we shall compare t_B with t_R, the length of the optimal Random Rosca. Unlike Autarky and the Random Rosca, each individual receives a different consumption path under a Bidding Rosca. However the general pattern is similar with an accumulation phase followed by a phase in which agents consume all of their incomes. Hence the allocation of non–durable consumption is described by $c_B(\alpha,\tau)=c_B(\alpha)$, for $\tau\epsilon[0,t_B]$ and $c_B(\alpha,\tau)=y$, for $\tau\epsilon(t_B,T]$.

IV.4 The Market

The final institution that we consider is a credit market. In the present context, we specify the behavior of such a market in the following way. Let $r(\tau)$ denote the market interest rate at time τ. It is convenient to define $\delta(\tau)$ as the present value of a dollar at time τ, i.e. $\delta(\tau) \equiv \exp(-\int_0^\tau r(z)dz)$ and to think of the market as determining a sequence of present value prices $\{\delta(\tau):\tau\epsilon[0,T]\}$ at which the supply of and demand for loanable funds are equated. Hence, an individual who buys the durable good at time s pays $\delta(s)B$ for it. Given the price path, an individual chooses a purchase time $s(\alpha)$ and a consumption path $\{c(\alpha,\tau):\tau\epsilon[0,T]\}$ to maximize utility.

Hence, the optimization problem solved by each individual is given by

$$(4.12)\qquad \begin{aligned} &\underset{\{c(\cdot),s\}}{\text{Max}} \int_0^T v(c(\tau))d\tau + \xi(T-s) \\ &\text{subject to } \int_0^T \delta(\tau)c(\tau)d\tau + \delta(s)B \leq y\int_0^T \delta(\tau)d\tau,\ s \in [0,T]. \end{aligned}$$

This assumes that each individual decides to purchase the durable at some time. Once again, this will require that ξ be big enough, given T, y and B. We shall assume that this is true in what follows.[20] Hence, we define a *market equilibrium* to be an allocation $\langle s_M,c_M\rangle$ and a price path $\delta(\cdot)$ satisfying two conditions: first, $\langle s_M(\alpha),c_M(\alpha,\tau)\rangle$ must be a solution to (4.12) for all $\alpha\epsilon[0,1]$, and second, at each date $t\epsilon[0,T]$,

$$(4.13)\qquad \int_0^t [y - \int_0^1 c_M(\alpha,\tau)d\alpha]d\tau = N(t;s_M)\cdot B.$$

Since individuals are identical, the first condition implies that, in equilibrium, all individuals are indifferent between durable purchase times. The second condition just says that savings equals investment at all points in time. We use W_M to denote the equilibrium level of utility enjoyed by agents in a market setting. Below, we show that this can be written in a form analogous to (4.3), (4.6) and (4.11).

Direct computation of competitive equilibrium prices and the associated allocation is difficult, even in the simple case of logarithmic utility explored in section VI. However, we are able to infer the existence and some of the properties of competitive equilibrium in our model by using the fact that, with identical agents, it must coincide with a Pareto efficient allocation which gives equal utility to every agent. Hence, explaining properties of the Market allocation must await consideration of efficient allocations more generally.

This completes our description and preliminary analysis of the different institutional frameworks which are dealt with in this paper. In section VI we shall make detailed comparisons of them. Our next task, however, is to discuss the implications of Pareto efficiency for the model described so far. This will aid the study of competitive allocations, while also providing insight into the basic resource allocation problem posed here by the existence of an indivisible durable good.

V. Efficient and *Ex Ante* Optimal Allocations

We have now shown how various institutions for mobilizing savings produce particular feasible allocations and associated distributions of utility in the population. To evaluate the allocative consequences of alternative institutions for financial intermediation we require some welfare criteria. Two such criteria are natural here. The first is *ex post* Pareto efficiency, or more simply, *efficiency*. We consider an allocation to have been *efficient* if there is no alternative feasible allocation which makes a non–negligible set of individuals strictly better off, while leaving all but a negligible set of individuals at least as well off.

The second criterion is defined in terms of *ex ante* expected utility. The allocation $<s,c>$ is better than $<s',c'>$, in this sense if $\int_0^1 u(\alpha;<s,c>)d\alpha > \int_0^1 u(\alpha;<s',c'>)d\alpha$. The preferred allocation gives a higher expected value of the lifetime utility of a type α individual, with α regarded as a random variable uniformly distributed on the unit interval. This is a "representative man" criterion which asks: which allocation would be preferable if one did not know to which position in the queue one would be assigned? We are particularly interested, therefore, in that allocation which yields the highest level of expected utility. We call this the *ex ante optimal allocation.* Notice that, since all individuals enjoy the same level of utility under Autarky, the Bidding Rosca, and the Market, the two criteria coincide when applied to allocations emerging from any one of these institutional forms. This is not the case for the Random Rosca. It is possible that an allocation generated by a Random Rosca might be Pareto dominated, and yet itself be preferred to some Pareto efficient allocation in terms of the *ex ante* optimality criterion. Indeed, we will present an example in which precisely this reversal occurs.

In what follows, we characterize efficient allocations for our economy and the *ex ante* optimal allocation. While this section is the analytic cornerstone of the paper, the reader whose primary interest lies in the results may wish just to skim the next few pages, focusing particularly on Corollary 1, which describes the essential properties of efficient allocations and the discussion of the optimal allocation following Theorem 2.

V.1 Efficient Allocations

To characterize efficient allocations, we introduce weights $\theta(\alpha) > 0$ for each agent $\alpha \epsilon [0,1]$, normalized so that $\int_0^1 \theta(\alpha) d\alpha = 1$. Define the set of all such weights: $\Theta \equiv \{\theta : [0,1] \to \mathbb{R}_{++} \mid \theta \text{ integrable and } \int_0^1 \theta(\alpha) d\alpha = 1\}$. By analogy with a standard argument in welfare economics, an efficient allocation should maximize a weighted sum of individuals' utilities. Hence, if $<s',c'>$ is efficient then there must be a set of weights in Θ such that $\int_0^1 \theta(\alpha) u(\alpha; <s',c'>) d\alpha \geq \int_0^1 \theta(\alpha) u(\alpha; <s,c>) d\alpha$, for all feasible allocations, $<s,c>$. In order to investigate the properties of efficient allocations we therefore study, for fixed $\theta \epsilon \Theta$, the problem:

$$(5.1) \qquad \max_{<s,\, c> \epsilon F} W(\theta; <s,c>) \equiv \int_0^1 \theta(\alpha) u(\alpha; <s,c>) d\alpha,$$

for $u(\alpha;<s,c>)$ as defined in (3.1).[21] Let $<s_\theta,c_\theta>$ denote an allocation which solves this problem, and $W_\theta \equiv W(\theta;<s_\theta,c_\theta>)$ denote the maximized value of the objective function.

Although (5.1) may appear to be a "non–standard" optimization problem, it can be solved explicitly under our simplifying assumptions on preferences. The solution is exhibited in Theorem 1 below. We develop the Theorem and discuss its implications at some length in what follows. The proof of this result, presented in the Appendix, exposes the nature of the resource allocation problem which any institution for financial intermediation must confront in our economy.

The first point to note about the efficiency problem is that it can be solved in two stages, loosely corresponding to static and dynamic efficiency. The first requires that any level of aggregate consumption be optimally allocated across individuals, i.e. maximizes the weighted sum of instantaneous utility at each date, while the second determines the optimal acquisition path for the durable good. We shall focus, once again, on the case in which everyone receives the durable at the optimum.

Let $\bar{c}_\theta(\tau)$ denote aggregate consumption in period τ, i.e. $\bar{c}_\theta(\tau) \equiv \int_0^1 c_\theta(\alpha,\tau)d\alpha$. A brief inspection of the efficiency problem should convince the reader that this should be distributed among the different types of individuals so as to maximize the weighted sum of utilities from consumption in period τ. To state this more precisely, define the problem

$$\max_{\chi(\cdot)} \int_0^1 \theta(\alpha)v(\chi(\alpha))d\alpha \tag{5.2}$$

$$\text{subject to} \int_0^1 \chi(\alpha)d\alpha = w$$

for all $w > 0$. Let $\chi_\theta(\cdot,w)$ denote the solution and let $V_\theta(w)$ denote the value of the objective function. Then a type α individual's consumption at time $\tau\epsilon[0,T]$ is given by $c_\theta(\alpha,\tau)=\chi_\theta(\alpha,\bar{c}_\theta(\tau))$, and total weighted consumption utility is given by $V_\theta(\bar{c}_\theta(\tau))$. It remains, therefore, to determine $s_\theta(\alpha)$ and $\bar{c}_\theta(\tau)$.

Note first that since preferences are strictly monotonic, the feasibility constraint that $<s,c> \epsilon F$ [see (3.2)] may without loss of generality be written as:

$$\int_0^\tau [y - \int_0^1 c(\alpha,\tau)d\alpha]d\tau = N(\tau;s)B, \ \forall\tau\epsilon[0,T]. \tag{5.3}$$

That is, all savings must be put immediately to use. Moreover, in the absence of discounting, if the flow of aggregate savings $\int_0^1[y-c(\alpha,\tau)]d\alpha$ equals zero at some date τ, then efficiency demands that it must also be zero at any later date $\tau'>\tau$. Otherwise, by simply moving the later savings forward in time one could assign some agents an earlier receipt date for the durable without reducing anyone's utility from non-durable consumption. The foregoing implies that any assignment function solving (5.1) must be continuous, increasing, and satisfy $s(0)=0$. This in turn implies that such an assignment function will be invertible and differentiable almost everywhere. We can use these facts to write $N(\tau;s) = s^{-1}(\tau)$, for $\tau\epsilon[0,s(1)]$, and $N(\tau;s) = 1$, for $\tau\epsilon(s(1),T]$. Substituting this expression for $N(t;s)$ in (5.3) and differentiating with respect to τ, we find that for all $\tau\epsilon[0,s(1))$ we have $y-\bar{c}(\tau)=B/s'(s^{-1}(\tau))$, and for all $\tau\epsilon(s(1),T]$ we have $y-\bar{c}(\tau)=0$. Therefore, we may conclude that the following condition must be satisfied by any solution to (5.1):

$$s'(\alpha) = B/[y-\bar{c}(s(\alpha))], \text{ for all } \alpha\epsilon[0,1); \text{ and, } \bar{c}(\tau)=y, \text{ for all } \tau\epsilon(s(1),T]. \tag{5.4}$$

Equations (5.4) are the analogue of "production efficiency" in our model, i.e. there can be no outright waste of resources. Hence, for part two of the solution, we are interested in the problem of choosing functions $s_\theta(\alpha)$ and $\bar{c}_\theta(\tau)$ to maximize

$$(5.5)\qquad \int_0^T V_\theta(\bar{c}(\tau))d\tau + \xi T - \xi\int_0^1 \theta(\alpha)s(\alpha)d\alpha$$

subject to (5.4). By analogy, with our earlier analysis we define

$$(5.6)\qquad \mu_\theta(\gamma) \equiv \min_{0\leq\sigma\leq y}\left[\frac{V_\theta(y)-V_\theta(\sigma)+\gamma}{y-\sigma}\right],\ \gamma\geq 0,$$

and denote the solution by $\sigma_\theta(\gamma)$. We now have:

Theorem 1: Let <s,c> be an efficient allocation with $s^{-1}(\nu)=\emptyset$, and let $\theta\epsilon\Theta$ be the weights for which <s,c> provides a solution in (5.1). Then the maximized value can be written in the form

$$W_\theta = T\cdot[V_\theta(y)+\xi] - B\cdot\int_0^1 \mu_\theta(\xi\textstyle\int_x^1\theta(z)dz)\ dx$$

and the assignment function satisfies:

$$s(\alpha) = B\cdot\int_0^\alpha [y-\sigma_\theta(\xi\textstyle\int_x^1\theta(z)dz)]^{-1}dx,\ \ \alpha\epsilon[0,1].$$

Moreover, for all $\alpha\epsilon[0,1]$, non–durable consumption obeys

$$c(\alpha,s(x)) = \chi_\theta(\alpha,\sigma_\theta(\xi\textstyle\int_x^1\theta(z)dz)),\text{ for } x\epsilon[0,1];\text{ and } c(\alpha,\tau) = \chi_\theta(\alpha,y),\text{ for } \tau\epsilon(s(1),T].$$

Proof: See the Appendix.

As noted, Pareto efficiency requires two conditions beyond the absence of physical waste of resources. First, any given aggregate level of non–durable consumption should be allocated efficiently among individuals and second, it must optimally manage the intertemporal trade–off between aggregate consumption of the non–durable and faster diffusion of durable ownership. We discussed above how $V_\theta(\cdot)$ summarized the first of these stages. More needs to be said about the dynamic efficiency considerations – in particular, the relevance of the minimization conducted in (5.6).

To see this consider the expression for W_θ in Theorem 1. This welfare measure is the difference of two terms. The first, $T\cdot[V_\theta(y)+\xi]$, would be the maximal weighted utility sum if the durable were a free good. The second term is, therefore, the (utility equivalent) cost of acquiring the durable. It is this cost which is minimized in (5.6). It has two competing components: non–durable consumption foregone in the process of acquiring the durable (since $\bar{c}(s(\alpha)) < y$) and durable services foregone in allowing some non–durable consumption (since $s'(\alpha) < B/y$). Consider a small interval of time $\tilde{\alpha}\epsilon(\alpha,\alpha+d\alpha)$, then the sum of these two components is approximately $[V_\theta(y)-V_\theta(\bar{c}(s(\alpha))+\xi\int_\alpha^1\theta(z)dz]$, while the duration of the time interval is $s'(\alpha)d\alpha=Bd\alpha/[y-\bar{c}(s(\alpha))]$. Efficient accumulation therefore means minimizing the product of these terms at each $\alpha\epsilon[0,1]$. This is precisely the problem described by (5.6).

A geometric treatment of the minimization problem (5.6) may also be helpful (see Figure 2). The function $V_\theta(\cdot)$ is smooth, increasing and strictly concave because we have assumed that $v(\cdot)$ has these properties. Therefore, choosing σ to minimize the ratio $[V_\theta(y)-V_\theta(\sigma)+\gamma]/[y-\sigma]$ means finding that point $(\sigma,V_\theta(\sigma))$ on the graph of $V_\theta(\cdot)$ such that the straight line containing it, and containing the point $(y,V_\theta(y)+\gamma)$, is tangent to the graph of $V_\theta(\cdot)$. Notice from the diagram that $\sigma_\theta(\gamma)$ must be decreasing, rising to y as γ falls to 0.

This observation, together with Theorem 1, permits us to deduce some useful properties of efficient allocations:

Corollary 1: Let $<s,c>$ be an efficient allocation with $s^{-1}(\nu)=\emptyset$. Then

(i) the assignment function $s(\cdot)$ is increasing, strictly convex and satisfies

$$\lim_{\alpha\to 1} s'(\alpha) = +\infty, \text{ and}$$

(ii) for all $\alpha\epsilon[0,1]$, $c(\alpha,\cdot)$ is increasing on the interval $[0,s(1)]$, and constant thereafter.

Proof: (i) In view of (5.4) and Theorem 1, we know that any efficient allocation <s,c> satisfies $s'(\alpha) = B/[y-\sigma_\theta(\xi\int_\alpha^1\theta(z)dz)]$, for some $\theta\epsilon\Theta$. As noted above $\sigma_\theta(\gamma)$ is decreasing and approaches y as γ falls to 0. Hence, the result.

(ii) This follows immediately from Theorem 1 after noting that $\chi_\theta(\alpha,\cdot)$ is increasing and that $\sigma_\theta(\gamma)$ is decreasing. □

The properties of the assignment function imply that, in an efficient allocation, the fraction of the population who have received the durable by time τ is increasing and strictly concave. In addition, the rate of accumulation (the time derivative of $N(\tau;s)$) approaches zero as τ goes to s(1).

Finally, it is worth drawing attention to the relationship between the characterization in Theorem 1 and the welfare expressions found for Autarky in (4.3), for the Random Rosca in (4.6) and for the Bidding Rosca in (4.11). These expressions all take the same general form, allowing the intuitive interpretation that welfare is the hypothetical utility achieved when the durable is a free good, net of the implicit utility cost of acquiring the durable's services. This observation is the basis for most of the results in section VI.

V.2 The Optimal Allocation

The optimal allocation is that efficient allocation in which individuals are equally weighted. Since individuals are identical and are assigned types randomly, this solution maximizes the *ex ante* expected utility of a representative agent. Hence, noting that $1\epsilon\Theta$, we can write *ex ante* expected utility as $W(1;<s,c>)$, and the *optimal allocation* $<s_0,c_0>$ must satisfy: $W(1;<s_0,c_0>) \geq W(1;<s,c>)$, $\forall <s,c>\epsilon F$.

Notice also that $V_1(w)\equiv v(w)$, and $\chi_1(\alpha,w)\equiv w$, for all (α,w). This implies that if all agents have equal weights and utility is strictly concave, then aggregate non–durable consumption should be allocated equally among all agents. It is also useful to note that $\mu_1(q)\equiv\mu(q)$, where $\mu(\cdot)$ is defined in (4.2), and that $\mu'(\gamma)=[y-\sigma_1(\gamma)]^{-1}$, using the Envelope Theorem. These facts, together with Theorem 1, yield:

Theorem 2: Let $<s_0,c_0>$ be the optimal allocation. Then, ex ante expected utility can be written in the form

$$W(1;<s_0,c_0>) \equiv W_0 = T\cdot[v(y)+\xi] - B\cdot\int_0^1 \mu(\xi(1-\alpha))d\alpha,$$

and the optimal assignment function satisfies

$$s_0(\alpha) = B\cdot\int_0^\alpha \mu'(\xi(1-x))dx.$$

Moreover, for all $\alpha\epsilon[0,1]$, non–durable consumption obeys

$$c_0(\alpha,s_0(x)) = y-1/\mu'(\xi(1-x)) \text{ for } x\epsilon[0,1], \text{ and } c_0(\alpha,\tau) = y \text{ for } \tau\epsilon(s_0(1),T].$$

Proof: Provided that $s_0^{-1}(\nu)=\emptyset$, the Theorem follows at once from Theorem 1 and the foregoing observations, after noting that $\int_\alpha^1\theta(z)dz=\int_\alpha^1 1(z)dz=1-\alpha$. To prove that $s_0^{-1}(\nu)=\emptyset$, we show that welfare under the optimum is increasing in the fraction of the population who ever get the durable, $\overline{\alpha}$. Hence, let $W_0(\overline{\alpha})$ denote maximal expected utility when a fraction $\overline{\alpha}$ of the population get the durable. Arguing as in the proof of Theorem 1, we obtain:

$$W_0(\overline{\alpha}) = T\cdot[v(y)+\overline{\alpha}\xi] - B\cdot\int_0^{\overline{\alpha}} \mu(\xi(\overline{\alpha}-\alpha))d\alpha.$$

Differentiating with respect to $\overline{\alpha}$, yields

$$W_0'(\overline{\alpha}) = T\xi + B\cdot\int_0^{\overline{\alpha}} \mu'(\xi(\overline{\alpha}-\alpha))\xi d\alpha - B\mu(0),$$

which, after using the Fundamental Theorem of Calculus, simplifies to

$$W_0'(\overline{\alpha}) = T\xi - B\cdot\mu(\xi\overline{\alpha}).$$

By Assumption 2, we know that $T\xi \geq B\mu(\xi)$. Since $\mu(\cdot)$ is increasing, it follows that $W_0'(\overline{\alpha}) > 0$. □

In addition to the properties outlined in Corollary 1, therefore, the optimal allocation also gives individuals identical consumption paths. Theorem 2 also tells us that $\bar{c}_0(0)$ equals c_A, the consumption level in the accumulation phase under Autarky[22]. The pattern of consumption and the fraction of the population which has acquired the durable good in the optimal allocation are illustrated, as a function of time, in Figure 1. Each individual has an identical consumption path beginning at c_A and rising smoothly to y at the end of the accumulation phase. The fraction of the population with the durable is increasing and concave over the interval of accumulation.

It is helpful to note the relationship between the problem solved by the optimal allocation and that of accumulating a perfectly divisible good under autarky. As noted earlier, if the durable good were perfectly divisible, then there would be no gains from trade and Autarky would be optimal.[23] It is precisely the indivisibility of the durable which creates the problem. Nonetheless, the economy may approximately replicate the situation under perfect divisibility, even in the presence of indivisibilities, by randomly assigning individuals to positions in the queue at the initial date. This is tantamount to granting each individual a "share" of the aggregate amount of the durable good available at any subsequent date.

The exact link between the two problems is spelled out in the following lemma. Here the function $K(\tau)$ is to be interpreted as the stock of the divisible asset the individual holds at time τ.

Lemma 2: The optimal aggregate consumption path $\{\bar{c}_0(\tau):\tau\epsilon[0,T]\}$ solves the problem:

$$\underset{c(\cdot)}{\text{Max}} \int_0^T [v(c(\tau)) + \xi(T-\tau)K'(\tau)]\, d\tau$$

$$\text{subject to } B\cdot K'(\tau)=y-c(\tau);\ \ K(0)=0;\ \ K(T)=1;\ \ 0\leq c(\tau)\leq y.$$

Proof: See the Appendix.

Thus the consumption path of non–durable consumption is precisely that attained by an individual accumulating a perfectly divisible durable good. In the absence of trade in durable services, financial intermediation provides the only means to overcome the limitations imposed on the agents by the constraint of indivisibility. In the optimal allocation it is completely overcome in the sense that *ex ante* expected utility is just the same as it would be under perfect divisibility.

While we have assumed that preferences are quasi–linear in deriving our results, there will be gains from randomization (on the *ex ante* expected utility criterion) in any *equilibrium* with indivisibilities. This is a general point, and it is key to many of the results of section VI. It is important therefore to understand why it is true. Assumptions about preferences affect the extent to which randomization provides a perfect substitute for divisibility, but not whether it improves over deterministic, equilibrium outcomes in the presence of indivisibility. From an *ex ante* viewpoint, the representative agent who faces the deterministic assignment function $s(\cdot)$ without knowing his types α is already, in effect, bearing risk. He sees a probability $N(\tau;s)$ of enjoying the durable's services beginning on or before τ, and evaluates his expected utility accordingly.

By contrast, adopting an *ex post* viewpoint on any *equilibrium*, an agent knowing his type α and facing assignment function $s(\cdot)$, must be willing to accept his assignment – that is, he must not want to be any other type.[24] This is what the substance of the equal utility conditions of the Bidding Rosca and the market equilibrium. Thus in the latter two institutional settings, only those allocations which satisfy feasibility *and* equal utility can be considered. This amounts to an additional constraint on the problem of maximizing *ex ante* expected utility.

This completes our characterization of efficient and optimal allocations. Having understood their properties, we are ready to examine how well the various institutions that we have discussed perform relative to these criteria.

VI. The Performance of Roscas

The purpose of this section is to analyze the allocative performance of Roscas in detail. We shall be interested in understanding how they perform relative to the efficiency and expected utility criteria, as well as in establishing additional features of the resource allocation which they produce. In order to do this, it is helpful to set out concisely some of the results of the previous two sections. This we do in Table 1. To keep the notation consistent we let t_0 denote the date at which all have acquired the durable under the optimum, i.e. $t_0 \equiv s_0(1)$.

TABLE 1

Institution	Welfare	Date by which all have acquired the durable
Autarky	$W_A=T[v(y)+\xi]-B\mu(\xi)$	$t_A=B\mu'(\xi)$
Random Rosca	$W_R=T[v(y)+\xi]-B\mu(\frac{\xi}{2})$	$t_R=B\mu'(\frac{\xi}{2})$
Bidding Rosca	$W_B=T[v(y)+\xi]-B\mu_B(\xi)$	$t_B=B/(y-\lambda_B)$
Optimum	$W_0=T[v(y)+\xi]-B\int_0^1\mu(\alpha\xi)d\alpha$	$t_0=B\int_0^1\mu'(\alpha\xi)d\alpha$

We begin by discussing the efficiency of resource allocation in Roscas.

Proposition 1: The allocations achieved by Bidding and Random Roscas are inefficient.

Proof: By Corollary 1(i) efficient allocations have *strictly convex* assignment functions, while the analysis of sections IV.2 and IV.3 showed that Roscas, with uniformly spaced meeting dates and constant contribution rates, lead to *linear* assignment functions. □

The constraint that contributions occur at a constant rate during the life of a Rosca is intended to reflect an important feature of these institutions as observed in practice – their simplicity. A result of this simple representation is the inefficiency noted above. The convexity of efficient assignment functions is a consequence of the fact that, as the remaining horizon becomes shorter, the value of the durable good to an agent who acquires it diminishes, so the amount of current consumption foregone to finance diffusion of durable goods should also decline. Roscas, as we have modeled them, cannot achieve this subtle intertemporal shift in resource allocation. Notwithstanding, the best Random Rosca does yield maximal *ex ante* expected utility to its members subject to the constraint of the assignment function being linear. Moreover, the best Bidding Rosca generates the highest *common* level of utility for its members, among all feasible allocations satisfying the linear assignment function requirement.

Our next result concerns the welfare comparisons.

Proposition 2: (a) The optimal allocation yields the highest level of *ex ante* expected utility and Autarky the lowest. (b) The Random Rosca dominates the Bidding Rosca. Hence, a representative agent, using the *ex ante* welfare criterion, would rank the institutions in the following way: $W_A \le W_B < W_R < W_o$.

Proof: Recall that $\mu(\xi) \equiv \text{Min}\{\frac{v(y)-v(c)+\xi}{y-c}\}$ is positive, increasing and concave, $\xi \geq 0$. Moreover, it is easy to show that $\mu'(\xi)$ is decreasing and convex, under our assumption that $v''' > 0$. Recall also the definitions $\mu_B(\xi) \equiv \min\{\frac{v(y)-v(c)+x\xi}{y-\lambda(c,x)}\}$, any $x \in [0,1]$, $\xi \geq 0$, where $\lambda(c,x) \equiv \int_0^1 v^{-1}(v(c)-\xi(x-\alpha))d\alpha$. Because $v(\cdot)$ is increasing and strictly concave we know that $v^{-1}(\cdot)$ is increasing and strictly convex. Therefore $\lambda(c,1) < c$, and (by Jensen's Inequality) $\lambda(c,\frac{1}{2}) > c$. Thus, taking $x=1$ we see that $\mu_B(\xi) < \mu(\xi)$, while taking $x=\frac{1}{2}$ allows us to see that $\mu_B(\xi) > \mu(\frac{\xi}{2})$. Hence, $\mu(\xi) > \mu_B(\xi) > \mu(\frac{\xi}{2})$. The result then follows immediately by reference to Table 1. □

In light of our earlier remarks, it seems obvious that Autarky must be the worst, and the *ex ante* expected utility maximizing allocation must be the best, of the alternatives being considered here. What is most noteworthy about Proposition 2 is the ranking of the Bidding and Random Roscas. Individuals, given our assumptions, would always prefer to use a savings association that allocates access to the accumulating resources by lot, rather than by competitive bidding. We shall return to the reason for this momentarily. At present, it should be noted that the assumption of homogeneous agents is crucial to what we have found. Bidding could serve as a means for individuals to credibly convey information about their differences to other Rosca members. Notice, though, that those differences would have to be *unobservable to the other agents* to obviate our conclusion above. Otherwise the self–selection would occur prior to the formation of the Roscas, resulting in homogeneous associations which would perform better with a random allocation of positions in the queue for funds than with bidding.[25]

Formally, it can be seen that the Bidding Rosca solves a more constrained problem than does a Random Rosca: equilibrium of the bidding process requires that lifetime utilities be equal, which precludes the equality of marginal utilities across agents which is necessary for *ex ante* expected utility maximization.[26] This point is also made in the related analysis of Bergstrom (1986).[27] Figure 4 provides a diagrammatic exposition of the main idea. It depicts a two person economy wherein individuals live for two periods. We will assume that parameter values are such that it is efficient to allocate the durable to only one individual in each time period. In such an economy Roscas can achieve efficiency since the assignment function must be linear, i.e. the same number of individuals must receive the good in each period. There are two utility possibility frontiers and which is relevant depends upon who gets the durable first. With a Random Rosca, non–durable consumption is equally allocated. Hence, the utility allocations will be at A if individual one wins the pot in period 1 and at B if individual 2 wins it. At either of these points, utility is allocated unequally. Since both utility allocations have equal probability, each individual's *expected* utility will be at point C. Under a Bidding Rosca, utility is equal and hence the allocation occurs at the intersection of the utility possibility frontiers. It is clear that the Random Rosca results in a higher level of expected utility than the Bidding Rosca. Indeed, in this simple economy, the Random Rosca achieves the optimal allocation.

For exactly the same reason, the competitive equilibrium allocation, constrained by definition to provide agents with equal utilities, generates lower *ex ante* expected utility than the optimal allocation $<s_O,c_O>$.[28] This can also be seen from Figure 3 since in a two person, two period world the Bidding Rosca and the Market result in identical allocations. In general, however, the Market is superior to a Bidding Rosca. To understand this, recall that, in addition to being constrained to provide agents equal utilities, the Bidding Rosca is also constrained to have a linear assignment function. We summarize these observations in

Proposition 3: While being inferior to the optimal allocation, the Market is preferred to a Bidding Rosca, i.e. $W_O > W_M > W_B$.

Proof: Since each agents' utility is constant in both $<s_M,c_M>$ and $<s_B,c_B>$, and since

$<s_B,c_B>$ is Pareto inefficient while by the First Fundamental Theorem of Welfare Economics $<s_M,c_M>$ is Pareto efficient, we must have $W_M>W_B$. Moreover, the constancy of agents' utility in a competitive equilibrium implies:

$$\text{(6.1)} \qquad \forall\theta\epsilon\Theta\text{:}\ W_\theta \equiv W(\theta;<s_\theta,c_\theta>) \geq W(\theta;<s_M,c_M>) = W(\theta_M;<s_M,c_M>) \equiv W_M.$$

where θ_M are the weights associated with the competitive allocation. The inequality in (6.1) reflects the fact that $<s_\theta,c_\theta>$ maximizes the weighted sum of utilities with weights θ; the equality is due to the fact that the weighted average of a constant function does not depend on the weights. So $W_M=\underset{\theta\epsilon\Theta}{\mathrm{Min}}\{\underset{<s,c>\epsilon F}{\mathrm{Max}} W(\theta;<s,c>)\}$! The competitive equilibrium solves an elegant mini–max problem. Thus, not only is $W_M<W_o$ (equality is impossible since then, by the strict concavity of $v(\cdot)$ and the fact that $c_M\neq c_o$, a strict convex combination of $<s_M,c_M>$ and $<s_o,c_o>$ would be feasible and would dominate $<s_o,c_o>$), but W_M is less than *any* maximized weighted sum of utilities. □

This proof demonstrates that the competitive equilibrium uses weights which minimize W_θ. This is key to our constructive demonstration, in Proposition 4 below, that there exist circumstances under which the competitive allocation is strictly dominated, in terms of *ex ante* expected utility, by the optimal Random Rosca. Hence, our final result on welfare comparisons shows that the "equal utility" constraint can be more of an impediment to generating *ex ante* welfare than the "linear assignment function" constraint. As already mentioned, W_R is maximal *ex ante* welfare subject to having a linear assignment function, while W_M maximizes the same criterion subject to the constraint that utilities are equal. The question naturally arises whether one can prove a general result on the relation of these values. One might have suspected that under some plausible conditions the competitive allocation would dominate the inefficient Random Rosca.

However, this is not the case. What follows is an illustration of the fact that a simple institution of financial intermediation, allocating its funds by lot, can actually out perform a perfectly competitive credit market.

Proposition 4: In the case of logarithmic utility, there exists a $\bar{\xi}$ such that for all $\xi > \bar{\xi}$, a Random Rosca dominates the Market; i.e. $W_R > W_M$.

Proof: See the Appendix.

The technique of proof is indirect, since explicit representation of competitive allocations, even in the case of logarithmic utility, seems intractable. We use the fact, from the proof of Proposition 3, that the market gives the least maximized weighted utility sum, over all possible weights. We then construct a set of weights whose maximized utility sum is less than W_R, to infer the result. Intuitively the result may be understood by recognizing that, when ξ is very large, respecting the equal utility constraint means those receiving the durable early must get much lower non–durable consumption than those acquiring it late. Therefore, the agents' marginal utilities from non–durable consumption will be very unequal, causing their *ex ante* expected utilities to be relatively low. Since the magnitude of ξ does not effect the utility cost of the non–optimal intertemporal allocation of non–durable consumption characteristic of Roscas, when ξ is sufficiently big the Random Rosca dominates.

Another difference between the institutions concerns the length of their accumulation periods. Our next task, therefore, is to explore some facts concerning these. In what follows we shall refer to the date by which all have acquired the durable as the *terminal time.* We now have:

Proposition 5: The terminal times under Autarky, the Random Rosca, and the Optimum can be ordered as follows: $t_R/2 < t_A < t_R < t_O$.

Proof: Considering the formulae in Table 1, and the fact that $\mu'(\cdot)$ is under our assumptions a decreasing, strictly convex function, the last two inequalities above follow at once. Let $c^*(\xi)$ give the minimum in (4.2). Then $\forall\xi \geq 0$: $\mu(\xi) = v'(c^*(\xi))$ and $\mu'(\xi) = [y - c^*(\xi)]^{-1}$. Some straightforward algebra shows that $c^*(\cdot)$ is decreasing and, given $v''' > 0$, strictly convex. Now from Table 1, $t_R < 2t_A$ if and only if $c^*(\frac{\xi}{2}) < [c^*(\xi)+y]/2 = [c^*(\xi)+c^*(0)]/2$, which follows from the strict convexity of $c^*(\cdot)$. □

The moral of this proposition is, roughly speaking, that *the more efficient the financial intermediation, the more protracted the period of accumulation.* Since under Autarky all savings lie idle, it stands to reason that the accumulation period would be relatively short. The *ex ante* optimum, on the other hand, allocates resources as if the services of the durable, as acquired, were immediately available to all agents on a prorated basis (see the discussion in Section V.2). We also know from Corollary 1 that the rate of acquisition must slow to zero as cumulative diffusion approaches unity. The Random Rosca is somewhere between these two extremes, but has the property that on the average its members acquire the durable good earlier than would be the case if they saved in isolation.

Note also that the results on the terminal dates can be used to infer something about the average rates of consumption ($\overline{c}$), and therefore saving, during the accumulation phase for the different institutions after remembering that $t = B/y - \overline{c}$. Hence a corollary of Proposition 5 is that the average savings rate is greatest under Autarky and lowest under the optimal allocation.

The optimal length of the period of accumulation under the Bidding Rosca (and, it would appear, also under the market mechanism) is more difficult to characterize, because it is not related to the function $\mu_B(\cdot)$ in any simple way. We have however been able to obtain the following result.

Proposition 6: (i) If $1/v'(\cdot)$ is concave, then the terminal time in a Bidding Rosca is later than under Autarky, i.e. $t_B \geq t_A$. (ii) If $1/v'(\cdot)$ is convex, then the terminal time under a Random Rosca is later than with a Bidding Rosca, i.e. $t_B \leq t_R$.

Proof: See the Appendix.

Whether $1/v'(c)$ is concave or convex does not follow directly from any well known property of the utility function. For iso–elastic utility functions, $v(y) = y^{1-\rho}/(1-\rho)$, $1/v'(c)$ is convex if $\rho > 1$ and concave if $\rho < 1$. It is interesting to note that in the borderline case, $v(c) = \ln(c)$, implying that $1/v'(c)$ is linear in c, both parts of the Proposition are satisfied, and $t_A < t_B < t_R$.

VII. Conclusions and Suggestions for Further Research

This paper has investigated the economic role and performance of Roscas. We have sought their rationale in the fact that some goods are indivisible, a fact which makes autarkic saving inefficient. While Rosca allocations are not Pareto efficient, they are superior to Autarky. Moreover, a Random Rosca may yield a higher level of expected utility than in a perfect credit market. The latter result is more striking still when taken in conjunction with the fact that Roscas are such a simple institution – a factor surely significant in many contexts in which they are adopted.

A number of issues remain outstanding and we hope to consider them in future work. The most important concerns the sustainability of Roscas. There is a strong incentive for those who win the pot early to stop contributing to the Rosca, although as we noted in the introduction, this does not appear to be a serious problem in practice. The kinds of primitive societies which spawn Roscas are comparatively rich in the ability to use social sanctions to enforce contractual performance. This ability may however deteriorate in the process of economic development causing problems for Roscas. Understanding these issues better would, we conjecture, further enhance our understanding of when Roscas are likely to be the most "appropriate" savings institution for an economy.

References

Ardener, S., 1964, The Comparative Study of Rotating Credit Associations, *Journal of the Royal Anthropological Society of Great Britain and Ireland* ?, 201–229.

Arnott, R. and J. Riley, 1977, Asymmetrical Production Possibilities, the Social Gains from Inequality and the Optimum Town, *Scandinavian Journal of Economics* 79, 301–311.

Begashaw, G., 1978, The Economic Role of Traditional Savings Institutions in Ethiopia, *Savings and Development* 2, 249--262.

Bergstrom, T., 1986, Soldiers of Fortune? in Heller, W.P., R.M. Starr and D.A. Starrett, (eds), *Equilibrium Analysis: Essays in Honor of Kenneth J. Arrow*, Cambridge: Cambridge University Press.

Bouman, F.J.A., 1977, Indigenous Savings and Credit Societies in the Third World: A Message, *Savings and Development* 1, 181–218.

Fernando, E., 1986, Informal Credit and Savings Organizations in SriLanka: The Cheetu, *Savings and Development* 10, 253--263.

Geertz, C., 1962, The Rotating Credit Association: a 'Middle Rung' in Development, *Economic Development and Cultural Change* 1, 241–263.

Goffman, C. and G. Pedrick, 1965, *First Course in Functional Analysis*, Englewood Cliffs, N.J., Prentice Hall.

Grossman, R., 1989, Deposit Insurance, Regulation, and Moral Hazard in the Thrift Industry: Evidence from the 1930s, Mimeo.

Hillman, A. and P. Swan, 1983, Participation Rules for Pareto Optimal Clubs, *Journal of Public Economics* 20, 55–76.

Leland, H.E., 1968, Saving and Uncertainty: The Precautionary Demand for Saving, *Quarterly Journal of Economics* 82, 465–473.

Light, I., 1972, *Ethnic Enterprise in America*, Berkeley, University of California Press.

Mirrlees, J.A., 1972, The Optimum Town, *Scandinavian Journal of Economics* 74, 115–135.

Osuntogun, A., and N. Adeyemo, 1981, Mobilization of Rural Savings and Credit Extension in Pre–Cooperative Organizations in South–Western Nigeria, *Savings and Development* 5, 247–261.

Pratt, J. and R. Zeckhauser, 1987, Proper Risk Aversion, *Econometrica* 55, 143–154.

Rhadhakrishnan, S. and others, 1975, *Chit Funds*, Institute for Financial Management and Research, Madras.

Symons, E. and J. White, 1984, *Banking Law*, 2nd Edition, ?

Appendix

Proof of Lemma 1: To prove that the optimal Random Rosca will be fully funded, we must first extend our model of a Random Rosca to allow for partial funding. To do this all we have to do is to let $N\epsilon(0,1]$ be the limiting ratio of meetings to members, but otherwise adopt the model of the optimal Random Rosca with a continuum of agents as it was described in the text. Then if the Rosca has length t, members save at the rate $N\cdot B/t$, and face probability N of ever winning the pot. Conditional on winning, the receipt date α is uniform on $[0,1]$. Reasoning in like manner to the arguments for (4.3) and (4.6), and denoting by $W_R(N)$ the maximal expected utility of the members of this Rosca, we have:

$$W_R(N) = \mathrm{Max}[tv(c)+(T-t)v(y)+N\cdot(T-\tfrac{t}{2})\xi] \text{ s.t. } t\cdot(y-c)=NB,\ 0\leq c\leq y-NB/T.$$

As before, if the optimal length $t<T$ then (4.2) may be employed to write:

$$W_R(N) = T\cdot[v(y)+N\xi] - NB\cdot\mu(N\xi/2) \tag{A.1}$$

as the welfare associated with that optimal "partially funded" Random Rosca which provides its members a probability N of ever obtaining the pot. "Full funding" of the Rosca is optimal iff $W_R \equiv W_R(1) \geq W_R(N)$, $\forall N\epsilon(0,1]$. That is, combining (4.6) and the above, iff:

$$\frac{T\cdot\xi}{B} \geq \frac{\mu(\xi/2) - N\mu(N\xi/2)}{1-N}, \ \forall N\epsilon(0,1]. \tag{A.2}$$

That this condition is stronger than the desirability requirement of Assumption 2 may be seen as follows: For $N \approx 1$ the RHS of the equation above $\approx \mu(\xi/2) + (\xi/2)\mu'(\xi/2) \geq \mu(\xi)$, by concavity. So if the inequality holds for $N \in (1-\epsilon,1]$, ϵ near zero, we may conclude that $T/B \geq \mu(\xi)/\xi$, which is Assumption 2. On the other hand, if Assumption 2 fails then, although a Random Rosca which guarantees eventual receipt of the durable may be preferred to doing without it, one which provides something less than certainty of receipt $(N<1)$ would be even better. Moreover, since $\xi\mu(\xi)$ is not generally a convex function, the above inequality could hold for $N \approx 1$, but fail for $N < 1$. Thus, without imposing some additional assumption we cannot assure even the local optimality of a "fully funded" Random Rosca.

We now show that (A.2) holds, given Assumption 2$'$. Strong desirability implies $\xi T/B \geq 2\mu(\xi/2)$. Let $x>0$, and $c^*(x)$ give the minimum in (4.2) when $\xi=x$. Then:

$$x\mu'(x) = \frac{x}{y-c^*(x)} < \frac{v(y)-v(c^*(x))+x}{y-c^*(x)} \equiv \mu(x).$$

Therefore, use (A.1), set $x=N\xi/2$ above, and recall that μ is strictly increasing, to get:

$$\frac{dW_R(N)}{dN} = T\xi - B[\mu(\tfrac{N\xi}{2}) + \tfrac{N\xi}{2}\cdot\mu'(\tfrac{N\xi}{2})] > B\cdot[\tfrac{\xi T}{B} - 2\mu(\tfrac{N\xi}{2})] \geq B\cdot[\tfrac{\xi T}{B} - 2\mu(\tfrac{\xi}{2})] \geq 0,$$

for $0<N\leq 1$. So $W_R(1)\geq W_R(N)$, $\forall N\epsilon(0,1]$, and (A.2) obtains.

To prove the remaining part of the lemma, consider the choice problem faced by the member of the optimal "fully funded" Random Rosca who wins the pot at date $\tau\epsilon[0,t]$. He must continue to pay into the Rosca at rate $y - B/T$ on the interval $(\tau,t]$. If he ever acquires the durable he does best to buy it immediately, in which case his continuation payoff is:

$$W^*(\tau) \equiv (t-\tau)\cdot v(y-\frac{B}{t}) + (T-t)\cdot v(y) + (T-\tau)\xi.$$

If he never acquires it, he should consume for the rest of his life at the maximal constant rate consistent with available resources, which generates the continuation payoff:

$$\overline{W}(\tau) \equiv (T-\tau)\cdot v\left[\frac{(t-\tau)(y-\frac{B}{t}) + (T-t)y + B}{T-\tau}\right].$$

We need to show that $W^*(\tau)\geq\overline{W}(\tau)$, $\forall\tau\in[0,t]$. It may be easily verified that this is true if and only if $W^*(t)\geq\overline{W}(t)$. (That is, if any member will want not to invest in the durable, it will be the last one to receive the pot.) Thus, the proof will be complete if we can show that $v(y) + \xi \geq v(y+\frac{B}{T-t})$, for t the optimal length of this Random Rosca. But it follows from (4.2), (4.4) and (4.5) that $t=B/[y-c^*(\xi/2)]=B\mu'(\xi/2)$, while (4.2) implies $v'(y)=\mu(0)$. So by the concavity of $v(\cdot)$, the properties of $\mu(\cdot)$ derived above, and in light of Assumption 2′:

$$v(y+\frac{B}{T-t})-v(y) \leq v'(y)\cdot(\frac{B}{T-t}) = \mu(0)/[\frac{T}{B}-\mu'(\xi/2)] \leq \frac{\mu(\xi/2)-(\xi/2)\cdot\mu'(\xi/2)}{\frac{2\mu(\xi/2)}{\xi} - \mu'(\xi/2)} = \frac{\xi}{2} < \xi.$$

Indeed, the same argument can be used to show that, under Strong Desirability, the last winner of the "partially funded" Random Rosca (N<1) will also choose to acquire the durable, as presumed. This completes the proof. □

Proof of Theorem 1: In view of the discussion preceding the statement of the Theorem:

$$W_\theta \equiv \text{Max}\{\int_0^{s(1)} V_\theta(\bar{c}(\tau))d\tau + (T-s(1))\cdot V_\theta(y) + \xi T-\xi\int_0^1 \theta(\alpha)s(\alpha)d\alpha\}$$

subject to

$$s'(\alpha) = B/[y-\bar{c}(s(\alpha))], \quad \alpha\in[0,1).$$

Now employ the change of variables: $\tau=s(\alpha)$, $d\tau=s'(\alpha)d\alpha$, $\tau\in[0,s(1)]$; note that $s(1) = \int_0^1 s'(\alpha)d\alpha$; and use (5.4) and the definition in (5.6) to get the following:

$$\begin{aligned} W_\theta &= \text{Max}\{\int_0^{s(1)} V_\theta(\bar{c}(\tau))d\tau + (T-s(1))\cdot V_\theta(y) + \xi\cdot(T-\int_0^1 \theta(\alpha)s(\alpha)d\alpha)\} \\ &= \text{Max}\{T[V_\theta(y)+\xi] - \int_0^1 s'(\alpha)\cdot[V_\theta(y)-V_\theta(\bar{c}(s(\alpha))+\xi\textstyle\int_\alpha^1\theta(z)dz]d\alpha\} \\ &= T[V_\theta(y)+\xi] - \text{Min}\{B\int_0^1\{[V_\theta(y)-V_\theta(\bar{c}(s(\alpha)))+\xi\textstyle\int_\alpha^1\theta(z)dz]/[y-\bar{c}(s(\alpha))]\}d\alpha\} \\ &= T[V_\theta(y)+\xi] - B\int_0^1 \mu_\theta(\xi\textstyle\int_\alpha^1\theta(z)dz)d\alpha. \end{aligned}$$

This proves (i). Note that the minimization above is pointwise, with respect to $\bar{c}(s(\alpha))$, at each $\alpha\epsilon[0,1]$. So it implies that for $\alpha\epsilon[0,1]$, $\bar{c}(s(\alpha))=\sigma_\theta(\xi\int_\alpha^1\theta(z)dz)$. In view of this and (5.4) we conclude that $s(\alpha)=\int_0^\alpha s'(x)dx$ satisfies (ii). Now also from (5.4) we know that $\bar{c}(\tau)=y$, for $\tau>s(1)$, and we noted earlier in the text that $c(\alpha,t)=\chi_\theta(\alpha,\bar{c}(t))$, $\forall\alpha\epsilon[0,1]$, $\forall t\epsilon[0,T]$. Taken together, these prove (iii). □

Proof of Lemma 2: From (5.4), (5.5), and the fact that $V_1(w)=v(w)$, we know that the functions $\bar{c}_0(\tau)$ and $s_0(\alpha)$ maximize:

$$\int_0^T v(\bar{c}(\tau))d\tau + \xi T - \xi\int_0^1 s(\alpha)d\alpha$$

subject to:

$$s'(\alpha) = B/[y-\bar{c}(s(\alpha))], \text{ for all } \alpha\epsilon[0,1); \text{ and, } \bar{c}(\tau)=y, \text{ for all } \tau\epsilon(s(1),T].$$

Now, for any feasible $\bar{c}(\tau)$ and $s(\alpha)$, define the function $K(\cdot)$ as follows: $K(\tau)=s^{-1}(\tau)$ for $\tau\epsilon[0,s(1))$ and $K(\tau)=1$ for $\tau\epsilon[s(1),T]$. Notice that $K'(\tau)=(y-\bar{c}(\tau))/B$ for $\tau\epsilon[0,s(1))$ and $K'(\tau)=0$ for $\tau\epsilon[s(1),T]$. It follows that we may rewrite the constraints as:

$$B\cdot K'(\tau)=y-c(\tau);\ \ K(0)=0;\ \ K(T)=1;\ \ 0\le c(\tau)\le y.$$

Now note that using the change of variables $\tau=s(\alpha)$, $d\tau=s'(\alpha)d\alpha$ we may write $\int_0^1 s(\alpha)d\alpha=\int_0^T\tau K'(\tau)d\tau$, and since $K(0)=0$ and $K(1)=1$, we have that $T=T\int_0^T K'(\tau)d\tau$. Thus the objective function may be written as:

$$\int_0^T[v(c(\tau)) + \xi(T-\tau)K'(\tau)]d\tau. \qquad \square$$

Proof of Proposition 4: When $v(c)=\ln(c)$, welfare under the Random Rosca is:

$$W_R = T[\ln(y)+\xi] - \frac{B}{y}[1+\chi(\zeta/2)]$$

where $\chi(\xi)-\ln(1+\chi(\xi))\equiv\xi$, $\xi\ge 0$, defines $\chi(\cdot)$. Similarly, for the Market we have

$$W_M = T[\ln(y)+\xi+\int_0^1\theta_M(\alpha)\ln(\theta_M(\alpha))d\alpha] - \frac{B}{y}\int_0^1[1+\chi(\xi\int_x^1\theta_M(z)dz)]dx,$$

for some $\theta_M\epsilon\Theta$. We know from the proof of Proposition 3 that $W_M = \min_{\theta\epsilon\Theta} W_\theta$. Hence, $W_R>W_M$ if and only if $\exists\theta\epsilon\Theta$ such that $W_R>W_\theta$. The proof constructs some weights for which this is so. First we need:

Lemma 3: Let $f(\cdot)$ be an increasing, strictly concave function satisfying $f(0) = 0$, and let $g(\cdot)$ be a function on [0,1], strictly decreasing satisfying $g(1) = 0$ and $g(0) = 1$. Then

$$\int_0^1 f(g(x))dx > f(1)\int_0^1 g(x)dx.$$

Proof: Let $\tilde{x}$ be a random variable which is uniformly distributed on [0,1]. Define $\tilde{y} = g(\tilde{x})$, and $\tilde{z} = 1$, if $\tilde{x} \le \int_0^1 g(x)dx$, and $\tilde{z} = 0$, if $\tilde{x} > \int_0^1 g(x)dx$. Then

$$E(\tilde{y}) = \int_0^1 g(x)dx = E(\tilde{z}),$$

where $E(\cdot)$ denotes the expectations operator. Moreover, $\tilde{z}$ is riskier than $\tilde{y}$ in the sense of second order stochastic dominance. Therefore, since $f(\cdot)$ is strictly concave:

$$E(f(\tilde{y})) = \int_0^1 f(g(x))dx > E(f(\tilde{z})) = f(1)\int_0^1 g(x)dx.$$

This proves the lemma. □

This lemma implies that

$$\int_0^1 \chi(\xi\int_x^1 \theta(z)dz)dx > \chi(\xi)\int_0^1 (\int_x^1 \theta(z)dz)dx.$$

This in turn implies that

$$W_\theta < T[\ln(y)+\xi+\int_0^1 \theta(x)\ln(\theta(x))dx] - \frac{B}{y}[1+\left[\int_0^1 x\theta(x)dx\right]\cdot\chi(\xi)],$$

where we have also used the fact that $\int_0^1\int_x^1 \theta(z)dzdx = \int_0^1 x\theta(x)dx$.

Hence, a sufficient condition for $W_R > W_M$, is that:

$$\exists\theta\epsilon\Theta \text{ such that } \frac{B}{y}\chi(\xi/2) > \int_0^1 \theta(x)\ln(\theta(x))dx - \frac{B}{y}\chi(\xi)\int_0^1 x\theta(x)dx.$$

Now define $E(\overline{\theta}) \equiv \left[\min_{\theta\epsilon\Theta} \int_0^1 \theta(x)\ln(\theta(x))dx \text{ s.t. } \int_0^1 x\theta(x)dx=\overline{\theta}\right]$. Then $W_R>W_M$ if:

$\exists\overline{\theta}\in(0,1)$ such that $(\frac{B}{Ty})\cdot[\overline{\theta}\chi(\xi)-\chi(\xi/2)] > E(\overline{\theta})$.

Let $\frac{B}{Ty}\equiv\gamma\in(0,1)$, and consider the problem: $\max_{\overline{\theta}\in[0,1]}\{\gamma\chi(\xi)\overline{\theta} - E(\overline{\theta})\}\equiv\Omega^*$. We conclude that it is sufficient for $W_R>W_M$ that $\Omega^* > \gamma\chi(\xi/2)$.

Lemma 4: (i) $E(\overline{\theta})$ is strictly convex, and $E(\overline{\theta})\geq E(\frac{1}{2})$, $\forall\overline{\theta}\in[0,1]$; and, (ii) if $E'(\overline{\theta})=\lambda$, then $E(\overline{\theta})=\lambda\overline{\theta}+\ln(\lambda(e^\lambda-1)^{-1})$.

Proof: Define the Lagrangean

$$L = \int_0^1 \theta(x)\ln(\theta(x))dx + \lambda[\overline{\theta}-\int_0^1 x\theta(x)dx] + \mu[1-\int_0^1 \theta(x)dx].$$

The first order condition with respect to $\theta(x)$ is: $\ln(\theta(x))+1-\lambda x-\mu=0$, $x\in[0,1]$. Inverting and integrating this condition, using the constraint, yields: $e^{\mu-1}\left[\frac{e^\lambda-1}{\lambda}\right] = 1$. Solving this for μ, substituting into the first order condition, multiplying by $\theta(x)$ and integrating yields (ii). To prove (i) observe that integrating the first order condition, after inverting and multiplying through by x, and using the above derived expression for μ yields:

$$\int_0^1 x\theta(x)dx \equiv \overline{\theta} = e^{\mu-1}\int_0^1 xe^{\lambda x}dx = \left[\frac{\lambda e^\lambda-(e^\lambda-1)}{\lambda^2}\right]/\left[\frac{e^\lambda-1}{\lambda}\right] = e^\lambda/(e^\lambda-1) - \lambda^{-1} \equiv \phi(\lambda).$$

It is straightforward now to see that $\phi(\lambda) \to 0$ as $\lambda \to -\infty$; $\phi(\lambda) \to 1$ as $\lambda \to +\infty$ and $\phi(\lambda) \to 1/2$ as $\lambda \to 0$. Part (i) is now proved by noting that $E'(\overline{\theta}) = \phi^{-1}(\overline{\theta})$, from the envelope condition. □

This result and simple calculation reveals: $\Omega^* > \gamma\chi(\xi/2)$ if and only $\frac{e^{\gamma\chi(\xi)}-1}{\gamma\chi(\xi)} > e^{\gamma\chi(\xi/2)}$. Note also that, from the definition of $\chi(\cdot)$, that $\chi'(\xi) = (1+\chi(\xi))/\chi(\xi)) > 1$. Thus $\chi(\xi) > \chi(\xi/2) + \xi/2$. Moreover, $(e^z-1)/z = \int_0^1 e^{zx}dx$ is a strictly increasing function. So:

$$\frac{e^{\gamma\chi(\xi)}-1}{\gamma\chi(\xi)} > \frac{e^{\gamma\chi(\frac{\xi}{2})}e^{\gamma(\frac{\xi}{2})}-1}{\gamma\chi(\xi/2)+\gamma\xi/2} = e^{\gamma\chi(\frac{\xi}{2})}\left[\frac{e^{\gamma\xi/2}}{\gamma\chi(\xi/2)+\gamma\xi/2}\right] - [\gamma(\chi(\xi/2)+\xi/2)]^{-1}.$$

The first of the two terms on the right hand side grows unboundedly as $\xi \to \infty$. Moreover, for a sufficiently large ξ, the second term vanishes. Hence, for large enough ξ, $[e^{\gamma\chi(\xi)}-1]/\gamma\chi(\xi) > e^{\gamma\chi(\xi/2)}$ and $W_R > W_M$. (Note that $\overline{\xi}$, the critical value of ξ, depends only on $\gamma\equiv B/Ty$.) □

Proof of Proposition 6: (i) Suppose that $1/v'(c)$ is a concave function of c. We will show that $t_B \geq t_A$. We will use the following notation. Let c_B be the consumption rate of the lowest bidder in the Rosca i.e., $c_B \equiv c_B(1)$. Define: $\hat{c}(\alpha) \equiv v^{-1}(v(c_B) - \xi(1-\alpha))$; $\bar{c} \equiv \int_0^1 \hat{c}(\alpha)d\alpha = \lambda(c_B,1))$; $c^1 \equiv \hat{c}(1) = c_B$, and $c^0 = \hat{c}(0)$. From the discussion of section IV.3 we know that $\hat{c}(\alpha)$ must equal the consumption level of individuals receiving the durable at date αt_B (i.e., $\hat{c}(\alpha) = c_B(\alpha)$), and $\bar{c}$ is the population average rate of consumption at each moment over the life of the Rosca. Notice that $\hat{c}'(x) = \xi/v'(\hat{c}(x))$. We will use the following Lemma.

Lemma 5: $1/v'(c)$ concave implies $v'(\bar{c}) \leq \xi/[c^1 - c^0]$, while $1/v'(c)$ convex implies $v'(\bar{c}) \geq \xi/[c^1 - c^0]$.

Proof: By the Fundamental Theorem of Calculus

$$\left[\frac{\xi}{c^1 - c^0}\right] = \left[\int_0^1 \frac{d\alpha}{v'(\hat{c}(\alpha))}\right]^{-1},$$

while by definition $[v'(\bar{c})]^{-1} = \left[v'(\int_0^1 \hat{c}(\alpha)d\alpha\right]^{-1}$. Now Jensen's inequality implies

$$\int_0^1 \frac{d\alpha}{v'(\hat{c}(\alpha))} \leq (\geq) \left[v'(\int_0^1 \hat{c}(\alpha)d\alpha\right]^{-1} = [v'(\bar{c})]^{-1},$$

when $1/v'(c)$ is concave (convex). □

We will use this lemma as follows. Note that $t_B \geq t_A$ if and only if $\bar{c} \geq c_A$, while c_A satisfies the first order condition:

$$[v(y) + \xi - v(c_A)] - v'(c_A)(y - c_A) = 0.$$

Moreover, using (4.10) (with x=1) one may calculate that the first order condition determining c_B is:

$$[v(y) + \xi - v(c_B)] - \left[\frac{\xi}{c^1 - c^0}\right][y - \bar{c}] = 0.$$

Let $\Delta(c) \equiv v(y) + \xi - v(c)$ and $\psi(c) \equiv \Delta(c) - v'(c)(y - c)$. Then $\bar{c} < c_B$, $\Delta'(c) < 0$ and $\psi'(c) \geq 0$. Therefore, when $1/v'(c)$ is concave, we have:

$$\psi(\bar{c}) \equiv \Delta(\bar{c}) - v'(\bar{c})(y - \bar{c}) \geq \Delta(c_B) - v'(\bar{c})(y - \bar{c})$$

$$= \left[\frac{\xi}{c^1-c^0} - v'(\bar{c})\right](y - \bar{c}) \geq 0 = \psi(c_A).$$

This proves (i).

To prove (ii) note that $t_R \geq t_B$ if and only if $c_R \geq \bar{c}$. With c^1, c^0, $\hat{c}(\cdot)$ as defined in the proof of (i) above, let $\bar{c}_B$ denote the consumption of a median bidder in the optimal Rosca, who receives the durable at date $t_B/2$ (i.e., $\bar{c}_B = \hat{c}(1/2)$). Bidding equilibrium requires: $v(\bar{c}_B) = v(c_B) + \xi/2$. The first order condition for the optimality of the Bidding Rosca may thus be rewritten as:

$$[v(y) + \xi/2 - v(\bar{c}_B)] = \left[\frac{\xi}{c^1-c^0}\right](y - \bar{c}),$$

and the function $\hat{c}(\alpha)$ now satisfies:

$$\hat{c}(\alpha) = v^{-1}(v(\bar{c}_B) - (\tfrac{1}{2} - \alpha)\xi), \; \alpha \in [0,1].$$

Notice that Jensen's inequality and the convexity of $v^{-1}(\cdot)$ imply:

$$\bar{c} \equiv \int_0^1 \hat{c}(\alpha)d\alpha = \int_0^1 v^{-1}(v(\bar{c}_B) - (\tfrac{1}{2} - \alpha)\xi)d\alpha \geq \bar{c}_B,$$

moreover, the first-order condition determining c_R is:

$$[v(y) + \xi/2 - v(c_R)] = v'(c_R)[y - c_R].$$

With $\Delta(c)$ and $\psi(c)$ defined as before we may use the Lemma to conclude that if $1/v'(c)$ is convex, then

$$\psi(\bar{c}) \equiv \Delta(\bar{c}) - v'(\bar{c})(y - \bar{c}) \leq \Delta(\bar{c}_B) - v'(\bar{c})(y - \bar{c}) = \left[\frac{\xi}{c^1-c^0} v'(\bar{c})\right] \cdot [y - \bar{c}] \leq 0 = \psi(c_R).$$

This completes the proof. □

End Notes

[1]Bouman (1977) provides a comprehensive listing of all the countries in which Roscas have been observed as well as some sense of their quantitative importance. He reports, for example, that an estimated 60% of the population in Addis Ababa belong to a Rosca of some form. Light (1972) discusses their use by ethnic minorities in the US. Additional specific country studies are provided by Begashaw (1978), Fernando (1986), and Osuntogun and Adeyemo (1981). The nineteenth century savings and loan associations in the US also appeared to have worked as Roscas – see Grossman (1989) and Symons and White (1984), page 65. Roscas have long been popular in India where they travel under the name of *Chit Funds*. Radhakrishnan *et al.* (1975) describes them in detail. In Kerala, for example, they report that Roscas are used in preference to banks by industrialists, educational institutions and churches. In 1967, there were 12491 registered Chit Funds in this state of India alone.

[2]The classic anthropological studies of Roscas are by Geertz (1962) and Ardener (1964). The latter paper is particularly recommmended as an introduction to the literature.

[3]While bidding and drawing lots seem to be the two most common ways of allocating the pot, it is also sometimes allocated according to need or known criteria such as age or kinship seniority. The reader is referred to Ardener (1964) for a more detailed discussion.

[4]This was clearly recognized by Ardener (1964). "The most obvious function of these associations is that they assist in small–scale capital–formation, or more simply, they create savings. Members could save their contributions themselves at home and accumulate their own "funds", but this would withdraw money from circulation: in a rotating credit association capital need never be idle." p. 217.

[5]See, for example, the discusion in Ardener (1964) p. 216.

[6]Note that the quantity B is a stock, while y and c are rates of flow.

[7]This is consonant with the anthropological stories. Individuals do not, typically, share the fruit from Rosca winnings. In cases such as the purchase a new roof for one's home, this is not typically feasible anyway.

[8]The function $c:[0,1]\times[0,T] \to \mathbb{R}_+$ is required to be such that $c(\cdot,\tau)$ and $c(\alpha,\cdot)$ are (Lebesgue) integrable for all (α,τ).

[9]That is, $N(\tau;s)$ is the measure of the set $s^{-1}([0,\tau])\equiv\{\alpha\epsilon[0,1] \mid s(\alpha)\leq\tau \;\&\; s(\alpha)\neq\nu\}$.

[10]The problem makes no economic sense otherwise, since $t_A=T$ implies $W_A<T\cdot v(y)$. We provide a condition which is both necessary and sufficient for $W_A\geq T\cdot v(y)$ in Assumption 2 below.

[11]It is trivial to show that $\mu(\cdot)$ is increasing and strictly positive. To prove strict concavity, let $\xi_1 \neq \xi_2$, define $\xi_\alpha \equiv \alpha\xi_1 + (1-\alpha)\xi_2$ and note that

$$\mu(\xi_\alpha) = \alpha\frac{v(y)-v(c_\alpha)-\xi_1}{y - c_\alpha} + (1-\alpha)\frac{v(y)-v(c_\alpha)-\xi_2}{y - c_\alpha},$$

where c_α is the optimal consumption level at ξ_α. Note also that

$$\mu(\xi_i) < \frac{v(y)-v(c_\xi)-\xi_i}{y - c_\xi} \text{ for } \xi_i = \{\xi_1,\xi_2\}.$$

Hence, $\mu(\xi_\alpha) > \alpha\mu(\xi_1) + (1-\alpha)\mu(\xi_2)$, as claimed.

[12]From the properties of $\mu(\cdot)$ stated in the text, we know that $\mu(\xi)/\xi$ is decreasing for $\xi \geq 0$. Moreover, $\mu(\xi) \geq v'(y)$; so the Envelope Theorem implies $\frac{\partial\mu}{\partial y} = \frac{v'(y)-\mu}{y-c} < 0$. Thus there exists a minimal $\underline{\xi}>0$, which is a decreasing function of both y and of $\frac{T}{B}$, such that it is desirable to acquire the durable under Autarky if and only if $\xi \geq \underline{\xi}$.

[13]If desirability fails, then $s_A \equiv \nu$ and $c_A(\alpha,\tau) \equiv y$, for all agents. Note also that we are using the symbol "c_A" to denote both the function which designates consumption levels for the agents at various dates, and the value of that function during the period of accumulation. No confusion should result.

[14]We assume for the moment that the winner will choose to buy the durable. This assumption is discussed further below.

[15]Notice that the particular limit obtained depends upon the assumption that the spacing of meeting dates is uniform and the contribution at each meeting is constant. By having the meetings occur with different frequencies at different times during the life of the Rosca, and by varying the rate of contribution across meeting dates, it may be shown that one can, in the limit, generate <u>every</u> feasible allocation $<s,c>$ in which the consumption paths $c(\alpha,\tau)$ are constant in α, as the *ex post* outcome of a Random Rosca. (Moreover, one can in similar fashion generate every feasible allocation $<s,c>$ in which the utility $u(\alpha;<s,c>)$ is constant in α, as the limiting outcome of a Bidding Rosca.) However, we do not attempt to exploit these fact in this paper. To do so would run contrary to the spirit of our analysis. The point of this exercise is to see how <u>simple</u> institutions of financial intermediation compare with each other and with a more complicated market mechanism. We believe, upon our reading of the anthropological literature, that the simplicity and minimal informational requirements of Roscas, as represented here and as observed in practice, are important factors explaining their popularity around the world. We therefore restrict ourselves to the study of Roscas which can in effect be fully and simply characterized in terms of two parameters — the frequency of meetings, and the contribution required per meeting. It is of some interest that, despite this restriction, we nevertheless find that Roscas can compare favorably with more complicated mechanisms of resource allocation.

[16]It is worth noting that the larger the number of individuals in a Rosca, the earlier is the average receipt date of the durable. ($E(\tilde{\tau}) = t\cdot[1 + 1/n]/2$.) Thus our formulation implies that individual participants are better off when more members are added to the Rosca. In practice, at least two factors are likely to work against this. First, a larger membership requires more meeting dates and there are likely to be transactions costs associated with these meetings. Second, the more members the more impersonal the relationship between participants, and hence the greater the likelihood of incentive problems. The general issue of the optimal number of members is an interesting subject for further research.

[17]Unless the optimal accumulation period is strictly shorter than the planning horizon, our model of the Random Rosca breaks down. Those winning the pot near the end of the Rosca would not rationally choose to buy a durable good if there were no time left to enjoy its services. Assumption 2′ introduced below is sufficient to insure that acquisition of the

durable by the winner is always rational, and so $t_R<T$ at the optimum. See Lemma 1.

[18]From (4.2), (4.4) and (4.6) it should also be noted that since $t_R = B\cdot\mu'(\xi/2)$, we can write the utility level of a type α individual as

$$u(\alpha;<s_R,c_R>) = T[v(y) + \xi] - B\cdot[\mu(\xi/2) - (1-2\alpha)(\xi/2)\mu'(\xi/2)].$$

Since $\mu(\cdot)$ is concave, $\mu(\xi/2) + (\xi/2)\mu'(\xi/2) > \mu(\xi)$, so that $u(1;<s_R,c_R>) < W_A$. Thus in the optimal Random Rosca the last recipient of the pot is worse–off than he would have been under Autarky.

[19]Both of these can be verified straightforwardly by noting that the first order condition for $c_B(x)$ can be written as $1 = \mu_B(\xi)/\int_0^1 c'(\alpha)d\alpha$.

[20]As in the case of the Bidding Rosca, we have been unable to establish whether Assumption 2 is sufficient to guarantee that all individuals will purchase the durable in a market equilibrium. Again, intuitively, it seems that it should be. We know that the market equilibrium is efficient and that all individuals have equal utilities. These two facts together with Assumption 2 should, one would imagine, imply that all individuals will receive the durable. For, if a group of individuals never received the durable, then, by the equal utilities requirement, they would have to be compensated by having higher consumption levels. Given desirability, however, it should be cheaper, in terms of the loss of the rest of the population's utility, to provide them with the durable.

[21]As justification for the characterization in (5.1) consider the "separating hyperplane" argument sketched as follows. Each feasible allocation $<s,c>\in F$ gives rise to an allocation of utility $u(\alpha;<s,c>)$, $\alpha\in[0,1]$. Let $U \equiv \{ u(\cdot;<s,c>) \mid <s,c>\in F \}$. Given the convention of labelling agents according to their order of receipt of the durable, U constitutes the utility possibility set for our model. Under our assumptions U is a convex, bounded subset of the space of Lebesgue integrable functions on the unit interval, $L^1([0,1])$, with non–empty interior, closed in the norm topology. Convexity is assured by the fact [see equation (5.4) below] that on the efficient frontier of F $s'(\alpha)$ is inversely proportional to the aggregate savings rate $[y-\int c(x,s(\alpha))dx]$. Thus the convex combination of two consumption allocations allows receipt dates for every agent which are less than the same convex combination of the corresponding assignment functions.

Now an efficient allocation $<s',c'>\in F$ generates an allocation of utility $u'\in U$ satisfying (suppress dependence of u on $<s,c>$ hereafter): $u(\alpha) > u'(\alpha)$ a.e. if and only if $u\notin U$. The Hahn–Banach Theorem implies a continuous linear functional $p:L^1([0,1]) \to R$ exists, such that $p(u') \geq p(u)$, $\forall u\in U$. It is well known that the dual of $L^1([0,1])$ may be identified with the set of bounded, measurable functions on [0,1]. (See, e.g., Goffman and Pendrick (1965), Theorem 1, p.147). Therefore, there exists such a function, ϕ, satisfying:

$$p(u') = \int_0^1 u'(\alpha)\phi(\alpha)d\alpha \geq \int_0^1 u(\alpha)\phi(\alpha)d\alpha = p(u),\ \forall\ u\in U.$$

Obviously $\phi(\cdot)$ must be non–negative, a.e. Moreover, if u' corresponds to an allocation in which all agents enjoy positive utility from the flow consumption good then, given any subset A of agents of measure strictly less than one, there is an alternative feasible allocation making all agents in A strictly better off, and all agents in $[0,1]\backslash A$ strictly worse off. Therefore ϕ must be strictly positive, a.e. The weights $\theta(\cdot)$ correspond to the function $\phi(\cdot)$ normalized to integrate to one.

[22]To see this, note that $c_0(\alpha,s_0(0)) = y - 1/\mu'(\xi) = c_A$.

[23]Even with indivisibility the allocation problem reduces in this way if the durable's services were fungible across agents – if there were, e.g., a perfect rental market for its services. There are, of course, good (adverse selection/moral hazard) reasons why such trade in durable services might not obtain, especially in a LDC setting. Moreover, some reports on the use of Roscas stress their role in financing personal expenditures (daughter's wedding, feast for fellow villagers, tin roof for house) which, though not producing a fungible asset, generate private consumption benefits lasting for some time that are not transferable to others.

[24]The similarity of this discussion to classical incentive compatibility considerations may be misleading, in that there is really only one "type" of agent in our model. Our results are based on incentive considerations only in the most primitive way – the equal utility requirement of equilibrium forces an allocation which is in the "non–convex part" of the overall utility possibility frontier for the economy. So the gains from randomization are induced by the indivisibility in the model. The non-convexity is due to the possibility of reassigning agents to different orders of receipt of the durable good. As discussed in the note justifying (5.1), given an ordering of the agents, as repesented by the convention that $s(\alpha)$ must be non–decreasing, utility possibilities are convex. By relabelling people at the same physical allocation one obtains a different distribution of utility among them. The union of utility possibilities over all possible relabellings is not convex. (See Figure 3.) The equilibrium incentive constraint of equal utility forces a utility distribution which is in the intersection of all utility possibility frontiers generated by a relabelling of individuals. As the Figure shows, the overall utility possibility set must be locally non–convex near such a point.

[25]This point can be made formally precise. Suppose that an individual who has the durable obtains instantaneous utility $v(c)+\omega\xi$ where $\omega\in[\underline{\omega},\overline{\omega}]$ is an unobservable, individual specific, taste parameter. Let $F(\omega)$ denote the fraction of individuals who have a valuation less than or equal to ω and let $\hat{\omega}$ denote the mean value of ω. Then welfare under the optimal Bidding Rosca, when bids (b) and receipt dates (s) are functions $<b(\omega),s(\omega)>$ of announced "type" constrained to be incentive compatible, can be written:

$$W_B = T[v(y)+\xi]-B\mu_B(\xi\hat{\omega}(1-G)),$$

where G is the Gini coefficient associated with the distribution $F(\omega)$. Thus welfare under a Bidding Rosca depends positively upon the extent of the dispersion in agents' valuations of the durable as measured by the Gini coefficient. By constrast, welfare under the Random Rosca which treats all agents alike remains: $W_R=T[v(y)+\xi]-B\mu(\xi\hat{\omega}/2)$. The Bidding Rosca can dominate with sufficient inequality in the distribution of valuations. A proof of this result is available from the authors on request.

[26]This does, of course, depend on the form of the utility function. However, it may be checked that the *only* case in which equality across agents of the level of lifetime utilities, and equality of the rate of marginal utilities of both goods at all dates, does not conflict is when utility may be written: $u(\alpha;<s,c>)=V[\int c(\alpha,t)dt + \xi(T-s(\alpha))]$. That is, the two goods must be perfect substitutes for one another between any two dates.

[27]The failure of the market to achieve maximal expected utility parallels results obtained in other literatures where indivisibilities are important, such as location models (Mirrlees (1972) and Arnott and Riley (1977)), club membership (Hillman and Swan (1986)) and conscription (Bergstrom 1986).

[28]An income lottery, with competitive equilibrium in a market for consumption loans occuring after the realization of random incomes becomes known, would allow a decentralized realization of the *ex ante* expected utility maximum, W_O.

End Notes

[1]Bouman (1977) provides a comprehensive listing of all the countries in which Roscas have been observed as well as some sense of their quantitative importance. He reports, for example, that an estimated 60% of the population in Addis Ababa belong to a Rosca of some form. Light (1972) discusses their use by ethnic minorities in the US. Additional specific country studies are provided by Begashaw (1978), Fernando (1986), and Osuntogun and Adeyemo (1981). The nineteenth century savings and loan associations in the US also appeared to have worked as Roscas – see Grossman (1989) and Symons and White (1984), page 65. Roscas have long been popular in India where they travel under the name of *Chit Funds*. Radhakrishnan *et al.* (1975) describes them in detail. In Kerala, for example, they report that Roscas are used in preference to banks by industrialists, educational institutions and churches. In 1967, there were 12491 registered Chit Funds in this state of India alone.

[2]The classic anthropological studies of Roscas are by Geertz (1962) and Ardener (1964). The latter paper is particularly recommmended as an introduction to the literature.

[3]While bidding and drawing lots seem to be the two most common ways of allocating the pot, it is also sometimes allocated according to need or known criteria such as age or kinship seniority. The reader is referred to Ardener (1964) for a more detailed discussion.

[4]This was clearly recognized by Ardener (1964). "The most obvious function of these associations is that they assist in small–scale capital–formation, or more simply, they create savings. Members could save their contributions themselves at home and accumulate their own "funds", but this would withdraw money from circulation: in a rotating credit association capital need never be idle." p. 217.

[5]See, for example, the discusion in Ardener (1964) p. 216.

[6]Note that the quantity B is a stock, while y and c are rates of flow.

[7]This is consonant with the anthropological stories. Individuals do not, typically, share the fruit from Rosca winnings. In cases such as the purchase a new roof for one's home, this is not typically feasible anyway.

[8]The function $c:[0,1]\times[0,T]\to\mathbb{R}_+$ is required to be such that $c(\cdot,\tau)$ and $c(\alpha,\cdot)$ are (Lebesgue) integrable for all (α,τ).

[9]That is, $N(\tau;s)$ is the measure of the set $s^{-1}([0,\tau])\equiv\{\alpha\epsilon[0,1]\,|\,s(\alpha)\leq\tau\ \&\ s(\alpha)\neq\nu\}$.

[10]The problem makes no economic sense otherwise, since $t_A=T$ implies $W_A<T\cdot v(y)$. We provide a condition which is both necessary and sufficient for $W_A\geq T\cdot v(y)$ in Assumption 2 below.

[11]It is trivial to show that $\mu(\cdot)$ is increasing and strictly positive. To prove strict concavity, let $\xi_1\neq\xi_2$, define $\xi_\alpha\equiv\alpha\xi_1+(1-\alpha)\xi_2$ and note that

$$\mu(\xi_\alpha)=\alpha\frac{v(y)-v(c_\alpha)-\xi_1}{y-c_\alpha}+(1-\alpha)\frac{v(y)-v(c_\alpha)-\xi_2}{y-c_\alpha},$$

where c_α is the optimal consumption level at ξ_α. Note also that

$$\mu(\xi_i) < \frac{v(y)-v(c_\xi)-\xi_i}{y - c_\xi} \text{ for } \xi_i = \{\xi_1,\xi_2\}.$$

Hence, $\mu(\xi_\alpha) > \alpha\mu(\xi_1) + (1-\alpha)\mu(\xi_2)$, as claimed.

[12]From the properties of $\mu(\cdot)$ stated in the text, we know that $\mu(\xi)/\xi$ is decreasing for $\xi \geq 0$. Moreover, $\mu(\xi) \geq v'(y)$; so the Envelope Theorem implies $\frac{\partial\mu}{\partial y} = \frac{v'(y)-\mu}{y-c} < 0$. Thus there exists a minimal $\underline{\xi}>0$, which is a decreasing function of both y and of $\frac{T}{B}$, such that it is desirable to acquire the durable under Autarky if and only if $\xi \geq \underline{\xi}$.

[13]If desirability fails, then $s_A \equiv \nu$ and $c_A(\alpha,\tau) \equiv y$, for all agents. Note also that we are using the symbol "c_A" to denote both the function which designates consumption levels for the agents at various dates, and the value of that function during the period of accumulation. No confusion should result.

[14]We assume for the moment that the winner will choose to buy the durable. This assumption is discussed further below.

[15]Notice that the particular limit obtained depends upon the assumption that the spacing of meeting dates is uniform and the contribution at each meeting is constant. By having the meetings occur with different frequencies at different times during the life of the Rosca, and by varying the rate of contribution across meeting dates, it may be shown that one can, in the limit, generate <u>every</u> feasible allocation <s,c> in which the consumption paths $c(\alpha,\tau)$ are constant in α, as the *ex post* outcome of a Random Rosca. (Moreover, one can in similar fashion generate every feasible allocation <s,c> in which the utility $u(\alpha;<s,c>)$ is constant in α, as the limiting outcome of a Bidding Rosca.) However, we do not attempt to exploit these fact in this paper. To do so would run contrary to the spirit of our analysis. The point of this exercise is to see how <u>simple</u> institutions of financial intermediation compare with each other and with a more complicated market mechanism. We believe, upon our reading of the anthropological literature, that the simplicity and minimal informational requirements of Roscas, as represented here and as observed in practice, are important factors explaining their popularity around the world. We therefore restrict ourselves to the study of Roscas which can in effect be fully and simply characterized in terms of two parameters – the frequency of meetings, and the contribution required per meeting. It is of some interest that, despite this restriction, we nevertheless find that Roscas can compare favorably with more complicated mechanisms of resource allocation.

[16]It is worth noting that the larger the number of individuals in a Rosca, the earlier is the average receipt date of the durable. ($E(\tilde{\tau}) = t\cdot[1 + 1/n]/2$.) Thus our formulation implies that individual participants are better off when more members are added to the Rosca. In practice, at least two factors are likely to work against this. First, a larger membership requires more meeting dates and there are likely to be transactions costs associated with these meetings. Second, the more members the more impersonal the relationship between participants, and hence the greater the likelihood of incentive problems. The general issue of the optimal number of members is an interesting subject for further research.

[17]Unless the optimal accumulation period is strictly shorter than the planning horizon, our model of the Random Rosca breaks down. Those winning the pot near the end of the Rosca would not rationally choose to buy a durable good if there were no time left to enjoy its services. Assumption 2′ introduced below is sufficient to insure that acquisition of the

durable by the winner is always rational, and so $t_R<T$ at the optimum. See Lemma 1.

[18]From (4.2), (4.4) and (4.6) it should also be noted that since $t_R = B\cdot\mu'(\xi/2)$, we can write the utility level of a type α individual as

$$u(\alpha;<s_R,c_R>) = T[v(y) + \xi] - B\cdot[\mu(\xi/2) - (1-2\alpha)(\xi/2)\mu'(\xi/2)].$$

Since $\mu(\cdot)$ is concave, $\mu(\xi/2) + (\xi/2)\mu'(\xi/2) > \mu(\xi)$, so that $u(1;<s_R,c_R>) < W_A$. Thus in the optimal Random Rosca the last recipient of the pot is worse–off than he would have been under Autarky.

[19]Both of these can be verified straightforwardly by noting that the first order condition for $c_B(x)$ can be written as $1 = \bar{\mu}_B(\xi)/\int_0^1 c'(\alpha)d\alpha$.

[20]As in the case of the Bidding Rosca, we have been unable to establish whether Assumption 2 is sufficient to guarantee that all individuals will purchase the durable in a market equilibrium. Again, intuitively, it seems that it should be. We know that the market equilibrium is efficient and that all individuals have equal utilities. These two facts together with Assumption 2 should, one would imagine, imply that all individuals will receive the durable. For, if a group of individuals never received the durable, then, by the equal utilities requirement, they would have to be compensated by having higher consumption levels. Given desirability, however, it should be cheaper, in terms of the loss of the rest of the population's utility, to provide them with the durable.

[21]As justification for the characterization in (5.1) consider the "separating hyperplane" argument sketched as follows. Each feasible allocation $<s,c>\epsilon F$ gives rise to an allocation of utility $u(\alpha;<s,c>)$, $\alpha\epsilon[0,1]$. Let $U \equiv \{ u(\cdot;<s,c>) \mid <s,c>\epsilon F \}$. Given the convention of labelling agents according to their order of receipt of the durable, U constitutes the utility possibility set for our model. Under our assumptions U is a convex, bounded subset of the space of Lebesgue integrable functions on the unit interval, $L^1([0,1])$, with non–empty interior, closed in the norm topology. Convexity is assured by the fact [see equation (5.4) below] that on the efficient frontier of F $s'(\alpha)$ is inversely proportional to the aggregate savings rate $[y-\int c(x,s(\alpha))dx]$. Thus the convex combination of two consumption allocations allows receipt dates for every agent which are less than the same convex combination of the corresponding assignment functions.

Now an efficient allocation $<s',c'>\epsilon F$ generates an allocation of utility $u'\epsilon U$ satisfying (suppress dependence of u on $<s,c>$ hereafter): $u(\alpha) > u'(\alpha)$ a.e. if and only if $u\notin U$. The Hahn–Banach Theorem implies a continuous linear functional $p:L^1([0,1]) \to R$ exists, such that $p(u') \geq p(u)$, $\forall u\epsilon U$. It is well known that the dual of $L^1([0,1])$ may be identified with the set of bounded, measurable functions on [0,1]. (See, e.g., Goffman and Pendrick (1965), Theorem 1, p.147). Therefore, there exists such a function, ϕ, satisfying:

$$p(u') = \int_0^1 u'(\alpha)\phi(\alpha)d\alpha \geq \int_0^1 u(\alpha)\phi(\alpha)d\alpha = p(u),\ \forall\ u\epsilon U.$$

Obviously $\phi(\cdot)$ must be non–negative, a.e. Moreover, if u' corresponds to an allocation in which all agents enjoy positive utility from the flow consumption good then, given any subset A of agents of measure strictly less than one, there is an alternative feasible allocation making all agents in A strictly better off, and all agents in $[0,1]\backslash A$ strictly worse off. Therefore ϕ must be strictly positive, a.e. The weights $\theta(\cdot)$ correspond to the function $\phi(\cdot)$ normalized to integrate to one.

[22]To see this, note that $c_0(\alpha,s_0(0)) = y - 1/\mu'(\xi) = c_A$.

[23]Even with indivisibility the allocation problem reduces in this way if the durable's services were fungible across agents – if there were, e.g., a perfect rental market for its services. There are, of course, good (adverse selection/moral hazard) reasons why such trade in durable services might not obtain, especially in a LDC setting. Moreover, some reports on the use of Roscas stress their role in financing personal expenditures (daughter's wedding, feast for fellow villagers, tin roof for house) which, though not producing a fungible asset, generate private consumption benefits lasting for some time that are not transferable to others.

[24]The similarity of this discussion to classical incentive compatibility considerations may be misleading, in that there is really only one "type" of agent in our model. Our results are based on incentive considerations only in the most primitive way – the equal utility requirement of equilibrium forces an allocation which is in the "non–convex part" of the overall utility possibility frontier for the economy. So the gains from randomization are induced by the indivisibility in the model. The non-convexity is due to the possibility of reassigning agents to different orders of receipt of the durable good. As discussed in the note justifying (5.1), given an ordering of the agents, as repesented by the convention that $s(\alpha)$ must be non–decreasing, utility possibilities are convex. By relabelling people at the same physical allocation one obtains a different distribution of utility among them. The union of utility possibilities over all possible relabellings is not convex. (See Figure 3.) The equilibrium incentive constraint of equal utility forces a utility distribution which is in the intersection of all utility possibility frontiers generated by a relabelling of individuals. As the Figure shows, the overall utility possibility set must be locally non–convex near such a point.

[25]This point can be made formally precise. Suppose that an individual who has the durable obtains instantaneous utility $v(c)+\omega\xi$ where $\omega\in[\underline{\omega},\overline{\omega}]$ is an unobservable, individual specific, taste parameter. Let $F(\omega)$ denote the fraction of individuals who have a valuation less than or equal to ω and let $\hat{\omega}$ denote the mean value of ω. Then welfare under the optimal Bidding Rosca, when bids (b) and receipt dates (s) are functions $<b(\omega),s(\omega)>$ of announced "type" constrained to be incentive compatible, can be written:

$$W_B = T[v(y)+\xi]-B\mu_B(\xi\hat{\omega}(1-G)),$$

where G is the Gini coefficient associated with the distribution $F(\omega)$. Thus welfare under a Bidding Rosca depends positively upon the extent of the dispersion in agents' valuations of the durable as measured by the Gini coefficient. By constrast, welfare under the Random Rosca which treats all agents alike remains: $W_R=T[v(y)+\xi]-B\mu(\xi\hat{\omega}/2)$. The Bidding Rosca can dominate with sufficient inequality in the distribution of valuations. A proof of this result is available from the authors on request.

[26]This does, of course, depend on the form of the utility function. However, it may be checked that the *only* case in which equality across agents of the level of lifetime utilities, and equality of the rate of marginal utilities of both goods at all dates, does not conflict is when utility may be written: $u(\alpha;<s,c>)=V[\int c(\alpha,t)dt + \xi(T-s(\alpha))]$. That is, the two goods must be perfect substitutes for one another between any two dates.

[27]The failure of the market to achieve maximal expected utility parallels results obtained in other literatures where indivisibilities are important, such as location models (Mirrlees (1972) and Arnott and Riley (1977)), club membership (Hillman and Swan (1986)) and conscription (Bergstrom 1986).

[28]An income lottery, with competitive equilibrium in a market for consumption loans occurring after the realization of random incomes becomes known, would allow a decentralized realization of the *ex ante* expected utility maximum, W_0.

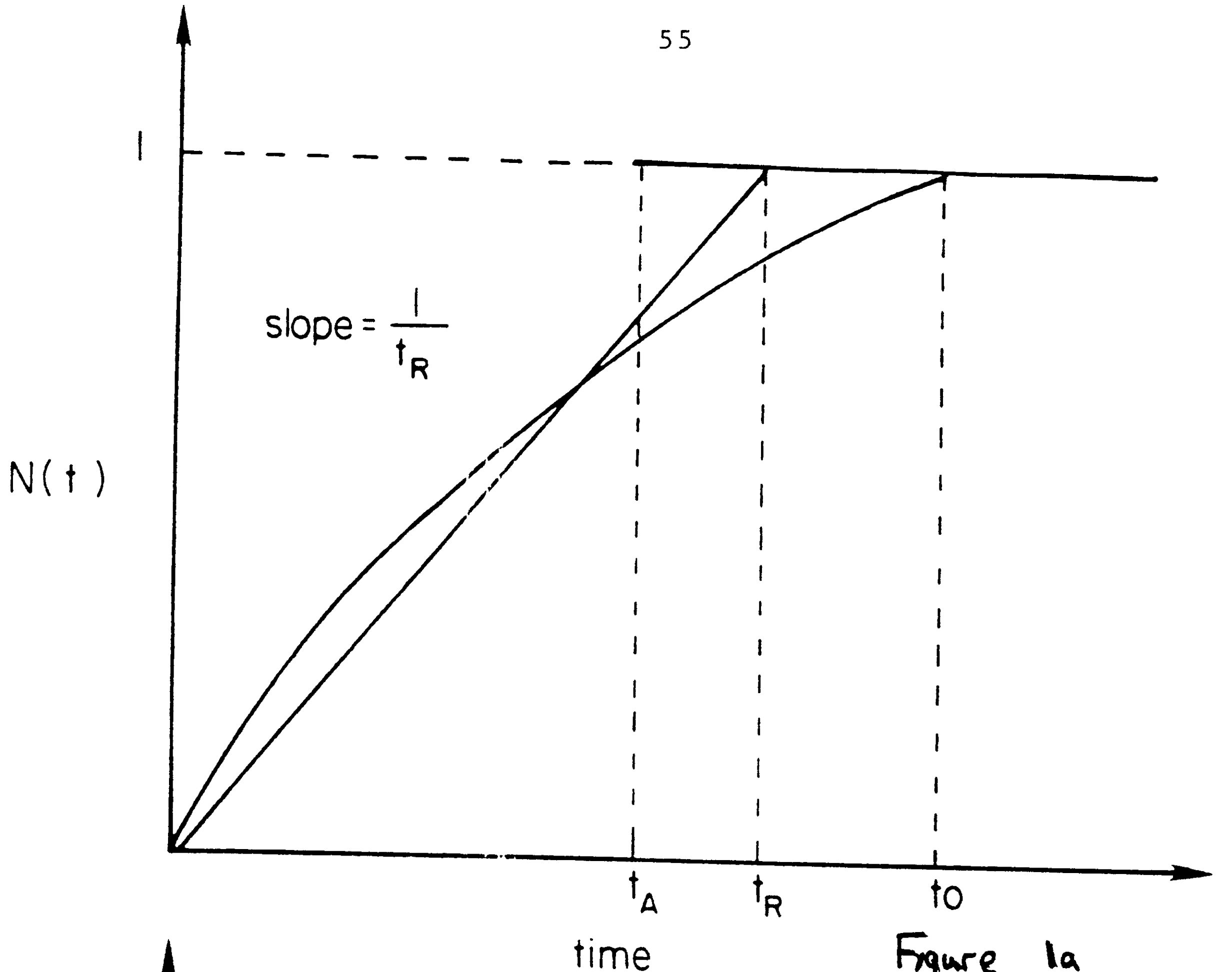


Figure 1a

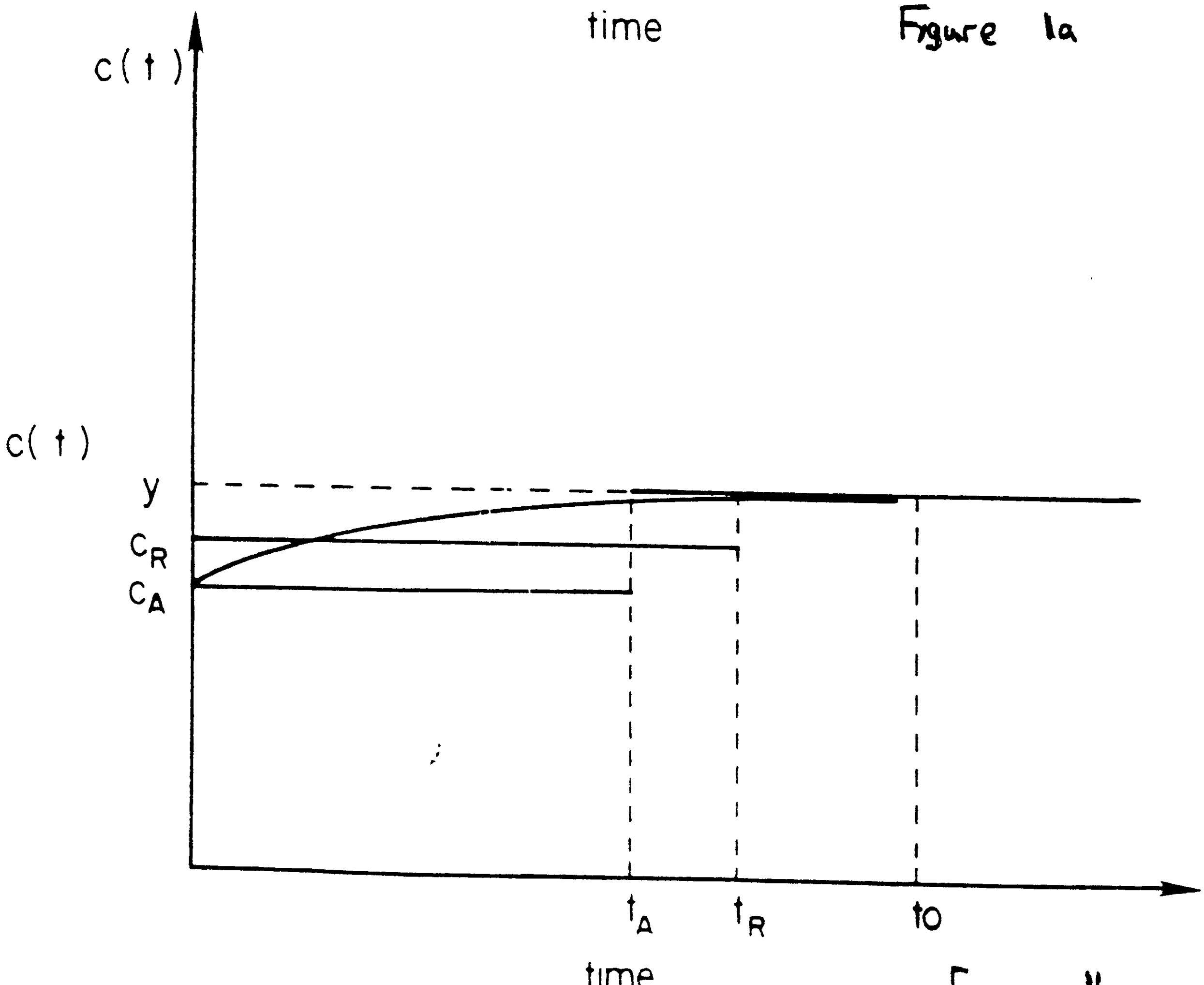
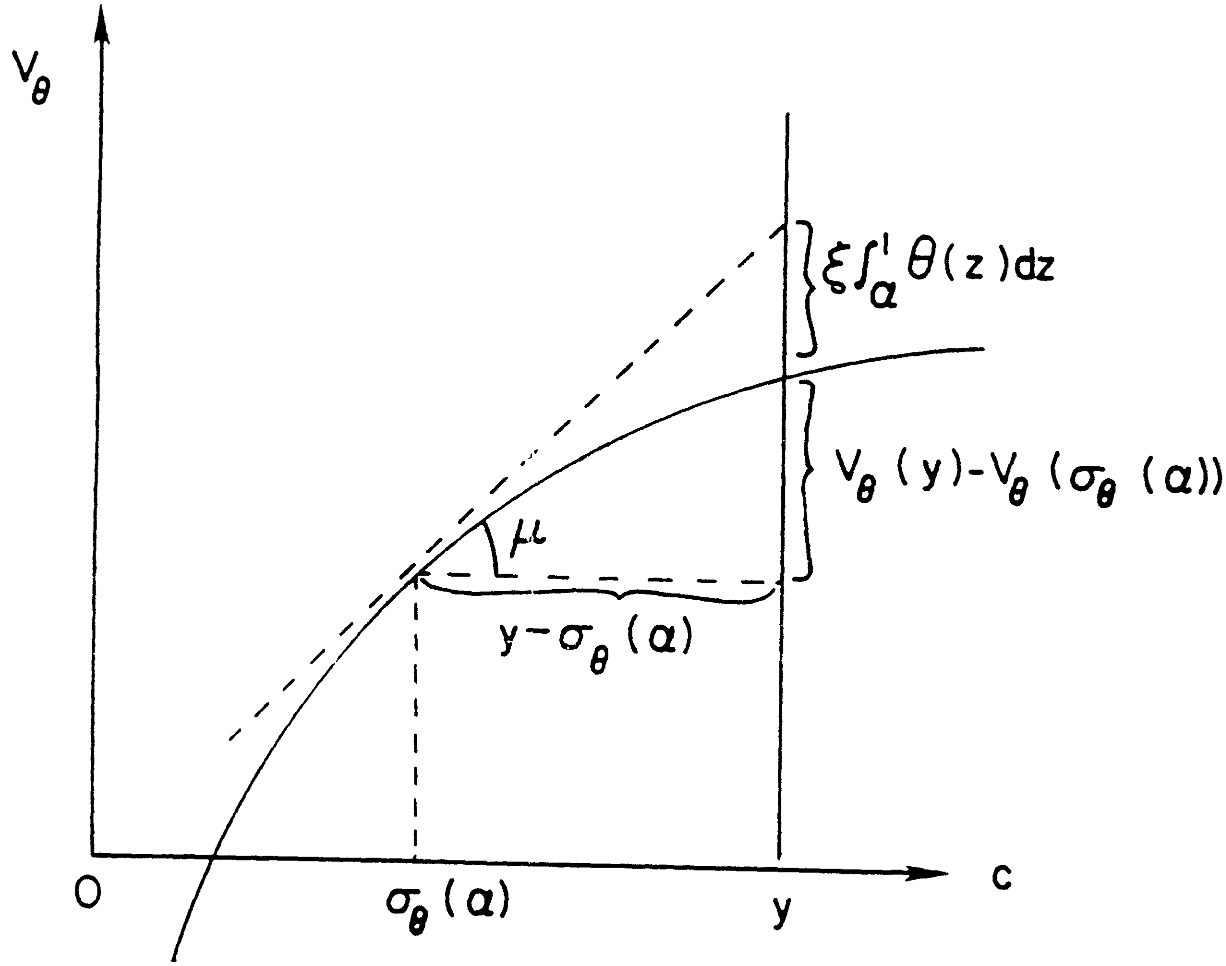


Figure 1b



V_θ
$\xi\int_\alpha^1 \theta(z)dz$
$V_\theta(y)-V_\theta(\sigma_\theta(\alpha))$
μ
$y-\sigma_\theta(\alpha)$
O
$\sigma_\theta(\alpha)$
y
c

Figure 2

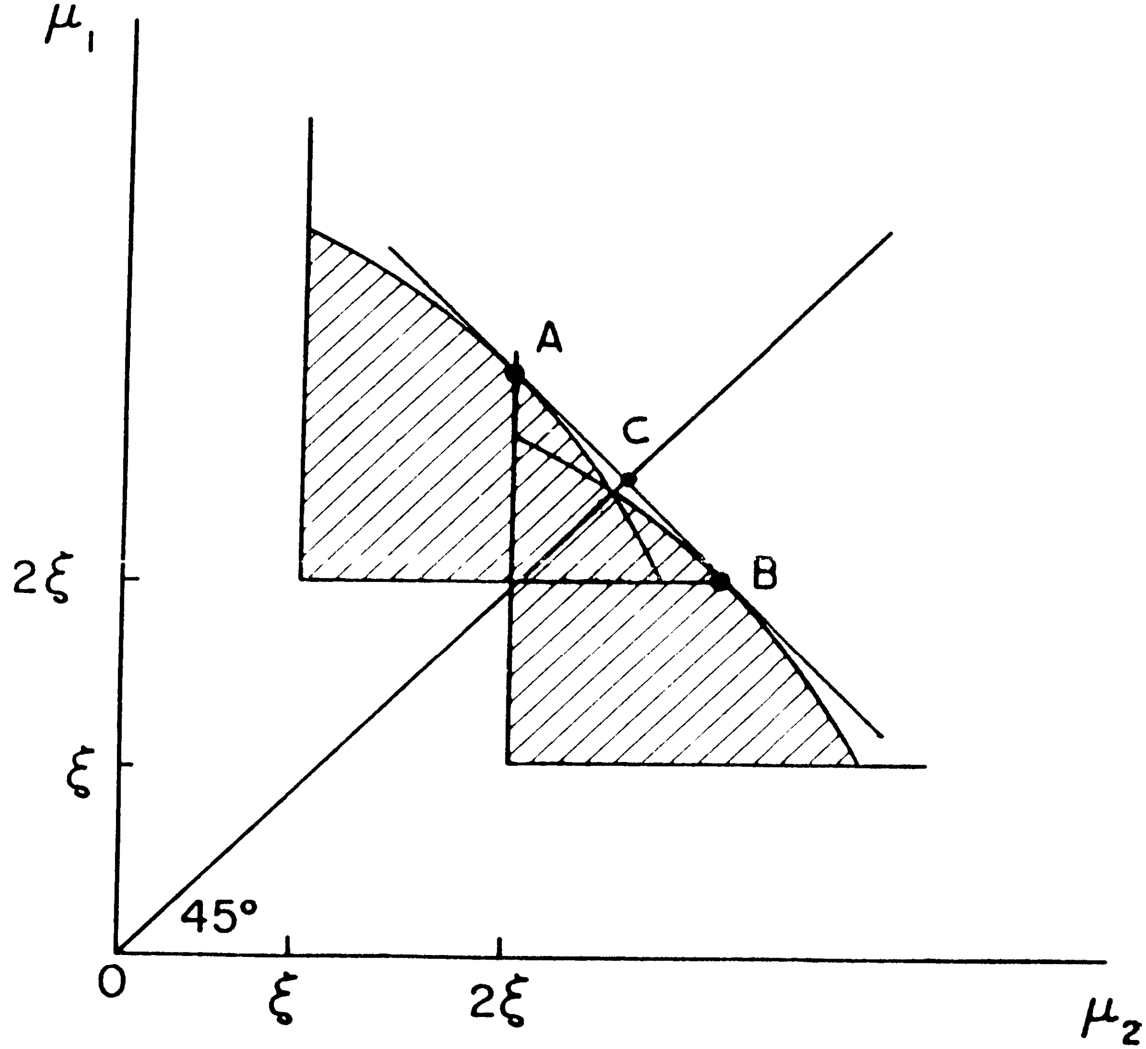


Figure 3