Neutralino dark matter in the MSSM with CP violation

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Abstract

In the MSSM, the presence of CP phases can have a significant impact on the neutralino relic abundance. These phase effects are on the one hand due to shifts in the masses, on the other hand due to modifications of the couplings. Typical variations in Ωh^2 solely from modifications in the couplings are $\mathcal{O}(10\%–100\%)$.

1   Introduction

With the conclusive evidence for a significant component of cold dark matter (CDM) in the Universe, there is considerable interest, both at the theoretical and experimental level, to identify this CDM and analyse its properties. In particular, if the CDM consists of a new weakly interacting massive particle (WIMP), for example the lightest neutralino in supersymmetric models, the next generation of colliders has good prospects to discover it and determine its properties. If the parameters of the underlying model can be measured with sufficient precision, it may be possible to predict the annihilation cross sections hence the thermal relic density of the CDM candidate. The aim is to reach an accuracy comparable to the one from cosmological measurements to be able to perform consistency checks between a particular model of new physics and cosmology [1, 2]. The recent data from WMAP and SDSS imply a value for the relic density of cold dark matter $\Omega h^2 = 0.105 \pm 0.008$ [3] at 2σ [2].

Most of the studies that have either analysed the constraints on the parameter space of the MSSM arising from the relic density measurement, or that have analysed how to extract precise information on the model parameters at future colliders have assumed that CP is conserved. However CP-violating phases are generic in the MSSM. Furthermore CP violation may help to generate the correct baryon asymmetry in the Universe in scenarios of electroweak baryogenesis [7].

CP phases can have a strong impact on the relic density of neutralino dark matter, both due to modifications in the sparticle couplings and due to changes in the physical

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[2]In the figures we will rather use the 2σ range as of 2003, $0.0945 < \Omega h^2 < 0.1287$ [4,5] and refer to this as the WMAP range. Note that the exact value and uncertainty of Ωh^2 extracted from cosmological data depend on both the precise data sets used and the assumptions about other parameters [6].

masses. Here we highlight the impact of CP phases in a few typical scenarios for which the LSP is a 'good' CDM candidate [8]. Some of the largest effects are in fact due to kinematics. This should be expected as the relic density is often very sensitive to masses, in particular to the exact mass difference between the LSP and NLSP in coannihilation processes, or in the case of annihilation near a s-channel Higgs resonance, to the difference between twice the LSP mass and the mass of the Higgs. In these scenarios, setting apart the purely kinematic effects hence somewhat tames the huge effects due to phases found in some of the early studies [9]. On the other hand, there are other cases where the phase dependence of masses and couplings work against each other. Taking out the kinematic effects actually enhances the phase dependence of the number density. Since what are relevant to experiments are rather the physical observables (masses, branching ratios, etc.) than the underlying parameters, we take special care to disentangle effects arising from changes in the couplings from purely kinematic effects.

2 The model

In the MSSM, the parameters that can have CP phases are the gaugino and Higgsino mass parameters, $M_i = |M_i|e^{i\phi_i}$ and $\mu = |\mu|e^{i\phi_\mu}$, and the trilinear sfermion-Higgs couplings, $A_f = |A_f|e^{i\phi_f}$. For the relic density of the LSP the relevant phases are those of the neutralino sector (only ϕ_1 and ϕ_μ since ϕ_2 can be rotated away) and the phase of the trilinear coupling A_t which affects the Higgs sector. The parameters that will be allowed to vary are those of the MSSM defined at the weak scale

$$|M_1|, \ |\mu|, \ \tan\beta, \ m_{H^+}, \ |A_t|, \ M_S, \ \phi_1, \ \phi_\mu, \ \phi_t, \ \phi_l. \tag{1}$$

where universality at the GUT scale for the gaugino masses is assumed. M_S is the common mass for third generation sfermion, the common mass for the sfermions of the first and second generation is set to $m_{\tilde{f}_{L,R}} = 10$ TeV and all trilinear couplings with the exception of A_t are set to $|A_f| = 1$ TeV. The phase of the selectron coupling ϕ_e, although irrelevant for the relic density has to be taken into account since it contributes to electric dipole moments (EDMs).

Allowing for CP-violating phases induces a mixing between the two CP-even states h^0, H^0 and the CP-odd state A^0 of the MSSM. The resulting mass eigenstates h_1, h_2, h_3 ($m_{h_1} < m_{h_2} < m_{h_3}$) are no longer eigenstates of CP, therefore it is preferable to use the charged Higgs mass, m_{H^+}, as an independent parameter. The Lagrangian for the interaction of the lightest neutralino with Higgs bosons which governs the neutralino annihilation cross section via Higgs exchange, writes

$$\mathcal{L}_{\tilde{\chi}_1^0\tilde{\chi}_1^0 h_i} = -\frac{g}{2}\sum_{i=1}^3 \overline{\tilde{\chi}_1^0}(g^S_{h_i\tilde{\chi}_1^0\tilde{\chi}_1^0} + i\gamma_5 g^P_{h_i\tilde{\chi}_1^0\tilde{\chi}_1^0})\tilde{\chi}_1^0 h_i \tag{2}$$

with the scalar and pseudoscalar couplings corresponding to the real and imaginary part of the same expression, see [8]. These couplings depend on the neutralino mixing, hence on phases in the neutralino sector, as well as on phases that enter the Higgs mixing, for example ϕ_t. Indeed, in the MSSM, the Higgs CP mixing is induced by loops involving top squarks and is proportional to $Im(A_t\mu)/(m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2)$ [10]. Thus a large mixing is expected when $Im(A_t\mu)$ is large as compared to the square of the stop masses.

3 Relic density of dark matter

The results presented here have been obtained with the new implementation of the CPV-MSSM within `micrOMEGAs`2.0 [11]. In this code, CP phases are taken into account consistently in all annihilation and coannihilation channels. The computation of masses, mixings and effective couplings in the Higgs sector relies on `CPsuperH` [13]. Standard `micrOMEGAs` routines are used to calculate the relic density of dark matter [12].

The cross sections for the annihilation and coannihilation processes will depend on phases, and so will the thermally-averaged cross section [8]. Part of this is due to changes in the physical masses, leading to huge variations in the relic density especially when coannihilation processes are important or when annihilation occurs near a resonance. We will therefore take special care to disentangle the effects from kinematics and couplings.

At vanishing relative velocity, $v \to 0$, neutralino annnihilation through s-channel scalar exchange is p-wave suppressed; the annihilation proceeds strictly through pseudoscalar exchange. Nevertheless when performing the thermal averaging, the scalar exchange cannot be neglected altogether. In the MSSM with real parameters it can amount to $\mathcal{O}(10\%)$ of the total contribution. In the presence of phases, all the neutral Higgs bosons can acquire a pseudoscalar component (that is $g^P_{h_i \tilde{\chi}^0_1 \tilde{\chi}^0_1} \neq 0$) and hence significantly contribute to neutralino annihilation even at small v. There is a kind of sum rule that relates the couplings squared of the Higgses to neutralinos. Therefore, for the two heavy eigenstates which are in general close in mass, we do not expect a large effect on the resulting relic density from Higgs mixing alone. A noteworthy exception occurs when, for kinematical reason, only one of the resonances is accessible to neutralino annihilation. That is for example the case when $m_{h_2} < 2m_{\tilde{\chi}^0_1} \simeq m_{h_3}$.

4 Results

We now turn to the numerical analysis and present results for two typical scenarios for which the relic density is in agreement with WMAP: the mixed bino-Higgsino LSP that annihilates into gauge bosons and the rapid annihilation through a Higgs resonance, the so-called Higgs funnel. More comprehensive results can be found in [8]. We always impose a constraint from the eEDM, $d_e < 2.2 \times 10^{-27}$e cm [14], including one- and two-loop contributions. When $\tan\beta$ is not too large and sfermions of the first and second generations are heavy, this constraint usually forces $\sin\phi_\mu \approx 10^{-2}$. In addition, the free parameter ϕ_e is adjusted so that this constraint is satisfied.

4.1 The mixed bino-Higgsino LSP

When all scalars except the light Higgs are heavy, $M_S = m_{H^+} = 1$ TeV, the most efficient annihilation of a pair of neutralinos with a mass of the order of 100 GeV is through its Higgsino component. In the real MSSM, one needs a Higgsino admixture of roughly 25%–30% for the relic density to be within the WMAP range [15, 16]. In terms of fundamental MSSM parameters this means $M_1 \approx \mu$. Figure 1a displays the allowed 2σ WMAP band in the $M_1 - \mu$ plane. The main annihilation mechanisms then are $\tilde{\chi}^0_1 \tilde{\chi}^0_1 \to WW/ZZ$ through t-channel chargino and neutralino exchange, as well as $\tilde{\chi}^0_1 \tilde{\chi}^0_1 \to t\bar{t}$ when kinematically allowed. The latter proceeds through s-channel Z or h_1 exchange. The LSP Higgsino
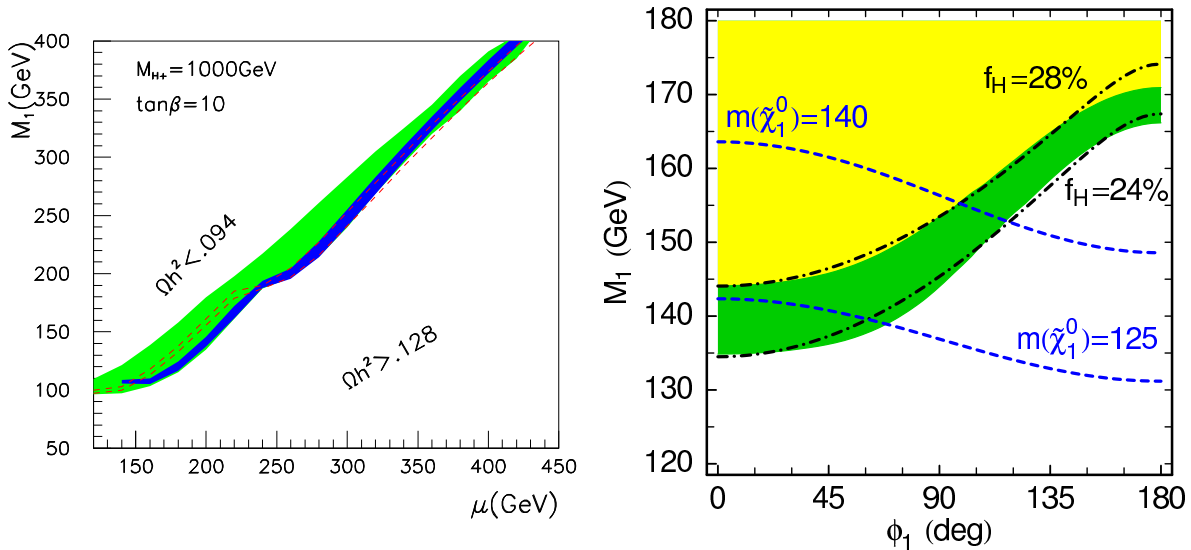
Figure 1: (a) The 2σ WMAP bands for $\tan\beta = 10$, $m_{H^+} = M_S = 1$ TeV a) in the M_1–μ plane for $A_t = 1$ TeV and for all phases zero (blue), for $\phi_\mu = 180°$ (or $\mu < 0$) and all other phases zero (dashed red lines) and for arbitrary phases (green). , (b) M_1–ϕ_1 plane for $\mu = 200$ GeV, $A_t = 1.2$ TeV, $\phi_t = \phi_\mu = 0$. In the yellow region Ωh^2 is below the WMAP bound, and in the white region it is too large. Superimposed are contours of constant LSP mass (dashed lines) and contours of constant LSP Higgsino fraction (dash-dotted lines).

fraction determines the size of the annihilation cross-section because it directly enters both the $\tilde\chi_1^0\tilde\chi_i^\pm W^\mp$ and $\tilde\chi_1^0\tilde\chi_j^0 Z$ vertices.

When allowing all phases to vary arbitrarily, thus increasing the number of free parameters, it is natural to expect a widening of the allowed band, see the green (light grey) band in Fig. 1a where we display models for which all constraints are satisfied for at least one combination of phases. Since the eEDM constraint strongly constrains ϕ_μ, the widening of the allowed band is mostly due to ϕ_1.

Figure 1b shows the WMAP band in the M_1–ϕ_1 plane for $\mu = 200$ GeV, $\tan\beta = 10$, $A_t = 1$ TeV and all other phases set to zero. Also shown are contours of constant LSP mass as well as contours of constant LSP Higgsino fraction f_H. We can make several observations. First, the mass of the LSP increases with ϕ_1. On the one hand this induces a decrease in the LSP pair-annihilation cross-sections. On the other hand, since the chargino mass is independent of ϕ_1, the NLSP–LSP mass splitting is reduced, making coannihilation processes with $\tilde\chi_1^+$ (and also $\tilde\chi_2^0$) more important. Second, the Higgsino fraction decreases with increasing ϕ_1. This modifies the LSP couplings to gauge bosons and leads to a decrease in the dominant $\tilde\chi_1^0\tilde\chi_1^0 \to WW/ZZ$ cross-sections and thus a higher value for the relic density. This phase dependence in the couplings is predominantly what determines the shift in the value of the relic density, to wit the almost perfect match between contours of constant Ωh^2 and those of constant f_H in Fig. 1b. The deviation near $\phi_1 \sim 180°$ comes from chargino/neutralino coannihilations. To isolate the effect that comes solely from modifications in the couplings, we display in Fig. 2 the variation of Ωh^2 as function of ϕ_1 for constant LSP mass, as compared to the variation of Ωh^2 for constant M_1. The former is more pronounced reaching almost an order of magnitude. In this scenario with a mixed bino-Higgsino LSP, the dependence of masses and couplings
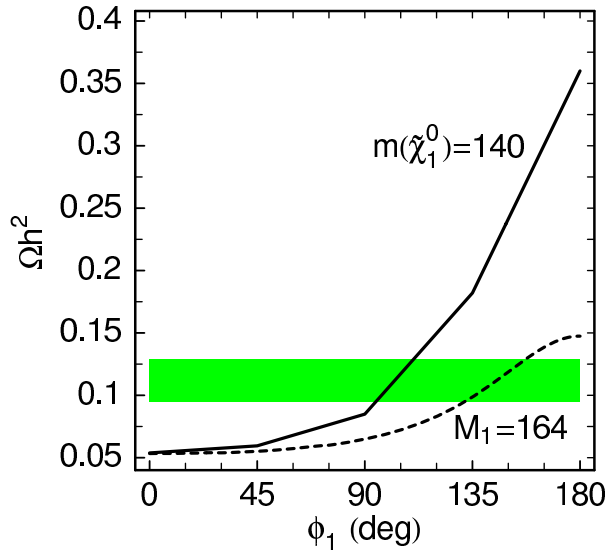
Figure 2: Ωh^2 as a function of ϕ_1 for the parameters of Fig. 1b; the dashed line is for fixed $M_1 = 164$ GeV, while for the full line M_1 is adjusted such that $m_{\tilde{\chi}_1^0} = 140$ GeV. The green (grey) band shows the 2σ WMAP-allowed range.

on ϕ_1 work against each other, so that taking out the kinematic effects actually enhances the variation of Ωh^2.

4.2   Annihilation through Higgs

In the Higgs sector, nonzero phases can induce scalar-pseudoscalar mixing as well as important changes in the masses. Moreover, the scalar and pseudoscalar couplings of the Higgses to the LSP depend on the phases in the neutralino sector. One can therefore expect large differences between the real and complex MSSM in the Higgs-funnel region. Since the relic density is very sensitive to the mass difference $\Delta m_{\tilde{\chi}_1^0 h_i} = m_{h_i} - 2m_{\tilde{\chi}_1^0}$ [1], it is important to disentangle the phase effects in kinematics and in couplings. For a case study of the Higgs funnel, we choose

$$M_1 = 150 \text{ GeV}, \ \tan\beta = 5, \ M_S = 500 \text{ GeV}, \ A_t = 1200 \text{ GeV}, \ \mu = 500 \text{ GeV}. \quad (3)$$

leading to a dominantly bino LSP and small mixing in the Higgs sector for $\phi_t \neq 0$.

In this scenario with relatively small μ, varying the phase of ϕ_t does not have a large impact since the scalar-pseudoscalar mixing never exceeds 8%. The main effect of ϕ_t on the value of the relic density can be explained by shifts in the physical masses and position of the resonance. On the other hand, the phase ϕ_1 changes the neutralino masses and mixing, and hence also the neutralino–Higgs couplings, Eq. (2). In Fig. 3a we show the WMAP-allowed regions in the $m_{H^+}-\phi_1$ plane for $\phi_t = 0$. For a given value of m_{H^+}, when increasing ϕ_1, the relic density drops. This is because the mass of the neutralino increases slowly, resulting in a smaller $\Delta m_{\tilde{\chi}_1^0 h_2}$. Adjusting $m_{\tilde{\chi}_1^0}$ or m_{h_2} such that the mass difference stays constant, we find rather that the relic density increases with ϕ_1, Fig. 3b. The maximum deviation which comes purely from modifications in the couplings can reach 70%. This can be readily understood from the phase dependence of the couplings of $h_{2,3}$ to the LSP. For $\phi_1 = 0$, the coupling of h_2 is predominantly pseudoscalar and thus
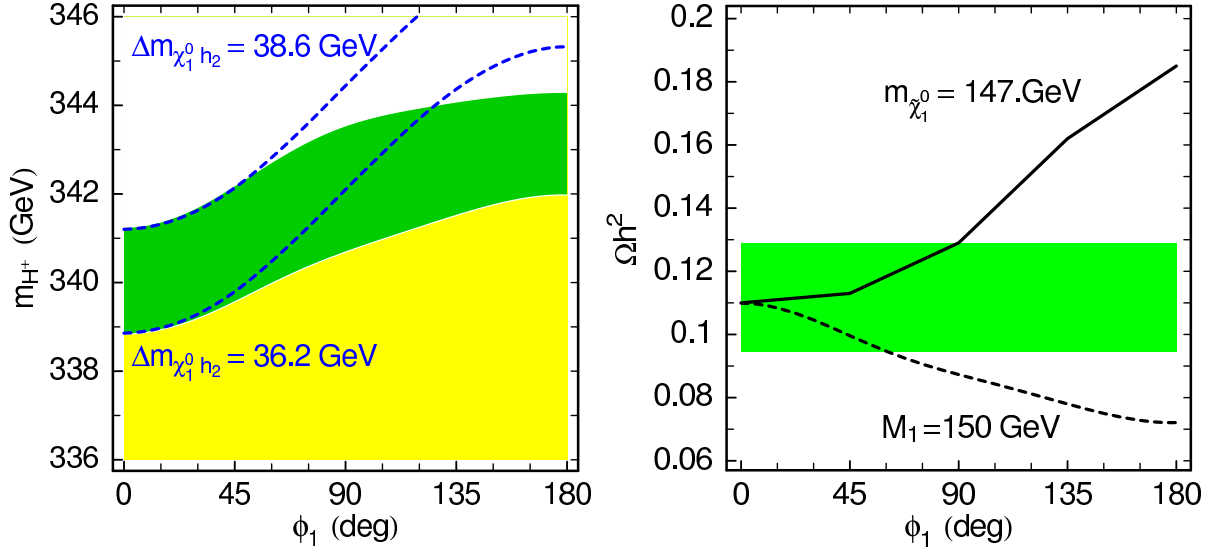
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Figure 3: a) The 2σ WMAP bands (green/dark grey) in the $m_{H^+}-\phi_1$ planes for the parameters of Eq. (3) and $\phi_t = 0$. Contours of constant mass differences $\Delta m_{\tilde{\chi}_1^0 h_2} = m_{h_2} - 2m_{\tilde{\chi}_1^0}$ are also displayed. In the yellow (light grey) region, Ωh^2 is below the WMAP range. b) Ωh^2 as a function of ϕ_1 for $m_{H^+} = 340$ GeV and the value of M_1 adjusted so that $\Delta m_{\tilde{\chi}_1^0 h_2}$ stays constant (full line). For comparison, the variation of Ωh^2 for fixed $M_1 = 150$ GeV is shown as a dashed line.

gives the dominant contribution to neutralino annihilation. For $\phi_1 = 90°$, the coupling of h_3 has a large pseudoscalar component and also contributes to the annihilation. When increasing ϕ_1 further (up to 180°), h_2 exchange again dominates, however with a coupling to neutralinos smaller than for $\phi_1 = 0$. Thus one needs a smaller mass splitting $\Delta m_{\tilde{\chi}_1^0 h_2}$ for Ωh^2 to fall within the WMAP range, see Fig. 3a.

When the mixing is large in the Higgs sector, say for $\mu = 1$TeV, the phase ϕ_t affects the scalar-pseudoscalar mixing as well as the mass splitting between h_2, h_3. The latter can reach several GeV's. This means that ϕ_t can induce very large shifts in the relic density which very sensitively depends on the mass difference between the LSP and the predominantly pseudoscalar Higgs. Isolating the phase dependence of Ωh^2 due to the scalar-pseudoscalar mixing by keeping $\Delta m_{\tilde{\chi}_1^0 h_3}$ constant, we found an increase in Ωh^2 relative to the $\phi_t = 0$ case by almost an order of magnitude [8]. This is however far less than the huge shifts of several orders of magnitude found in Ref. [9] for fixed values of m_{H^+} when a Higgs pole is passed.

5 Conclusions

We have shown here that in the CPV-MSSM, the relic density could be quite different as compared to that in the MSSM with vanishing phases, the variations in Ωh^2 often exceeding the $\sim 10\%$ range of the WMAP bound. In many cases, a large part of the phase dependence can be explained by changes in the masses of the involved particles. However, in some cases disentangling the kinematic effects also leads to an enhancement of the phase dependence. This happens, for instance, in the case of a mixed bino-Higgsino LSP, where we have found effects of almost an order of magnitude from modifications in

the couplings due to nonzero CP phases.

When aiming at a precise prediction of the neutralino relic density from collider measurements, it is clear that one does not only need precise sparticle spectroscopy — one also has to precisely measure the relevant couplings and this certainly includes the determination of possible CP phases. Whether parameters of the CPV-MSSM can be determined with sufficient precision at the LHC or at a future linear collider requires careful investigation.

Acknowledgements

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