HEAVY QUARKS FROM QCD SPECTRAL SUM RULES

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We summarize the recent developments on the extraction of the dynamical properties of the heavy quarks from QCD spectral sum rules.

1. Introduction

We have been living with QCD spectral sum rules (QSSR) (or QCD sum rules, or ITEP sum rules, or hadronic sum rules...) for 15 years, within the impressive ability of the method for describing the complex phenomena of hadronic physics with the few universal "fundamental" parameters of the QCD Lagrangian (QCD coupling α_s, quark masses and vacuum condensates built from the quarks and/or gluon fields, which parametrize the non-perturbative phenomena). The approach might be very close to the lattice calculations as it also uses the first principles of QCD, but unlike the case of the lattice, which is based on sophisticated numerical simulations, QSSR is quite simple as it is a semi-analytic approach based on a semiperturbative expansion and Feynman graph techniques implemented in an Operator Product Expansion (OPE), where the condensates contribute as higher-dimension operators. The QCD information is transmitted to the data via a dispersion relation obeyed by the hadronic correlators, in such a way that in this approach, one can *really control and in some sense localize* the origin of the numbers obtained from the analysis. With this simplicity, QSSR can describe in an elegant way the complexity of the hadron phenomena, without waiting for a complete understanding of the confinement problem.

One can fairly say that QCD spectral sum rules already started, before QCD, at the time of current algebra, in 1960, when different *ad hoc* super-convergence sum rules, especially the Weinberg and Das–Mathur–Okubo sum rules, were proposed but they came under control only with the advent of QCD [1]. However, the main flow comes from the classic paper of Shifman–Vainshtein–Zakharov [2] (hereafter referred to as SVZ), which goes beyond the *naïve* perturbation theory thanks to the inclusion of the vacuum condensate effects in the OPE (more details and more complete discussions of QSSR and its various applications to hadron physics can be found, for instance, in [3]).

In this talk, I shall present aspects of QSSR in the analysis of the properties of heavy flavours. As I am limited in space-time (an extended and updated version of this talk will be published elsewhere [4]), I cannot cover in detail here all QSSR applications to the heavy-quark physics. I will only focus on the following topics, which I think are important in the development of the understanding of the heavy-quark properties in connection with the progress done recently in the heavy quark effective theory (HQET) and in Lattice calculations:

– heavy-quark masses,

– pseudoscalar decay constants and the bag parameter B_B,

– heavy-to-light semileptonic and radiative decay form factors,

– $SU(3)$ breaking in $\bar{B}/D \to Kl\bar{\nu}$ and determination of V_{cd}/V_{cs},

– slope of the Isgur-Wise (IW) function and determination of V_{cb},

– properties of hybrids and B_c-like mesons.

2. QCD spectral sum rules

In order to illustrate the QSSR method in a pedagogical way, let us consider the two-point correlator:

$$
\begin{aligned}
\Pi_b^{\mu\nu} &\equiv i \int d^4x \; e^{iqx} \; \langle 0|\mathcal{T} J_b^\mu(x)\, (J_b^\nu(o))^\dagger\, |0\rangle \\
&= -\left(g^{\mu\nu}q^2 - q^\mu q^\nu\right) \Pi_b(q^2, M_b^2), \qquad (1)
\end{aligned}
$$

where $J_b^\mu(x) \equiv \bar{b}\gamma^\mu b(x)$ is the local vector current of the b-quark. The correlator obeys the well-known Källen–Lehmann dispersion relation:

$$
\Pi_b(q^2, M_b^2) = \int_{4M_b^2}^{\infty} \frac{dt}{t - q^2 - i\epsilon}\, \frac{1}{\pi}\, \mathrm{Im}\Pi_b(t) + ..., (2)
$$

which expresses in a clear way the *duality* between the spectral function Im $\Pi_b(t)$, which can be measured experimentally, as here it is related to the e^+e^- into Υ-like states total cross-section, while $\Pi_b(q^2, M_b^2)$ can be calculated directly in QCD, even at $q^2 = 0$, thanks to the fact that $M_b^2 - q^2 \gg \Lambda^2$. The QSSR is an improvement on the previous dispersion relation.

On the QCD side, such an improvement is achieved by adding to the usual perturbative expression of the correlator, the non-perturbative contributions as parametrized by the vacuum condensates of higher and higher dimensions in the OPE [2]:

$$
\Pi_b(q^2, M_b^2) \simeq \sum_{D=0,2,4,...} \frac{1}{\left(M_b^2 - q^2\right)^{D/2}}
$$

$$
\cdot \sum_{dimO=D} C^{(J)}(q^2, M_b^2, \mu)\langle O(\mu)\rangle, \tag{3}
$$

where μ is an arbitrary scale that separates the long- and short-distance dynamics; $C^{(J)}$ are the Wilson coefficients calculable in perturbative QCD by means of Feynman diagrams techniques: $D = 0$ corresponds to the case of the naïve perturbative contribution; $\langle O \rangle$ are the non-perturbative condensates built from the quarks or/and gluon fields. For $D = 4$, the condensates that can be formed are the quark $M_i\langle\bar{\psi}\psi\rangle$ and gluon $\langle\alpha_s G^2\rangle$ ones; for $D = 5$, one can have the mixed quark-gluon condensate $\langle\bar{\psi}\sigma_{\mu\nu}\lambda^a/2G_a^{\mu\nu}\psi\rangle$, while for $D = 6$ one has, for instance, the triple gluon $gf_{abc}\langle G^a G^b G^c\rangle$ and the four-quark $\alpha_s\langle\bar{\psi}\Gamma_1\psi\bar{\psi}\Gamma_2\psi\rangle$, where Γ_i are generic notations for any Dirac and colour matrices. The validity of this expansion has been understood formally, using renormalon techniques (IR renormalon ambiguity is absorbed into the definitions of the condensates) [5] and by building renormalization-invariant combinations of the condensates (Appendix of [6] and references therein). The SVZ expansion is phenomenologically confirmed from the unexpected accurate determination of the QCD coupling α_s from semi-inclusive tau decays [6,7]. In the present case of heavy-heavy correlators the OPE is much simpler, as one can show [8,9] that the heavy-quark condensate effects can be included into those of the gluon condensates, so that, up to $D \leq 6$, only the G^2 and G^3 condensates appear in the OPE. Indeed, SVZ have, originally, exploited this feature for their first estimate of the gluon condensate value.

For the phenomenological side, the improvement comes from the uses of either a finite number of derivatives and finite values of q^2 (moment sum rules):

$$
\begin{aligned}
\mathcal{M}^{(n)} &\equiv \frac{1}{n!} \frac{\partial^n \Pi_b(q^2)}{(\partial q^2)^n}\bigg|_{q^2=0} \\
&= \int_{4M_b^2}^{\infty} \frac{dt}{t^{n+1}}\, \frac{1}{\pi}\, \mathrm{Im}\Pi_b(t), \tag{4}
\end{aligned}
$$

or an infinite number of derivatives and infinite values of q^2, but keeping their ratio fixed as $\tau \equiv n/q^2$ (Laplace or Borel or exponential sum rules):

$$\mathcal{L}(\tau, M_b^2) = \int_{4M_b^2}^{\infty} dt \, \exp(-t\tau) \, \frac{1}{\pi} \, \mathrm{Im}\Pi_b(t). \qquad (5)$$

There also exist non-relativistic versions of these two sum rules, which are convenient quantities to work with in the large-quark-mass limit. In these cases, one introduces non-relativistic variables E and τ_N:

$$t \equiv (E + M_b)^2 \quad \text{and} \quad \tau_N = 4M_b\tau. \qquad (6)$$

In the previous sum rules, the gain comes from the weight factors, which enhance the contribution of the lowest ground-state meson to the spectral integral. Therefore, the simple duality ansatz parametrization:

"one resonance"$\delta(t - M_R^2)$ +
"QCD continuum"$\Theta(t - t_c)$, $\qquad (7)$

of the spectral function, gives a very good description of the spectral integral, where the resonance enters via its coupling to the quark current. In the case of the Υ, this coupling can be defined as:

$$\langle 0|\bar{b}\gamma^\mu b|\Upsilon\rangle = \sqrt{2}\frac{M_\Upsilon^2}{2\gamma_\Upsilon}. \qquad (8)$$

The previous feature has been tested in the light-quark channel from the $e^+e^- \to I = 1$ hadron data and in the heavy-quark ones from the $e^+e^- \to \Upsilon$ or ψ data, within a good accuracy. To the previous sum rules, one can also add the ratios:

$$\mathcal{R}^{(n)} \equiv \frac{\mathcal{M}^{(n)}}{\mathcal{M}^{(n+1)}} \quad \text{and} \quad \mathcal{R}_\tau \equiv -\frac{d}{d\tau} \log \mathcal{L}, \quad (9)$$

and their finite energy sum rule (FESR) variants, in order to fix the squared mass of the ground state. In principle, the pairs (n, t_c), (τ, t_c) are free external parameters in the analysis, so that the optimal result should be insensitive to their variations. Stability criteria, which are equivalent to the variational method, state that the best results should be obtained at the minimas or at the inflexion points in n or τ, while stability in t_c is useful to control the sensitivity of the result in the changes of t_c values. To these stability criteria are added constraints from local duality FESRs, which correlate the t_c value to those of the ground state mass and coupling [10]. Stability criteria have also been tested in models such as the harmonic oscillator, where the exact and approximate solutions are known [11]. The *most conservative optimization criteria*, which include various types of optimizations in the literature, are the following: the *optimal* result is obtained in the region, starting at the beginning of τ/n stability (this corresponds in most of the cases to the so-called plateau often discussed in the literature, but in my opinion, the interpretation of this nice plateau as a sign of a good continuum model is not sufficient, in the sense that the flatness of the curve extends in the uninteresting high-energy region where the properties of the ground state are lost), until the beginning of the t_c stability, where the value of t_c more or less corresponds to the one fixed by FESR duality constraints. The earlier *sum rule window* introduced by SVZ, stating that the optimal result should be in the region where both the non-perturbative and continuum contributions are *small*, is included in the previous region. Indeed, at the stability point, we have an equilibrium between the continuum and non-perturbative contributions, which are both small, while the OPE is still convergent at this point.

3. The heavy-quark-mass values

Here, we will summarize the recent results obtained in [12], where an improvement and an update of the existing results have been done, with the emphasis that the apparent discrepancy encountered in the literature is mainly due to the different values of α_s used by various authors. Using the *world average* value $\alpha_s(M_Z) = 0.118 \pm 0.006$ [13], the *first* determination of the

running mass to two loops, from the Ψ and Υ systems, is:

$$
\begin{aligned}
\overline{m}_c(M_c^{PT2}) &= (1.23^{+0.02}_{-0.04} \pm 0.03) \text{ GeV} \\
\overline{m}_b(M_b^{PT2}) &= (4.23^{+0.03}_{-0.04} \pm 0.02) \text{ GeV}, \quad (10)
\end{aligned}
$$

where the errors are respectively due to α_s and to the gluon condensate. One can transform this result into the *perturbative* pole mass and obtain, to two-loop accuracy:

$$
\begin{aligned}
M_c^{PT2} &= (1.42 \pm 0.03) \text{ GeV} \\
M_b^{PT2} &= (4.62 \pm 0.02) \text{ GeV}. \quad (11)
\end{aligned}
$$

It is informative to compare these values with the ones from the pole masses from non-relativistic sum rules to two loops:

$$
\begin{aligned}
M_c^{NR} &= (1.45^{+0.04}_{-0.03} \pm 0.03) \text{ GeV} \\
M_b^{NR} &= (4.69^{+0.02}_{-0.01} \pm 0.02) \text{ GeV}, \quad (12)
\end{aligned}
$$

where one may interpret the small mass difference, less than 70 MeV as the size of the renormalon effect into the pole mass. A similar comparison can be done at three-loop accuracy. One obtains:

$$
\begin{aligned}
M_c^{PT3} &= (1.62 \pm 0.07 \pm 0.03) \text{ GeV} \\
M_b^{PT3} &= (4.87 \pm 0.05 \pm 0.02) \text{ GeV}, \quad (13)
\end{aligned}
$$

to be compared with the *dressed mass*:

$$
M_b^{nr} = (4.94 \pm 0.10 \pm 0.03) \text{ GeV}, \quad (14)
$$

obtained from a non-relativistic Balmer formula based on a $\bar{b}b$ Coulomb potential and including higher order α_s-corrections [14]. Here, one still has the 70 MeV mass difference, which reinforces our interpretation that it is due to the renormalon effect (for an exposition of this effect, see e.g. [16]). One can also use the previous results, in order to deduce the mass-difference between the b and c (non)-relativistic pole masses:

$$
M_b - M_c = (3.22 \pm 0.03) \text{ GeV}, \quad (15)
$$

in good agreement with potential model expectations [15].

An extension of the previous analysis to the case of the B and B^* mesons leads to the value $M_b^{PT2} = (4.63 \pm 0.08)$ GeV, in good agreement with the previous results. The meson-quark mass difference has been also directly estimated in the large mass limit. By combining the result from HQET [17] with the one from the full QCD spectral sum rules [18,19], one can deduce the *weighted average*:

$$
\begin{aligned}
\bar{\Lambda} \equiv \delta M_b^\infty &\equiv (M_B - M_b^{NR})_\infty \\
&= (0.58 \pm 0.05) \text{ GeV}, \quad (16)
\end{aligned}
$$

of the quark-meson mass difference, which is in agreement with the previous findings, but less accurate.

Using the previous result in (10) and the expression of the running mass to two-loops, one also obtains at 1 GeV:

$$
\begin{aligned}
\overline{m}_c(1 \text{ GeV}) &= (1.46^{+0.09}_{-0.05} \pm 0.03) \text{ GeV} \\
\overline{m}_b(1 \text{ GeV}) &= (6.37^{+0.64}_{-0.39} \pm 0.07) \text{ GeV}, \quad (17)
\end{aligned}
$$

By combining the previous value of the running b-quark mass with the s-quark one evaluated at 1 GeV, which we take from $\overline{m}_s(1 \text{ GeV}) = 150$ MeV [20] until 230 MeV [21], one obtains the scale-independent ratio:

$$
\overline{m}_b/\overline{m}_s \simeq 33.5 \pm 7.6, \quad (18)
$$

a result of great interest for model-building and GUT-phenomenology.

4. The pseudoscalar decay constants and the bag parameter B_B

The decay constants f_P of a pseudoscalar meson P are defined as:

$$
(m_q + M_Q)\langle 0|\bar{q}(i\gamma_5)Q|P\rangle \equiv \sqrt{2}M_P^2 f_P, \quad (19)
$$

where in this normalization $f_\pi = 93.3$ MeV. A *rigorous* upper bound on these couplings can be derived from the second-lowest superconvergent moment:

$$\mathcal{M}^{(2)} \equiv \frac{1}{2!} \frac{\partial^2 \Psi_5(q^2)}{(\partial q^2)^2}\bigg|_{q^2=0}, \qquad (20)$$

where Ψ_5 is the two-point correlator associated to the pseudoscalar current. Using the positivity of the higher-state contributions to the spectral function, one can deduce [22]:

$$f_P \leq \frac{M_P}{4\pi} \left\{ 1 + 3\frac{m_q}{M_Q} + 0.751\bar{\alpha}_s + ... \right\}, \qquad (21)$$

where one should not misinterpret the mass-dependence in this expression compared to the one expected from heavy-quark symmetry. Applying this result to the D meson, one obtains:

$$f_D \leq 2.14 f_\pi. \qquad (22)$$

Although presumably quite weak, this bound, when combined with the recent determination to two loops [23]:

$$\frac{f_{D_s}}{f_D} \simeq (1.15 \pm 0.04) f_\pi, \qquad (23)$$

implies

$$f_{D_s} \leq (2.46 \pm 0.09) f_\pi, \qquad (24)$$

which is useful for a comparison with the recent measurement of f_{D_s} from WA75: $f_{D_s} \simeq (1.76 \pm 0.52) f_\pi$ and from CLEO: $f_{D_s} \simeq (2.61 \pm 0.49) f_\pi$. One cannot push, however, the uses of the moments to higher n values in this D channel, in order to minimize the continuum contribution to the sum rule with the aim to derive an estimate of the decay constant because the QCD series will not converge at higher n values. In the D channel, the most appropriate sum rule is the Laplace

sum rule. The results from different groups are consistent for a given value of the c-quark mass. Using the table in [23] and the value of the perturbative pole mass obtained previously, one obtains to two loops:

$$\begin{aligned} f_D &\simeq (1.35 \pm 0.04 \pm 0.06) f_\pi &\Rightarrow \\ f_{D_s} &\simeq (1.55 \pm 0.10) f_\pi. \end{aligned} \qquad (25)$$

For the B meson, one can either work with the Laplace, the moments or their non-relativistic variants. Given the previous value of M_b, these different methods give consistent values of f_B, though the one from the non-relativistic sum rule is very inaccurate due to the huge effect of the radiative corrections in this method. The best value comes from the Laplace sum rule; from the table in [23], one obtains:

$$f_B \simeq (1.49 \pm 0.06 \pm 0.05) f_\pi, \qquad (26)$$

while [23]:

$$\frac{f_{B_s}}{f_B} \simeq 1.16 \pm 0.04, \qquad (27)$$

where the most accurate estimate comes from the "relativistic" Laplace sum rule. One could notice, since the *first* result $f_B \simeq f_D$ of [24], a large violation of the scaling law expected from heavy-quark symmetry. Indeed, this is due to the large $1/M_b$-correction found from the HQET sum rule [17] and from the one in full QCD [19,18]:

$$\begin{aligned} f_B \sqrt{M_b} &\simeq (0.42 \pm 0.07) \text{ GeV}^{3/2} \\ &\cdot \left\{ 1 - \frac{(0.88 \pm 0.18) \text{ GeV}}{M_b} \right\}, \end{aligned} \qquad (28)$$

which is due to the meson-mass gap $\delta M \equiv M_B - M_b$ [17] and to the continuum energy E_c [19,25] ($E_c \simeq \frac{3}{4}\delta M$ [18]):

$$f_B \sqrt{M_b} \simeq \frac{1}{\pi} E_c^{3/2} \left\{ 1 - \frac{\delta M}{M_b} - \frac{3}{2}\frac{E_c}{M_b} + ... \right\}. \qquad (29)$$

The apparent disagreement among different existing QSSR numerical results in the literature is due mainly to the different values of the quark masses used because the decay constants are very sensitive to that quantity as shown explicitly in [23].

Finally, let me mention that we have also tested the validity of the vacuum saturation $B_B = 1$ of the bag parameter, using a sum rule analysis of the four-quark two-point correlator to two loops [26]. We found that the radiative corrections are quite small. Under some physically reasonable assumptions for the spectral function, we found that the vacuum saturation estimate is only violated by about 15%, giving:

$$B_B \simeq 1 \pm 0.15. \tag{30}$$

By combining this result with the one for f_B, we deduce:

$$f_B \sqrt{B_B} \simeq (197 \pm 18) \text{ MeV}, \tag{31}$$

if we use the normalization $f_\pi = 132$ MeV, which is $\sqrt{2}$ times the one defined in (18), in excellent agreement with the present lattice calculations [27].

5. Heavy-to-light semileptonic and radiative decay form factors

One can extend the analysis done for the two-point correlator to the more complicated case of three-point function, in order to study the form factors related to the $B \to K^* \gamma$ and $B \to \rho/\pi$ semileptonic decays. In so doing, one can consider the generic three-point function:

$$V(p, p', q^2) \equiv -\int d^4x \, d^4y \, e^{i(p'x - py)}$$
$$\langle 0 | \mathcal{T} J_L(x) O(0) J_B^\dagger(y) | 0 \rangle, \quad (32)$$

where J_L, J_B are the currents of the light and B mesons; O is the weak operator specific for each

process (penguin for the $K^*\gamma$, weak current for the semileptonic); $q \equiv p - p'$. The vertex obeys the double dispersion relation :

$$\begin{aligned} V(p^2, p'^2) &\simeq \int_{M_b^2}^\infty \frac{ds}{s - p^2 - i\epsilon} \\ &\quad \int_{m_L^2}^\infty \frac{ds'}{s' - p'^2 - i\epsilon} \frac{1}{\pi^2} \, \mathrm{Im} V(s, s') \end{aligned} \tag{33}$$

As usual, the QCD part enters in the LHS of the sum rule, while the experimental observables can be introduced through the spectral function after the introduction of the intermediate states. The improvement of the dispersion relation can be done in the way discussed previously for the two-point function. In the case of the heavy-to-light transition, the only possible improvement with a good M_b behaviour at large M_b (convergence of the QCD series) is the so-called hybrid sum rule (HSR) corresponding to the uses of the moments for the heavy-quark channel and to the Laplace for the light one [18,28]:

$$\begin{aligned} \mathcal{H}(n, \tau') &= \frac{1}{\pi^2} \int_{M_b^2}^\infty \frac{ds}{s^{n+1}} \\ &\quad \int_0^\infty ds' \, e^{-\tau's'} \, \mathrm{Im} V(s, s'). \end{aligned} \tag{34}$$

We have studied analytically the different form factors entering the previous processes [29]. They are defined as:

$$\langle \rho(p') | \bar{u} \gamma_\mu (1 - \gamma_5) b | B(p) \rangle = (M_B + M_\rho) A_1 \epsilon^*_\mu -$$
$$\frac{A_2}{M_B + M_\rho} \epsilon^* p'(p + p')_\mu + \frac{2V}{M_B + M_\rho} \epsilon_{\mu\nu\rho\sigma} p^\rho p'^\sigma,$$
$$\langle \pi(p') | \bar{u} \gamma_\mu b | B(p) \rangle = f_+(p + p')_\mu + f_-(p - p')_\mu, \tag{35}$$

and:

$$\langle \rho(p') | \bar{s} \sigma_{\mu\nu} \left(\frac{1 + \gamma_5}{2} \right) q^\nu b | B(p) \rangle$$
$$= i \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p^\rho p'^\sigma F_1^{B \to \rho} +$$

6

$$\left\{\epsilon^*_\mu(M_B^2 - M_\rho^2) - \epsilon^* q(p+p')_\mu\right\}\frac{F_1^{B\to\rho}}{2}. \tag{36}$$

We found that they are dominated, for $M_b \to \infty$, by the effect of the light-quark condensate, which dictates the M_b behaviour of the form factors to be typically of the form:

$$F(0) \sim \frac{\langle\bar{d}d\rangle}{f_B}\left\{1 + \frac{\mathcal{I}_F}{M_b^2}\right\}, \tag{37}$$

where $\mathcal{I}_F$ is the integral from the perturbative triangle graph, which is constant as $t'^2_c E_c / \langle\bar{d}d\rangle$ (t'_c and E_c are the continuum thresholds of the light and b quarks) for large values of M_b. It indicates that at $q^2 = 0$ and to leading order in $1/M_b$, all form factors behave like $\sqrt{M_b}$, although, in most cases, the coefficient of the $1/M_b^2$ term is large. The study of the q^2 behaviours of the form factors shows that, with the exception of the A_1 form factor, their q^2 dependence is only due to the non-leading (in $1/M_b$) perturbative graph, so that for $M_b \to \infty$, these form factors remain constant from $q^2 = 0$ to q^2_{max}. The resulting M_b behaviour at q^2_{max} is the one expected from the heavy quark symmetry. The numerical effect of this q^2-dependence at finite values of M_b is a polynomial in q^2 (which can be resummed), which mimics quite well the usual pole parametrization for a pole mass of about 6–7 GeV. The situation for the A_1 is drastically different from the other ones, as here the Wilson coefficient of the $\langle\bar{d}d\rangle$ condensate contains a q^2 dependence with a *wrong* sign and reads:

$$A_1(q^2) \sim \frac{\langle\bar{d}d\rangle}{f_B}\left\{1 - \frac{q^2}{M_b^2}\right\}, \tag{38}$$

which, for $q^2_{max} \equiv (M_B - M_\rho)^2$, gives the expected behaviour:

$$A_1(q^2_{max}) \sim \frac{1}{\sqrt{M_b}}. \tag{39}$$

It can be noticed that the q^2 dependence of A_1 is in complete contradiction with the pole behaviour

due to its wrong sign. This result explains the numerical analysis of [30]. It is urgent and important to test this feature experimentally. It should be finally noticed that owing to the overall $1/f_B$ factor, all form factors have a large $1/M_b$ correction.

In the numerical analysis, we obtain at $q^2 = 0$, the value of the $B \to K^*\gamma$ form factor:

$$\begin{aligned} F_1^{B\to\rho} &\simeq 0.27 \pm 0.03, \\ \frac{F_1^{B\to K^*}}{F_1^{B\to\rho}} &\simeq 1.14 \pm 0.02, \end{aligned} \tag{40}$$

which leads to the branching ratio $(4.5 \pm 1.1) \times 10^{-5}$, in perfect agreement with the CLEO data and with the estimate in [31]. One should also notice that, in this case, the coefficient of the $1/M_b^2$ correction is very large, which makes dangerous the extrapolation of the c-quark results to higher values of the quark mass. This extrapolation is often done in some lattice calculations.

For the semileptonic decays, QSSR give a good determination of the ratios of the form factors with the values [28]:

$$\begin{aligned} \frac{A_2(0)}{A_1(0)} &\simeq \frac{V(0)}{A_1(0)} \simeq 1.11 \pm 0.01 \\ \frac{A_1(0)}{F_1^{B\to\rho}(0)} &\simeq 1.18 \pm 0.06 \\ \frac{A_1(0)}{f_+(0)} &\simeq 1.40 \pm 0.06. \end{aligned} \tag{41}$$

Combining these results with the "world average" value of $f_+(0) = 0.25 \pm 0.02$ and the one of $F_1^{B\to\rho}(0)$, one can deduce the rate and polarization:

$$\begin{aligned} \Gamma_\pi &\simeq (4.3 \pm 0.7)|V_{ub}|^2 \times 10^{12} \text{ s}^{-1} \\ \Gamma_\rho/\Gamma_\pi &\simeq 0.9 \pm 0.2 \\ \Gamma_+/\Gamma_- &\simeq 0.20 \pm 0.01 \\ \alpha &\equiv 2\frac{\Gamma_L}{\Gamma_T} - 1 \simeq -(0.60 \pm 0.01). \end{aligned} \tag{42}$$

These results are much more precise than the ones from a direct estimate of the absolute values of the form factors due to the cancellation of systematic errors in the ratios. They indicate that, we are on the way to reach V_{ub} with a good accuracy from the exclusive modes. Also here, mainly because of the non-pole behaviour of A_1^B, the ratio between the widths into ρ and into π is about 1, while in different pole models, it ranges from 3 to 10. For the asymmetry, one obtains a large negative value of α, contrary to the case of the pole models.

6. $SU(3)$ breaking in $\bar{B}/D \to Kl\bar{\nu}$ and determination of V_{cd}/V_{cs}

We extend the previous analysis for the estimate of the $SU(3)$ breaking in the ratio of the form factors:

$$R_P \equiv f_+^{P \to K}(0)/f_+^{P \to \pi}(0), \qquad (43)$$

where $P \equiv \bar{B}, \pi$. Its analytic expression is given in [32], which leads to the numerical result:

$$R_B = 1.007 \pm 0.020 \qquad R_D = 1.102 \pm 0.007, (44)$$

where one should notice that for $M_b \to \infty$, the $SU(3)$ breaking vanishes, while its size at finite mass is typically of the same order as the one of f_{D_s} or of the $B \to K^* \gamma$ discussed before. What is more surprising is the fact that using the previous value of R_D with the present value of CLEO data [33]:

$$\frac{Br(D^+ \to \pi^0 l\nu)}{Br(D^+ \to \bar{K}^0 l\nu)} = (8.5 \pm 2.7 \pm 1.4)\%, \qquad (45)$$

one deduces:

$$V_{cd}/V_{cs} = 0.322 \pm 0.056, \qquad (46)$$

which is much larger than the value 0.226 ± 0.005 derived from the unitarity of the CKM matrix.

This can mean either that the CLEO data are wrong (recall that MARKIII data [34] would imply a value 0.25 ± 0.15, in agreement with unitarity, but less accurate), or that the unitarity constraint is in trouble. It is difficult to see how the QSSR result is wrong as other predictions derived in the same way (see e.g f_{B_s} and $F_1^{B \to K^*}$) agree successfully with results from alternative approaches.

7. Slope of the Isgur–Wise function and determination of V_{cb}

Let me now discuss the slope of the Isgur–Wise function. Taron–de Rafael [35] have exploited the analyticity of the elastic b-number form factor F defined as:

$$\langle B(p')|\bar{b}\gamma^\mu b|B(b)\rangle = (p + p')^\mu F(q^2), \qquad (47)$$

which is normalized as $F(0) = 1$ in the large mass limit $M_B \simeq M_D$. Using the positivity of the vector spectral function and a mapping in order to get a bound on the slope of F outside the physical cut, they obtained a rigorous but weak bound:

$$F'(vv' = 1) \geq -6. \qquad (48)$$

Including the effects of the Υ states below $\bar{B}B$ thresholds by assuming that the $\Upsilon\bar{B}B$ couplings are of the order of 1, the bound becomes stronger:

$$F'(vv' = 1) \geq -1.5. \qquad (49)$$

Using QSSR, we can estimate the part of these couplings entering in the elastic form factor. We obtain the value of their sum [36]:

$$\sum g_{\Upsilon\bar{B}B} \simeq 0.34 \pm 0.02. \qquad (50)$$

In order to be conservative, we have multiplied the previous estimate by a factor 3 larger. We thus obtained the improved bound

$$F'(vv' = 1) \geq -1.34, \qquad (51)$$

but the gain over the previous one is not much. Using the relation of the form factor with the slope of the Isgur–Wise function, which differs by $-16/75 \log \alpha_s(M_b)$ [37], one can deduce the final bound:

$$\zeta'(1) \geq -1.04. \tag{52}$$

However, one can also use the QSSR expression of the Isgur–Wise function from vertex sum rules [17] in order to extract the slope *analytically*. To leading order in 1/M, the *physical* IW function reads:

$$
\begin{aligned}
\zeta_{phys}(y \equiv vv') &= \left(\frac{2}{1+y}\right)^2 \Bigg\{ 1 + \frac{\alpha_s}{\pi} f(y) \\
&\quad - \langle \bar{d}d \rangle \tau^3 g(y) + \langle \alpha_s G^2 \rangle \tau^4 h(y) \\
&\quad + g \langle \bar{d}Gd \rangle \tau^5 k(y) \Bigg\},
\end{aligned}
\tag{53}
$$

where τ is the Laplace sum rule variable and f, h and k are analytic functions of y. From this expression, one can derive the analytic form of the slope [36]:

$$\zeta'_{phys}(y=1) \simeq -1 + \delta_{pert} + \delta_{NP}, \tag{54}$$

where at the τ-stability region: $\delta_{pert} \simeq -\delta_{NP} \simeq -0.04$, which shows the near-cancellation of the non-leading corrections. Adding a generous 50% error of 0.02 for the correction terms, we finally deduce to leading order result in 1/M:

$$\zeta'_{phys}(y=1) \simeq -1 \pm 0.02, \tag{55}$$

Using this result in different existing model parametrizations, we deduce the value of the mixing angle:

$$
\begin{aligned}
V_{cb} &\simeq \left(\frac{1.48 \text{ ps}}{\tau_b}\right)^{1/2} \times \\
&\quad . \quad (37.3 \pm 1.2 \pm 1.4) \times 10^{-3},
\end{aligned}
\tag{56}
$$

where the first error comes from the data and the second one from the model dependence.

Let us now discuss the effects due to the 1/M corrections. It has been argued recently (but the situation is still controversial [38]) that the $1/M^2$ effect can lower the Isgur–Wise function to 0.91 ± 0.03 at $y = 1$, which is a compromise value between the ones in [38], such that the extracted value of V_{cb} using an extrapolation until this particular point will also increase by 11%. However, the data from different groups near this point are very inaccurate and lead to an inaccurate, though a model-independent result. Moreover, in order to see the effect of the 1/M correction, one can combine this previous result at $y = 1$ with the sum rule estimate of the relevant form factor at $q^2 = 0$, which is about 0.53 ± 0.09 [28], just on top of the CLEO data [39]. Notice that this result has been obtained without doing a 1/M expansion. With these two extremal boundary conditions and using the linear parametrization, which also agrees with the data [39]:

$$\zeta = \zeta_0 + \zeta'(y - 1), \tag{57}$$

we can deduce the slope:

$$\zeta' \simeq -(0.76 \pm 0.2). \tag{58}$$

It indicates that the 1/M correction tends also to decrease ζ', which implies that, for larger values of y where the data are more accurate, the increase of V_{cb} is weaker (+ 3.7%) than the one at $y = 1$. This leads to the *final* estimate:

$$
\begin{aligned}
V_{cb} &\simeq \left(\frac{1.48 \text{ ps}}{\tau_b}\right)^{1/2} \times \\
&\quad . \quad (38.8 \pm 1.2 \pm 1.5 \pm 1.5) \times 10^{-3},
\end{aligned}
\tag{59}
$$

where the new last error is induced by the error from the slope. This result is more precise than the one obtained at $y = 1$, while the model-dependence only brings a relatively small error.

It also shows that the value from the exclusive channels is lower than that from the inclusive one, which is largely affected by the large uncertainty in the mass definition which enters in its fifth power. Previous results for the slope and for V_{cb} are in good agreement with the new CLEO data presented at this meeting [39].

8. Properties of hybrid and B_c-like mesons

Let me conclude this talk by shortly discussing the masses of the hybrid $\bar{Q}GQ$ and the mass and decays of the $B_c$like-mesons. Hybrid mesons are interesting because of their exotic quantum numbers. Moreover, it is not clear if these states are true resonances or if they only manifest themselves as a wide continuum instead. The lowest $\bar{c}Gc$ states appear to be a 1^{+-} of mass around 4.1 GeV [3]. The available sum-rule analysis of the 1^{-+} state is not very conclusive due to the absence of stability for this channel. However, the analysis indicates that the spin-one states are in the range 4.1–4.7 GeV. Their characteristic decays should occur via the η' $U(1)$-like particle produced together with a ψ or an η_c. However, the phase-space suppression can be quite important for these reactions. The sum rule predicts that the 0^{--}, 0^{++} $\bar{c}Gc$ states are in the range 5–5.7 GeV, i.e. about 1 GeV above the spin one.

We have estimated the B_c-meson mass and coupling by combining the results from potential models and QSSR [9]. We predict from potential models:

$$
\begin{aligned}
M_{B_c} &= (6255 \pm 20) \text{ MeV}, \\
M_{B_c^*} &= (6330 \pm 20) \text{ MeV}, \\
M_{\Lambda(bcu)} &= (6.93 \pm 0.05) \text{ GeV}, \\
M_{\Omega(bcs)} &= (7.00 \pm 0.05) \text{ GeV}, \\
M_{\Xi^*(ccu)} &= (3.63 \pm 0.05) \text{ GeV}, \\
M_{\Xi^*(bbu)} &= (10.21 \pm 0.05) \text{ GeV}, \quad (60)
\end{aligned}
$$

which are consistent with, but more precise than, the sum-rule results. The decay constant of the B_c meson is better determined from QSSR. The average of the sum rules with the potential model results reads:

$$
f_{B_c} \simeq (2.94 \pm 0.12) f_\pi, \quad (61)
$$

which leads to the leptonic decay rate into $\tau\nu_\tau$ of about $(3.0 \pm 0.4) \times (V_{cb}/0.037)^2 \times 10^{10}$ s^{-1}. We have also studied the semileptonic decay of the B_c mesons and the q^2-dependence of the form factors. We found that, in all cases, the QCD predictions increase faster than the usual pole dominance ones. The behaviour can be fitted with an effective pole mass of about 4.1–4.6 GeV instead of the 6.3 GeV expected from a pole model. Basically, we also found that each exclusive channel has almost the same rate which is about 1/3 of the leptonic one. Detection of these particles in the next B-factory machine will serve as a stringent test of the results from the potential models and QSSR analysis.

9. Conclusion

We have shortly presented different results from QCD spectral sum rules in the heavy-quark sector, which are useful for further theoretical studies and complement the results from lattice calculations or/and heavy-quark symmetry. From the experimental point of view, QSSR predictions agree with available data, but they also lead to some new features, which need to be tested in forthcoming experiments.

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