13 HIGGS TRIPLETS

13.1 Introduction

John F. Gunion and Chris Hays

Even though the Standard Model (SM) of the strong and electroweak interactions has proven enormously successful, it need not be the case that the Higgs sector consists solely of a single $SU(2)_L$ Higgs doublet field, that is a field with total weak isospin $T = \frac{1}{2}$ having two members with $T_3 = +\frac{1}{2}$ and $T_3 = -\frac{1}{2}$ and $U(1)$ hypercharge $Y = 1$. The inclusion of additional doublets as well as singlets (i.e., fields with $T = Y = 0$) is a frequently considered possibility (and at least two doublets are required in the supersymmetric context). The next logical step is to consider the inclusion of one or more triplet $SU(2)_L$ representations (i.e. a $T = 1$, three-component field with $T_3 = +1, 0, -1$ members). The purpose of this section is to review the phenomenology of a Higgs sector which contains both doublet and triplet fields. Surprisingly attractive models, fully consistent with all existing experimental constraints, can be constructed. These yield many exotic features and unusual experimental signatures. In exploring the physics of electroweak symmetry breaking at future colliders, it will be important to consider the alternative possibilities characteristic of this and other non-minimal Higgs sectors. Our discussion here will focus primarily on models in which only the Higgs sector of the SM is extended via the addition of triplet representation(s) with hypercharge $Y = 0$ and/or $Y = \pm 2$ (which are real and complex, respectively).

There are many models in which both the Higgs sector and the gauge sector are expanded that provide a natural setting for Higgs triplet fields, as for example the left-right (LR) symmetric models with extended gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [1–6] (see also Section 6.1). Supersymmetric left-right (SUSYLR) symmetric models with triplets can also be constructed and have many attractive features [7–14]. One of the primary motivations for left-right symmetric models with triplets in the Higgs sector is that they provide a natural setting for the see-saw mechanism of neutrino mass generation. The minimal Higgs sector contains a bi-doublet Higgs field, an $SU(2)_R$ triplet (employed for the see-saw) and its LR partner $SU(2)_L$ triplet. For the phenomenology of such models in the absence of supersymmetry, useful starting references are [15–17]. A brief outline of the SUSYLR Higgs scenarios is given at the end of this review.

We will focus on the $Y = 0$ and $Y = \pm 2$ triplet models. Only the $Y = 0$ and $Y = \pm 2$ triplet representations have a neutral member which, if it acquires a non-zero vacuum expectation value (vev), can influence electroweak symmetry breaking and give rise to non-zero VV'-Higgs vertices (where $V, V' = W, Z, \gamma$). Triplet models with still larger even-integer values of Y do not have a neutral member. We also do not consider triplet models with an odd-integer value of Y. The Higgs fields of such a model would have fractional charge. Table 13.1 lists the triplet models we consider and establishes some notation for the Higgs bosons appearing in the various models, including those with custodial or left-right symmetry.

Before zeroing in on triplet models, we make a few more general remarks regarding Higgs representations with weak isospin $T \geq 1$, generically denoting the Higgs fields and bosons by δ. Let us focus on two particular hallmark signatures, both of which require the presence of a Higgs representation with $T \geq 1$ that contains a neutral field member *with non-zero vacuum expectation value (vev)*.

(1) Verification of the presence of a non-zero $\delta^{++}W^-W^-$ tree-level strength interaction. In the $T = 1$ context, this would require the $Y = \pm 2$ representation.
This vertex would be detected by observation of the single-δ^{++} fusion production process $W^+W^+ \delta^{++}$ and/or via the presence of $\delta^{++} \to W^+W^+$ decays.

(2) Verification of the existence of a tree-level $\delta^- W^+ Z$ vertex. For $T = 1$, this can occur for either a $Y = 0$ or $Y = \pm 2$ representation (or if both are present).
Detection of this vertex would be via single δ^+ production, e.g. $Z^* \to \delta^+ W^-$, or decays such as

Table 13.1: Notation for different Higgs representations and models. Without superscripts, T and T_3 refer to the SM $SU(2)_L$ group. $SU(2)_C$ refers, when relevant, to the custodial symmetry group of the model. For left-right symmetric models, we distinguish T^L and T_3^L of $SU(2)_L$ from T^R and T_3^R of $SU(2)_R$. In the absence of the $SU(2)_R$ group the charge formula is $Q = T_3 + \frac{1}{2}$. In left-right symmetric models $Q = T_3^L + T_3^R + \frac{1}{2}(B-L)$. The listed couplings are those present at tree-level. Without subscripts V and V' refer to the usual $W^\pm, Z$. For left-right symmetric models, the $V_L = W_L, Z_L$ of $SU(2)_L$ and the $V_R = W_R, Z_R$ of $SU(2)_R$ are distinguished. In the limit where no triplet neutral field has a vev, the doublet and triplet fields separate and the couplings of the triplets to VV' are absent.

Generic Higgs field

General (T,Y)	$\phi_{T,Y}$	Couplings

Complex doublet Higgs field with neutral member

$(T,Y) = (1/2, \pm 1)$	$\phi^\pm, \phi^0$	VV' and $f\overline{f}$

Generic Triplet Higgs fields

$T = 1$, Y arbitrary, δ fields	$\delta^{\pm\pm}, \delta^\pm, \delta^0, \ldots$	

Complex triplet Higgs representation with neutral member

$(T,Y) = (1, \pm 2)$, χ fields	$\chi^{\pm\pm}, \chi^\pm, \chi^0$	VV' and $\ell\ell$

Real triplet Higgs representation with neutral member

$(T,Y) = (1, 0)$, ξ fields	$\xi^\pm, \xi^0$	VV' only

Model with one doublet and one real triplet

mix of $\phi = (1/2, \pm 1)$ and $\xi = (1, 0)$	$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix}$	
Mass eigenstates for $\langle \xi^0 \rangle \neq 0$	$h^\pm, h^0, k^0$	VV' and $f\overline{f}$

Triplet model with tree-level custodial $SU(2)_C$ symmetry

mix of $\phi = (1/2, \pm 1)$, $\xi = (1, 0)$ and $\chi = (1, \pm 2)$	$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ \chi^- & \xi^0 & \chi^+ \\ \chi^{--} & \xi^- & \chi^0 \end{pmatrix}$	
$SU(2)_C$ decomposition for $\langle \xi^0 \rangle = \langle \chi^0 \rangle \neq 0$	Mass eigenstates	Couplings
$SU(2)_C$ 5-plet: $(1, \pm 2)$ and $(1, 0)$ mix	$H_5^{\pm\pm}, H_5^\pm, H_5^0$	VV' and $\ell\ell$
$SU(2)_C$ 3-plet: $(1/2, \pm 1)$, $(1, \pm 2)$ and $(1, 0)$ mix	$H_3^\pm, H_3^0$	$f\overline{f}$ only
$SU(2)_C$ singlet #1: pure $(1/2, 0)$	H_1^0	VV' and $f\overline{f}$
$SU(2)_C$ singlet #2: $(1, \pm 2), (1, 0)$ mixture	$H_1^{0\prime}$	VV' and $\ell\ell$

Left-right symmetric models

Bi-doublet, ϕ, $(T^L, T^R, B-L) = (\frac{1}{2}, \frac{1}{2}, 0)$	$\begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}$	$V_{L,R} V'_{R,L}$ and $f\overline{f}$
Left triplet, Δ_L, $(T^L, T^R, B-L) = (1, 0, 2)$	$\begin{pmatrix} \frac{\delta_L^+}{\sqrt{2}} & \delta_L^{++} \\ \delta_L^0 & -\frac{\delta_L^+}{\sqrt{2}} \end{pmatrix}$	$V_L V'_L$ and $\ell\ell$
Right triplet, Δ_R, $(T^L, T^R, B-L) = (0, 1, 2)$	$\begin{pmatrix} \frac{\delta_R^+}{\sqrt{2}} & \delta_R^{++} \\ \delta_R^0 & -\frac{\delta_R^+}{\sqrt{2}} \end{pmatrix}$	$V_R V'_R$ and $\ell\ell$

$\delta^+ \to W^+ Z$.

Neither signature would be observed for any doublet model. For example, although a doublet representation with $Y = 3$ has a doubly-charged Higgs boson, it does not contain a neutral member and the doubly-charged Higgs could only be pair produced at a hadron or e^+e^- collider. (Note that to preserve $U(1)_{EM}$, any Higgs potential involving such exotic representations must be constructed so that vevs do not develop for the charged fields.) Nonetheless, it should also be kept in mind that such exotic doublet representations *can* influence gauge coupling running and, therefore, coupling unification.

An example of a Higgs sector with $T > 1$ that can give rise to the two hallmark signatures is the $T = 2$, $Y = 0$ Higgs representation, which contains $\delta^{\pm\pm}$, $\delta^\pm$ and δ^0 fields. If the neutral member has non-zero vev then non-zero $\delta^{\pm\pm}W^\mp W^\mp$ and $\delta^\pm W^\mp Z$ vertices will both be generated at tree-level. Both signatures also arise for non-zero neutral field vev in the case of the $T = 3, Y = 4$ representation that is the next simplest beyond the doublet representation to have a built-in custodial symmetry that guarantees a tree-level value of $\rho = \frac{m_W{}^2}{m_Z{}^2 \cos^2\theta_W} = 1$ with *finite* radiative corrections.

For any $T \geq 1$ model, it is entirely possible and, in the absence of a built-in custodial symmetry, probably most natural for the vev of the neutral field to be zero. In particular, only in this way can we guarantee $\rho = 1$ at tree-level and that radiative corrections to ρ will be finite. Thus, we will devote considerable discussion in our triplet review to the dramatic alterations in phenomenology that arise in such a case as compared to the case when the neutral-field vev is non-zero. The most obvious phenomenological consequence is that for zero neutral-field vev all $VV'\delta$ vertices are zero at tree-level. The single δ production processes and $\delta \to VV'$ decays that rely on such vertices are then highly suppressed or absent altogether.

Of course, couplings of Higgs bosons to the SM fermions are also a crucial ingredient for phenomenology. In fact, subject to the exception discussed below, only doublet Higgs bosons can have couplings to SM fermions. If a $T = 1$ representation has a non-zero neutral field vev, then the physical neutral and singly-charged Higgs eigenstates will typically be mixtures of doublet and triplet members and will have fermionic couplings proportional to their doublet components. If the neutral field vev is zero, then the triplet fields will not mix with the doublet fields and they will form their own separate set of physical mass eigenstates and these will not have couplings to SM fermions. The only exception to this statement is the following. In the case of the $Y = \pm 2$ triplet representation there is the possibility of $\delta^{++}l^-l^-$, $\delta^+l^-\nu_l$ and $\delta^0\nu_l\nu_l$ (Majorana-like) couplings, where the l^-'s and ν_l's are the left-handed objects with $Y = -1$. Analogous couplings are not possible in the case of the $Y = 0$ triplet representation since the right-handed leptons and neutrinos have $T = 0$. If Higgs-lepton-lepton couplings are present for the $T = 1, Y = \pm 2$ case, they play a particularly prominent phenomenological role when the vev for the neutral field is zero (or very small).

We wish to note that the discussions presented here for purely Higgs sector additions to the SM require some modification in the context of the Little Higgs models which also contain Higgs triplets. In particular, custodial symmetry issues become much more complicated, and their implications are closely tied to the ultraviolet cutoff of the effective theory. For example, in a model with an ultraviolet cutoff it is natural to allow for the presence of "non-renormalizable" effective operators suppressed by some inverse power of the cutoff that could affect such observables as ρ at tree-level.

13.1.1 Model Considerations

Higgs triplets can, in principle, carry any hypercharge Y. The real triplet with $Y = 0$ and the complex triplet with $Y = \pm 2$ both contain a neutral Higgs field, namely that component with $T_3 + \frac{1}{2}Y = 0$. For these cases, a non-zero vev for the neutral component (generically denoted by $\phi^0_{T=1}$) leads to a deviation in the tree-level prediction of $\rho = \frac{m_W{}^2}{m_Z{}^2 \cos^2\theta_W} \sim 1$, whereas $\rho \sim 1$ is automatic for doublets (plus possible singlets). For triplet models, the simplest possibility will therefore be that $\langle\phi^0_{T=1}\rangle = 0$. If $\langle\phi^0_{T=1}\rangle \neq 0$, one can avoid large corrections to the electroweak ρ parameter at tree level by either (i) choosing $\langle\phi^0_{T=1}\rangle$ very small compared to the vevs of neutral doublet fields or (ii) arranging the triplet fields and the vevs of their neutral members so that a custodial $SU(2)$ symmetry, $SU(2)_C$, is maintained. A number of models of type (ii), with a custodial $SU(2)_C$ symmetry, have been proposed in the literature. The most popular is that containing a real $Y=0$ triplet and a $Y=\pm2$ complex triplet as proposed in [18] and later considered in greater depth in [19, 20], with further follow-up in [21]. The Higgs potential for the model can be constructed in such a way that it preserves the tree-level $SU(2)_C$ symmetry. For such a model, the $SU(2)_C$ is maintained after higher-order loop corrections from Higgs self-interactions.

However, in all triplet models with $\langle\phi^0_{T=1}\rangle\neq0$, the presence of interactions of the $U(1)$ B field with the Higgs sector necessarily violates $SU(2)_C$. This is because the $U(1)$ hypercharge operator corresponds to the T_3^C of the would-be custodial $SU(2)_C$ and the non-zero vev then explicitly breaks the $SU(2)_C$ T_3^C symmetry. As a result, loop corrections to the W and Z masses are infinite [22]. In fact, corrections to ρ are quadratically divergent and achieving $\rho=1$ is at least as unnatural in the presence of such quadratic divergences as is achieving a low mass for the Higgs boson in the presence of quadratic divergences due to SM particle loops. Just as in the SM, these quadratic divergences can be ignored and computations can be carried out using standard renormalization procedures, where a set of experimental observables (measured in some appropriate way) are input and other observables are computed in terms of them. In the case of triplet models with $\langle\phi^0_{T=1}\rangle\neq0$, ρ must be renormalized, implying that $\rho=1$ is no longer a prediction of the theory but rather the observed value of ρ (or some other related electroweak parameter, often chosen to be $\sin\theta_W$ as defined via the Zee coupling, $-i\overline{e}(v_e+\gamma_5a_e)\gamma_\mu eZ^\mu$, where $1-4\sin^2\theta_W=\mathrm{Re}v_e/\mathrm{Re}a_e$, θ_W being the Weinberg angle) must be considered as an additional experimental input. Other observables in the electroweak sector can then be computed in terms of the observed value of ρ [23–25]. To obtain $\rho=1$ up to only *finite* radiative corrections, implying that $\rho\sim1$ is a natural prediction of the model, $\langle\phi^0_{T=1}\rangle=0$ is required. This implies an extra custodial symmetry of the theory such that the triplet fields generate only finite loop corrections to ρ. The phenomenology associated with this class of triplet models is very different from that which arises in models with $\langle\phi^0_{T=1}\rangle\neq0$. We will consider the two possibilities in turn.

13.1.2 Phenomenology when a neutral triplet field has non-zero vev ($\langle\phi^0_{T=1}\rangle\neq0$)

The general tree-level expression for ρ is [26]

$$\rho=\frac{\sum_{T,Y}[4T(T+1)-Y^2]|V_{T,Y}|^2c_{T,Y}}{\sum_{T,Y}2Y^2|V_{T,Y}|^2}\,, \tag{13.1}$$

where $\langle\phi^0_{T,Y}\rangle=V_{T,Y}$ ($\phi^0_{T,Y}$ being the neutral field in a given T,Y representation) and $c_{T,Y}=1\ (1/2)$ for a complex (real) representation. If we consider a Higgs sector with one $Y=1$ doublet and one $Y=0$ or $Y=\pm2$ triplet, and define $r_{1,0}=V_{1,0}/V_{1/2,1}$ and $r_{1,2}=V_{1,\pm2}/V_{1/2,1}$, we obtain

$$\rho=\begin{cases}1+2r_{1,0}^2\,, & Y=0\\ (1+2r_{1,2}^2)(1+4r_{1,2}^2)^{-1}\,, & Y=\pm2\end{cases} \tag{13.2}$$

so that $\rho-1>0\ (<0)$ for the $T,Y=1,0\ (1,\pm2)$ case. If there is more than one $Y=1$ doublet field, the above results can be generalized by replacing $V^2_{1/2,1}\to\sum_k V^2_{k\,1/2,1}$. The notation we will employ for the $Y=1$ doublet and the $Y=0$ and $Y=\pm2$ triplets is:

$$\phi_{T=1/2,Y=1}=\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}\,,\quad \phi_{T=1,Y=0}=\begin{pmatrix}\xi^+\\ \xi^0\\ \xi^-\end{pmatrix}\,,\quad \phi_{T=1,Y=2}=\begin{pmatrix}\chi^{++}\\ \chi^+\\ \chi^0\end{pmatrix}\,. \tag{13.3}$$

Of course, if the neutral fields have non-zero vevs, the above refers to the quantum fluctuations of the fields relative to these vevs. In the conventions we employ, $\phi^{+\,*}=-\phi^-$, $(\xi^0)^*=\xi^0$, $(\xi^+)^*=-\xi^-$, $(\chi^{++})^*=\chi^{--}$ and $(\chi^+)^*=-\chi^-$.

We have already noted the two experimental signatures that would immediately signal a Higgs sector with representations beyond the usual $T = 1/2, Y = 1$ doublets (at least one of which will always be assumed to be present) and $T = 0, Y = 0$ singlets. First, as exemplified by the $T = 1, Y = \pm 2$ representation, there can be doubly-charged Higgs bosons. More exotic choices of T and Y can yield Higgs bosons with still larger integer charge or even fractional charge. Second, triplet models that have a non-zero vev for a neutral field member typically predict a non-zero $ZW^\pm H^\mp$ vertex, where $H^\pm$ is some charged Higgs (or linear combination of charged Higgs) of the model. The general result (allowing for any T, Y and assuming only neutral fields have vevs) is[1]

$$\mathcal{L}_{H^\pm W^\mp Z} = -\frac{g^2}{2c_W}\kappa\left[W^+{}_\mu Z^\mu H^- + \text{h.c.}\right] , \tag{13.4}$$

where

$$\kappa^2 = \sum_{T,Y} Y^2\left[4T(T+1) - Y^2\right]|V_{T,Y}|^2 - \frac{\left\{\sum_{T,Y} 2Y^2|V_{T,Y}|^2\right\}^2}{\sum_{T,Y}\left[4T(T+1) - Y^2\right]|V_{T,Y}|^2 c_{T,Y}} \tag{13.5}$$

This formula has many implications. For instance, if $\rho = 1$ we can use Eq. (13.1) to simplify Eq. (13.5) to obtain:

$$\kappa^2 \stackrel{\rho=1}{=} \frac{\sum_{T,Y}\left[4T(T+1) - Y^2 - 2\right]|V_{T,Y}|^2}{\sum_{T,Y} 2Y^2|V_{T,Y}|^2} \geq \frac{\sum_{T,Y}\left[4T - 2\right]|V_{T,Y}|^2}{\sum_{T,Y} 2Y^2|V_{T,Y}|^2} , \tag{13.6}$$

where to obtain the 2nd equality one must note that $|Y/2| = |T_3|$ is required for the neutral field with non-zero $V_{T,Y}$ and that $|T_3| \leq T$. Eq. (13.6) shows that if $\rho = 1$ then $\kappa^2 = 0$ is only possible if all representations with non-zero $V_{T,Y}$ have $T = 1/2$ (i.e., if they are doublets). In other words, any model containing triplet or higher Higgs representations with a neutral field member that has a non-zero vacuum expectation value, and that simultaneously yields $\rho = 1$ at tree-level, must have at least one charged Higgs with non-zero coupling to the WZ channel [26].

It will be useful to discuss several specific triplet models in order to illustrate some of the many subtleties. We will consider two models: a) the model with one $T = 1/2, Y = 1$ doublet plus one $T = 1, Y = 0$ triplet; and b) the model with one $T = 1/2, Y = 1$ doublet, one $T = 1, Y = 0$ triplet and one $T = 1, Y = \pm 2$ triplet with Higgs potential such that $\langle\xi^0\rangle = \langle\chi^0\rangle$ so that there is a custodial symmetry at tree-level implying $\rho(\text{tree}) = 1$. We will end with a discussion of the implications of Higgs-lepton-lepton couplings for a $Y = 2$ triplet when $\langle\chi^0\rangle \neq 0$.

13.1.2.1 The model with one $T = 1/2, Y = 1$ doublet plus one $T = 1, Y = 0$ triplet

The model with one $T = 1/2, Y = 1$ doublet plus one $T = 1, Y = 0$ triplet model illustrates many important aspects of triplet models. For this model, we will write, using the notation of Eq. (13.3), $\langle\phi^0\rangle = v/\sqrt{2}$ and $\langle\xi^0\rangle = v' = \frac{1}{2}v\tan\beta$, where β is the mixing angle that isolates the charged Goldstone boson absorbed in giving mass to the W (see below). Then at tree level, $\rho = 1/c_\beta^2$. (We will write $t_\beta \equiv \tan\beta$, $c_\beta = \cos\beta$, and so forth). The physical Higgs bosons comprise the h^0, the k^0 and the $h^\pm$. In general, the physical eigenstate h^0 is a mixture of $\text{Re}\phi^0/\sqrt{2}$ and ξ^0 (since ξ^0 is a real field, the Goldstone boson eaten by the Z is $\text{Im}\phi^0/\sqrt{2}$) and h^+ is a mixture of ϕ^+ and ξ^+ (the orthogonal g^+ being eaten by the W^+). The mixings are specified by two angles, β and γ, according to:

$$\begin{pmatrix} g^+ \\ h^+ \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}\begin{pmatrix} \phi^+ \\ \xi^+ \end{pmatrix} , \quad \begin{pmatrix} h^0 \\ k^0 \end{pmatrix} = \begin{pmatrix} c_\gamma & s_\gamma \\ -s_\gamma & c_\gamma \end{pmatrix}\begin{pmatrix} \text{Re}\phi^0/\sqrt{2} \\ \xi^0 \end{pmatrix} . \tag{13.7}$$

In general, m_{h^0}, m_{k^0} and $m_{h^\pm}$ can be adjusted independently of one another. However, if the Higgs potential trilinear and quartic couplings are kept finite (they should not be too big in order to avoid a

[1] We follow [26], correcting a small error.

non-perturbative regime) then in the limit of $t_\beta \to 0$ the h^0 approaches the SM Higgs boson while the k^0 and $h^\pm$ have $m_{k^0} \sim m_{h^\pm}$ and decouple. The limit $t_\beta \to 0$ requires $\lambda_4 \to 0$ in the $\lambda_4 \phi^\dagger \sigma^a \phi \xi^a$ [$a = 1, 2, 3$, where, e.g., $\xi^+ = \frac{1}{\sqrt{2}}(\xi^1 + i\xi^2)$] Higgs potential term that explicitly mixes the doublet and triplet fields. It is also possible to choose the Higgs potential so that it has an extra custodial $SU(2)_C$ symmetry from the beginning by setting $\lambda_4 = 0$. In this case, $v' = 0$ and $m_{h^\pm} = m_{k^0}$ and the triplet Higgs sector gives no correction to ρ at any order. As a final theoretical point, we note that the unitarity constraints for $W^+W^- \to W^+W^-$ scattering and/or perturbativity for the Higgs potential parameters imply that if $\tan\beta$ is not small then all the physical Higgs states should have mass below ~ 1 TeV.

We now turn to the prediction for ρ in the $T = 1, Y = 0$ model just described. The prediction must be compared to the standard precision electroweak constraints as encapsulated, for instance, in the S, T, U parameters of [27]. In particular, it is useful to note that $\alpha T = \rho - 1$. The current S, T plot, based purely on Z-pole data, from the LEP Electroweak Working Group (LEWWG) [28] is shown as the left, top plot in Fig. 13.1. Note that the data have a somewhat positive S and positive T relative to the $m_{h_{SM}} = 114$ GeV prediction. If one includes NuTeV, atomic parity violation and SLAC results on Moller scattering as well, one finds the S, T ellipse [29] (with updated S, T plot provided by P. Langacker) appearing at the bottom of Fig. 13.1. The center of the ellipse shifts to slightly negative S and T values. Assuming $U = 0$ the center is at (assuming $m_{h_{SM}} = 117$ GeV and $m_t = 172.6 \pm 2.9$ GeV)

$$S = -0.07 \pm 0.09\,, \quad T = -0.03 \pm 0.09\,. \tag{13.8}$$

These latter values are completely consistent with $S = T = 0$ for new physics contributions. The value of ρ corresponding to the above T is $\rho = 0.9990 \pm 0.0009$, leaving very little room for new physics effects. At tree-level, the fit to the pure Z-pole LEWWG ellipse can be improved by using a fairly heavy SM Higgs with $m_{h_{SM}} \sim 500$ GeV along with a $T = 1, Y = 0$ triplet with $r_{1,0} \sim 0.03$, corresponding to $\beta \sim 0.045$ radians. The large $m_{h_{SM}}$ value moves the prediction towards positive S and negative T. This is compensated by the correction from the above $r_{1,0}$ value which gives a positive T contribution that places the net prediction more or less at the center of the ellipse. If one employs all available data as represented by the bottom-center S, T ellipse, $\beta \sim 0$ is preferred, but $\beta \sim 0.045$ and a heavy Higgs is still a possibility, giving a prediction in the upper right-hand corner of the PDG ellipse.

Of course, loop corrections from the triplet sector should also be included [23, 30]. For small β (as above), and assuming for simplicity that parameters are chosen so that there is no mixing between the doublet $\mathrm{Re}\phi^0/\sqrt{2}$ and the triplet ξ^0 ($\gamma = 0$), the one-loop triplet contributions give [30]

$$S_{1,0} = 0, \quad T_{1,0} \sim \frac{1}{6\pi}\frac{1}{s_W^2 c_W^2}\frac{\Delta m^2}{m_Z{}^2}, \quad U_{1,0} = \frac{\Delta m}{3\pi m_{h^\pm}}, \tag{13.9}$$

where $\Delta m = m_{k^0} - m_{h^\pm}$, and s_W and c_W are the sine and cosine of the standard electroweak angle, respectively. If the λ_i parameters of the most general Higgs potential are kept fixed (in particular, with $\lambda_4 \neq 0$) and $r_{1,0} \to 0$, then $m_{k^0}, m_{h^\pm} \to \infty$ while $\Delta m \to 0$. In this limit, the triplet decouples from the precision electroweak parameters. A small value for Δm is thus a natural possibility.

In a fully general treatment of ρ at one loop, we have already noted that ρ (or some related parameter) must be input to the renormalization scheme as an additional observable. In [25, 31], a study of the m_t dependence of the precision electroweak constraints deriving from $G_\mu = \frac{\pi\alpha}{\sqrt{2} m_W{}^2 \sin^2\theta_W}(1 + \Delta R)$ and Γ_Z is performed in the $\tan\beta \neq 0$ context. In their approach, the value of $\sin\theta_W$ from the Zee vertex is input as the additional observable. An interesting phenomenon emerges: the sensitivity of ΔR (through fixing $\sin\theta_W$) to m_t is greatly reduced. In the SM, ΔR depends on m_t quadratically, whereas in the triplet model with $\sin\theta_W$ as the additional experimental input parameter, renormalization proceeds differently, and ΔR is only logarithmically sensitive to m_t. This weak dependence is shown in the left plot of Fig. 13.2. Additional flexibility in the predictions arises if one allows for a range of m_{k^0} and $m_{h^\pm}$ values — a large selection of models are consistent with current data.

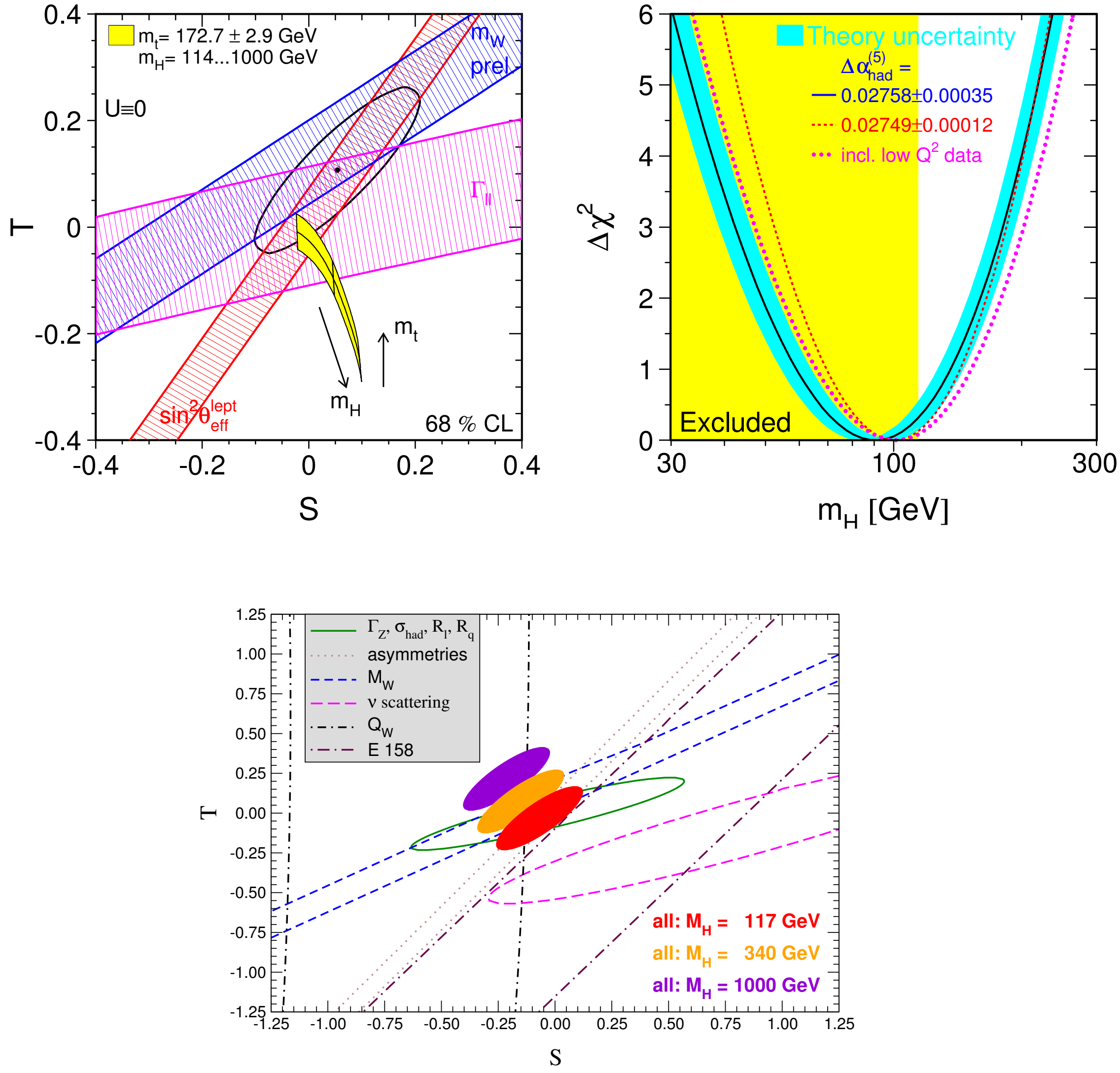

Fig. 13.1: Top left: The latest S, T ellipse for Z-pole data only from the LEWWG is shown. The reference SM Higgs and top masses employed are $m_{h_{SM}} = 150$ GeV and $m_t = 175$ GeV and $U = 0$ is assumed. The SM prediction as a function of $m_{h_{SM}}$ from $m_{h_{SM}} = 114$ GeV to 1 TeV with $m_t = 172.7 \pm 2.9$ GeV is shown. Top right: The blueband plot showing that $m_{h_{SM}} \sim 100$ GeV provides the best SM fit. Bottom center: The latest S, T ellipse to appear in the PDG (thanks to P. Langacker) which includes Z-pole data as well as parity violation and NuTeV data.

Other schemes are also possible. Instead of inputting $\sin\theta_W$ as the extra observable, one could directly input T as the extra observable and simply fix it to agree with the value at the center of the ellipse. Then, $\sin\theta_W$ would acquire logarithmic sensitivity to m_t.

Of course, other observables also depend upon m_t, most notably the Z boson width Γ_Z which has a strong m_t dependence from vertex corrections to the $Z \to b\bar{b}$ decay width — Γ_Z decreases rapidly with increasing m_t as shown in the right-hand plot of Fig. 13.2. Combining the m_W and Γ_Z sensitivity to m_t gives a prediction for m_t within about 30 GeV. Whatever scheme is employed, the important consequence is that the SM *prediction* of the top mass from precision electroweak data is considerably weakened in triplet models with non-zero vev for the neutral field.

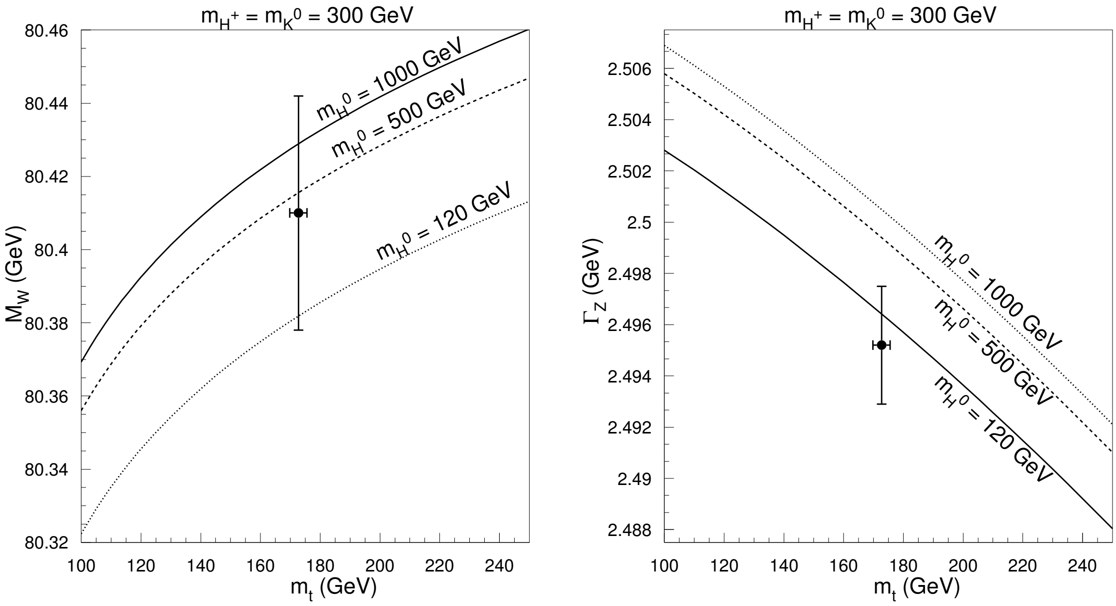

Fig. 13.2: Left: Prediction for m_W as a function of m_t in the $Y = 0$ triplet model (TM). Right: The prediction of the TM for Γ_Z as a function of m_t. The 1σ error bars are shown in both plots. Results shown are for $m_{h^+} = m_{k^0} = 300$ GeV and for m_{h^0} values as indicated by the different lines.

Of course, we now have a very accurate measurement of m_t. We have already noted that this measurement plus the various LEP and other precision measurements very strongly constrain ρ, allowing only very small deviations from $\rho = 1$ corresponding to a small non-zero vev for ξ^0 and requiring m_{h^0} to be near 100 GeV.

Some final notes are the following. Although there is no doubly-charged Higgs boson in this model, for $\tan\beta \neq 0$ there is a non-zero H^+W^-Z vertex specified by setting $\kappa = v\sin\beta$ in Eq. (13.4). Experimental probes of this vertex will be discussed shortly. It is also quite amusing to note [32] that a model containing two $Y = 1$ doublets and one $Y = 0$ triplet leads to gauge coupling unification at a scale of $M_U \sim 1.6 \times 10^{14}$ GeV, for $\alpha_s(m_Z) \sim 0.115$. While full gauge group unification cannot occur at this M_U without encountering difficulties with proton decay, coupling unification without gauge group unification is a feature of certain string models.

13.1.2.2 The model with one $T = 1/2, Y = 1$ doublet, one $T = 1, Y = 0$ triplet and one $T = 1, Y = \pm 2$ triplet with $\rho(\mathrm{tree}) = 1$

If one wishes to construct a triplet model with non-zero neutral triplet vevs and $\rho = 1$ at tree-level, despite the fact that at the one-loop level ρ is infinitely renormalized and simply must be treated as an input parameter, then one can consider the model of [18] containing one doublet, one $Y = 0$ triplet and one $Y = 2$ triplet. The following outlines the full analysis of this model presented in [20]. A convenient representation of the Higgs boson sector is

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} , \quad \chi = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ \chi^- & \xi^0 & \chi^+ \\ \chi^{--} & \xi^- & \chi^0 \end{pmatrix} . \tag{13.10}$$

By taking $\langle \chi^0 \rangle = \langle \xi^0 \rangle = b$ and $\langle \phi^0 \rangle = a/\sqrt{2}$, at tree-level one finds $\rho = 1$ with $m_W{}^2 = m_Z{}^2 \cos^2\theta_W = \frac{1}{4} g^2 v^2$ where $v^2 \equiv a^2 + 8b^2$. The amount of vev carried by the doublet sector is then characterized by $c_H \equiv \cos\theta_H = a/v$ and that in the triplet sector is then given by $s_H \equiv \sin\theta_H = 2\sqrt{2}b/v$. Thus,

$t_H \equiv \tan\theta_H$ characterizes the amount of the W mass coming from the doublet vs. the triplet fields. To fit a deviation from $\rho = 1$ at tree-level, the $\langle\chi^0\rangle = \langle\xi^0\rangle$ equality could be slightly broken. The one-loop corrections to S, T, U have not been computed in this model, but the Higgs sector parameters could probably be adjusted to allow for $\rho = 1$ at tree level along with a fairly heavy SM-like Higgs and an appropriate positive addition to T so as to remain within the current S, T ellipse of the last plot in Fig. 13.1.

If the Higgs potential preserves the custodial $SU(2)_C$, as desirable to minimize deviations from $\rho \sim 1$ arising from Higgs loops, then the physical states of this model can be classified according to their transformation properties under the tree-level custodial $SU(2)_C$. One finds a five-plet $H_5^{\pm\pm}, H_5^{\pm}, H_5^0$, a three-plet $H_3^{\pm}, H_3^0$, and two singlets, H_1^0 and $H_1^{0\prime}$. All members of a given multiplet are degenerate in mass at tree-level and only the H_1^0 and $H_1^{0\prime}$ can mix. For simplicity, we will present a discussion in which this mixing is absent and the H_1^0 and $H_1^{0\prime}$ are mass eigenstates. The phenomenology of the model reveals many new features and corresponding phenomenological possibilities.

Several features of the VV and $f\overline{f}$ couplings should be noted. First, ignoring the HV and HH type channels, at tree level the H_5's couple and decay only to vector boson pairs (we return later to the possibility of $U(1)$-conserving Higgs-lepton-lepton couplings), while the H_3's couple and decay only to fermion-antifermion pairs. Second, we observe that the SM is regained in the limit where $s_H \to 0$, in which case the H_1^0 plays the role of the SM Higgs and has SM couplings and the H_3's decouple from fermions. However, in this model with custodial $SU(2)_C$ symmetry, there is no intrinsic need for s_H to be small in order to keep ρ near 1 at tree-level. When $s_H \neq 0$, many new phenomenological features emerge. First, using the setup giving $\rho = 1$ at tree-level, there is a non-zero $H_5^+W^-Z$ coupling specified by $\kappa = s_H v$, where $\frac{1}{2}gv = m_W$. The second highly distinctive feature is the presence of a doubly-charged H_5^{++} with coupling to W^+W^+ given by $\sqrt{2}gm_W s_H$. The effects of both these new couplings are maximized in the $c_H \to 0$ limit where all the electroweak symmetry breaking resides in the triplets. We note that if c_H is small then the couplings of the doublet to the fermions must be much larger than in the SM in order to obtain the experimentally determined quark masses. Then, the Higgs bosons that do couple to fermions have much larger fermion-antifermion pair couplings and decay widths than in the SM.

Constraints on the model are significant. First, there is unitarity for vector-boson scattering. For $t_H \neq 0, \infty$, many of the Higgs bosons contribute to preserving unitarity for the various VV scattering processes. For example, unitarity for $ZW^- \to ZW^-$ requires the presence of H_1^0, $H_1^{0\prime}$, and H_5^0 in t-channel graphs and H_5^- in s- and u-channel graphs. The masses of all four must lie below ~ 1 TeV in order to avoid unitarity violation. In $W^+W^+ \to W^+W^+$, the H_1^0, $H_1^{0\prime}$ and H_5^0 appear in t-channel and u-channel graphs while the H_5^{++} appears in the s-channel. Again, all masses must lie below ~ 1 TeV to avoid unitarity violation. Note that for $s_H \to 1$ the $H_1^{0\prime}$ can have W^+W^-, ZZ couplings that are larger than in the SM so long as canceling contributions from exchanges of one of the H_5 states are present. Of course, if $s_H \sim 0$, then the masses of the triplet Higgs states (i.e., the $H_1^{0\prime}$ and the H_5 states) are unconstrained by unitarity, while if $c_H \sim 0$ then $m_{H_1^0}$ can be arbitrarily large.

We now discuss the general phenomenology when s_H is not near zero. In this case, the Higgs-lepton-lepton couplings discussed later do not play a role. Due to space limitations, we mainly restrict the discussion to a very brief outline of the phenomenology of the H_5^{++}, the hallmark state of this model. Quantum numbers imply that the only tree-level decays of the H_5^{++} are to virtual or real $H_3^+W^+$, W^+W^+ or $H_3^+H_3^+$ channels. When the H_5's are sufficiently light both of the bosons must be virtual. This results in four-body states with two non-resonant fermion-antifermion pairs. In practice, in the four-body region only $W^{+*}W^{+*}$ diagrams are important due to the weak coupling of the H_3^+ to $f\bar{f}$ states. As a result, the H_5^{++} has a very long lifetime at low mass. For larger m_{H_5}, mixed real-virtual channels take over. The final state then consists of a real V or H plus a $f\bar{f}$ pair. The possibilities include $H_3^+W^{+*}$ ($W^+H_3^{+*}$ and $H_3^+H_3^{+*}$ contributions are much smaller) and W^+W^{+*}. At still higher m_{H_5}, we enter the region where at least one of the two-body modes — W^+W^+, $H_3^+W^+$ and $H_3^+H_3^+$ — is

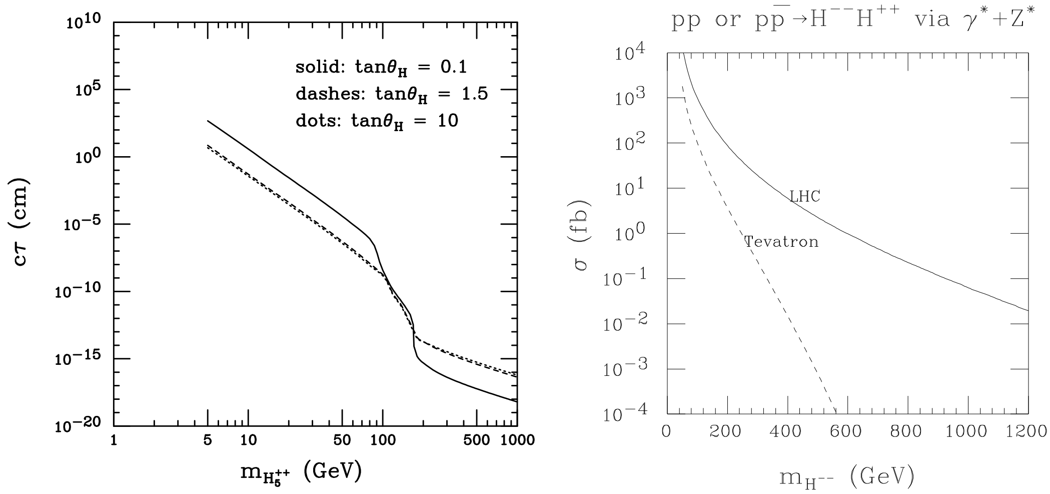

Fig. 13.3: Left: The lifetime $c\tau$ (in cm) of the H_5^{++} as a function of m_{H_5}, assuming $m_{H_3} = m_W$. Right: The Drell-Yan cross section for $H_5^{++}H_5^{--}$ pair production as a function of m_{H_5} (labeled as $m_{H^{--}}$) at the LHC and Tevatron, taken from [33].

allowed. In all cases, secondary H_3^+ and W^+ bosons typically decay to an $f\bar{f}$ final state. In Fig. 13.3 we present the lifetime $c\tau$ for the H_5^{++} as a function of m_{H_5}. We have chosen $m_{H_3} = m_W$ for this plot. For $\tanh = 0.1$, the $W^+W^+ \to H_5^{++}$ coupling, given by $\sqrt{2}gm_W s_H$, is suppressed and $c\tau$ can be larger than a μm for $m_{H_5} \lesssim 45$ GeV.

Regarding the experimental implications, we note that LEP Z-pole data would have revealed an extra width contribution coming from $Z \to H_5^{++}H_5^{--}$ for $m_{H_5} \lesssim 40$ GeV. For higher m_{H_5}, continuum pair production is the most relevant process, yielding (independent of t_H)

$$\sigma(e^+e^- \to H_5^{++}H_5^{--}) \overset{\sqrt{s}\gg m_{H_5}}{\to} \frac{1 + 4\sin^4\theta_W}{2\sin^4 2\theta_W}\,\sigma(e^+e^- \to \mu^+\mu^-) \tag{13.11}$$

which corresponds to slightly more than 1 unit of R. For $m_{H_5} \gtrsim 45$ GeV, the H_5^{++} decay becomes prompt and only the two fermion-antifermion pairs emerging from the decay are visible. Although the cross section above is substantial, we are not aware of LEP $\sqrt{s} \sim 200 - 210$ GeV analyses that exclude $H_5^{++}H_5^{--}$ production with cascade decays to such complicated final states.

Turning to H_5^{++} production at hadron colliders, there are two basic processes. First, there is $H_5^{++}H_5^{--}$ Drell-Yan pair production via γ^*, Z^*. The cross section for this process is independent of t_H and is presented for the LHC and Tevatron in Fig. 13.3. These tree-level cross sections are increased significantly (20% to 30%) by QCD radiative corrections [34]. Second, there is $W^+W^+ \to H_5^{++}$ fusion production, which is large for large t_H, but is quickly suppressed as $t_H \to 0$. An estimate for the cross section is easily obtained starting with the fact that

$$\gamma(H_5^{++} \to W^+W^+) = 2s_H^2\gamma(h_{SM} \to W^+W^-)\,. \tag{13.12}$$

For *pp* collisions, the W^+W^+ luminosity is slightly larger than the W^+W^- luminosity, but after adding in the $ZZ \to h_{SM}$ fusion processes one obtains at LHC energies and moderate m_{H_5}

$$\sigma(W^+W^+ \to H_5^{++}) \sim s_H^2\,\sigma(W^+W^-, ZZ \to h_{SM}) \tag{13.13}$$

for $m_{H_5} = m_{h_{SM}}$. As regards the H_5^{++} decays, the main decay other than $H_5^{++} \to W^+W^+$ is likely to be $H_5^{++} \to H_3^+W^+$. In the limit where $m_{H_5} \gg m_{H_3}, m_W$, the widths of the two modes are in the ratio $\frac{\gamma(H_5^{++}\to W^+W^+)}{\gamma(H_5^{++}\to H_3^+W^+)} \to 2t_H^2$. Thus, for $t_H \gtrsim 1$ a highly distinctive signature for the H_5^{++} would arise via the process $W^+W^+ \to W^+W^+$ with $W^+ \to l^+\nu_l$ decays [35]. To cleanly observe the s-channel H_5^{++} exchange as a peak in $M_{W^+W^+}$, given the presence [20] of t- and u-channel graphs with exchanges of H_5^0, H_1^0 and $H_1^{0\prime}$, would require using the mode where one $W^+ \to l^+\nu_l$ while the second decays via $W^+ \to q'\overline{q}$. The charge conjugate process, $W^-W^- \to W^-W^-$, will also be present at a somewhat lower rate. At the Tevatron, the rate for $W^+W^+ \to H_5^{++}$ will be rather small. Only searches based on Drell-Yan production are likely to be fruitful. There are currently no Tevatron searches for $H_5^{++}H_5^{--}$ pair production with $m_{H_5} > 2m_W$ based on $H_5^{++} \to W^+W^+$ and $H_5^{--} \to W^-W^-$.

At a linear collider it is possible to operate in the e^-e^- mode, in which case $W^-W^- \to W^-W^-$ scattering will take place [20, 36]. Using the $W^- \to q\overline{q}'$ decay modes, the W^-W^- mass can be reconstructed. If there is an H_5^{--} present in the s-channel, sizable bumps in the $M_{W^-W^-}$ distribution will emerge for $t_H = 1$ if $m_{H_5} \sim 200 - 300$ GeV, assuming $\sqrt{s} = 500$ GeV. Another interesting possibility is $W^-W^- \to H_3^-H_3^-$, with H_5^{--} exchange in the s-channel [37]. The reaction $W^-W^- \to H_5^-H_5^-$ occurs via t- and u-channel Higgs exchanges. Although there is no s-channel resonance, the size of the cross section depends strongly on t_H and the masses of the exchanged H_5^0 and H_3^0.

It is also interesting to note that the $H_1^{0\prime}$ can be quite light and at tree-level would only decay via the s_H suppressed $H_1^{0\prime} \to W^{-*}W^{+*} \to fermions$. As pointed out in [38], see also [21], the $\gamma\gamma$ loop-induced decay can be quite competitive in such an instance and some experimental limits may be applicable [39], depending on the t_H value.

As already noted, triplet models with $\rho = 1$ at tree-level and non-zero neutral field vevs will yield a non-zero charged-Higgs-ZW vertex. In general, observation of such an interaction would be an immediate signal for a Higgs sector with $SU(2)_L$ representations beyond the doublet. In the present model, for $s_H \neq 0$ there is a non-zero $H_5^+ZW^-$ vertex given by $\kappa = s_H v$. In the $T = 1/2, Y = 1$ plus $T = 1, Y = 0$ model there was a non-zero h^+ZW^- vertex with $\kappa = s_\beta v$. This kind of coupling, especially if suppressed by small s_H, s_β or their equivalents, is not easy to probe experimentally. Possibilities include $e^+e^- \to Z^* \to \chi^\mp W^\pm$ [40, 41], $pp \to Z^* \to \chi^\pm W^\mp$, and $pp \to W^{\pm *} \to Z\chi^\pm$ [42]. Constraints on a charged-Higgs-ZW vertex from the static electromagnetic properties of the W boson are discussed in [43].

13.1.2.3 $Y = 2$ triplets with non-zero Higgs-lepton-lepton coupling

Let us finally return to the Higgs-lepton-lepton couplings of a $Y = 2$ triplet. These can be written in the form

$$\mathcal{L} = ih_{ij}\left(\psi_{i\,L}^T C\tau_2 \Delta \psi_{j\,L}\right) + h.c. \;, \tag{13.14}$$

where $\psi_{i\,L}$ is the usual two-component leptonic doublet field, $\psi_{i\,L} = \begin{pmatrix} \nu_{l_i\,L} \\ l_{i\,L} \end{pmatrix}$, Δ is a 2×2 representation of the $Y = 2$ complex triplet field,

$$\Delta \equiv \begin{pmatrix} \frac{\chi^+}{\sqrt{2}} & \chi^{++} \\ \chi^{0*} & -\frac{\chi^+}{\sqrt{2}} \end{pmatrix} \;, \tag{13.15}$$

and i, j are family indices. Expanding out this Yukawa interaction, we find Majorana mass terms for the neutrinos of the form

$$m_{ij} = 2h_{ij}\langle \chi^0 \rangle = \frac{h_{ij} s_H v}{\sqrt{2}}. \tag{13.16}$$

If we assume that this matrix is diagonal, then the strongest limit on the Majorana mass is that for ν_e deriving from neutrinoless double-beta decay ($\beta\beta_{0\nu}$). From this we obtain $h_{ee} \lesssim 5.75 \times 10^{-12}/s_H$. For the muon and tau neutrinos, there are the usual limits from μ and τ decays. But, WMAP data, especially in combination with results from SDSS and/or 2dFGRS, imply [44] a much stronger upper bound of roughly 1 eV on the largest of the neutrino masses, corresponding to $h \lesssim 1 \times 10^{-11}/s_H$. Neutrino oscillation data provide further constraints on the h's. Indeed, we know that there is mixing among the neutrinos and that at least two of the neutrinos must have some Majorana mass. This could arise entirely from a see-saw mechanism and all the h_{ij} could be zero. Lower bounds on the h_{ij} arise if the entire Majorana neutrino mass is assumed to come from the triplet vev. The combination of $\Delta m^2_{\rm atm} \sim 2.4 \times 10^{-3}$ eV^2 and $\Delta m^2_{\rm solar} \sim 8 \times 10^{-5}$ eV^2 imply that at least one of the neutrinos (which depends upon whether we have a normal or inverted hierarchy) should have a Majorana mass of order 0.05 eV and a second should have a mass of at least 0.009 eV for normal hierarchy or again of order 0.05 eV for the inverted hierarchy case. A Majorana mass of ~ 0.05 eV corresponds to $h \sim 3 \times 10^{-13}/s_H$. Whether or not couplings that saturate these limits can be phenomenologically relevant is determined by the extent to which lepton-lepton channels can be of significance in the decays of the Higgs bosons. (The limits above clearly imply that the couplings are not useful for Higgs boson production.) For any $Y = \pm 2$ triplet Higgs boson with decay mediated by an h_{ij}, such as the decay $H_5^{++} \to l^+ l^+$, the relevant Feynman rule coupling for the decay is easily obtained from Eq. (13.14) and takes the form $-2h_{ll} v^T(k) C P_L v(l)$, where $P_L \equiv (1-\gamma_5)/2$, C is the usual charge conjugation matrix, and k and l are the momenta of the two final state leptons. The resulting decay width for a generic χ is

$$\Gamma(\chi \to l l') = \frac{|h_{ll'}|^2}{8\pi} m_\chi, \tag{13.17}$$

where l, l' might be either charged leptons or neutrinos. In the present $Y = 0$ plus $Y = \pm 2$ triplet model, the small size of the $h_{ll'}$ imply that these decays are rather unlikely to be phenomenologically important unless s_H is very small, a limit to which we will now turn.

This completes our summary of results applicable when the neutral member of a triplet has a substantial vev and thus makes a substantial contribution to electroweak symmetry breaking. We next turn to triplet models in which the triplet(s) play little or no role in electroweak symmetry breaking.

13.1.3 Triplet models with no or forbidden triplet vev ($\langle \phi^0_{T=1} \rangle = 0$)

From the perspective of the preceding section, this would seem a very special case. However, the $\langle \phi^0_{T=1} \rangle = 0$ limit of a triplet model is the point at which custodial $SU(2)_C$ is an unbroken symmetry to all orders. One obtains $\rho = 1$ at tree-level with *finite* radiative corrections. It is no longer necessary to input ρ as an additional observable as part of the renormalization procedure. However, at least in the $Y = 0$ TM, the SM Higgs must then be fairly light. The $\langle \phi^0_{T=1} \rangle = 0$ choice also has the advantage of restoring the prediction that $m_t \sim 174$ GeV in order to agree with precision electroweak data.

If $\langle \phi^0_{T=1} \rangle = 0$, then the triplet Higgs boson(s) will not have any couplings to purely SM particle final states (leaving aside the lepton-lepton coupling possibility for the moment). In addition, all couplings of triplets to the SM-like Higgs will be ones in which two triplet Higgs of the same type appear — these also do not allow for decay to the SM Higgs which would in turn decay as usual. To explore the non-Higgs-diagonal Higgs-Higgs-V couplings, we first turn to the model containing one $Y = 0$ and one $Y = \pm 2$ triplet. An important issue is whether the Higgs-Higgs-V couplings could allow a cascade decay of the H_5^{++}. For the model in question, for $s_H = 0$ there are non-zero $H_5^{++} H_3^- W^-$ and $H_3^+ H_1^{0\prime} W^-$ couplings. Further, for $s_H = 0$ we have ${m_{H_5}}^2 = 3{m_{H_3}}^2$ and $m_{H_1^{0\prime}} = 0$ (at tree-level). As a result, there will be a rapid cascade of $H_5^{++} \to H_3^+ W^+ \to H_1^{0\prime} W^+ W^+$. The $H_1^{0\prime}$, being stable and having no interactions with SM particles, would lead to missing energy. (Of course, one or more of the above particles could be virtual.) Thus, we would have a very distinctive H_5^{++} decay chain. A final state of four W's plus missing energy coming from the production of an $H_5^{++} H_5^{--}$ pair would be hard to miss if the rate is adequate.

In the case of the simpler single $Y = 0$ triplet, $\langle \xi^0 \rangle = 0$ implies that the k^0 and $h^\pm$ are degenerate. Presumably this degeneracy would be slightly broken by electromagnetic interactions, resulting in a larger mass for the $h^\pm$. Generically speaking, these corrections would be expected to yield $m_{h^\pm} - m_{k^0}$ of order few$\times m_\pi$, in which case the $h^\pm$ decay would eventually take place, but perhaps not in a typical detector (see below). The k^0 would be stable.

If all triplet neutral Higgs fields have zero vev, then the lightest of the associated Higgs bosons could well be absolutely stable and would then provide an excellent dark matter candidate. For example, in the case of the $T = 1, Y = 0$ representation if $\langle \xi^0 \rangle = 0$ then the k^0 is expected to be lighter than the $h^\pm$ and would be absolutely stable against decay to a purely SM particle state by virtue of the custodial $SU(2)_C$ (a direct $\nu_L \nu_L$ coupling being forbidden by Y conservation). Annihilation would proceed via $k^0 k^0 \to h^0$. In the model with one $Y = 0$ and one $Y = \pm 2$ triplet, the very light $H_1^{0\prime}$ (which is massless at tree-level) would be stable. Annihilation in the early universe would proceed via $H_1^{0\prime} H_1^{0\prime} \to H_1^0$, where H_1^0 is the SM Higgs boson when $s_H = 0$. A consistent description of the observed dark matter density would require an appropriate choice of $m_{H_1^0}$ relative to $2m_{H_1^{0\prime}}$. In the case where only a $T = 1, Y = 2$ Higgs representation is added to the SM, the χ^0 would similarly provide a good dark matter candidate if the (allowed by Y) coupling to $\nu_L \nu_L$ is absent, i.e., $h_{\nu\nu} = 0$.

13.1.4 Triplet Higgs bosons with large $c\tau$

Let us first discuss the situation when there are no Majorana couplings leading to decays of $Y = 2$ triplet Higgs bosons to lepton-lepton channels. In general, see the examples above, any triplet model with $\langle \phi^0_{T=1} \rangle = 0$ will have a custodial $SU(2)_C$ which guarantees the absence of all other decays for the lightest neutral triplet Higgs boson. Custodial $SU(2)_C$ is easily imposed in the models considered above by requiring that the Higgs potential be invariant under an appropriate discrete symmetry. Avoiding decays of a charged Higgs boson, at least within a detector size, is a far trickier business, For example, we have already noted that in the $T = 1, Y = 0$ plus $T = 1, Y = \pm 2$ Higgs model described above, ${m_{H_5}}^2 = 3{m_{H_3}}^2$ and the H_5^+ and H_5^{++} would quickly chain decay down to the $H_1^{0\prime}$. Single triplet representation theories are safer against such chain decays. For example, the h^+ of the single $Y = 0$ representation triplet model would be split from the k^0 by electromagnetic radiative corrections by an amount of order a few times m_π. Thus, it would decay via the far off-shell doubly virtual $h^+ \to k^{0*} W^{+*}$ process which would yield a long path length (given the small mass splitting) resulting in stability of the h^+ within the detector. Similarly, if one employed a single $Y = 2$ triplet representation, the χ^{++} and χ^+ would be split from one another and from the χ^0 by electromagnetic amounts only (for the custodial $SU(2)_C$ symmetry limit of $\langle \chi^0 \rangle = 0$) and would be stable within the detector.

For $Y = \pm 2$ triplet Higgs bosons, the $h_{ll'}$ couplings can dramatically alter the above conclusions. (In the following, we do not assume, except where noted, that the h_{ll}, $h_{l\nu_l}$ and $h_{\nu_l \nu_l}$ couplings are related by the Clebsch-Gordon factors predicted by Eq. (13.14). This allows for model independent statements. However, the reader should keep in mind that they are most probably fixed relative to one another.) For non-zero $h_{ll'}$, we would have neutral triplet Higgs decaying to $\nu\nu'$, singly $+$-charged triplet Higgs decaying to $\nu' l^+$ and doubly $++$-charged triplet Higgs decaying to $l^+ l'^+$. Rewriting Eq. (13.17) in terms of the corresponding $c\tau$ yields (here l and l' refer to either charged leptons or neutrinos that could be in the same family or different families)

$$c\tau(\chi \to ll') = 0.5\ \mu\mathrm{m} \left(\frac{100\ \mathrm{GeV}}{m_\chi} \right) \left(\frac{10^{-5}}{h_{ll'}} \right)^2 = 1\ \mathrm{m} \left(\frac{100\ \mathrm{GeV}}{m_\chi} \right) \left(\frac{0.7 \times 10^{-8}}{h_{ll'}} \right)^2 . \tag{13.18}$$

For $m_\chi = 100$ GeV, the decay is very prompt unless all $h_{ll'}$'s are considerably smaller than 10^{-5}, detector sized decay lengths being reached for $h < 0.7 \times 10^{-8}$. What are the constraints? First, *for zero triplet vev, there are no constraints on the $h_{ll'}$ couplings arising from neutrino mass limits.* Limits on diagonal ee and $\mu\mu$ couplings come from $e^+e^- \to e^+e^-$, $\frac{1}{2}(g-2)_\mu$, and muonium to anti-muonium conversion; limits on lepton flavor-violating couplings derive from $\mu \to e\gamma$, $\mu \to ee\bar{e}$ and $\tau \to l_i l_j \bar{l}_j (i, j = e, \mu)$. Theoretical formalism for these decays focused on triplet Higgs models appears in [16,45]. It is particularly interesting to note that the contributions to Δa_μ from a χ^- and a χ^{--} are

$$\Delta a_\mu(\chi^-) = -\frac{m_\mu^2}{48\pi m_{\chi^-}^2} \sum_{j=e,\mu,\tau} h_{\mu j}^2 , \quad \Delta a_\mu(\chi^{--}) = -\frac{m_\mu^2}{6\pi m_{\chi^{--}}^2} \sum_{j=e,\mu,\tau} h_{\mu j}^2 , \tag{13.19}$$

i.e., both are opposite in sign to the observed experimental deviation. Current experimental limits on the h's are reviewed in the separate experimental section. One finds that all of the diagonal limits are well above the $h = 10^{-5}$ prompt decay range. Thus, in direct collider searches all possibilities ranging from prompt to long-lived decays must be considered.

As reviewed in the experimental section, there are substantial direct limits from LEP and Tevatron experiments on doubly-charged Higgs bosons that either have a very long path length or decay to like-sign dileptons. A long-lived heavily ionizing χ^{++} track is easily seen, while in the prompt decay limit the $l^+l^+ + l^-l^-$ events have very small background. LEP limits include those from [46–48]. Tevatron limits have been obtained in [49–51]. Very roughly, current limits are of order 120 GeV, and will be extended to ~ 250 GeV by the end of the Tevatron running. At the LHC, the heavily ionizing track or $l^+l^+ + l^-l^-$ events would again stand out and limits on $m_{\chi^{++}}$ of order 1 TeV will be achieved [33,52]. Backgrounds for singly-charged Higgs bosons that decay to $l\nu$ are much larger and Tevatron results for this case have not been presented. The neutral triplet Higgs bosons are produced entirely by $Z^* \to \chi^0\chi^0$ and decay only to $\nu\nu$. At hadron colliders, the events would only be observable through the initial state radiation of photons or gluons. Such Tevatron analyses have been performed, but have not been interpreted in this context.

At a linear collider, one can probe the triplet Higgs with only ll' couplings via $e^+e^- \to \gamma^*, Z^* \to \chi\overline{\chi}$ pair production. (Recall that for $\langle\phi^0_{T=1}\rangle = 0$ there are no $ZZ\chi^0$ or $ZW^+\chi^-$ coupling.) Searches for long-lived track pairs or $l^+l^+ + l^-l^-$ events will be sensitive to m_χ values up to nearly $\frac{1}{2}\sqrt{s}$. Should a doubly-charged Higgs boson decaying to like-sign leptons be seen either at the LHC or in e^+e^- collisions, operation of the linear collider in the e^-e^- collision mode will be very highly motivated. Very small values of h_{ee} can be probed using s-channel resonance production $e^-e^- \to \chi^{--} \to l^-l^-$. This would provide the best means for actually determining h_{ee}. This is reviewed in [32,53]. The alternative processes of $e^-e^- \to \chi^{--}Z^0$ and $e^-e^- \to \chi^-W^-$ are much less sensitive [54], as are $\gamma e^- \to l^+\chi^{--}$ and $e^+e^- \to e^+l^+\chi^{--}$ [55–57].

13.1.5 Left-Right Symmetric (LR) and Supersymmetric Left-Right (SUSYLR) Models

In this section, we provide a very brief overview of models based on left-right symmetry with an extended gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [1–6] and the role therein of triplet Higgs bosons. Supersymmetric left-right symmetric models have some especially attractive features [7–14, 58]. One of the primary motivations for left-right symmetric models is that they provide a natural setting for the see-saw mechanism of neutrino mass generation. For many years, the preferred means of implementing the see-saw has been to employ a $SU(2)_R$ Higgs triplet representation (which requires the presence also of an $SU(2)_L$ triplet in order to implement the LR symmetry). We shall denote our triplet members in the LR case by $\delta^{++}_{L,R}$, $\delta^+_{L,R}$ and $\delta^0{}_{L,R}$. In addition to the triplet Higgs fields, the LR models typically contain a bi-doublet Higgs representation for generating the usual Dirac quark and lepton masses. The minimal Higgs field content of the LR model is then

$$\phi = \begin{pmatrix} \phi^0_1 & \phi^+_1 \\ \phi^-_2 & \phi^0_2 \end{pmatrix}, \quad \Delta_L = \begin{pmatrix} \delta^+_L/\sqrt{2} & \delta^{++}_L \\ \delta^0_L & -\delta^+_L/\sqrt{2} \end{pmatrix}, \quad \Delta_R = \begin{pmatrix} \delta^+_R/\sqrt{2} & \delta^{++}_R \\ \delta^0_R & -\delta^+_R/\sqrt{2} \end{pmatrix}. \tag{13.20}$$

A non-zero neutral field triplet vev, $\langle\delta^0_R\rangle = v_R/\sqrt{2}$, breaks $SU(2)_R \times U(1)_{B-L}$ down to $U(1)_Y$. The non-zero vevs in the bi-doublet, $\langle\phi^0_{1,2}\rangle = \kappa_{1,2}/\sqrt{2}$, break the $SU(2)_L \times U(1)_Y$ to $U(1)_{EM}$ as usual. The neutrino see-saw operates as follows. First, LR symmetry requires the presence of a ν_R as well as the usual ν_L. Quantum numbers allow a Majorana style $\delta^0_R\nu_R\nu_R$ coupling, see Eq. (13.14). For $\langle\delta^0_R\rangle = v_R/\sqrt{2}$ a Majorana mass of order $v_R h_{\nu\nu}$ (LR symmetry requires $h_{\nu_R\nu_R} = h_{\nu_L\nu_L}$ and so we drop the R, L subscript). For large v_R and small Dirac neutrino masses, if $h_{\nu\nu}$ is not extremely small then the see-saw mechanism takes place. The LR models can be constructed either with the requirement that $\langle\delta^0_L\rangle = v_L/\sqrt{2}$ be zero or non-zero. In general, at least some of the additional Higgs bosons of the LR models can be light; their phenomenology was first studied in [15–17] and basic results are summarized in [26]. Further results appear in [21, 55, 57, 59–73]. Here, we confine ourselves to a very few remarks regarding the status of Higgs triplets in LR and SUSYLR models.

Precision electroweak constraints are most easily satisfied if $v_L = 0$, but can also be satisfied with $v_L \neq 0$ [25, 74, 75]. Renormalization proceeds much as in the $T = 1/2, Y = 1$ plus $T = 1, Y = 0$ triplet model; for $v_L \neq 0$ the experimental value of $\sin\theta_W$ (or ρ or, equivalently, T) becomes an input rather than a prediction. Using the input-$\sin\theta_W$ scheme, one again finds a greatly reduced sensitivity of ΔR to m_t, in this case because the $\delta\sin^2\theta_W/\sin^2\theta_W$ contribution to ΔR, though quadratic in m_t, is proportional to $m_{W_1}^2/m_{W_2}^2$, where m_{W_1} must be very close to the observed W mass. Thus, this contribution to ΔR vanishes as $m_{W_2} \to \infty$.

In the simplest LR model, with Higgs content sketched above, minimization of the Higgs potential results in surprisingly strong constraints. In [5, 16, 17] it was shown that there is a "vev see-saw" relation that reads (following [17])

$$(2\rho_1 - \rho_3)v_L v_R = \beta_2\kappa_1^2 + \beta_1\kappa_1\kappa_2 + \beta_3\kappa_2^2 \tag{13.21}$$

where the β_i and ρ_i are certain Higgs potential parameters. Thus, for generic Higgs potential parameter choices, if $v_L \ll \kappa_{1,2}$ (where $\kappa_1^2 + \kappa_2^2$ is of order the usual 246 GeV), then $v_R \gg \kappa_{1,2}$ is required by the minimization. In [17], it was shown that the only phenomenologically acceptable solutions are $\beta_2 = 0$ (as required if we demand that the Higgs potential be invariant under $\phi \to i\phi$, which also cures certain FCNC problems of the model) with $v_L = \kappa_2 = 0$, $\kappa_1, v_R \neq 0$ and $\rho_{diff} \equiv 2\rho_1 - \rho_3 \neq 0$. If ρ_{diff} is of order 1, then all Higgs bosons other than a single SM-like Higgs boson will have masses of order v_R. Interesting new Higgs phenomenology at the TeV-scale would require a very small value for ρ_{diff}. In fact, an additional symmetry can be imposed on the Higgs potential that guarantees $\rho_{diff} = 0$. However, $\rho_{diff} = 0$ implies that the Higgs bosons residing in the real and imaginary parts of δ_L^0 are massless at tree-level; this is inconsistent with constraints from the Z. Thus, this symmetry must be slightly broken at the v_R scale by effective operators. Assuming very small ρ_{diff}, we would have the following. After removing the usual Goldstone bosons, the δ_L^{++} and δ_L^+ states have masses of order v_L, the $\mathrm{Im}\delta_L^0/\sqrt{2}$ and $\mathrm{Re}\delta_L^0/\sqrt{2}$ states have masses of order $\sqrt{\rho_{diff}}v_R$, while the δ_R^{++}, δ_R^+ and $\mathrm{Im}\phi_2^0/\sqrt{2}$ states have masses of order v_R. The residual h^+ state that is a combination of ϕ_1^+ and δ_R^+ is heavy, as is one combination, called H^0, of $\mathrm{Re}\phi_1^0/\sqrt{2}$ and δ_R^0, while the orthogonal combination (h^0) plays the role of a light SM-like Higgs boson. In the end, the TeV-scale phenomenology has many similarities to that of a one-doublet + one $Y = 2$ triplet model, including the presence of ll couplings for the δ_L states (a remnant of the LR symmetry and the see-saw neutrino mass generation mechanism). The detailed phenomenology of this model can be found in [15–17].

The SUSYLR models have some important attractive features. In particular, it is possible to construct them so that both the strong CP problem and the SUSY CP problem (i.e., the generic problem of SUSY phases giving large EDM's unless cancellations are carefully arranged) are automatically solved [7–9, 11]. If LR symmetry and SUSY are implemented in the triplet-Higgs context, then one needs additional triplet fields; in the SUSY extension of the triplet model discussed above, these would be the conjugates of the Δ_R and Δ_L. In addition, as we sketch below, one also needs to include singlet superfields. Before symmetry breaking, both the δ_R^{++} Higgs bosons (there are now two) and their higgsino partners are massless due to the existence of a flat direction associated with rotations in $\langle\delta_R^0\rangle$–$\langle\delta_R^{++}\rangle$ space. If the scale of supersymmetry breaking, m_{SUSY} is above v_R, then after SUSY the theory lives in a charge-violating vacuum unless non-renormalizable operators suppressed by $1/M_{\mathrm{P}}$ involving the singlet field(s) are included. After including these operators, the δ_R^{++} Higgs bosons typically acquire only a small mass. If $m_{\mathrm{SUSY}} < v_R$, then the renormalizable theory may live in a charge-conserving vacuum and the δ_R^{++} Higgs bosons pick up a mass of order m_{SUSY}. Now, however, the corresponding higgsinos are very light since the breaking of supersymmetry is assumed to be soft. Non-renormalizable operators are now needed to give the higgsinos sufficient mass to avoid current experimental constraints. Minimization of the Higgs potential after including the Δ_L and $\overline{\Delta}_L$ fields, and the associated Higgs bosons, have not be carefully studied in this context.

More recently, an alternative SUSYLR model has emerged in which Higgs triplet fields do not play a role [58]. The $SU(2)_R \times U(1)_{B-L}$ symmetry is broken down to $U(1)_Y$ in the supersymmetric limit by $B-L = \pm 1$ doublet scalar fields, namely the right-handed doublet denoted by $\chi^c(1,2,-1)$ accompanied by its left-handed partner $\chi(2,1,1)$, where the items in parenthesis indicate the $SU(2)_L$, $SU(2)_R$ and $B-L$ representations, respectively. Anomaly cancellation requires the presence of the charge conjugate fields, $\overline{\chi}^c(1,2,1)$ and $\overline{\chi}(2,1,-1)$, as well. The vevs $\langle\chi^c\rangle = \langle\overline{\chi}^c\rangle = v_R$ break the left-right symmetry group down to the MSSM gauge symmetry. The only Higgs bosons of the resulting model with masses at the TeV scale (rather than at scale v_R) correspond to the H_u and H_d doublet fields of the MSSM. There are some additional singlet Higgs fields with masses of order v_R, but no triplet Higgs fields are employed. The main advantage of this model over SUSYLR models with triplets is that the SUSY phase problem is solved based on requiring LR parity symmetry alone as opposed to requiring charge conjugation symmetry as well. In addition, introduction of non-renormalizable effective interactions is not required in order to guarantee a charge-conserving vacuum. However, non-renormalizable operators suppressed by $1/M_{\rm P}$, as well as both a visible sector singlet field and a hidden sector singlet field, *are* required in order to generate an effective soft-supersymmetry breaking $B\mu$ term. A non-renormalizable operator form $(fLL\chi\chi + f^* L^c L^c \chi^c \chi^c)/M_{\rm P}$ is also employed to produce Majorana masses for the ν_R's of size $v_R^2/M_{\rm P}$. For $v_R \sim 10^{14} - 10^{16}$ GeV, the predicted Majorana masses are in the right ball park to explain solar and atmospheric oscillation data. Overall, the model is not very simple and the canonical see-saw with Majorana masses of order $(246~{\rm GeV})^2/v_R$ is totally abandoned. Thus, the SUSYLR models with triplet Higgs should certainly not be ignored.

13.1.6 Experiment

A wide variety of experimental searches and standard model tests probe the existence of Higgs triplets. The possibilities depend strongly on the type of triplet model. In this section, we will focus on a single $T = 1, Y = \pm 2$ triplet addition to the SM doublet with Higgs bosons denoted as previously by χ^0, $\chi^\pm$, and $\chi^{\pm\pm}$. For this case, if we ignore loop corrections then $V_{1,\pm 2} \equiv \langle\chi^0\rangle$ is constrained to be small ($\langle\chi^0\rangle < 8$ GeV) from the measurements of the W mass and other electroweak parameters — see, for example, Eq. (13.2) and note that $\rho < 1$ is predicted at tree-level whereas data suggest a small positive value for $\rho - 1$. Radiative corrections for this case have not been worked out, but it seems safe to say that even after their inclusion $\langle\chi^0\rangle$ would have to be quite small. Our discussion will be based on this approximation. Among other things, it implies that there is rather small mixing between the doublet Higgs bosons and the triplet Higgs bosons. Thus, we will speak of the χ^0 and $\chi^\pm$ as though they are unmixed states with phenomenology determined by perturbative corrections associated with the small non-zero value of $\langle\chi^0\rangle$. Of course, the χ^{++} and χ^{--} are pure states.

The parameters determining the sensitivity of a given experiment are the Higgs mass and couplings. The expected phenomena depend on whether the lightest triplet member is χ^0, $\chi^\pm$ or $\chi^{\pm\pm}$. If there were no mixing between the χ^0 and the ϕ^0 nor between the χ^+ and the ϕ^+, then it would be most probable that the χ^0 would be the lightest state. The effects of introducing mixing terms into the Higgs Lagrangian have not been worked out for this case, but it seems possible for the mass ordering of the states to be altered. We will discuss various signatures for each of the states in turn assuming that the state in question is the lightest of the triplet states.

We will begin with the $\chi^{\pm\pm}$. We focus on various extreme possibilities.

- The $\chi^{\pm\pm}$ has significant couplings to leptons.
- The $\chi^{\pm\pm}$ has negligible couplings to leptons and W bosons.
- The $\chi^{\pm\pm}$ has negligible couplings to leptons and small but significant couplings to W bosons.
- The $\chi^{\pm\pm}$ is a member of a supersymmetric triplet with R-parity-conserving interactions.

13.1.6.1 $\chi^{\pm\pm}$ with leptonic couplings

The richest triplet phenomenology occurs for light doubly charged Higgs bosons with significant leptonic couplings. This is possible without conflicting with neutrino masses if either $\langle\chi^0\rangle = 0$ (as would be more or less required if h_{ll}, l being the charged lepton, is substantial and $h_{\nu\nu} \sim h_{ll}$, as predicted by Eq. (13.14)) or if only h_{ll} is substantial. In any case, it is important to obtain limits on h_{ll} without introducing any model-dependent inputs. The effects of the χ^{++} through h_{ll} couplings can be observed indirectly through rare leptonic decays or conversion processes, or directly through production at lepton and hadron colliders.

Including flavor changing possibilities, there are six $\chi^{\pm\pm}$ leptonic couplings, which we denote by h_{ij}. These are undetermined parameters, so there is no theoretical guidance to whether a particular leptonic decay is preferred, and if so, which one. Off-diagonal couplings lead to lepton-flavor-violating processes such as $\mu \to 3e$ [16], $\tau \to 3l$ [16], and $\mu \to e\gamma$ [76], while diagonal couplings contribute to the Bhabha scattering cross section [16, 77, 78] the muon anomalous magnetic moment [16, 79], and muonium to antimuonium conversion (Figs. 13.4 and 13.5) [77]. Table 13.2 shows the coupling limits for $m_{\chi^{\pm\pm}} = 100$ GeV from searches for, or measurements of, these processes. Future data to be taken by the BELLE and MEG collaborations will improve coupling sensitivity by about an order of magnitude from $\tau \to 3l$ and $\mu \to e\gamma$ searches, respectively.

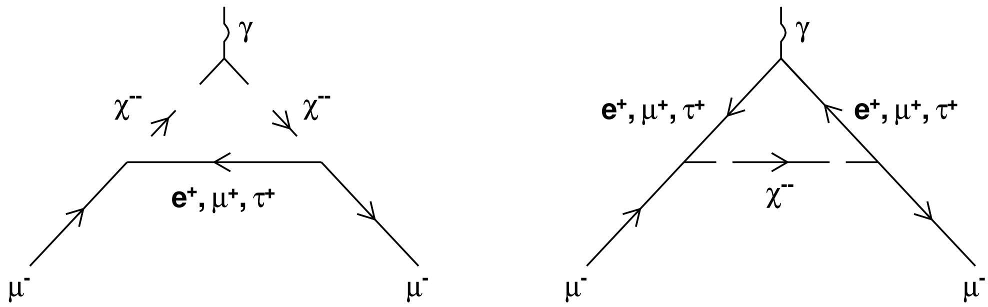

Fig. 13.4: The $\chi^{\pm\pm}$ contributions to the μ^- anomalous magnetic moment. Equivalent charge-conjugate diagrams exist for the μ^+. The same diagrams with the outgoing muons replaced by electrons result in the decay $\mu \to e\gamma$.

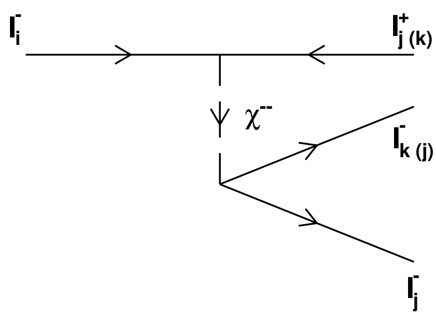

Fig. 13.5: χ^{--}-mediated $l_i^- \to l_j^- l_{k(j)}^- l_{j(k)}^+$ decay. A corresponding charge-conjugate diagram mediates $l_i^+ \to l_j^+ l_{k(j)}^+ l_{j(k)}^-$ decay.

While indirect studies probe the lepton-coupling-to-$\chi^{\pm\pm}$-mass ratio in the form $c_{ij} \equiv h_{ij}^2/m^2_{\chi^{\pm\pm}}$, direct searches are sensitive to a given mass for couplings spanning several orders of magnitude. Pairs of $\chi^{\pm\pm}$ bosons are produced in e^+e^- and hadron collisions through Z/γ^* exchange and for $m_{\chi^{++}} \gtrsim 100$ GeV decay promptly ($c\tau < 10$ microns) if $\sum h_{ij} > 10^{-5}$ — see Eq. (13.18) — even if $\langle\chi^0\rangle = 0$ (so that $\chi^{\pm\pm} \to W^\pm W^\pm$ decays are absent). Searches for $\chi^{++}\chi^{--}$ pair production have excluded

Process	Limit
$e^+e^- \to e^+e^-$	$h_{ee} < 0.15$ [80]
$\frac{1}{2}(g-2)_\mu$	$h_{\mu\mu} < 0.22$ [81]
$M \to \bar{M}$	$h_{ee}h_{\mu\mu} < 2.0 \times 10^{-3}$ [82]
$\mu \to e\gamma$	$h_{e\mu}, h_{e\tau}, h_{\tau\mu} < 4.5 \times 10^{-3}$ [83]
$\mu \to eee$	$h_{ee}h_{\mu e} < 2 \times 10^{-7}$ [84]
$\tau \to l_i l_j l_j,\ i,j = e,\mu$	$h_{ij}h_{j\tau} < 6 \times 10^{-2}$ [85, 86]

Table 13.2: The Yukawa coupling limits on $\chi^{\pm\pm}$ for $m_{\chi^{\pm\pm}} = 100$ GeV/c^2. The h_{ij} limits increase linearly with increasing $\chi^{\pm\pm}$ mass. Any assumptions made on the relative couplings have been chosen to produce conservative (i.e., higher) limits.

the $\chi^{\pm\pm}$ if its mass is below 100-135 GeV (Fig. 13.6), provided decay channels other than the dilepton channels have small branching ratio (as would be true if $\langle\chi^0\rangle$ is tiny or zero or if one or more of the h_{ij} are large). The limits depend on the dominant h_{ij} coupling and on whether the $\chi^{\pm\pm}$ has left-handed or right-handed[2] couplings. The limits also assume $m_{\chi^\pm} >> m_{\chi^{\pm\pm}}$, and become stronger if $m_{\chi^\pm} \approx m_{\chi^{\pm\pm}}$ (see Section 13.2). Ongoing $p\bar{p}$ data collected at the Tevatron will increase the mass sensitivity to ~ 250 GeV, and future pp data from the LHC will further increase the sensitivity to ~ 1 TeV. For an analysis of doubly charged Higgs bosons in the left-right symmetric model at the LHC, see Section 6.4.

13.1.6.2 Long-Lived $\chi^{\pm\pm}$

If the $\chi^{\pm\pm}$ leptonic couplings are significantly suppressed, and it has no other significant decay channels (requiring very small or zero $\langle\chi^0\rangle$ and very small or negative $m_{\chi^{++}} - m_{\chi^+}$), then the $\chi^{\pm\pm}$ is likely to be long-lived ($c\tau > 10$ m). In this case, $\chi^{\pm\pm}$ phenomena will be limited to direct production at lepton and hadron colliders. Current mass limits range from 110-135 GeV (Fig. 13.6), depending on whether the doubly charged Higgs has Majorana couplings to the left- or right-handed leptons. The full Tevatron data set will extend the sensitivity to $\sim$ 250 GeV, and the LHC pp collisions will make observation of ~ 1 TeV $\chi^{\pm\pm}$ bosons possible.

13.1.6.3 $\chi^{\pm\pm}$ with W couplings

Doubly charged Higgs couplings to W bosons are determined by the vacuum expectation value of the neutral Higgs field in the triplet. If $\langle\chi^0\rangle$ takes its maximum allowed value, then pp collisions at the LHC will produce an observable rate of single χ^{++} bosons produced both in association with a W boson and also via WW fusion processes such as $W^+W^+ \to \chi^{++}$. The sensitivity of the Tevatron data to these topologies has not yet been determined.

The $W^-\chi^{++} + W^+\chi^{--}$ final state would result in low-background signatures of like-sign leptons $+\not{E}_T + X$ in hadron and e^+e^- colliders assuming that the h_{ll} couplings are large enough that the $\chi^{++} \to l^+l^+$ decay is dominant over decays such as $\chi^{++} \to W^+\phi^+$ which would typically be present for $\langle\chi^0\rangle \neq 0$. Note, however, that this scenario with both large h_{ll} and large $\langle\chi^0\rangle$ is inconsistent with neutrino masses unless $h_{\nu\nu} \ll h_{ll}$, in contradiction to the $SU(2)_L$ invariant interaction form of Eq. (13.14).

13.1.6.4 $\chi^{\pm\pm}$ with SUSY couplings

In the supersymmetric extension of the $T = 1, Y = \pm 2$ model, many additional possibilities emerge. First, there would be SUSY-determined analogues of the h_{ij} specifying the coupling of the χ^{++} to slepton pairs $\widetilde{l}^+\widetilde{l}^+$. These would give rise to a like-sign slepton signal for the χ^{++}. In addition, there

[2] In considering the case of couplings to right-handed charged leptons, we are implicitly extending our considerations to a left-right symmetric model with a $\chi^{\pm\pm}$ $SU(2)_R$ triplet member.