Appendix

For the convenience of the reader, let us collect basic material on the following topics:

- notation,
- physical units, the Planck system, and the energetic system of units,
- the Gaussian system of units and the Heaviside system, and
- the method of dimensional analysis – a magic wand of physicists.

A comprehensive table on the units of the most important physical quantities can be found at the end of the Appendix on page 967.

A.1 Notation

Sets and mappings. The abbreviation 'iff' stands for 'if and only if'. To formulate definitions, we use the symbol ':='. For example, we write

$$f(x) := x^2$$

iff the value $f(x)$ of the function f at the point x is equal to x^2, by definition.

The symbol $U \subseteq V$ (resp. $U \subset V$) means that U is a subset (resp. a proper subset) of V. This convention resembles the symbols $x \le y$ (resp. $x < y$) for real numbers. A map

$$f : X \to Y$$

sends each point x living in the set X to an image point $f(x)$ living in the set Y. The set X is also called the domain of definition, $\mathrm{dom}(f)$, of the map f. By definition, the image, $\mathrm{im}(f)$, of the map f is the set of all image points $f(x)$. Furthermore, the set

$$f(U) := \{f(x) : \ x \in U\}$$

is called the image of the set U by the map f. In other words, by definition, the set $f(U)$ contains precisely all the points $f(x)$ with the property that x is an element of the set U. The set

$$f^{-1}(V) := \{x \in X : f(x) \in V\}$$

is called the pre-image of the set V by the map f.

- The map f is called *surjective* iff each point of the set Y is an image point. In this case, we also say that f maps the set X 'onto' the set Y. The French word 'sur' means 'onto'.
- The map f is called *injective* iff $x_1 \neq x_2$ always implies $f(x_1) \neq f(x_2)$. Such maps are also called 'one-to-one'.

E. Zeidler, *Quantum Field Theory I: Basics in Mathematics and Physics*,

- The map f is called *bijective* iff it is both surjective and injective. Precisely in this case, the inverse map $f^{-1} : Y \to X$ exists.

For each given point y in the set Y, consider the equation

$$\boxed{f(x) = y, \qquad x \in X,} \tag{A.1}$$

that is, we are looking for a solution x in the set X. Observe that the map f is surjective (resp. bijective) iff the equation (A.1) has always at least one (resp. precisely one) solution. The map f is injective iff the equation has always at most one solution.

Inverse map. If the map $f : X \to Y$ is bijective, then the inverse map

$$f^{-1} : Y \to X$$

is defined by $f^{-1}(y) := x$ iff $f(x) = y$.

Sets of numbers. The symbol $\mathbb{K}$ always stands either for the set $\mathbb{R}$ of real numbers or the set $\mathbb{C}$ of complex numbers. The real number x is called positive, negative , nonnegative, non-positive iff

$$x > 0, \qquad x < 0, \qquad x \geq 0, \qquad x \leq 0,$$

respectively. The symbols

$$\mathbb{R}^{\times}, \quad \mathbb{R}_{>}, \quad \mathbb{R}_{<}, \quad \mathbb{R}_{\geq}, \quad \mathbb{R}_{\leq}$$

denote the set of nonzero real numbers, positive real numbers, negative real numbers, nonnegative real numbers, non-positive real numbers, respectively.[9] Concerning the sign of a real number, we write $\operatorname{sgn}(x) := 1, -1, 0$ if $x > 0,\ x < 0,\ x = 0$, respectively.

For a given complex number $z = x + y\mathrm{i}$, we introduce both the conjugate complex number $z^\dagger := x - y\mathrm{i}$ and the modulus

$$\boxed{|z| := \sqrt{zz^\dagger} = \sqrt{x^2 + y^2}.}$$

The real (resp. imaginary) part of z is denoted by $\Re(z) := x$ (resp. $\Im(z) := y$). The definition of the principal argument, $\arg(z)$, of the complex number z can be found on page 211. Traditionally,

- the symbol $\mathbb{Z}$ denotes the set of integers $0, \pm 1, \pm 2, \ldots,$
- the symbol $\mathbb{N}$ denotes the set of nonnegative integers $0, 1, 2, \ldots$ (also called natural numbers),[10]
- the symbol $\mathbb{N}^{\times}$ denotes the set of positive integers $1, 2, \ldots$, and
- the symbol $\mathbb{Q}$ denotes the set of rational numbers.

For closed, open, and half-open intervals, we use the notation

$$[a, b] := \{x \in \mathbb{R} : a \leq x \leq b\}, \qquad]a, b[:= \{x \in \mathbb{R} : a < x < b\},$$

and $]a, b] := \{x \in \mathbb{R} : a < x \leq b\}$, as well as $[a, b[:= \{x \in \mathbb{R} : a \leq x < b\}$.

The Landau symbols. Around 1900 the following symbols were introduced by the number theorist Edmund Landau (1877–1938). We write

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[9] For the closed half-line $\mathbb{R}_{\geq}$, one also uses the symbol $\mathbb{R}_{+}$.

[10] For the set $\mathbb{N}$, one also uses the symbol $\mathbb{Z}_{\geq}$.

$$\boxed{f(x) = o(g(x)) \qquad \text{as} \quad x \to a}$$

iff $f(x)/g(x) \to 0$ as $x \to a$. For example, $x^2 = o(x)$ as $x \to 0$. The symbol

$$f(x) = O(g(x)) \qquad \text{as} \quad x \to a \tag{A.2}$$

tells us that $|f(x)| \leq \text{const}\ |g(x)|$ in a sufficiently small, open neighborhood of the point $x = a$. For example, $2x = O(x)$ as $x \to 0$. We write

$$f(x) \simeq g(x), \qquad x \to a$$

iff $f(x)/g(x) \to 1$ as $x \to a$. For example,

$$\sin x \simeq x, \qquad x \to 0.$$

Relativistic physics. In an inertial system, we set

$$x^1 := x, \quad x^2 := y, \quad x^3 := z, \quad x^0 := ct$$

where x, y, z are right-handed Cartesian coordinates, t is time, and c is the velocity of light in a vacuum. Generally,

- Latin indices run from 1 to 3 (e.g., $i, j = 1, 2, 3$), and
- Greek indices run from 0 to 3 (e.g., $\mu, \nu = 0, 1, 2, 3$).

In particular, we use the Kronecker symbols

$$\delta_{ij} = \delta^{ij} = \delta^i_j := \begin{cases} 1 & \text{if} \quad i = j, \\ 0 & \text{if} \quad i \neq j, \end{cases} \tag{A.3}$$

and the Minkowski symbols

$$\eta_{\mu\nu} = \eta^{\mu\nu} := \begin{cases} 1 & \text{if} \quad \mu = \nu = 0, \\ -1 & \text{if} \quad \mu = \nu = 1, 2, 3, \\ 0 & \text{if} \quad \mu \neq \nu. \end{cases} \tag{A.4}$$

Einstein's summation convention. In the Minkowski space-time, we always sum over equal upper and lower Greek (resp. Latin) indices from 0 to 3 (resp. from 1 to 3). For example, for the position vector, we have

$$\mathbf{x} = x^j \mathbf{e}_j := \sum_{j=1}^{3} x^j \mathbf{e}_j,$$

where $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ are orthonormal basis vectors of a right-handed orthonormal system. Moreover,

$$\eta_{\mu\nu} x^\nu := \sum_{\nu=0}^{3} \eta_{\mu\nu} x^\nu.$$

Greek indices are lowered and lifted with the help of the Minkowski symbols. That is,

$$x_\mu := \eta_{\mu\nu} x^\nu, \qquad x^\mu = \eta^{\mu\nu} x_\nu.$$

Hence

$$\boxed{x_0 = x^0, \qquad x_j = -x^j, \qquad j = 1, 2, 3.}$$

For the indices $\alpha, \beta, \gamma, \delta = 0, 1, 2, 3$, we introduce the antisymmetric symbol $\epsilon^{\alpha\beta\gamma\delta}$ which is normalized by

$$\epsilon^{0123} := 1, \tag{A.5}$$

and which changes sign if two indices are transposed. In particular, $\epsilon^{\alpha\beta\gamma\delta} = 0$ if two indices coincide. For example, $\epsilon^{0213} = -1$ and $\epsilon^{0113} = 0$. Lowering of indices yields $\epsilon_{\alpha\beta\gamma\delta} := -\epsilon^{\alpha\beta\gamma\delta}$. For example, $\epsilon_{0123} := -1$.

The Minkowski metric. Unfortunately, there exist two different conventions in the literature, namely, the so-called west coast convention (W) which uses the following Minkowski metric,

$$\boxed{\eta_{\mu\nu} x^\mu x^\nu = c^2 t^2 - x^2 - y^2 - z^2,} \tag{A.6}$$

and the east coast convention (E) based on $-c^2t^2 + x^2 + y^2 + z^2$. (This refers to the east and west coast of the United States of America.) From the mathematical point of view, the east coast convention has the advantage that there does not occur any sign change when passing from the Euclidean metric

$$x^2 + y^2 + z^2$$

to the Minkowski metric. From the physical point of view, the west coast convention has the advantage that the Minkowski square of the momentum-energy 4-vector $(\mathbf{p}, E/c)$ is positive,

$$\eta_{\mu\nu} p^\mu p^\nu = \frac{E^2}{c^2} - \mathbf{p}^2 = m_0^2 c^2. \tag{A.7}$$

Here, m_0 denotes the rest mass of the particle. Since most physicists and physics textbooks use the west coast convention, we will follow this tradition, which dates back to Einstein's papers, Dirac's 1930 monograph *Foundations of Quantum Mechanics* and Feynman's papers. Concerning elementary particles, we use the same terminology as in the standard textbook by Peskin and Schroeder (1995). One can easily pass from our convention to the east coast convention by using the replacements

$$\eta_{\mu\nu} \mapsto -\eta_{\mu\nu}, \qquad \gamma^\mu \mapsto -\mathrm{i}\gamma^\mu$$

for the Minkowski metric and the Dirac-Pauli matrices, γ^μ, from the Dirac equation (A.20), respectively.[11]

A.2 The International System of Units

The ultimate goal of physicists is to measure physical quantities in physical experiments. To this end, physicists have to compare the quantity under consideration with appropriate standard quantities. For example, the measurement of the length of a distance can be obtained by comparing the length with the standard length m (meter). This procedure leads to systems of physical units.

The SI system. In the international system of units, SI (for Système International in French), the following basic units are used:

[11] For example, the east coast convention is used in Misner, Thorne, and Wheeler (1973), and in Weinberg (1995).

Table A.1. Prefixes in the SI system

10^{-1}	deci	d	10	deka	D
10^{-2}	centi	c	10^{2}	hecto	H
10^{-3}	milli	m	10^{3}	kilo	K
10^{-6}	micro	μ	10^{6}	mega	M
10^{-9}	nano	n	10^{9}	giga	G
10^{-12}	pico	p	10^{12}	tera	T
10^{-15}	femto	f	10^{15}	peta	P

- length: m (meter),
- time: s (second),
- energy: J (Joule),
- electric charge: C (Coulomb),
- temperature: K (Kelvin).

Each physical quantity q can be uniquely represented as

$$\boxed{q = q_{\mathrm{SI}} \cdot \mathrm{m}^{\alpha}\mathrm{s}^{\beta}\mathrm{J}^{\gamma}\mathrm{C}^{\mu}\mathrm{K}^{\nu}.} \tag{A.8}$$

Here, q_{SI} is a real number, and the exponents $\alpha, \beta, \gamma, \mu, \nu$ are rational numbers. Physicists say that the physical quantity q has the dimension

$$(\text{length})^{\alpha}(\text{time})^{\beta}(\text{energy})^{\gamma}(\text{electric charge})^{\mu}(\text{temperature})^{\nu}.$$

Let us consider a few examples.

- The unit of mass is the kilogram, $\mathrm{kg} := \mathrm{Js}^2\mathrm{m}^{-2}$.
- The unit of force is the Newton, $\mathrm{N} = \mathrm{Jm}^{-1}$.
- The unit of electric current strength is the Ampere, $\mathrm{A} := \mathrm{Cs}^{-1}$.

The physical dimensions of the most important physical quantities in the SI system can be found in Table A.4 on page 967. Instead of meter one also uses kilometer, nanometer, femtometer, and so on, which corresponds to

$$1000\mathrm{m}, \quad 10^{-9}\mathrm{m}, \quad 10^{-15}\mathrm{m},$$

respectively (see Table A.1).

The universal character of the SI system. Unfortunately, for historical reasons, there exist many different systems of units used by physicists. In what follows we want to help the reader to understand the relations between the different systems. Let us explain the following.

> *If one knows the physical dimension of some quantity in the SI system, then one can easily pass to every other system used in physics.*

In particular, we will discuss

- the natural SI system,
- the Planck system, and
- the energetic system.

The Planck system has the advantage that the fundamental physical constants $G, \hbar, c, \varepsilon_0, \mu_0, k$ do not appear explicitly in the basic equations (e.g., in elementary particle physics and cosmology). In this system, all the physical quantities are dimensionless.

The energetic system is mainly used in elementary particle physics. In this system, all of the physical quantities are measured in powers of energy, and the physical constants $\hbar, c, \varepsilon_0, \mu_0, k$ do not appear explicitly.

A.3 The Planck System

> All the systems of units which have hitherto been employed owe their origin to the coincidence of accidental circumstances, inasmuch as the choice of the units lying at the base of every system has been made, not according to general points of view, but essentially with reference to the special needs of our terrestrial civilization...
> In contrast with this it might be of interest to note that we have the means of establishing units which are independent of special bodies or substances. The means of determining the units of length, mass, and time are given by the action constant h, together with the magnitude of the velocity of propagation of light in a vacuum c, and that of the constant of gravitation G... These quantities must be found always the same, when measured by the most widely differing intelligences according to the most widely differing methods.
>
> Max Planck, 1906
> *The Theory of Heat Radiation*[12]

Fundamental constants. There exist the following universal constants in nature:

- G (gravitational constant),
- c (velocity of light in a vacuum),
- h (Planck's quantum of action),
- ε_0 (electric field constant of a vacuum),
- k (Boltzmann constant).

The explicit numerical values of these fundamental constants can be found in Table A.3 on page 965. We also use the constants

- $\hbar := h/2\pi$ (reduced Planck's quantum of action), and
- $\mu_0 := 1/\varepsilon_0 c^2$ (magnetic field constant of vacuum).

Basic laws in physics. These universal constants enter the following six basic laws of physics.

(i) Einstein's equivalence between rest mass m_0 and rest energy E of a particle: $E = m_0 c^2$.
(ii) Energy E of a photon with frequency ν: $E = h\nu$.
(iii) Gravitational force F between two masses M_1 and M_2 at distance r:

$$F = \frac{GM_1M_2}{r^2}.$$

[12] M. Planck, Theorie der Wärmestrahlung, Barth, Leipzig 1906. Reprinted by Dover Publications, 1991.

Table A.2. SI system

$1\ \text{m} = 0.63 \cdot 10^{35}\mathbf{m}$	$1\ \mathbf{m} = l = 1.6 \cdot 10^{-35}\text{m}$
$1\ \text{s} = 0.19 \cdot 10^{44}\mathbf{s}$	$1\ \mathbf{s} = 5.3 \cdot 10^{-44}\text{s}$
$1\ \text{J} = 0.51 \cdot 10^{-9}\mathbf{J}$	$1\ \mathbf{J} = 1.97 \cdot 10^{9}\text{J}$
$1\ \text{kg} = 0.48 \cdot 10^{8}\mathbf{kg}$	$1\ \mathbf{kg} = 2.1 \cdot 10^{-8}\text{kg}$
$1\ \text{C} = 0.19 \cdot 10^{19}\mathbf{C}$	$1\ \mathbf{C} = 5.34 \cdot 10^{-19}\text{C}$
$1\ \text{K} = 0.71 \cdot 10^{-32}\mathbf{K}$	$1\ \mathbf{K} = 1.4 \cdot 10^{32}\text{K}$
$1\ \text{GeV} = 10^9\ \text{eV} = 1.602 \cdot 10^{-10}\text{J}$ $1\ \text{GeV}/c^2 = 1.78 \cdot 10^{-27}\text{kg}$	

(iv) Electric force F between two electric charges Q_1 and Q_2 at distance r:

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}.$$

(v) Magnetic force F between two parallel electric currents of strength J_1 and J_2 in a wire of length L at distance r:

$$F = \frac{\mu_0 L J_1 J_2}{2\pi r}.$$

(vi) Mean energy E corresponding to one degree of freedom in a many-particle system at temperature T: $E = kT$.

In the SI system, the unit of electric current, called an ampere, is defined in such a way that the magnetic field constant of a vacuum is given by

$$\mu_0 = 4\pi \cdot 10^{-7}\frac{\text{N}}{\text{A}^2}.$$

By Table A.1 on prefixes, 1 MeV $=10^6$eV (mega electron volt).

Natural SI units. The five natural constants $G, c, \hbar, \varepsilon_0$, and k can be used to systematically replace the SI units m, s, J, C, K by the following so-called natural SI units:

- Planck length: $\mathbf{m} := l := \sqrt{\hbar G/c^3}$,
- Planck time: $\mathbf{s} := l/c$,
- Planck energy: $\mathbf{J} := \hbar c/l$,
- Planck charge: $\mathbf{C} := \sqrt{c\hbar\varepsilon_0}$,
- Planck temperature: $\mathbf{K} := \hbar c/kl$.

Parallel to kg $=$ Js^2/m^2, let us introduce the Planck mass

$$\mathbf{kg} := \mathbf{Js}^2/\mathbf{m}^2 = \hbar/cl.$$

The numerical values can be found in Table A.2. From (A.8) we obtain the representation

$$q = q_{\text{Pl}} \cdot \mathbf{m}^\alpha \mathbf{s}^\beta \mathbf{J}^\gamma \mathbf{C}^\mu \mathbf{K}^\nu \tag{A.9}$$

of the physical quantity q in natural SI units. Hence

$$q = q_{\rm Pl} \cdot l^\alpha \left(\frac{l}{c}\right)^\beta \left(\frac{\hbar c}{l}\right)^\gamma (c\hbar\varepsilon_0)^{\mu/2} \left(\frac{\hbar c}{kl}\right)^\nu .$$

This implies

$$\boxed{q = q_{\rm Pl} \cdot l^A c^B \hbar^C \varepsilon_0^D k^E.} \tag{A.10}$$

Explicitly,

$$A = \alpha + \beta - \gamma - \nu, \quad B = \gamma + \nu - \beta + \mu/2, \quad C = \gamma + \nu + \mu/2,$$

and $D = \mu/2,\ E = -\nu$.

The Planck system of units. In this system, we set

$$\boxed{l = c = \hbar = \varepsilon_0 = k := 1.}$$

In particular, for the gravitational constant, this implies $G = 1$. By (A.10), $q = q_{\rm Pl}$.

The Planck system is characterized by the fact that all the physical quantities are dimensionless and their numerical values coincide with the numerical values in natural SI units.

In order to go back from the Planck system to the SI system, one has to replace each physical quantity q by

$$\boxed{q \quad \Rightarrow \quad \frac{q}{l^A c^B \hbar^C \varepsilon_0^D k^E}} \tag{A.11}$$

according to (A.10). The corresponding exponents $A, B, \ldots$ follow from (A.9) and (A.10). These exponents can be found in Table A.4 on page 967.

Example 1. For the proton, we get

$$E = 0.77 \cdot 10^{-19}\mathbf{J} = 1.5 \cdot 10^{-10}\mathrm{J} = 0.938\,\mathrm{GeV} \qquad \text{(rest energy)}$$

along with

$$M = E/c^2 = 0.77 \cdot 10^{-19}\mathbf{kg} = 1.67 \cdot 10^{-27}\mathrm{kg} \qquad \text{(rest mass)}$$

and

$$e = \sqrt{4\pi\alpha}\,\mathbf{C} = 0.30\mathbf{C} = 1.6 \cdot 10^{-19}\mathrm{C} \qquad \text{(electric charge)}.$$

Therefore, $E_{\rm Pl} = M_{\rm Pl} = 0.77 \cdot 10^{-19}$, and $e_{\rm Pl} = 0.30$. In the Planck system, this implies

$$E = M = 0.77 \cdot 10^{-19} \quad \text{and} \quad e = 0.30.$$

Example 2. Consider the Einstein relation

$$E = m_0 c^2 \tag{A.12}$$

between the rest mass m_0 and the rest energy E of a free relativistic particle in the SI system. Letting $c := 1$, we obtain the corresponding equation

$$\boxed{E = m_0} \tag{A.13}$$

in the Planck system. Here, $E = E_{\rm Pl}$ and $m_0 = M_{\rm Pl}$. In order to go back from (A.13) to the SI system, one has to observe that

$$E = E_{\rm Pl} \cdot \mathbf{J}, \qquad m_0 = M_{\rm Pl} \cdot \mathbf{Js^2m^{-2}}$$

in natural SI units, by Table A.4 on page 967. Hence

$$E = E_{\mathrm{Pl}}\frac{\hbar c}{l}, \qquad m_0 = M_{\mathrm{Pl}}\frac{\hbar}{lc}.$$

Thus, we have to replace E and m_0 by

$$\frac{El}{\hbar c} \quad \text{and} \quad \frac{m_0 lc}{\hbar},$$

respectively. This way, we pass over from (A.13) to (A.12).

Example 3. In the SI system, the Maxwell equations in a vacuum are given by

$$\begin{aligned} \operatorname{div}\mathbf{D} &= \varrho, & \operatorname{div}\mathbf{B} &= 0, \\ \mathbf{curl}\,\mathbf{E} &= -\dot{\mathbf{B}}, & \mathbf{curl}\,\mathbf{H} &= \dot{\mathbf{D}} + \mathbf{j} \end{aligned} \tag{A.14}$$

along with $\mathbf{D} = \varepsilon_0\mathbf{E}$ and $\mathbf{B} = \mu_0\mathbf{H}$. Moreover, $c^2 = 1/\varepsilon_0\mu_0$. Alternatively,

$$\begin{aligned} \varepsilon_0 \operatorname{div}\mathbf{E} &= \varrho, & \operatorname{div}\mathbf{B} &= 0, \\ \mathbf{curl}\,\mathbf{E} &= -\dot{\mathbf{B}}, & c^2\,\mathbf{curl}\,\mathbf{B} &= \dot{\mathbf{E}} + \mu_0 c^2\mathbf{j}. \end{aligned} \tag{A.15}$$

Letting $\varepsilon_0 = \mu_0 = c := 1$, we obtain the corresponding Maxwell equations in the Planck system:

$$\begin{aligned} \operatorname{div}\mathbf{E} &= \varrho, & \operatorname{div}\mathbf{B} &= 0, \\ \mathbf{curl}\,\mathbf{E} &= -\dot{\mathbf{B}}, & \mathbf{curl}\,\mathbf{B} &= \dot{\mathbf{E}} + \mathbf{j}. \end{aligned} \tag{A.16}$$

In order to transform equation (A.16) back to the SI system, we replace the quantities $\mathbf{x}, t, \mathbf{E}, \mathbf{B}, \varrho, \mathbf{j}$ by

$$\frac{\mathbf{x}}{\mathbf{m}}, \quad \frac{t}{\mathbf{s}}, \quad \mathbf{E}\cdot\frac{\mathbf{mC}}{\mathbf{J}}, \quad \mathbf{B}\cdot\frac{\mathbf{m}^2}{\mathbf{sJ}}, \quad \varrho\cdot\frac{\mathbf{m}^3}{\mathbf{C}}, \quad \mathbf{j}\cdot\frac{\mathbf{m}^2}{\mathbf{Cs}}, \tag{A.17}$$

respectively, according to Table A.4 on page 967. In addition, the partial derivatives $\partial/\partial x^j$, $\partial/\partial t$ have to be replaced by

$$\mathbf{m}\cdot\frac{\partial}{\partial x^j}, \qquad \mathbf{s}\cdot\frac{\partial}{\partial t},$$

respectively. Finally, we set

$$\mathbf{m} := l, \quad \mathbf{s} := \frac{l}{c}, \quad \mathbf{C} := (c\hbar\varepsilon_0)^{1/2}, \quad \mathbf{J} := \frac{\hbar c}{l}.$$

This way, we get (A.14). In fact, for example, the first Maxwell equation div $\mathbf{E} = \varrho$ from (A.16) means explicitly

$$\partial_j E^j = \varrho,$$

in Cartesian coordinates. Here, $\partial_j = \partial/\partial x^j$, and we sum over $j = 1, 2, 3$. By (A.17), this is transformed into the equation

$$\beta\cdot\partial_j E^j = \varrho,$$

where $\beta := \mathbf{C}^2/\mathbf{Jm}$. Since $\beta = c\hbar\varepsilon_0/c\hbar = \varepsilon_0$, we obtain $\varepsilon_0 \operatorname{div}\mathbf{E} = \varrho$. This is the first Maxwell equation from (A.15).

Example 4. In the SI system, the Schrödinger equation reads as

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{\hbar^2}{2m_0}\Delta\psi + U\psi. \tag{A.18}$$

Here, m_0 and U denote the mass of the particle and the potential energy, respectively. Letting $\hbar = 1$, we arrive at the Schrödinger equation

$$\boxed{i\frac{\partial\psi}{\partial t} = \frac{\Delta\psi}{2m_0} + U\psi} \tag{A.19}$$

in the Planck system. In a Cartesian (x, y, z)-system, the Laplacian is defined by

$$\boxed{\Delta := -\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}.}$$

Note that our sign convention coincides with the use of the Laplacian in modern differential geometry (Riemannian geometry) and string theory.[13]

In order to go back from the Planck system to the SI system[14], we replace the quantities $\mathbf{x}, t, U, m_0, \psi$ by

$$\frac{\mathbf{x}}{\mathbf{m}}, \quad \frac{t}{\mathbf{s}}, \quad \frac{U}{\mathbf{J}}, \quad m_0 \cdot \frac{\mathbf{m}^2}{\mathbf{Js}^2}, \quad \psi \cdot \mathbf{m}^{3/2},$$

respectively. The Laplacian contains spatial derivatives of second order. Thus, the Laplacian Δ and the partial time derivative $\partial/\partial t$ have to be replaced by

$$\mathbf{m}^2 \cdot \Delta, \qquad \mathbf{s} \cdot \frac{\partial}{\partial t}.$$

Consequently, equation (A.19) is transformed into

$$i\mathbf{Js}\psi_t = \frac{(\mathbf{Js})^{\mathbf{2}}}{2m_0}\Delta\psi + U\psi.$$

Since $\mathbf{Js} = \hbar$, we get (A.18).

Example 5. Let us start with the Dirac equation

$$\boxed{i\gamma^\mu\partial_\mu\psi = m_0\psi} \tag{A.20}$$

for the relativistic electron of rest mass m_0 formulated in the Planck system. Here, $\partial_\mu = \partial/\partial x^\mu$. Recall that we sum over μ from 0 to 3, by Einstein's summation convention. The definition of the Dirac–Pauli matrices $\gamma^0, \gamma^1, \gamma^2, \gamma^3$ can be found on page 791. In order to pass over to the SI system, we replace the quantities x^μ, m_0, ψ by

$$\frac{x^\mu}{\mathbf{m}}, \qquad m_0 \cdot \frac{\mathbf{m}^2}{\mathbf{Js}^2}, \qquad \mathbf{m}^{3/2} \cdot \psi,$$

respectively, according to Table A.4 on page 967. Note that the dimension of the wave function ψ in the SI system is the same as in the case of the Schrödinger equation. Hence

$$i\gamma^\mu\partial_\mu\psi = \sigma m_0\psi$$

[13] In classic textbooks, one has to replace Δ by $-\Delta$.

[14] The normalization condition $\int_{\mathbb{R}^3} \psi\psi^\dagger d^3x = 1$ implies that the wave function ψ has the dimension $\mathrm{m}^{-3/2}$ in the SI system.

where $\sigma := \mathbf{m/s^2J}$. Since $\sigma = c/\hbar$, in the SI system the Dirac equation reads as

$$\boxed{\mathrm{i}\hbar\gamma^\mu\partial_\mu\psi = m_0 c\psi.} \tag{A.21}$$

The quantity $\lambda_e := h/cm_0$ is called the Compton wave length of the electron.

Classical systems of units. In the context of the Maxwell equations, physicists frequently use the Gaussian system or the Heaviside system, for historical reasons. Let us explain the relation of these two systems to the SI system. The idea is to measure all of the physical quantities by meter, second, kilogram, and Kelvin. That is, we do not introduce a specific unit for electric charge. In the Gaussian system, the electric force F between two electric charges Q_1 and Q_2 at distance r (Coulomb law) is given by

$$F = \frac{Q_1Q_2}{r^2}.$$

Moreover, we use the Gaussian definition of the magnetic field

$$\mathbf{H}_G := c\mathbf{B}.$$

This definition is motivated by the fact that the electric field $\mathbf{E}$ and the Gaussian magnetic field $\mathbf{H}_G$ possess the same physical dimension. In the Heaviside system, we use the Coulomb law

$$F = \frac{Q_1Q_2}{4\pi r^2}.$$

In contrast to the Gaussian system from the 1830s, the Heaviside system from the 1880s has the advantage that the factor 4π does not appear in the Maxwell equations.

The Heaviside system of units. We use the SI system and set

$$\boxed{\varepsilon_0 := 1.}$$

In the SI system, each physical quantity q can be written as

$$q = q_{\mathrm{Pl}} \cdot l^A c^B \hbar^C \varepsilon_0^D k^E,$$

by (A.10) on page 954. Letting $\varepsilon_0 := 1$, we get

$$q = q_{\mathrm{Pl}} \cdot l^A c^B \hbar^C k^E,$$

in the Heaviside system. Consequently,

$$q = q_{\mathrm{H}} \cdot \mathrm{m}^a \mathrm{s}^b \mathrm{kg}^c \mathrm{K}^d.$$

That is, each physical quantity can be described by powers of meter, second, kilogram, and Kelvin. In the Heaviside system, the Maxwell equations read as follows:

$$\begin{aligned} \operatorname{div}\mathbf{E} &= \varrho, & \operatorname{div}\mathbf{H}_G &= 0, \\ \mathbf{curl}\,\mathbf{E} &= -\frac{1}{c}\frac{\partial\mathbf{H}_G}{\partial t}, & \mathbf{curl}\,\mathbf{H}_G &= \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} + \frac{\mathbf{j}}{c}. \end{aligned} \tag{A.22}$$

To obtain this, start with the Maxwell equations in the SI system. By (A.15) along with $c^2 = 1/\varepsilon_0\mu_0$,

$$\begin{aligned} \varepsilon_0 \operatorname{div}\mathbf{E} &= \varrho, & \operatorname{div}(c\mathbf{B}) &= 0, \\ \mathbf{curl}\,\mathbf{E} &= -\frac{1}{c}\frac{\partial(c\mathbf{B})}{\partial t}, & \mathbf{curl}(c\mathbf{B}) &= \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} + \frac{\mathbf{j}}{\varepsilon_0 c}. \end{aligned}$$

Letting $\varepsilon_0 := 1$ and $\mathbf{H}_G := c\mathbf{B}$, we get (A.22).

The Gaussian system of units. Using the rescaling

$$\mathbf{E} \Rightarrow \frac{\mathbf{E}}{4\pi}, \qquad \mathbf{H}_G \Rightarrow \frac{\mathbf{H}_G}{4\pi},$$

the Heaviside system passes over to the Gaussian system. In particular, the Maxwell equations in the Gaussian system read as follows:

$$\begin{aligned} \operatorname{div} \mathbf{E} &= 4\pi\varrho, & \operatorname{div} \mathbf{H}_G &= 0, \\ \mathbf{curl}\, \mathbf{E} &= -\frac{1}{c}\frac{\partial \mathbf{H}_G}{\partial t}, & \mathbf{curl}\, \mathbf{H}_G &= \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi \mathbf{j}}{c}. \end{aligned} \tag{A.23}$$

Observe that the variants (A.22) and (A.23) of the Maxwell equations differ by the factor 4π. The Gaussian system is used in the 10-volume standard textbook on theoretical physics by Landau and Lifshitz (1982).

A.4 The Energetic System

The most important physical quantity in elementary particle physics is given by the energy of a particle accelerator. Therefore, particle physicists like to use energy as basic unit. Let us discuss this. In the SI system, an arbitrary physical quantity can be written as

$$q = q_{\mathrm{Pl}} \cdot l^A c^B \hbar^C \varepsilon_0^D k^E,$$

by (A.10). In the energetic system, we set[15]

$$\boxed{c = \hbar = \varepsilon_0 = k := 1.}$$

Hence

$$q = q_{\mathrm{Pl}} \cdot l^A.$$

Consequently, each physical quantity has the physical dimension of some power of length. In particular, for energy E we get

$$E = E_{\mathrm{Pl}} \cdot l^{-1}\hbar c,$$

in the SI system. Hence

$$E = E_{\mathrm{Pl}} \cdot l^{-1},$$

in the energetic system. That is, energy has the physical dimension of inverse length.

Conversely, length has the physical dimension of inverse energy in the energetic system of units.

In terms of natural SI units, each physical quantity can be written as

$$q = q_{\mathrm{Pl}} \cdot \mathbf{m}^\alpha \mathbf{s}^\beta \mathbf{J}^\gamma \mathbf{C}^\mu \mathbf{K}^\nu.$$

It follows from $c = \hbar = \varepsilon_0 = k := 1$ that

[15] In particular, this implies $\mu_0 = 1$.

$$\boxed{\mathbf{s} = \mathbf{m} = \mathbf{J}^{-1}, \qquad \mathbf{K} = \mathbf{J}, \qquad \mathbf{C} = 1.} \tag{A.24}$$

This implies

$$q = q_{\rm Pl} \cdot \mathbf{J}^A,$$

where $A = -\alpha - \beta + \gamma + \nu$. This way, each physical quantity can be expressed by powers of the Planck energy $\mathbf{J}$. Using Table A.4 on page 967 along with (A.24), we immediately obtain all the dimensions of important physical quantities in the energetic system. For example, velocity has the dimension

$$v = v_{\rm SI} \cdot {\rm ms}^{-1},$$

in the SI system. Thus, in natural SI units,

$$v = v_{\rm Pl} \cdot \mathbf{ms}^{-1}.$$

In the energetic system $\mathbf{m} = \mathbf{s}$, by (A.24). Hence

$$v = v_{\rm Pl},$$

that is, velocity is dimensionless. Note that this follows more simply from the fact that $c := 1$ in the energetic system; that is, the velocity of light is dimensionless. Similarly, using the dimensionless quantities $\hbar = \varepsilon_0 = \mu_0 = k := 1$ along with the basic physical laws (i)-(vi) on page 952, we encounter the following physical dimensions in the energetic system:

- [mass] = [momentum] = [temperature] = [energy],
- [length] = [time] = $[\text{energy}]^{-1}$,
- [cross section] = [area] = $[\text{length}]^2 = [\text{energy}]^{-2}$,
- [electric charge] = [velocity] = [action] = dimensionless,
- [force] = [electric field] = [magnetic field] = $[\text{energy}]^2$,
- [potential] = [vector potential] = [energy],
- the coupling constants of quantum electrodynamics, quantum chromodynamics, and electroweak interaction are dimensionless.

Since the electric charge and the coupling constants of the Standard Model in particle physics are dimensionless in the energetic system, these quantities are independent of the rescaling of energy.

Examples. The Einstein relation $E = m_0 c^2$ reads as

$$E = m_0$$

in the energetic system, since $c := 1$.

> *The Maxwell equations* (A.16)*, the Schrödinger equation* (A.19)*, and the Dirac equation* (A.20) *coincide in the Planck system and in the energetic system.*

In elementary particle physics, physicists like to use GeV (giga electron volt), where

$$\mathbf{J} = 1.98 \cdot 10^{19} \text{ GeV}.$$

This is called the Planck energy. Note that the rest energy of the proton is equal to 0.938 GeV. Consequently, from Table A.2 on page 953, we obtain the following conversion formulas between the SI system and the energetic system:

$$\begin{aligned} 1\,\mathrm{m} &\doteq 5 \cdot 10^{16}\,(\mathrm{GeV})^{-1}, \\ 1\,\mathrm{s} &\doteq 1.5 \cdot 10^{24}\,(\mathrm{GeV})^{-1}, \\ 1\,\mathrm{J} &\doteq 6.3 \cdot 10^{9}\,\mathrm{GeV}, \\ 1\,\mathrm{K} &\doteq 1.4 \cdot 10^{-13}\,\mathrm{GeV}, \\ 1\,\mathrm{C} &\doteq 1.9 \cdot 10^{18}. \end{aligned}$$

Depending on the energy scale, physicists also use mega electron volt, MeV. Here, 1 GeV $= 10^3$ MeV.

The physical dimension of cross sections. Observe that

$$\hbar c = 1.97327 \cdot 10^{-13}\ \mathrm{MeV} \cdot \mathrm{m}.$$

This implies

$$\mathrm{m}^2 = \frac{(\hbar c)^2}{(1.97327)^2} \cdot 10^{26} (\mathrm{MeV})^{-2}.$$

In the SI system, the cross section σ is measured in m^2. Setting

$$\hbar = c := 1,$$

we get the cross section in the energetic system measured in $(\mathrm{MeV})^{-2}$. Conversely, the passage from the energetic system to the SI system can be easily obtained by using the replacement

$$\boxed{\sigma \quad \Rightarrow \quad \frac{\sigma}{(\hbar c)^2}.}$$

In fact, if $\sigma = a$ in the energetic system, then $\sigma = (\hbar c)^2 a$ in the SI system.

A.5 The Beauty of Dimensional Analysis

Physicists use the dimensionality of physical quantities in order to get important information. Let us illustrate this by considering three examples: the pendulum, Newton's gravitational law, and Kolmogorov's law for turbulence.

The pendulum. Consider a pendulum of length l and mass m. We are looking for a formula for the period of oscillation, T, of the pendulum. We expect that T depends on l, m, and the gravitational acceleration g. Thus, we begin with the ansatz

$$T = C \cdot l^\alpha m^\beta g^\gamma$$

where C is a dimensionless constant. Passing to dimensions we get

$$\mathrm{s} = \mathrm{m}^\alpha \mathrm{kg}^\beta \mathrm{m}^\gamma \mathrm{s}^{-2\gamma}.$$

This implies $\beta = 0$, $\gamma = -\frac{1}{2}$, and $\alpha = -\gamma = \frac{1}{2}$, that is,

$$\boxed{T = C\sqrt{\frac{l}{g}}.} \tag{A.25}$$

The constant C has to be determined from experiment. The explicit solution of the problem via elliptic integrals shows that, for small pendulum motions, equation (A.25) is valid with $C = 2\pi$.

Newton's gravitational law. In 1619 Kepler discovered empirically that the motion of a planet satisfies the law

$$\frac{T^2}{a^3} = \text{const}$$

where T is the period of revolution, and a is the great semi-axis of the elliptic orbit. In order to guess Newton's gravitational law from this information, let us make the ansatz

$$m\ddot{\mathbf{x}} = C|\mathbf{x}|^{\mu}\mathbf{x}$$

for the motion $\mathbf{x} = \mathbf{x}(t)$ of the planet. Here, m is the mass of the planet, and C is a constant. We want to show that $\mu = -3$ is the only natural choice. To this end, consider the rescaled motion $\mathbf{y}(t) := \alpha\mathbf{x}(\beta t)$. Then

$$m\ddot{\mathbf{y}} = \beta^2\alpha^{-\mu}C|\mathbf{y}|^{\mu}\mathbf{y}.$$

We postulate that the equation of motion and the third Kepler law are independent of the rescaling. This means that $\beta^2\alpha^{-\mu} = 1$ and

$$(T\beta)^2/(\alpha a)^3 = T^2/a^3.$$

Hence $\mu = -3$. Summarizing, we obtain Newton's gravitational law

$$\boxed{m\ddot{\mathbf{x}} = \frac{C}{|\mathbf{x}|^2}\cdot\frac{\mathbf{x}}{|\mathbf{x}|}.}$$

The Kolmogorov law for energy dissipation in turbulent flows. It is a typical property of turbulent flow that there exist eddies of different diameters λ, where $\lambda_{\min} \leq \lambda \leq \lambda_{\max}$. One may think, for example, of clouds in the air or of nebulas in astronomy. One finds that the large eddies tend to break down into smaller eddies. This way, energy from large eddies flows to smaller eddies. Here, physicists assume that the energy of the smallest eddies with $\lambda = \lambda_{\min}$ is transformed into heat by friction (energy dissipation). Viscosity is of significance only for small eddies. We define

$$\varepsilon := \frac{\text{loss of energy by dissipation}}{\text{mass}\cdot\text{time}}.$$

This is the crucial physical quantity. Note that ε can be measured in experiments; it is equal to the produced heat. Using the method of dimensional analysis, Kolmogorov obtained the law

$$\varepsilon = \int_{\lambda_{\min}}^{\lambda_{\max}} s(\lambda)d\lambda$$

along with the spectral function

$$\boxed{s(\lambda) := C\left(\frac{\lambda}{\lambda_{\min}}\right)\frac{\eta\varepsilon^{2/3}}{\varrho}\cdot\frac{1}{\lambda^{7/3}}.}$$

Here, η and ϱ are viscosity and mass density, respectively. The function C is dimensionless. For values λ near $\lambda_{\min}$, the function C can be approximated by a constant. Therefore, physicists speak of Kolmogorov's 7/3-law. The proof can be found in Zeidler (1986), Vol. IV, p. 514.

It turns out that dimensional analysis represents a magic wand of physicists. In this setting, a minimum of hypotheses provides us a maximum of information.

A.6 The Similarity Principle in Physics

Rescaled SI units. Let us replace the SI units m, s, J, C, K with the rescaled units

$$\mathrm{m}_*, \mathrm{s}_*, \mathrm{J}_*, \mathrm{C}_*, \mathrm{K}_*,$$

where $\mathrm{m}_* = m_+ \cdot \mathrm{m}$, $\mathrm{s}_* = s_+ \cdot \mathrm{s}, \ldots$ with the real numbers $m_+, s_+, \ldots$ Then, each physical quantity q can be represented as

$$\boxed{q = q_* \cdot \mathrm{m}_*^\alpha \mathrm{s}_*^\beta \mathrm{J}_*^\gamma \mathrm{C}_*^\mu \mathrm{K}_*^\nu = q_* \cdot [q].}$$

The real number q_* is called the numerical value of q, and $[q]$ is called the dimension of q with respect to this system of units. In practice, one chooses $\mathrm{m}_*, \mathrm{s}_*$, ... in such a way that the numerical values of the physical quantities are neither too large nor too small. For example, if we want to study thin layers, then it is convenient to use $\mathrm{m}_* := 10^{-9}\mathrm{m} = 1\mathrm{nm}$ (nanometer). In astronomy, one uses light years for measuring distances, and so on.

The role of small quantities in physics. It is impossible to speak of a small length L in physics. In fact, if

$$L = 1 \text{ meter},$$

then passing to a new length scale, we get

$$L = 10^{15} \text{ femtometer}.$$

Therefore, it makes sense to speak about smallness only for dimensionless quantities. For example, choose the radius r_E of earth and the radius r_p of a proton. Then the dimensionless ratio

$$\frac{r_p}{r_E} = 6 \cdot 10^{-21}$$

is a small quantity compared with 1.

The experience of physicists shows that two different theories are good approximations of each other if suitable dimensionless quantities are small. Let us consider two crucial examples.

(i) Relativistic physics: Let v and c be the velocity of some particle and the velocity of light, respectively. If the dimensionless quotient

$$\frac{v}{c}$$

is sufficiently small, then the relativistic motion of the particle can be described approximately by Newton's classical mechanics. For example, the relativistic mass

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} = m_0 \left(1 - \frac{v^2}{2c^2} + o\left(\frac{v^2}{c^2}\right)\right), \qquad \frac{v^2}{c^2} \to 0$$

is approximately equal to the rest mass m_0 if the quotient v/c is sufficiently small.

(ii) Quantum mechanics: Let $S = E(t_1 - t_2)$ be the action for the motion of some particle with constant energy E during a fixed reasonable time interval $[t_1, t_2]$, say, one hour. If the dimensionless ratio

$$\frac{S}{\hbar}$$

is small, then the quantum motion of the particle can be approximately described by Newton's classical mechanics.

In (i) and (ii), corrections to classical mechanics can be obtained by perturbation theory if v/c and $S/\hbar$ are small. These are the post-Newtonian approximation and the WKB approximation, respectively.

The fundamental similarity principle in physics. We postulate that

Physical processes are described by equations which are invariant under rescaling of units. Explicitly, we demand that the laws of physics can be written in such a way that, in a fixed system of units, they only depend on the dimensionless quotients

$$\frac{q}{[q]}, \qquad \frac{r}{[r]}, \qquad \ldots$$

of all the physical quantities q, r, ...

A special role is played by those physical quantities which are dimensionless in the SI system. We expect that such quantities are related to important physical effects. The experience of physicists confirms this. For example, the so-called fine structure constant

$$\alpha := \frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{1}{137.04}$$

represents the most important dimensionless quantity that can be constructed from the universal constants. This constant measures the strength of the interaction between electrons, positrons, and photons in quantum electrodynamics. The smallness of α is responsible for the fact that perturbation theory can be successfully applied to quantum electrodynamics.

Example. Consider the Einstein relation

$$E = m_0c^2$$

between rest mass m_0 and rest energy E of a particle. In any rescaled SI system,

$$E = E_* \cdot \mathrm{J}_*, \qquad m_0 = (m_0)_* \cdot \mathrm{J}_*\mathrm{s}_*^2\mathrm{m}_*^{-2}, \qquad c = c_* \cdot \mathrm{m}_*\mathrm{s}_*^{-1}.$$

Hence $E_* = (m_0)_*c_*^2$. Moreover, $[E] = J_*$, and

$$[m_0][c]^2 = \mathrm{J}_*\mathrm{s}_*^2\mathrm{m}_*^{-2} \cdot \mathrm{m}_*^2\mathrm{s}_*^{-2} = \mathrm{J}_*.$$

This means that

$$\frac{E}{[E]} = \frac{m_0c^2}{[m_0][c]^2}.$$

Physicists frequently use such dimension tests in order to check the correctness of formulas.

Counterexample. Let x and t denote position and time, respectively. The equation

$$x = \sin t$$

is not allowed in the SI system, since it is not invariant under the rescaling $x \Rightarrow \alpha x$ and $t \Rightarrow \beta t$ for nonzero constants α and β. In contrast to this, the equation

$$\frac{x}{x_0} = \sin\left(\frac{t}{t_0}\right)$$

is admissible in any system of units if x and x_0 as well as t and t_0 possess the same dimensions.

Application to Reynolds numbers in turbulence. The motion of a viscous fluid in a 3-dimensional bounded domain $\mathcal{G}$ is governed by the so-called Navier–Stokes equations[16]

$$\begin{aligned} \varrho \mathbf{v}_t - \nu \Delta_{\mathbf{x}} \mathbf{v} + \varrho(\mathbf{v}\nabla_{\mathbf{x}})\,\mathbf{v} &= \mathbf{f} - \nabla_{\mathbf{x}}\, p \quad \text{on } \mathcal{G}, \\ \nabla_{\mathbf{x}}\mathbf{v} &= 0 \quad \text{on } \mathcal{G}, \\ \mathbf{v} &= 0 \quad \text{on } \partial\mathcal{G}. \end{aligned}$$

The symbols possess the following physical meaning: $\mathbf{v}$ velocity vector, ϱ mass density, $\mathbf{f}$ force density vector, p pressure, ν viscosity constant, $\mathbf{x}$ position vector, and t time. Set

$$\mathbf{x} = \mathbf{X} \cdot \mathrm{m}_*, \quad t = T \cdot \mathrm{s}_*, \quad \mathbf{v} = \mathbf{u} \cdot \mathrm{m}_* \mathrm{s}_*^{-1}, \quad \varrho = \Omega \cdot \mathrm{J}_* \mathrm{s}_*^2 \mathrm{m}_*^{-5}$$

and

$$\mathbf{f} = \mathbf{F} \cdot \mathrm{J}_* \mathrm{m}_*^{-4}, \quad p = P \cdot \mathrm{J}_* \mathrm{m}_*^{-3}, \quad \nu = N \cdot \mathrm{J}_* \mathrm{m}_*^{-3},$$

where the coefficients $\mathbf{X}, T, \ldots$ are dimensionless. Furthermore, let d and v denote the diameter of the domain $\mathcal{G}$ and a typical velocity of the fluid, respectively. Naturally enough, we choose

$$\mathrm{m}_* := d, \quad \mathrm{s}_* := dv^{-1}, \quad \mathrm{J}_* := \varrho v^2 d^{-3}.$$

This way, we obtain the rescaled dimensionless Navier–Stokes equations

$$\begin{aligned} \mathbf{u}_t - \mathrm{Re}^{-1} \Delta_{\mathbf{X}}\, \mathbf{u} + (\mathbf{u}\nabla_{\mathbf{X}})\,\mathbf{u} &= \mathbf{F} - \nabla_{\mathbf{X}}\, P \quad \text{on } \mathcal{H}, \\ \nabla_{\mathbf{X}}\mathbf{u} &= 0 \quad \text{on } \mathcal{H}, \\ \mathbf{u} &= 0 \quad \text{on } \partial\mathcal{H} \end{aligned}$$

with the dimensionless Reynolds number

$$\mathrm{Re} := \frac{\varrho v d}{\nu}.$$

The rescaled domain $\mathcal{H}$ is obtained from the original domain $\mathcal{G}$ by replacing the points $\mathbf{x}$ of $\mathcal{G}$ by $d^{-1}\mathbf{x}$. Physical experiments show that if the Reynolds number Re is sufficiently large, then turbulence occurs.

The rescaled dimensionless Navier–Stokes equations reflect an important similarity principle in hydrodynamics. Explicitly, if two physical situations in different regions are governed by the same rescaled dimensionless Navier–Stokes equations, then the physics is the same up to suitable similarity transformations.

Discovery of errors in physical computations. Physicists use physical dimensions in order to detect errors in their computations. To explain this with a simple example, suppose that we arrive at the equation

$$p = c^3 m_0 \tag{A.26}$$

after finishing some computation. Here, we use the following notation: p momentum, m_0 particle mass, c velocity of light. We want to check this. In the SI system, we have the following dimensions:

$$[p] = \mathrm{kg} \cdot \mathrm{ms}^{-1}, \quad [m_0] = \mathrm{kg}, \quad [c] = \mathrm{ms}^{-1}.$$

Hence $[p] = [c] \cdot [m_0]$. It follows from (A.26) that $[p] = [c]^3[m_0]$. This implies $[c]^2 = 1$, which is a contradiction. Consequently, our result (A.26) is wrong. The same argument can be used in the energetic system. However, we now have $[c] = 1$, which does not lead to any contradiction. In other words, the energetic system of units is too weak in order to detect that equation (A.26) is wrong, by checking physical dimensions.

[16] Navier (1785–1836), Stokes (1819–1903).

Table A.3. Fundamental constants in nature

fundamental constant	SI units	natural SI units
velocity of light in a vacuum	$c = 2.998 \cdot 10^8$ m/s	$c = \mathbf{m/s}$
Planck's action quantum	$h = 6.626 \cdot 10^{-34}$ Js $\hbar = h/2\pi$	$\hbar = \mathbf{Js}$
gravitational constant	$G = 6.673 \cdot 10^{-11}$ m^5/Js^4	$G = \mathbf{m^5/Js^4}$ $= l^2c^3/\hbar$
electric field constant	$\varepsilon_0 = 8.854 \cdot 10^{-12}$ C^2/Jm	$\varepsilon_0 = \mathbf{C^2/Jm}$
magnetic field constant	$\mu_0 = 1/\varepsilon_0 c^2$ $= 4\pi \cdot 10^{-7}$ Js^2/C^{2}m	$\mu_0 = \mathbf{Js^2/C^2m}$
Boltzmann constant	$k = 1.380 \cdot 10^{-23}$ J/K	$k = \mathbf{J/K}$
fine structure constant	$\alpha = e^2/4\pi c\hbar\varepsilon_0$ (dimensionless)	$\alpha = 1/137.04$
charge of the proton	$e = 1.602 \cdot 10^{-19}$ C	$e = \sqrt{4\pi\alpha}\,\mathbf{C}$ $= 0.30\,\mathbf{C}$
rest energy of the proton	$E_p = 1.5 \cdot 10^{-10}$ J $= 0.938$ GeV (giga electron volt)	$E_p = 0.77 \cdot 10^{-19}\,\mathbf{J}$
rest mass of the proton	$m_p = 1.672 \cdot 10^{-27}$ kg $= 0.938$ GeV/c^2	$m_p = 0.77 \cdot 10^{-19}\,\mathbf{kg}$
Compton wave length of the proton $\lambda_p = h/m_p c$	$\lambda_p = 1.32 \cdot 10^{-15}$ m $= 1.32$ fm (femtometer)	$\lambda_p = 0.83 \cdot 10^{20}\,\mathbf{m}$
rest energy of the electron	$E_e = 8.16 \cdot 10^{-14}$ J $= 0.511$ MeV (mega electron volt)	$E_e = E_p/1838.1$

Table A.3. (continued)

rest mass of the electron	$m_e = 0.91 \cdot 10^{-30}$ kg	$m_e = m_p/1838.1$
Compton wave length of the electron $\lambda_e = \hbar/m_p c$	$\lambda_e = 2.43 \cdot 10^{-12}$ m	$\lambda_e = 1838.1\,\lambda_p$
radius of the proton	$r_p = 1.3 \cdot 10^{-15}$ m $= 1.3$ fm (femtometer)	$r_p = 0.882 \cdot 10^{20}\, l$
fundamental constant	SI units	natural SI units
Bohr radius of the hydrogen atom	$r_B = 0.529 \cdot 10^{-10}$ m $= 5.29$ nm (nanometer)	$r_B = 40000\, r_p$
Bohr magneton	$\mu_B = -e\hbar/2m_e$ $= -9.27\, 10^{-24}$ mC/s	$\mu_B = -\,\mathbf{mC/s}$
magnetic moment of the electron	$\mu_e = \left(1 + \frac{\alpha}{2\pi} - ...\right)\mu_B$	$\mu_e = 1.001\mu_B$
nuclear magneton	$\mu_n = e\hbar/2m_p$ $= 5.05\, 10^{-27}$ mC/s	$\mu_B = 1836.1\mu_n$
magnetic moment of the proton	$\mu_p = 2.79\mu_n$	$\mu_p = 2.79\mu_n$

More precise values can be found in CODATA Bull. **63** (1986), and E. Cohen and B. Taylor, Review of Modern Physics **59**(4) (1986). A list of high-precision values can also be found in the Appendix to Zeidler, Oxford User's Guide to Mathematics, Oxford University Press, 2004. In the following Table A.4, observe that the two quantities **E** and c**B**, possess the same physical dimension in the SI system. The same is true for c**D** and **H**. Here, we use the notation:

- **E** electric field vector,
- **B** magnetic field vector,
- **D** electric field intensity vector,
- **H** magnetic field intensity vector.

In the literature, the terminology with respect to **E**, **B**, **D**, **H** is not uniform, for historical reasons. Since **E** and **B** generate the electromagnetic field tensor (see (14.51) on page 794), it follows from Einstein's theory of special relativity that the vector fields **E** and **B** (resp. **D** and **H**) form a unit. The mean magnetic field of earth has the strength $\mathbf{B}_{\text{earth}} = 0.5$ Gauss $= 0.5 \cdot 10^{-4}$ Tesla.

Table A.4. Units of physical quantities

Physical quantity	SI units $m^\alpha s^\beta J^\gamma C^\mu K^\nu$	natural SI units $\mathbf{m}^\alpha \mathbf{s}^\beta \mathbf{J}^\gamma \mathbf{C}^\mu \mathbf{K}^\nu = l^A c^B \hbar^C \varepsilon_0^D k^E$
length	m (meter)	$\mathbf{m} = l$ (Planck length)
time	s (second)	$\mathbf{s} = l/c$ (Planck time)
energy, work	J (Joule)	$\mathbf{J} = \hbar c/l$ (Planck energy)
electric charge	C (Coulomb)	$\mathbf{C} = (c\hbar\varepsilon_0)^{\frac{1}{2}}$ (Planck charge)
temperature	K (Kelvin)	$\mathbf{K} = \hbar c/lk$ (Planck temperature)
mass	$kg = Js^2/m^2$ (kilogram)	$\mathbf{J}\mathbf{s}^2/\mathbf{m}^2 = \hbar/cl$ (Planck mass)
electric current strength	$A = C/s$ (ampere)	$\mathbf{C}/\mathbf{s} = c^{\frac{3}{2}}(\hbar\varepsilon_0)^{\frac{1}{2}}/l$
voltage	$V = J/C$ (volt)	$\mathbf{J}/\mathbf{C} = (c\hbar)^{\frac{1}{2}}/l\varepsilon_0^{\frac{1}{2}}$
action (energy × time)	Js	$\mathbf{Js} = \hbar$
momentum (mass × velocity)	Js/m	$\mathbf{Js}/\mathbf{m} = \hbar/l$
power (energy/time)	$W = J/s$ (Watt)	$\mathbf{J}/\mathbf{s}$
force (energy/length)	$N = J/m$ (Newton)	$\mathbf{J}/\mathbf{m}$
frequency ν (number of oscillations/time)	1/s	$1/\mathbf{s}$
angular frequency $\omega = 2\pi\nu$	1/s	$\mathbf{1}/\mathbf{s}$
pressure (force/area)	$Pa = N/m^2 = J/m^3$	$\mathbf{J}/\mathbf{m}^3$
area, cross section	m^2	$\mathbf{m}^2 = l^2$
volume	m^3	$\mathbf{m}^3 = l^3$

Table A.4. (continued)

Physical quantity	SI units $\mathrm{m}^\alpha\mathrm{s}^\beta\mathrm{J}^\gamma\mathrm{C}^\mu\mathrm{K}^\nu$	natural SI units $\mathbf{m}^\alpha\mathbf{s}^\beta\mathbf{J}^\gamma\mathbf{C}^\mu\mathbf{K}^\nu = l^A c^B \hbar^C \varepsilon_0^D k^E$
velocity	m/s	$\mathbf{m/s} = c$
acceleration	$\mathrm{m/s^2}$	$\mathbf{m/s^2} = c^2/l$
mass density	$\mathrm{kg/m^3 = Js^2/m^5}$	$\mathbf{Js^2/m^5}$
electric charge density ρ (charge/volume)	$\mathrm{C/m^3}$	$\mathbf{C/m^3}$
electric current density vector $\mathbf{j} = \rho\mathbf{v}$	$\mathrm{C/m^2s}$	$\mathbf{C/m^2s}$
electric field vector $\mathbf{E}$ (force/charge)	$\mathrm{N/C = V/m = J/mC}$	$\mathbf{J/mC}$
magnetic field vector $\mathbf{B}$	$\mathrm{T = Vs/m^2 = Js/m^2C}$ (Tesla)	$\mathbf{Js/m^2C}$
magnetic flow $\int \mathbf{B}\, d\mathbf{f}$	$\mathrm{Wb = Vs = Js/C}$ (Weber)	$\mathbf{Js/C}$
electric field intensity vector $\mathbf{D}$	$\mathrm{C/m^2}$	$\mathbf{C/m^2}$
magnetic field intensity vector $\mathbf{H}$	$\mathrm{A/m = C/sm}$	$\mathbf{C/sm}$
electric dipole moment	Cm	$\mathbf{Cm}$
magnetic dipole moment	$\mathrm{Am^2 = m^2C/s}$	$\mathbf{m^2C/s}$
polarization $\mathbf{P}$ (electric dipole moment density), $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$	$\mathrm{C/m^2}$	$\mathbf{C/m^2}$
magnetization $\mathbf{M}$ (magnetic dipole moment density), $\mathbf{B} = \mu_0\mathbf{H} + \mathbf{M}$	C/ms	$\mathbf{C/ms}$
scalar potential U ($\mathbf{E} = -\,\mathbf{grad}\, U - \mathbf{A_t}$)	$\mathrm{V = J/C}$	$\mathbf{J/C}$
vector potential $\mathbf{A}$ ($\mathbf{B} = \mathbf{curl}\,\mathbf{A}$)	$\mathrm{Vs/m = Js/mC}$	$\mathbf{Js/mC}$
4-potential A^μ ($A^0 = U/c$, $\mathbf{A} = A^j\mathbf{e}_j$)	Js/mC	$\mathbf{Js/mC}$

Table A.4. (continued)

Physical quantity	SI units $m^{\alpha}s^{\beta}J^{\gamma}C^{\mu}K^{\nu}$	natural SI units $\mathbf{m}^{\alpha}\mathbf{s}^{\beta}\mathbf{J}^{\gamma}\mathbf{C}^{\mu}\mathbf{K}^{\nu}$ $= l^A c^B \hbar^C \varepsilon_0^D k^E$
electromagnetic field tensor $F_{\mu\nu}$ $(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu)$	$\mathrm{Vs/m^2} = \mathrm{Js/m^2C}$	$\mathbf{Js/m^2C}$
electric 4-current j^μ $(j^0 = c\rho,\ \mathbf{j} = j^k \mathbf{e}_k)$	$\mathrm{C/m^2s}$	$\mathbf{C/m^2s}$
Schrödinger function ψ in N space dimensions (solution of the Schrödinger equation)	$\mathrm{m}^{-\frac{N}{2}}$	$\mathbf{m}^{-\frac{N}{2}} = l^{-\frac{N}{2}}$
Dirac function ψ (solution of the Dirac equation, electron field, quark field, fermion fields)	$1/\mathrm{ms}^{\frac{1}{2}}$	$\mathbf{1/ms}^{\frac{1}{2}} = c^{\frac{1}{2}}/l^{\frac{3}{2}}$
Lagrangian L in classical mechanics action $= \int_{t_0}^{t_1} L(q, \dot{q}, t)\, dt$	J	$\mathbf{J} = \hbar c/l$
Lagrangian density $\mathcal{L}$ in relativistic field theory action $= \int_{\mathbb{R}^4} \mathcal{L}(\psi, \partial\psi, x)\, d^4x$	$\mathrm{Js/m^4}$	$\mathbf{Js/m^4} = \hbar/l^4$
Hamiltonian H in classical mechanics, $H = p\dot{q} - L$	J	$\mathbf{J} = \hbar c/l$
Hamiltonian density $\mathcal{H} = \pi\dot{\psi} - \mathcal{L}$	$\mathrm{Js/m^4}$	$\mathbf{Js/m^4} = \hbar/l^4$
4-potential B_μ of the gluon field in QCD, $(\mathrm{i}B_\mu \in SU(3))$	Js/m	$\mathbf{Js/m} = \hbar/l$
field tensor $G_{\mu\nu}$ of the gluon field $\big(G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + \mathrm{i}g_s[B_\mu, B_\nu]\big)$	$\mathrm{Js/m^2}$	$\mathbf{Js/m^2} = \hbar/l^2$

Epilogue

Mathematics is the gate and the key to the sciences.
Roger Bacon (1214–1294)

I love mathematics not only because it is applicable to technology but also because it is beautiful.
Rósza Péter (1905–1977)

The perfection of mathematical beauty is such whatsoever is most beautiful is also found to be most useful and excellent.
D'Arcy Wentworth Thompson (1860–1948)

The observation which comes closest to an explanation for the mathematical concepts cropping up in physics which I know is Einstein's statement that the only physical theories we are willing to accept are the beautiful ones.
Eugene Wigner (1902–1995)

A truly realistic mathematics should be conceived, in line with physics, as a branch of the theoretical construction of the one real world, and should adopt the same sober and cautious attitude toward hypothetic extensions of its foundations as is exhibited by physics.
Hermann Weyl (1885–1955)

The interplay between generality and individuality, deduction and construction, logic and imagination – this is the profound essence of live mathematics.
Any one or another of the aspects can be at the center of a given achievement. In a far-reaching development all of them will be involved. Generally speaking, such a development will start from the "concrete ground," then discard ballast by abstraction and rise to the lofty layers of thin air where navigation and observations are easy; after this flight comes the crucial test of landing and reaching specific goals in the newly surveyed low plains of individual "reality."
In brief, the flight into abstract generality must start from and return to the concrete and specific.[17]
Richard Courant (1888–1972)

[17] Mathematics in the modern world, Scientific American **211**(3) (1964), 41–49 (reprinted with permission).

There are mathematicians who reject a binding of mathematics to physics, and who justify mathematical work solely by aesthetical satisfaction which, besides all the difficulty of the material, mathematics is able to offer. Such mathematicians are more likely to regard mathematics as a form of art than science, and this point of view of mathematical unselfishness can be characterized by the slogan "l'art pour l'art".
On the other hand, there are physicists who regret that their science is so much related to mathematics. They fear a loss of intuition in the natural sciences. They consider the intimate relation with nature, the finding of ideas in nature itself, which was given to Goethe (1749–1832) in such a high degree, as being destroyed by mathematics, and their anger or sorrow is the more serious the more they are forced to realize the inevitability of mathematics.
Both points of view deserve serious consideration; because not only people with narrow minds have expressed such opinions. Yes, one can say that such a radical inclination to one side or the other, if not caused by a lack of talent, is sometimes evidence of a deeper perception of science, as if someone is interested in both sciences, but at the same time is satisfied with obvious connections between mathematics and physics...
Mathematics is an organ of knowledge and an infinite refinement of language. It grows from the usual language and world of intuitions as does a plant from the soil, and its roots are the numbers and simple geometrical intuitions. We do not know which kind of content mathematics (as the only adequate language) requires; we cannot imagine into what depths and distances this spiritual eye will lead us.[18]

Erich Kähler (1906–2000)

The most vitally characteristic fact about mathematics, in my opinion, is its quite peculiar relationship to the natural sciences, or more generally, to any science which interprets experience on a higher than purely descriptive level...
I think that this is a relatively good approximation to truth – which is much too complicated to allow anything but approximations – that mathematical ideas originate in *empirical facts*, although the genealogy is sometimes long and obscure. But, once they are so conceived, the subject begins to live a peculiar life of its own and is better compared to a creative one, governed by almost entirely aesthetic motivations, than to anything else and, in particular, to an empirical science.
But there is a grave danger that the subject will develop along the line of least resistance, that the stream, so far from its source, will separate into a multitude of insignificant tributaries, and that the discipline will become a disorganized mass of details and complexities. In other words, at a great distance from its empirical sources or after much *abstract inbreeding*, a mathematical object is in danger of degeneration. At the inception, the style is usually classical; when it shows signs of becoming *baroque*, then the danger signal is up...
Whenever this stage is reached, then the only remedy seems to be a rejuvenating return to the source: the re-injection of more or less directly *empirical ideas.* I am convinced that this is a necessary condition to con-

[18] On the relations of mathematics to physics and astronomy (in German), Jahresberichte der Deutschen Mathematiker-Vereinigung **51** (1941), 52–63 (reprinted with permission).

serve the freshness and the vitality of the subject and that this will remain equally true in the future.[19]

John von Neumann (1903–1957)

I want to say a word about the communication between mathematicians and physicists.
It has been very bad in the past, and some of the blame is doubtless to be laid on the physicist's shoulders. We tend to be very vague, and we don't know what the problem is until we have already seen how to solve it. We drive mathematicians crazy when we try to explain what our problems are. When we write articles we don't do a good enough job of specifying how certain we are about our statements; we do not distinguish guesses from theorems.
On the other hand, since I have said a lot of nice things about mathematics, I have to say that the mathematicians carry an even greater burden of guilt for this communication problem, largely because of their elitism. They often have, it seems to me, as their ideal the savant who is understandable only to a few co-specialists and who writes articles that one has to spend years to try to fathom.
When physicists write articles, they generally start them with a paragraph saying, "Up until now, this has been thought to be the case. Now, so – and – so has pointed out this problem. In this article, we are going to try to suggest a resolution of this difficulty." On the other hand, I have seen books of mathematics, not just articles but books, in which the first sentence in the preface was, "Let H be a nilpotent subgroup of..." These books are written in what I would call a lapidary style. The idea seems to be that there should be no word in the book that is not absolutely necessary, that is inserted merely to help the reader to understand what is going on.

I think this is getting much better. I find it is wonderful how mathematicians these days are willing to explain their field to interested physicists. This situation is improving, partly because as Iz Singer mentioned, we realize now that in certain areas we have much more in common than we had thought, but I think a lot more has to be done. There is still too much mathematics written which is not only not understandable to experimental or theoretical physicists, but is not even understandable to mathematicians who are not the graduate students of the author.[20]

Steven Weinberg (born 1933)

Relations between mathematics and physics vary with time. Right now, and for the past few years, harmony reigns and a honeymoon blossoms. However, I have seen other times, times of divorce and bitter battles, when the sister sciences declared each other as useless – or worse. The following exchange between a famous theoretical physicist and an equally famous mathematician might have been typical, some fifteen or twenty years ago: Says the physicist: "I have no use for mathematics. All the mathematics I ever need, I invent in one week."

[19] The Mathematician. In: The Works of the Mind, Vol. 1, pp. 180–196. Edited by R. Heywood, University of Chicago Press, 1947 (reprinted with permission).

[20] Mathematics: The unifying thread in science: Notices Amer. Math. Soc. **33** (1986), 716–733 (reprinted with permission).

Answers the mathematician: “You must mean the seven days it took the Lord to create the world.”
A slightly more reliable document is found in the preface of the first edition of Hermann Weyl’s book on group theory and quantum mechanics from 1928. He writes: “I cannot abstain from playing the role of an (often unwelcome) intermediary in this drama between mathematics and physics, which fertilize each other in the dark, and deny and misconstrue one another when face to face.”
This dramatic situation, described here by one of the great masters in both sciences, is a result of recent times. At the time of Newton (1643–1727) disharmony between mathematics and physics seemed unthinkable and unnatural, since both were his brainchildren; and close symbiosis persisted through the whole of the eighteenth century. The rift arose around 1800 and was caused by the development of pure mathematics (represented by number theory) on the one hand, and of a new kind of physics, independent of mathematics, which developed out of chemistry, electricity and magnetism on the other. This rift was widened in Germany under the influence of Goethe (1749–1832) and his followers, Schelling (1775–1854) and Hegel (1770–1831) and their “Naturphilosophie”.
Our protagonists are Carl Friedrich Gauss (1777–1855), as the creator of modern number theory, and Michael Faraday (1791-1867) as the inventor of physics without mathematics (in the strict sense of the word).
It would be foolish, of course, to claim the nonexistence of number theory before Gauss. An amusing document may illustrate the historical development. Erich Hecke’s famous *Lectures on the Theory of Algebraic Numbers* has on its last page a “timetable”, which chronologically lists the names and dates of the great number theoreticians, starting with Euclid (300 B.C.) and ending with Hermann Minkowski (1864–1909). As a physicist, I am impressed to find so many familiar names in this Hall of Fame: Fermat (1601–1665), Euler (1707–1783), Lagrange (1736–1813), Legendre (1752–1833), Fourier (1768–1830), and Gauss. In fact, we cannot find a single great number theoretician before Gauss, whom we would not count among the great physicists, provided we disregard antiquity. Specialization starts after 1800 with names like Kummer (1810–1891), Galois (1811–1832), and Eisenstein (1823–1852); who were all under the great influence of Gauss’ *Disquisitiones arithmeticae* from 1801. In this specific sense, Gauss’ book marks the dividing line between mathematics as a universal science and mathematics as a union of special disciplines, and between the “géomètre” as a universal “savant” in the sense of the eighteenth century and the specialized “mathématicien” of modern times. As is typical for a man of transition, Gauss does not belong to either category, he was universal and specialized. The struggle raged within him – and made him suffer.

Res Jost (1918–1990)
Mathematics and physics since 1800: discord and sympathy[21]

By a particular prerogative, not only does each man advance day by day in the sciences, but all men together make continual progress as the universe ages... Thus, the entire body of mankind as a whole, over many centuries, must be considered as a single man, who lives forever and continues to learn.

Blaise Pascal (1623–1662)

[21] In: R. Jost, The Fairy Tale about the Ivory Tower, essays and lectures. Edited by K. Hepp, W. Hunziker, and W. Kohn, Springer, Berlin, pp. 219–240 (reprinted with permission).

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[1] Hints for further reading can be found in Chap. 17. The author's homepage contains a complete list of the references to Volumes I through VI.
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List of Symbols

$f(x) := x^2$ (definition of f)
$f(x) \simeq g(x),\ x \to a$ (asymptotic equality); this means $\lim_{x\to a} \frac{f(x)}{g(x)} = 1$, 949
$f(x) = o(g(x)),\ x \to a$ (Landau symbol); this means $\lim_{x\to a} \frac{f(x)}{g(x)} = 0$, 949
$f(x) = O(g(x)),\ x \to a$, 949
$f(x) \sim \sum_{n=0}^{\infty} a_n x^n$ (asymptotic expansion), 308, 863

$\mathrm{sgn}(a)$ (sign of the real number a), 949
$[a,b],\ ]a,b[,\ ]a,b]$ (intervals), 949
$\sum_{n=-\infty}^{\infty} b_n$, 215
δ_{ij} (Kronecker symbol), 949
$\delta_{11} := 1,\ \delta_{12} := 0$
$\delta^{ij} = \delta_{ij} = \delta^i_j$, 949
$\delta_{\mathbf{pq}}$, 672
ε_{ij} (skew-symmetric symbol)
$\varepsilon_{12} = -\varepsilon_{21} = 1,\ \varepsilon_{11} = \varepsilon_{22} = 0$, 337

x, y, z (right-handed Cartesian coordinates)
$\mathbf{i}, \mathbf{j}, \mathbf{k}$ (right-handed orthonormal basis)
$\mathbf{x} := x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ (position vector)
$||\mathbf{x}||$ (length (norm) of the vector $\mathbf{x}$)

t (time)
$x^1 := x, x^2 := y, x^3 := z, x^0 := ct$ (space-time point in Minkowski space), 949
$\mu = 0, 1, 2, 3$ (indices for space-time variables in Minkowski space), 949
$j = 1, 2, 3$ (indices for spatial variables in Minkowski space), 949
$\eta_{\mu\nu}$ (Minkowski symbol), $\eta_{00} := 1$, $\eta_{11} := -1,\ \eta_{01} := 0$, 949
$\eta^{\mu\nu} = \eta_{\mu\nu} = \eta^\nu_\mu$, 949
$\epsilon^{\alpha\beta\gamma\delta}, \epsilon_{\alpha\beta\gamma\delta}$ (skew-symmetric symbol)
$\epsilon_{0123} := 1,\ \epsilon_{1023} := -1$, 950

$a_\mu b^\mu := \sum_{\mu=0}^{3} a_\mu b^\mu$ (Einstein's convention in Minkowski space), 950
$\zeta(s)$ (zeta function), 280
B_n (Bernoulli number), 280

$\theta(t)$ (Heaviside function), 92, 579
$\delta(t)$ (Dirac delta function), 593
δ, δ_x (Dirac delta distribution), 612
δ_μ (Dirac delta function with respect to the measure μ), 605
$\delta_{\Delta t}(t)$ (standard discrete Dirac delta function), 582
$\delta_{\Delta^4 x}, \delta_{\mathcal{C}(L)}, \delta_{\mathcal{G}(N)}, \delta_T, \delta_{\mathrm{dis}}$ (discrete Dirac delta functions), 443, 672
$\mathcal{C}(N)$ (cube in position space), 671
$\mathcal{G}(N)$ (grid in momentum space), 671
$\Delta^3 p$, 672
$\mathcal{V}$ (normalization volume), 671
$\delta(x^2 - a^2)$ (special distribution), 598
$\mathcal{P}\left(\frac{1}{x}\right)$ (special distribution), 621
$\frac{1}{x \pm 0_+ \mathrm{i}}$ (special distribution), 623
$\mathcal{P}(\ln|x|)$ (special distribution), 738

I, id (identity operator)
$x \in U$ (the point x is an element of U)
$U \subseteq V$ (U is a subset of V)
$U \subset V$ (U is a proper subset of V), 947
$U \cup V$ (the union of two given sets U and V)
$U \cap V$ (the intersection of two given sets U and V)
$U \setminus V$ (the difference of two sets U and V, i.e., the set of elements of U not belonging to V)
∂U (boundary of the set U)
$\mathrm{int}(U)$ (interior of U)
$\mathrm{cl}(U) \equiv U \cup \partial U$ (closure of U), 545
$\emptyset$ (empty set)
$\{x : x \text{ has the property } \mathcal{P}\}$ (the set of all things which have the property $\mathcal{P}$)

$\partial\!\!\!/$, $\nabla\!\!\!\!/$ (Feynman's slash symbols), 794

c (velocity of light in a vacuum), 965
h (Planck's quantum of action), $\hbar \equiv h/2\pi$, 965
k (Boltzmann constant), 965
G (gravitational constant), 965
ε_0 (electric field constant of a vacuum), 965
μ_0 (magnetic field constant of a vacuum), 965
e (electric charge of the proton), 965
$-e$ (electron charge)
m_e (electron mass), 965
α (fine structure constant), 965
λ_C (Compton wave length), 144
$\bar{\lambda}_C \equiv \lambda_C/2\pi$ (reduced Compton wavelength), 144

m (meter), 950
s (second), 950
J (Joule), 950
C (Coulomb), 950
K (Kelvin), 950
m, **s**, **J**, **C**, **K** (Planck units), 953
eV (electron volt), 953
MeV (mega electron volt), 953
GeV (giga electron volt), 953

$\hat{\varrho}$ (statistical operator), 760
ϱ (density operator), 760
$\beta \equiv 1/kT$, 760

$P(t,s)$ (propagator), 385
$P^+(t,s)$ (retarded propagator), 386
$P^-(t,s)$ (advanced propagator), 386
$S[\varphi]$ (action of the field φ), 754
$\mathsf{S}(s,t)$ (S-matrix on the time interval $[t,s]$), 392

$\mathsf{S}(T)$ (S-matrix on $[-\frac{T}{2},\frac{T}{2}]$), 824
$|0\rangle, \Phi_0$ (ground state of a free quantum field)
$|0_{\text{int}}\rangle, \Phi_{\text{int}}$ (ground state of a quantum field under interactions)
$\mathcal{T}$ (chronological operator), 390, 746
$:AB:$ (normal product), 824

$Z(J), C_n, \mathcal{C}_n, G_n$ (discrete model of a quantum field), 446
$Z(J,\varphi), Z_{\text{free}}(J,\varphi)$, 450
S_n, 452
φ_{mean}, 459
Z_{red}, 461
V_n, 462
$C_{n,\text{free}}$, 468

$Z(J)$ (full generating functional), 751
$Z_{\text{free}}(J)$ (free generating functional), 751
C_n (full n-point correlation function), 746
$C_{n,\text{free}}$ (free n-point correlation function), 745
G_n (full n-point Green's function), 746
$G_{n,\text{free}}$ (free n-point Green's function), 745

$\frac{1}{p^2-m_0^2+0_+\mathrm{i}}$ (special distribution), 782
$\mathcal{G}_{F,m_0}$(Feynman propagator for mesons), 777
G_{F,m_0} (Feynman propagator distribution for mesons), 780
$\mathsf{G}_F \equiv \mathsf{G}_{F,m_0=0}$ (Feynman propagator distribution for the wave equation)
$\mathsf{D}_F^{\alpha\beta} \equiv -\eta^{\alpha\beta}\mathsf{G}_F$ (Feynman propagator distribution for photons), 802
$\mathsf{S}_F \equiv (\mathrm{i}\gamma^\alpha\partial_\alpha + m_e)\mathsf{G}_{F,m_e}$ (Feynman propagator distribution for electrons), 802

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