$|V_{us}|$ from Strange Hadronic Tau Data [1]

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Abstract

We report on recent work to determine the CKM matrix element $|V_{us}|$ using strange hadronic τ decay data. We use the recent OPAL update of the strange spectral function, while on the theory side we update the dimension-two perturbative contribution including the recently calculated α_s^3 terms. Our result $|V_{us}| = 0.2220 \pm 0.0033$ is already competitive with the standard extraction from K_{e3} decays and other new proposals to determine $|V_{us}|$. The actual uncertainty on $|V_{us}|$ from τ data is dominated largely by experiment and will eventually be much reduced by B-factories and future τ-charm factory data, providing one of the most accurate determinations of this Standard Model parameter.

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1 Introduction and Motivation

BaBar and Belle are just starting to produce their first results on hadronic tau decays and in particular on Cabibbo-suppressed modes.[1] These results will eventually further increase the high precision status of observables such as

$$R_\tau \equiv \frac{\Gamma[\tau^- \to \text{hadrons } \nu_\tau(\gamma)]}{\Gamma[\tau^- \to e^- \overline{\nu}_e \nu_\tau(\gamma)]} \,, \tag{1}$$

attained by ALEPH and OPAL at LEP and CLEO at CESR. This status has been already exploited in a very successful determination of $a_\tau \equiv \alpha_s(M_\tau)/\pi$.[2, 3, 4, 5, 6] On the other hand, SU(3) breaking effects are sizeable in the semi-inclusive hadronic τ-decay width into Cabibbo-suppressed modes.[7, 8, 9, 10, 11]

These two facts make the strange hadronic τ decay data an ideal place for determining SU(3) breaking parameters such as $|V_{us}|$ and/or m_s. The obvious advantage of this procedure is that the experimental uncertainty will eventually be reduced at the B-factories and at future facilities like the τ-charm factory BEPCII at Beijing.

2 Theoretical Framework

Using analytic properties of two-point correlation functions for vector $(\mathcal{J} = \mathcal{V})$ and axial-vector $(\mathcal{J} = \mathcal{A})$ two quark-currents,

$$\begin{aligned}
\Pi^{\mu\nu}_{\mathcal{J},\rangle|}(q) &\equiv i \int \mathrm{d}^4 x \, e^{iq \cdot x} \langle 0|T[\mathcal{J}^{\mu\dagger}_{\rangle|}(\S) \mathcal{J}^{\nu}_{\rangle|}(\prime)]|\prime\rangle \\
&\equiv \left[q^\mu q^\nu - q^2 g^{\mu\nu} \right] \Pi^T_{\mathcal{J},\rangle|}(q^2) + q^\mu q^\nu \Pi^L_{\mathcal{J},\rangle|}(q^2) \,,
\end{aligned} \tag{2}$$

one can express R_τ as a contour integral running counter-clockwise around the circle $|s| = M_\tau^2$ in the complex s-plane:

$$\begin{aligned}
R_\tau \equiv -i\pi \oint_{|s|=M_\tau^2} \frac{\mathrm{d}s}{s} \left[1 - \frac{s}{M_\tau^2} \right]^3 \\
\times \left\{ 3 \left[1 + \frac{s}{M_\tau^2} \right] D^{L+T}(s) + 4 \, D^L(s) \right\} \,.
\end{aligned} \tag{3}$$

We have used integration by parts to rewrite R_τ in terms of the logarithmic derivatives

$$D^{L+T}(s) \equiv -s \frac{\mathrm{d}}{\mathrm{d}s} \Pi^{L+T}(s) \,;$$

$$D^L(s) \equiv \frac{s}{M_\tau^2} \frac{\mathrm{d}}{\mathrm{d}s} \left[s \Pi^L(s) \right] \,. \tag{4}$$

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Moreover, one can experimentally decompose R_τ into

$$R_\tau \equiv R_{\tau,V} + R_{\tau,A} + R_{\tau,S}, \tag{5}$$

according to the quark content

$$
\begin{aligned}
\Pi^J(s) &\equiv |V_{ud}|^2 \left\{ \Pi^J_{V,ud}(s) + \Pi^J_{A,ud}(s) \right\} \\
&+ |V_{us}|^2 \left\{ \Pi^J_{V,us}(s) + \Pi^J_{A,us}(s) \right\},
\end{aligned}
\tag{6}
$$

where $R_{\tau,V}$ and $R_{\tau,A}$ correspond to the first two terms in the first line and $R_{\tau,S}$ to the second line, respectively.

Additional information can be obtained from the measured invariant-mass distribution of the final hadrons, which defines the moments

$$R_\tau^{kl} \equiv \int_0^{M_\tau^2} ds \left(1 - \frac{s}{M_\tau^2} \right)^k \left(\frac{s}{M_\tau^2} \right)^l \frac{dR_\tau}{ds}. \tag{7}$$

At large enough Euclidean $Q^2 = -s$, both $\Pi^{L+T}(Q^2)$ and $\Pi^L(Q^2)$ can be organised in a dimensional operator series using well established QCD operator product expansion (OPE) techniques. One gets then

$$
\begin{aligned}
R_\tau^{kl} \equiv N_c S_{\mathrm{EW}} &\left\{ (|V_{ud}|^2 + |V_{us}|^2) \left[1 + \delta^{kl(0)} \right] \right. \\
&\left. + \sum_{D \geq 2} \left[|V_{ud}|^2 \delta^{kl(D)}_{ud} + |V_{us}|^2 \delta^{kl(D)}_{us} \right] \right\}.
\end{aligned}
\tag{8}
$$

The electroweak radiative correction $S_{\mathrm{EW}} = 1.0201 \pm 0.0003$ has been pulled out[12] explicitly and $\delta^{kl(0)}$ denotes the purely perturbative dimension-zero contribution. The symbols $\delta^{kl(D)}_{ij}$ stand for higher dimensional corrections in the OPE from dimension $D \geq 2$ operators, which contain implicit $1/M_\tau^D$ suppression factors.[2, 13, 16, 17] The most important being the operators m_s^2 with $D = 2$ and $m_s \langle \bar{q}q \rangle$ with $D = 4$.

In addition, the flavour SU(3)-breaking quantity

$$
\begin{aligned}
\delta R_\tau^{kl} &\equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} \\
&= N_c S_{EW} \sum_{D \geq 2} \left[\delta^{kl(D)}_{ud} - \delta^{kl(D)}_{us} \right]
\end{aligned}
\tag{9}
$$

enhances the sensitivity to the strange quark mass. The dimension-two correction $\delta^{kl(2)}_{ij}$ is known to order α_s^3 for both correlators, $J = L$ and $J = L+T$.[13, 16, 18]

In Ref. [13], an extensive analysis of this $D = 2$ correction was done and it was shown that the perturbative $J = L$ correlator behaves very badly. The $J = L+T$ correlator was also analysed there to order α_s^2 and showed a relatively good convergence. Here, we have included the recently calculated $O(\alpha_s^3)$ correction for $J = L + T$.[18] One can see that the $J = L + T$ series also starts to show its asymptotic character at this order, though it is still much better behaved than the $J = L$ component. These series show clearly an asymptotic behaviour and it does not make much sense to sum all known orders.

3 Determination of $|V_{us}|$

One can use the relation –and analogous relations for other moments–

$$|V_{us}|^2 = \frac{R^{00}_{\tau,S}}{\dfrac{R^{00}_{\tau,V+A}}{|V_{ud}|^2} - \delta R^{00}_{\tau,\mathrm{th}}} \tag{10}$$

to determine the Cabibbo–Kobayashi–Maskawa (CKM) matrix element $|V_{us}|$. Notice that, on the right-hand side of (10), the only theoretical input is $\delta R^{00}_{\tau,\mathrm{th}}$, which is around 0.25 and gets compared to the experimental quantity $R^{00}_{\tau,V+A}/|V_{ud}|^2$ which is around 3.7. Therefore, with a not so precise theoretical prediction for $\delta R^{00}_{\tau,\mathrm{th}}$ one can get a quite accurate value for $|V_{us}|$, depending on the experimental accuracy.

The very bad QCD behaviour of the $J = L$ component in $\delta R^{kl}_{\tau,\mathrm{th}}$ induces a large theoretical uncertainty, which can be reduced considerably using phenomenology for the scalar and pseudo-scalar correlators.[19, 20, 21] In particular, the pseudo-scalar spectral functions are dominated by far by the well-known kaon pole, to which we add suppressed contributions from the pion pole, as well as higher excited pseudo-scalar states whose parameters have been estimated in Ref. [22]. For the strange scalar spectral function, we take the result obtained[23] from a study of S-wave $K\pi$ scattering within resonance chiral perturbation theory,[24] which has been recently updated in Ref. [25].

The smallest theoretical uncertainty arises for the $kl = 00$ moment, for which we get

$$\begin{aligned} \delta R^{00}_{\tau,\mathrm{th}} &= 0.1544\,(37) + 9.3\,(3.4)\,m_s^2 \\ &+ 0.0034\,(28) = 0.240\,(32)\,, \end{aligned} \tag{11}$$

where m_s denotes the strange quark mass, in GeV units, in the $\overline{\mathrm{MS}}$ scheme at $\mu = 2$ GeV. The first term contains the phenomenological scalar and pseudo-scalar contributions, the second term contains the rest of the perturbative $D = 2$ contribution, while the last term stands for the rest of the contributions. Notice that the phenomenological contribution is more than 64% of the total, while the rest comes almost from the perturbative $D = 2$ contribution. Here, we update $\delta R^{00}_{\tau,\mathrm{th}}$ in Refs. [19, 20, 21] in various respects. First, we use the recently updated scalar spectral function[25]; second, we include the α_s^3 corrections to the $J = L + T$ correlator calculated in Ref. [18] and finally, we use an average of contour improved[26] and fixed order perturbation results for the asymptotically summed series. A detailed analysis will be presented elsewhere.[27]

For the m_s input value, we use the recent average $m_s(2\,\mathrm{GeV}) = (94 \pm 6)$ MeV,[25] which includes the most recent determinations of m_s from QCD sum rules and lattice QCD. The strange quark mass uncertainty corresponds to the most precise determination from the lattice.

Recently Maltman and Wolfe criticised the theory error we previously employed for the $D = 2$ OPE coefficient.[30] In our updated estimate (11), we have decided to include a more conservative estimate of unknown higher-order corrections by using an average of contour improved and fixed-order perturbation theory. Notice however, that $\delta R_{\tau,\text{th}}^{00}$ is dominated by the scalar and pseudo-scalar contributions which are rather well known from phenomenology, and that the larger perturbative uncertainty is compensated by the smaller m_s error, so that our final theoretical uncertainty is almost the same as in previous works.[19, 20, 21]

In order to finally determine $|V_{us}|$, we employ the following updates of the remaining input parameters: $|V_{ud}| = 0.97377 \pm 0.00027$,[28] the non-strange branching fraction $R_{\tau,V+A}^{00} = 3.471 \pm 0.011$,[11] as well as the strange branching fraction[11] $R_{\tau,S}^{00} = 0.1686 \pm 0.0047$ (see also Refs. [7] and [9]), which includes the theoretical prediction for the decay $B[\tau \to K\nu_\tau(\gamma)] = 0.715 \pm 0.003$ which is based on the better known $K \to \mu\nu_\mu(\gamma)$ decay rate. For $|V_{us}|$, we then obtain

$$|V_{us}| = 0.2220 \pm 0.0031_{\text{exp}} \pm 0.0011_{\text{th}} . \tag{12}$$

The experimental uncertainty includes a small component from the error in $|V_{ud}|$, but it is dominated by the uncertainty in $R_{\tau,S}^{00}$, while the theoretical error is dominated by the uncertainty in the perturbative expansion of the $D = 2$ contribution.

4   Results and Conclusions

High precision Cabibbo-suppressed hadronic tau data from ALEPH and OPAL at LEP and CLEO at CESR provide already a competitive result for $|V_{us}|$. As presented above and in Refs. [19, 20, 21], the final uncertainty in the τ determination of $|V_{us}|$ becomes an experimental issue and will eventually be much reduced with the new B-factories data[1] and further reduced at future τ facilities. A combined fit to determine both $|V_{us}|$ and m_s will then be possible. Hadronic τ decays have the potential to provide the most accurate measurement of $|V_{us}|$ and a very competitive m_s determination.

Acknowledgments

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