

Appendix

For the convenience of the reader, let us collect basic material on the following topics:

- notation,
- physical units, the Planck system, and the energetic system of units,
- the Gaussian system of units and the Heaviside system, and
- the method of dimensional analysis – a magic wand of physicists.

A comprehensive table on the units of the most important physical quantities can be found at the end of the Appendix on page 967.

A.1 Notation

Sets and mappings. The abbreviation ‘iff’ stands for ‘if and only if’. To formulate definitions, we use the symbol ‘:=’. For example, we write

$$f(x) := x^2$$

iff the value $f(x)$ of the function f at the point x is equal to x^2 , by definition.

The symbol $U \subseteq V$ (resp. $U \subset V$) means that U is a subset (resp. a proper subset) of V . This convention resembles the symbols $x \leq y$ (resp. $x < y$) for real numbers. A map

$$f : X \rightarrow Y$$

sends each point x living in the set X to an image point $f(x)$ living in the set Y . The set X is also called the domain of definition, $\text{dom}(f)$, of the map f . By definition, the image, $\text{im}(f)$, of the map f is the set of all image points $f(x)$. Furthermore, the set

$$f(U) := \{f(x) : x \in U\}$$

is called the image of the set U by the map f . In other words, by definition, the set $f(U)$ contains precisely all the points $f(x)$ with the property that x is an element of the set U . The set

$$f^{-1}(V) := \{x \in X : f(x) \in V\}$$

is called the pre-image of the set V by the map f .

- The map f is called *surjective* iff each point of the set Y is an image point. In this case, we also say that f maps the set X ‘onto’ the set Y . The French word ‘sur’ means ‘onto’.
- The map f is called *injective* iff $x_1 \neq x_2$ always implies $f(x_1) \neq f(x_2)$. Such maps are also called ‘one-to-one’.

- The map f is called *bijective* iff it is both surjective and injective. Precisely in this case, the inverse map $f^{-1} : Y \rightarrow X$ exists.

For each given point y in the set Y , consider the equation

$$\boxed{f(x) = y, \quad x \in X,} \tag{A.1}$$

that is, we are looking for a solution x in the set X . Observe that the map f is surjective (resp. bijective) iff the equation (A.1) has always at least one (resp. precisely one) solution. The map f is injective iff the equation has always at most one solution.

Inverse map. If the map $f : X \rightarrow Y$ is bijective, then the inverse map

$$f^{-1} : Y \rightarrow X$$

is defined by $f^{-1}(y) := x$ iff $f(x) = y$.

Sets of numbers. The symbol \mathbb{K} always stands either for the set \mathbb{R} of real numbers or the set \mathbb{C} of complex numbers. The real number x is called positive, negative, nonnegative, non-positive iff

$$x > 0, \quad x < 0, \quad x \geq 0, \quad x \leq 0,$$

respectively. The symbols

$$\mathbb{R}^\times, \quad \mathbb{R}_>, \quad \mathbb{R}_<, \quad \mathbb{R}_\geq, \quad \mathbb{R}_\leq$$

denote the set of nonzero real numbers, positive real numbers, negative real numbers, nonnegative real numbers, non-positive real numbers, respectively.⁹ Concerning the sign of a real number, we write $\text{sgn}(x) := 1, -1, 0$ if $x > 0, x < 0, x = 0$, respectively.

For a given complex number $z = x + yi$, we introduce both the conjugate complex number $z^\dagger := x - yi$ and the modulus

$$\boxed{|z| := \sqrt{zz^\dagger} = \sqrt{x^2 + y^2}.$$

The real (resp. imaginary) part of z is denoted by $\Re(z) := x$ (resp. $\Im(z) := y$). The definition of the principal argument, $\arg(z)$, of the complex number z can be found on page 211. Traditionally,

- the symbol \mathbb{Z} denotes the set of integers $0, \pm 1, \pm 2, \dots$,
- the symbol \mathbb{N} denotes the set of nonnegative integers $0, 1, 2, \dots$ (also called natural numbers),¹⁰
- the symbol \mathbb{N}^\times denotes the set of positive integers $1, 2, \dots$, and
- the symbol \mathbb{Q} denotes the set of rational numbers.

For closed, open, and half-open intervals, we use the notation

$$[a, b] := \{x \in \mathbb{R} : a \leq x \leq b\}, \quad]a, b[:= \{x \in \mathbb{R} : a < x < b\},$$

and $]a, b] := \{x \in \mathbb{R} : a < x \leq b\}$, as well as $[a, b[:= \{x \in \mathbb{R} : a \leq x < b\}$.

The Landau symbols. Around 1900 the following symbols were introduced by the number theorist Edmund Landau (1877–1938). We write

⁹ For the closed half-line \mathbb{R}_\geq , one also uses the symbol \mathbb{R}_+ .

¹⁰ For the set \mathbb{N} , one also uses the symbol \mathbb{Z}_\geq .

$$\boxed{f(x) = o(g(x)) \quad \text{as } x \rightarrow a}$$

iff $f(x)/g(x) \rightarrow 0$ as $x \rightarrow a$. For example, $x^2 = o(x)$ as $x \rightarrow 0$. The symbol

$$f(x) = O(g(x)) \quad \text{as } x \rightarrow a \tag{A.2}$$

tells us that $|f(x)| \leq \text{const } |g(x)|$ in a sufficiently small, open neighborhood of the point $x = a$. For example, $2x = O(x)$ as $x \rightarrow 0$. We write

$$f(x) \simeq g(x), \quad x \rightarrow a$$

iff $f(x)/g(x) \rightarrow 1$ as $x \rightarrow a$. For example,

$$\sin x \simeq x, \quad x \rightarrow 0.$$

Relativistic physics. In an inertial system, we set

$$x^1 := x, \quad x^2 := y, \quad x^3 := z, \quad x^0 := ct$$

where x, y, z are right-handed Cartesian coordinates, t is time, and c is the velocity of light in a vacuum. Generally,

- Latin indices run from 1 to 3 (e.g., $i, j = 1, 2, 3$), and
- Greek indices run from 0 to 3 (e.g., $\mu, \nu = 0, 1, 2, 3$).

In particular, we use the Kronecker symbols

$$\delta_{ij} = \delta^{ij} = \delta_j^i := \begin{cases} 1 & \text{if } i = j, \\ 0 & \text{if } i \neq j, \end{cases} \tag{A.3}$$

and the Minkowski symbols

$$\eta_{\mu\nu} = \eta^{\mu\nu} := \begin{cases} 1 & \text{if } \mu = \nu = 0, \\ -1 & \text{if } \mu = \nu = 1, 2, 3, \\ 0 & \text{if } \mu \neq \nu. \end{cases} \tag{A.4}$$

Einstein's summation convention. In the Minkowski space-time, we always sum over equal upper and lower Greek (resp. Latin) indices from 0 to 3 (resp. from 1 to 3). For example, for the position vector, we have

$$\mathbf{x} = x^j \mathbf{e}_j := \sum_{j=1}^3 x^j \mathbf{e}_j,$$

where $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ are orthonormal basis vectors of a right-handed orthonormal system. Moreover,

$$\eta_{\mu\nu} x^\nu := \sum_{\nu=0}^3 \eta_{\mu\nu} x^\nu.$$

Greek indices are lowered and lifted with the help of the Minkowski symbols. That is,

$$x_\mu := \eta_{\mu\nu} x^\nu, \quad x^\mu = \eta^{\mu\nu} x_\nu.$$

Hence

$$\boxed{x_0 = x^0, \quad x_j = -x^j, \quad j = 1, 2, 3.}$$

For the indices $\alpha, \beta, \gamma, \delta = 0, 1, 2, 3$, we introduce the antisymmetric symbol $\epsilon^{\alpha\beta\gamma\delta}$ which is normalized by

$$\epsilon^{0123} := 1, \quad (\text{A.5})$$

and which changes sign if two indices are transposed. In particular, $\epsilon^{\alpha\beta\gamma\delta} = 0$ if two indices coincide. For example, $\epsilon^{0213} = -1$ and $\epsilon^{0113} = 0$. Lowering of indices yields $\epsilon_{\alpha\beta\gamma\delta} := -\epsilon^{\alpha\beta\gamma\delta}$. For example, $\epsilon_{0123} := -1$.

The Minkowski metric. Unfortunately, there exist two different conventions in the literature, namely, the so-called west coast convention (W) which uses the following Minkowski metric,

$$\boxed{\eta_{\mu\nu}x^\mu x^\nu = c^2t^2 - x^2 - y^2 - z^2}, \quad (\text{A.6})$$

and the east coast convention (E) based on $-c^2t^2 + x^2 + y^2 + z^2$. (This refers to the east and west coast of the United States of America.) From the mathematical point of view, the east coast convention has the advantage that there does not occur any sign change when passing from the Euclidean metric

$$x^2 + y^2 + z^2$$

to the Minkowski metric. From the physical point of view, the west coast convention has the advantage that the Minkowski square of the momentum-energy 4-vector $(\mathbf{p}, E/c)$ is positive,

$$\eta_{\mu\nu}p^\mu p^\nu = \frac{E^2}{c^2} - \mathbf{p}^2 = m_0^2c^2. \quad (\text{A.7})$$

Here, m_0 denotes the rest mass of the particle. Since most physicists and physics textbooks use the west coast convention, we will follow this tradition, which dates back to Einstein's papers, Dirac's 1930 monograph *Foundations of Quantum Mechanics* and Feynman's papers. Concerning elementary particles, we use the same terminology as in the standard textbook by Peskin and Schroeder (1995). One can easily pass from our convention to the east coast convention by using the replacements

$$\eta_{\mu\nu} \mapsto -\eta_{\mu\nu}, \quad \gamma^\mu \mapsto -i\gamma^\mu$$

for the Minkowski metric and the Dirac-Pauli matrices, γ^μ , from the Dirac equation (A.20), respectively.¹¹

A.2 The International System of Units

The ultimate goal of physicists is to measure physical quantities in physical experiments. To this end, physicists have to compare the quantity under consideration with appropriate standard quantities. For example, the measurement of the length of a distance can be obtained by comparing the length with the standard length m (meter). This procedure leads to systems of physical units.

The SI system. In the international system of units, SI (for *Système International* in French), the following basic units are used:

¹¹ For example, the east coast convention is used in Misner, Thorne, and Wheeler (1973), and in Weinberg (1995).

Table A.1. Prefixes in the SI system

10^{-1}	deci	d	10	deka	D
10^{-2}	centi	c	10^2	hecto	H
10^{-3}	milli	m	10^3	kilo	K
10^{-6}	micro	μ	10^6	mega	M
10^{-9}	nano	n	10^9	giga	G
10^{-12}	pico	p	10^{12}	tera	T
10^{-15}	femto	f	10^{15}	peta	P

- length: m (meter),
- time: s (second),
- energy: J (Joule),
- electric charge: C (Coulomb),
- temperature: K (Kelvin).

Each physical quantity q can be uniquely represented as

$$q = q_{\text{SI}} \cdot \text{m}^{\alpha} \text{s}^{\beta} \text{J}^{\gamma} \text{C}^{\mu} \text{K}^{\nu}. \quad (\text{A.8})$$

Here, q_{SI} is a real number, and the exponents $\alpha, \beta, \gamma, \mu, \nu$ are rational numbers. Physicists say that the physical quantity q has the dimension

$$(\text{length})^{\alpha} (\text{time})^{\beta} (\text{energy})^{\gamma} (\text{electric charge})^{\mu} (\text{temperature})^{\nu}.$$

Let us consider a few examples.

- The unit of mass is the kilogram, $\text{kg} := \text{J s}^2 \text{m}^{-2}$.
- The unit of force is the Newton, $\text{N} = \text{J m}^{-1}$.
- The unit of electric current strength is the Ampere, $\text{A} := \text{C s}^{-1}$.

The physical dimensions of the most important physical quantities in the SI system can be found in Table A.4 on page 967. Instead of meter one also uses kilometer, nanometer, femtometer, and so on, which corresponds to

$$1000\text{m}, \quad 10^{-9}\text{m}, \quad 10^{-15}\text{m},$$

respectively (see Table A.1).

The universal character of the SI system. Unfortunately, for historical reasons, there exist many different systems of units used by physicists. In what follows we want to help the reader to understand the relations between the different systems. Let us explain the following.

If one knows the physical dimension of some quantity in the SI system, then one can easily pass to every other system used in physics.

In particular, we will discuss

- the natural SI system,
- the Planck system, and
- the energetic system.

The Planck system has the advantage that the fundamental physical constants $G, \hbar, c, \varepsilon_0, \mu_0, k$ do not appear explicitly in the basic equations (e.g., in elementary particle physics and cosmology). In this system, all the physical quantities are dimensionless.

The energetic system is mainly used in elementary particle physics. In this system, all of the physical quantities are measured in powers of energy, and the physical constants $\hbar, c, \varepsilon_0, \mu_0, k$ do not appear explicitly.

A.3 The Planck System

All the systems of units which have hitherto been employed owe their origin to the coincidence of accidental circumstances, inasmuch as the choice of the units lying at the base of every system has been made, not according to general points of view, but essentially with reference to the special needs of our terrestrial civilization. . .

In contrast with this it might be of interest to note that we have the means of establishing units which are independent of special bodies or substances. The means of determining the units of length, mass, and time are given by the action constant h , together with the magnitude of the velocity of propagation of light in a vacuum c , and that of the constant of gravitation G . . . These quantities must be found always the same, when measured by the most widely differing intelligences according to the most widely differing methods.

Max Planck, 1906

*The Theory of Heat Radiation*¹²

Fundamental constants. There exist the following universal constants in nature:

- G (gravitational constant),
- c (velocity of light in a vacuum),
- h (Planck's quantum of action),
- ε_0 (electric field constant of a vacuum),
- k (Boltzmann constant).

The explicit numerical values of these fundamental constants can be found in Table A.3 on page 965. We also use the constants

- $\hbar := h/2\pi$ (reduced Planck's quantum of action), and
- $\mu_0 := 1/\varepsilon_0 c^2$ (magnetic field constant of vacuum).

Basic laws in physics. These universal constants enter the following six basic laws of physics.

- (i) Einstein's equivalence between rest mass m_0 and rest energy E of a particle:
 $E = m_0 c^2$.
- (ii) Energy E of a photon with frequency ν : $E = h\nu$.
- (iii) Gravitational force F between two masses M_1 and M_2 at distance r :

$$F = \frac{GM_1 M_2}{r^2}.$$

¹² M. Planck, *Theorie der Wärmestrahlung*, Barth, Leipzig 1906. Reprinted by Dover Publications, 1991.

Table A.2. SI system

1 m = 0.63 · 10 ³⁵ m	1 m = <i>l</i> = 1.6 · 10 ⁻³⁵ m
1 s = 0.19 · 10 ⁴⁴ s	1 s = 5.3 · 10 ⁻⁴⁴ s
1 J = 0.51 · 10 ⁻⁹ J	1 J = 1.97 · 10 ⁹ J
1 kg = 0.48 · 10 ⁸ kg	1 kg = 2.1 · 10 ⁻⁸ kg
1 C = 0.19 · 10 ¹⁹ C	1 C = 5.34 · 10 ⁻¹⁹ C
1 K = 0.71 · 10 ⁻³² K	1 K = 1.4 · 10 ³² K
1 GeV = 10 ⁹ eV = 1.602 · 10 ⁻¹⁰ J 1 GeV/c ² = 1.78 · 10 ⁻²⁷ kg	

(iv) Electric force F between two electric charges Q_1 and Q_2 at distance r :

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}.$$

(v) Magnetic force F between two parallel electric currents of strength J_1 and J_2 in a wire of length L at distance r :

$$F = \frac{\mu_0 L J_1 J_2}{2\pi r}.$$

(vi) Mean energy E corresponding to one degree of freedom in a many-particle system at temperature T : $E = kT$.

In the SI system, the unit of electric current, called an ampere, is defined in such a way that the magnetic field constant of a vacuum is given by

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{N}}{\text{A}^2}.$$

By Table A.1 on prefixes, 1 MeV = 10⁶ eV (mega electron volt).

Natural SI units. The five natural constants G , c , \hbar , ϵ_0 , and k can be used to systematically replace the SI units m, s, J, C, K by the following so-called natural SI units:

- Planck length: **m** := $l := \sqrt{\hbar G/c^3}$,
- Planck time: **s** := l/c ,
- Planck energy: **J** := $\hbar c/l$,
- Planck charge: **C** := $\sqrt{c\hbar\epsilon_0}$,
- Planck temperature: **K** := $\hbar c/kl$.

Parallel to $\text{kg} = \text{Js}^2/\text{m}^2$, let us introduce the Planck mass

$$\mathbf{kg} := \mathbf{Js}^2/\mathbf{m}^2 = \hbar/cl.$$

The numerical values can be found in Table A.2. From (A.8) we obtain the representation

$$q = q_{\text{P1}} \cdot \mathbf{m}^\alpha \mathbf{s}^\beta \mathbf{J}^\gamma \mathbf{C}^\mu \mathbf{K}^\nu \tag{A.9}$$

of the physical quantity q in natural SI units. Hence

$$q = q_{\text{P1}} \cdot l^\alpha \left(\frac{l}{c}\right)^\beta \left(\frac{\hbar c}{l}\right)^\gamma (c\hbar\varepsilon_0)^{\mu/2} \left(\frac{\hbar c}{kl}\right)^\nu.$$

This implies

$$\boxed{q = q_{\text{P1}} \cdot l^A c^B \hbar^C \varepsilon_0^D k^E}. \tag{A.10}$$

Explicitly,

$$A = \alpha + \beta - \gamma - \nu, \quad B = \gamma + \nu - \beta + \mu/2, \quad C = \gamma + \nu + \mu/2,$$

and $D = \mu/2$, $E = -\nu$.

The Planck system of units. In this system, we set

$$\boxed{l = c = \hbar = \varepsilon_0 = k := 1}.$$

In particular, for the gravitational constant, this implies $G = 1$. By (A.10), $q = q_{\text{P1}}$.

The Planck system is characterized by the fact that all the physical quantities are dimensionless and their numerical values coincide with the numerical values in natural SI units.

In order to go back from the Planck system to the SI system, one has to replace each physical quantity q by

$$\boxed{q \Rightarrow \frac{q}{l^A c^B \hbar^C \varepsilon_0^D k^E}} \tag{A.11}$$

according to (A.10). The corresponding exponents A, B, \dots follow from (A.9) and (A.10). These exponents can be found in Table A.4 on page 967.

Example 1. For the proton, we get

$$E = 0.77 \cdot 10^{-19} \mathbf{J} = 1.5 \cdot 10^{-10} \mathbf{J} = 0.938 \text{ GeV} \quad (\text{rest energy})$$

along with

$$M = E/c^2 = 0.77 \cdot 10^{-19} \mathbf{kg} = 1.67 \cdot 10^{-27} \mathbf{kg} \quad (\text{rest mass})$$

and

$$e = \sqrt{4\pi\alpha} \mathbf{C} = 0.30 \mathbf{C} = 1.6 \cdot 10^{-19} \mathbf{C} \quad (\text{electric charge}).$$

Therefore, $E_{\text{P1}} = M_{\text{P1}} = 0.77 \cdot 10^{-19}$, and $e_{\text{P1}} = 0.30$. In the Planck system, this implies

$$E = M = 0.77 \cdot 10^{-19} \quad \text{and} \quad e = 0.30.$$

Example 2. Consider the Einstein relation

$$E = m_0 c^2 \tag{A.12}$$

between the rest mass m_0 and the rest energy E of a free relativistic particle in the SI system. Letting $c := 1$, we obtain the corresponding equation

$$\boxed{E = m_0} \tag{A.13}$$

in the Planck system. Here, $E = E_{\text{P1}}$ and $m_0 = M_{\text{P1}}$. In order to go back from (A.13) to the SI system, one has to observe that

$$E = E_{\text{P1}} \cdot \mathbf{J}, \quad m_0 = M_{\text{P1}} \cdot \mathbf{J} \mathbf{s}^2 \mathbf{m}^{-2}$$

in natural SI units, by Table A.4 on page 967. Hence

$$E = E_{\text{Pl}} \frac{\hbar c}{l}, \quad m_0 = M_{\text{Pl}} \frac{\hbar}{lc}.$$

Thus, we have to replace E and m_0 by

$$\frac{El}{\hbar c} \quad \text{and} \quad \frac{m_0 lc}{\hbar},$$

respectively. This way, we pass over from (A.13) to (A.12).

Example 3. In the SI system, the Maxwell equations in a vacuum are given by

$$\begin{aligned} \operatorname{div} \mathbf{D} &= \varrho, & \operatorname{div} \mathbf{B} &= 0, \\ \operatorname{curl} \mathbf{E} &= -\dot{\mathbf{B}}, & \operatorname{curl} \mathbf{H} &= \dot{\mathbf{D}} + \mathbf{j} \end{aligned} \tag{A.14}$$

along with $\mathbf{D} = \varepsilon_0 \mathbf{E}$ and $\mathbf{B} = \mu_0 \mathbf{H}$. Moreover, $c^2 = 1/\varepsilon_0 \mu_0$. Alternatively,

$$\begin{aligned} \varepsilon_0 \operatorname{div} \mathbf{E} &= \varrho, & \operatorname{div} \mathbf{B} &= 0, \\ \operatorname{curl} \mathbf{E} &= -\dot{\mathbf{B}}, & c^2 \operatorname{curl} \mathbf{B} &= \dot{\mathbf{E}} + \mu_0 c^2 \mathbf{j}. \end{aligned} \tag{A.15}$$

Letting $\varepsilon_0 = \mu_0 = c := 1$, we obtain the corresponding Maxwell equations in the Planck system:

$$\begin{aligned} \operatorname{div} \mathbf{E} &= \varrho, & \operatorname{div} \mathbf{B} &= 0, \\ \operatorname{curl} \mathbf{E} &= -\dot{\mathbf{B}}, & \operatorname{curl} \mathbf{B} &= \dot{\mathbf{E}} + \mathbf{j}. \end{aligned} \tag{A.16}$$

In order to transform equation (A.16) back to the SI system, we replace the quantities $\mathbf{x}, t, \mathbf{E}, \mathbf{B}, \varrho, \mathbf{j}$ by

$$\frac{\mathbf{x}}{\mathbf{m}}, \quad \frac{t}{\mathbf{s}}, \quad \mathbf{E} \cdot \frac{\mathbf{m} \mathbf{C}}{\mathbf{J}}, \quad \mathbf{B} \cdot \frac{\mathbf{m}^2}{\mathbf{s} \mathbf{J}}, \quad \varrho \cdot \frac{\mathbf{m}^3}{\mathbf{C}}, \quad \mathbf{j} \cdot \frac{\mathbf{m}^2}{\mathbf{C} \mathbf{s}}, \tag{A.17}$$

respectively, according to Table A.4 on page 967. In addition, the partial derivatives $\partial/\partial x^j, \partial/\partial t$ have to be replaced by

$$\mathbf{m} \cdot \frac{\partial}{\partial x^j}, \quad \mathbf{s} \cdot \frac{\partial}{\partial t},$$

respectively. Finally, we set

$$\mathbf{m} := l, \quad \mathbf{s} := \frac{l}{c}, \quad \mathbf{C} := (c\hbar\varepsilon_0)^{1/2}, \quad \mathbf{J} := \frac{\hbar c}{l}.$$

This way, we get (A.14). In fact, for example, the first Maxwell equation $\operatorname{div} \mathbf{E} = \varrho$ from (A.16) means explicitly

$$\partial_j E^j = \varrho,$$

in Cartesian coordinates. Here, $\partial_j = \partial/\partial x^j$, and we sum over $j = 1, 2, 3$. By (A.17), this is transformed into the equation

$$\beta \cdot \partial_j E^j = \varrho,$$

where $\beta := \mathbf{C}^2/\mathbf{Jm}$. Since $\beta = c\hbar\varepsilon_0/c\hbar = \varepsilon_0$, we obtain $\varepsilon_0 \operatorname{div} \mathbf{E} = \varrho$. This is the first Maxwell equation from (A.15).

Example 4. In the SI system, the Schrödinger equation reads as

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m_0} \Delta \psi + U \psi. \tag{A.18}$$

Here, m_0 and U denote the mass of the particle and the potential energy, respectively. Letting $\hbar = 1$, we arrive at the Schrödinger equation

$$\boxed{i \frac{\partial \psi}{\partial t} = \frac{\Delta \psi}{2m_0} + U \psi} \tag{A.19}$$

in the Planck system. In a Cartesian (x, y, z) -system, the Laplacian is defined by

$$\Delta := - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}.$$

Note that our sign convention coincides with the use of the Laplacian in modern differential geometry (Riemannian geometry) and string theory.¹³

In order to go back from the Planck system to the SI system¹⁴, we replace the quantities $\mathbf{x}, t, U, m_0, \psi$ by

$$\frac{\mathbf{x}}{\mathbf{m}}, \quad \frac{t}{\mathbf{s}}, \quad \frac{U}{\mathbf{J}}, \quad m_0 \cdot \frac{\mathbf{m}^2}{\mathbf{J}\mathbf{s}^2}, \quad \psi \cdot \mathbf{m}^{3/2},$$

respectively. The Laplacian contains spatial derivatives of second order. Thus, the Laplacian Δ and the partial time derivative $\partial/\partial t$ have to be replaced by

$$\mathbf{m}^2 \cdot \Delta, \quad \mathbf{s} \cdot \frac{\partial}{\partial t}.$$

Consequently, equation (A.19) is transformed into

$$i\mathbf{J}\mathbf{s}\psi_t = \frac{(\mathbf{J}\mathbf{s})^2}{2m_0} \Delta \psi + U \psi.$$

Since $\mathbf{J}\mathbf{s} = \hbar$, we get (A.18).

Example 5. Let us start with the Dirac equation

$$\boxed{i\gamma^\mu \partial_\mu \psi = m_0 \psi} \tag{A.20}$$

for the relativistic electron of rest mass m_0 formulated in the Planck system. Here, $\partial_\mu = \partial/\partial x^\mu$. Recall that we sum over μ from 0 to 3, by Einstein's summation convention. The definition of the Dirac–Pauli matrices $\gamma^0, \gamma^1, \gamma^2, \gamma^3$ can be found on page 791. In order to pass over to the SI system, we replace the quantities x^μ, m_0, ψ by

$$\frac{x^\mu}{\mathbf{m}}, \quad m_0 \cdot \frac{\mathbf{m}^2}{\mathbf{J}\mathbf{s}^2}, \quad \mathbf{m}^{3/2} \cdot \psi,$$

respectively, according to Table A.4 on page 967. Note that the dimension of the wave function ψ in the SI system is the same as in the case of the Schrödinger equation. Hence

$$i\gamma^\mu \partial_\mu \psi = \sigma m_0 \psi$$

¹³ In classic textbooks, one has to replace Δ by $-\Delta$.

¹⁴ The normalization condition $\int_{\mathbb{R}^3} \psi \psi^\dagger d^3x = 1$ implies that the wave function ψ has the dimension $\mathbf{m}^{-3/2}$ in the SI system.

where $\sigma := \mathbf{m}/\mathbf{s}^2\mathbf{J}$. Since $\sigma = c/\hbar$, in the SI system the Dirac equation reads as

$$\boxed{i\hbar\gamma^\mu\partial_\mu\psi = m_0c\psi.} \tag{A.21}$$

The quantity $\lambda_e := h/cm_0$ is called the Compton wave length of the electron.

Classical systems of units. In the context of the Maxwell equations, physicists frequently use the Gaussian system or the Heaviside system, for historical reasons. Let us explain the relation of these two systems to the SI system. The idea is to measure all of the physical quantities by meter, second, kilogram, and Kelvin. That is, we do not introduce a specific unit for electric charge. In the Gaussian system, the electric force F between two electric charges Q_1 and Q_2 at distance r (Coulomb law) is given by

$$F = \frac{Q_1Q_2}{r^2}.$$

Moreover, we use the Gaussian definition of the magnetic field

$$\mathbf{H}_G := c\mathbf{B}.$$

This definition is motivated by the fact that the electric field \mathbf{E} and the Gaussian magnetic field \mathbf{H}_G possess the same physical dimension. In the Heaviside system, we use the Coulomb law

$$F = \frac{Q_1Q_2}{4\pi r^2}.$$

In contrast to the Gaussian system from the 1830s, the Heaviside system from the 1880s has the advantage that the factor 4π does not appear in the Maxwell equations.

The Heaviside system of units. We use the SI system and set

$$\boxed{\varepsilon_0 := 1.}$$

In the SI system, each physical quantity q can be written as

$$q = q_{\text{P1}} \cdot l^A c^B \hbar^C \varepsilon_0^D k^E,$$

by (A.10) on page 954. Letting $\varepsilon_0 := 1$, we get

$$q = q_{\text{P1}} \cdot l^A c^B \hbar^C k^E,$$

in the Heaviside system. Consequently,

$$q = q_{\text{H}} \cdot \text{m}^a \text{s}^b \text{kg}^c \text{K}^d.$$

That is, each physical quantity can be described by powers of meter, second, kilogram, and Kelvin. In the Heaviside system, the Maxwell equations read as follows:

$$\begin{aligned} \operatorname{div} \mathbf{E} &= \varrho, & \operatorname{div} \mathbf{H}_G &= 0, \\ \operatorname{curl} \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{H}_G}{\partial t}, & \operatorname{curl} \mathbf{H}_G &= \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{\mathbf{j}}{c}. \end{aligned} \tag{A.22}$$

To obtain this, start with the Maxwell equations in the SI system. By (A.15) along with $c^2 = 1/\varepsilon_0\mu_0$,

$$\begin{aligned} \varepsilon_0 \operatorname{div} \mathbf{E} &= \varrho, & \operatorname{div}(c\mathbf{B}) &= 0, \\ \operatorname{curl} \mathbf{E} &= -\frac{1}{c} \frac{\partial(c\mathbf{B})}{\partial t}, & \operatorname{curl}(c\mathbf{B}) &= \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{\mathbf{j}}{\varepsilon_0 c}. \end{aligned}$$

Letting $\varepsilon_0 := 1$ and $\mathbf{H}_G := c\mathbf{B}$, we get (A.22).

The Gaussian system of units. Using the rescaling

$$\mathbf{E} \Rightarrow \frac{\mathbf{E}}{4\pi}, \quad \mathbf{H}_G \Rightarrow \frac{\mathbf{H}_G}{4\pi},$$

the Heaviside system passes over to the Gaussian system. In particular, the Maxwell equations in the Gaussian system read as follows:

$$\begin{aligned} \operatorname{div} \mathbf{E} &= 4\pi\rho, & \operatorname{div} \mathbf{H}_G &= 0, \\ \operatorname{curl} \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{H}_G}{\partial t}, & \operatorname{curl} \mathbf{H}_G &= \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi\mathbf{j}}{c}. \end{aligned} \tag{A.23}$$

Observe that the variants (A.22) and (A.23) of the Maxwell equations differ by the factor 4π . The Gaussian system is used in the 10-volume standard textbook on theoretical physics by Landau and Lifshitz (1982).

A.4 The Energetic System

The most important physical quantity in elementary particle physics is given by the energy of a particle accelerator. Therefore, particle physicists like to use energy as basic unit. Let us discuss this. In the SI system, an arbitrary physical quantity can be written as

$$q = q_{P1} \cdot l^A c^B \hbar^C \varepsilon_0^D k^E,$$

by (A.10). In the energetic system, we set¹⁵

$$\boxed{c = \hbar = \varepsilon_0 = k := 1.}$$

Hence

$$q = q_{P1} \cdot l^A.$$

Consequently, each physical quantity has the physical dimension of some power of length. In particular, for energy E we get

$$E = E_{P1} \cdot l^{-1} \hbar c,$$

in the SI system. Hence

$$E = E_{P1} \cdot l^{-1},$$

in the energetic system. That is, energy has the physical dimension of inverse length.

Conversely, length has the physical dimension of inverse energy in the energetic system of units.

In terms of natural SI units, each physical quantity can be written as

$$q = q_{P1} \cdot \mathbf{m}^\alpha \mathbf{s}^\beta \mathbf{J}^\gamma \mathbf{C}^\mu \mathbf{K}^\nu.$$

It follows from $c = \hbar = \varepsilon_0 = k := 1$ that

¹⁵ In particular, this implies $\mu_0 = 1$.

$$\boxed{\mathbf{s} = \mathbf{m} = \mathbf{J}^{-1}, \quad \mathbf{K} = \mathbf{J}, \quad \mathbf{C} = 1.} \tag{A.24}$$

This implies

$$q = q_{\text{P1}} \cdot \mathbf{J}^A,$$

where $A = -\alpha - \beta + \gamma + \nu$. This way, each physical quantity can be expressed by powers of the Planck energy \mathbf{J} . Using Table A.4 on page 967 along with (A.24), we immediately obtain all the dimensions of important physical quantities in the energetic system. For example, velocity has the dimension

$$v = v_{\text{SI}} \cdot \text{ms}^{-1},$$

in the SI system. Thus, in natural SI units,

$$v = v_{\text{P1}} \cdot \mathbf{ms}^{-1}.$$

In the energetic system $\mathbf{m} = \mathbf{s}$, by (A.24). Hence

$$v = v_{\text{P1}},$$

that is, velocity is dimensionless. Note that this follows more simply from the fact that $c := 1$ in the energetic system; that is, the velocity of light is dimensionless. Similarly, using the dimensionless quantities $\hbar = \varepsilon_0 = \mu_0 = k := 1$ along with the basic physical laws (i)-(vi) on page 952, we encounter the following physical dimensions in the energetic system:

- [mass] = [momentum] = [temperature] = [energy],
- [length] = [time] = [energy]⁻¹,
- [cross section] = [area] = [length]² = [energy]⁻²,
- [electric charge] = [velocity] = [action] = dimensionless,
- [force] = [electric field] = [magnetic field] = [energy]²,
- [potential] = [vector potential] = [energy],
- the coupling constants of quantum electrodynamics, quantum chromodynamics, and electroweak interaction are dimensionless.

Since the electric charge and the coupling constants of the Standard Model in particle physics are dimensionless in the energetic system, these quantities are independent of the rescaling of energy.

Examples. The Einstein relation $E = m_0 c^2$ reads as

$$E = m_0$$

in the energetic system, since $c := 1$.

The Maxwell equations (A.16), the Schrödinger equation (A.19), and the Dirac equation (A.20) coincide in the Planck system and in the energetic system.

In elementary particle physics, physicists like to use GeV (giga electron volt), where

$$\mathbf{J} = 1.98 \cdot 10^{19} \text{ GeV}.$$

This is called the Planck energy. Note that the rest energy of the proton is equal to 0.938 GeV. Consequently, from Table A.2 on page 953, we obtain the following conversion formulas between the SI system and the energetic system:

$$\begin{aligned}
 1 \text{ m} &\doteq 5 \cdot 10^{16} (\text{GeV})^{-1}, \\
 1 \text{ s} &\doteq 1.5 \cdot 10^{24} (\text{GeV})^{-1}, \\
 1 \text{ J} &\doteq 6.3 \cdot 10^9 \text{ GeV}, \\
 1 \text{ K} &\doteq 1.4 \cdot 10^{-13} \text{ GeV}, \\
 1 \text{ C} &\doteq 1.9 \cdot 10^{18}.
 \end{aligned}$$

Depending on the energy scale, physicists also use mega electron volt, MeV. Here, $1 \text{ GeV} = 10^3 \text{ MeV}$.

The physical dimension of cross sections. Observe that

$$\hbar c = 1.97327 \cdot 10^{-13} \text{ MeV} \cdot \text{m}.$$

This implies

$$\text{m}^2 = \frac{(\hbar c)^2}{(1.97327)^2} \cdot 10^{26} (\text{MeV})^{-2}.$$

In the SI system, the cross section σ is measured in m^2 . Setting

$$\hbar = c := 1,$$

we get the cross section in the energetic system measured in $(\text{MeV})^{-2}$. Conversely, the passage from the energetic system to the SI system can be easily obtained by using the replacement

$$\sigma \Rightarrow \frac{\sigma}{(\hbar c)^2}.$$

In fact, if $\sigma = a$ in the energetic system, then $\sigma = (\hbar c)^2 a$ in the SI system.

A.5 The Beauty of Dimensional Analysis

Physicists use the dimensionality of physical quantities in order to get important information. Let us illustrate this by considering three examples: the pendulum, Newton’s gravitational law, and Kolmogorov’s law for turbulence.

The pendulum. Consider a pendulum of length l and mass m . We are looking for a formula for the period of oscillation, T , of the pendulum. We expect that T depends on l , m , and the gravitational acceleration g . Thus, we begin with the ansatz

$$T = C \cdot l^\alpha m^\beta g^\gamma$$

where C is a dimensionless constant. Passing to dimensions we get

$$\text{s} = \text{m}^\alpha \text{kg}^\beta \text{m}^\gamma \text{s}^{-2\gamma}.$$

This implies $\beta = 0$, $\gamma = -\frac{1}{2}$, and $\alpha = -\gamma = \frac{1}{2}$, that is,

$$T = C \sqrt{\frac{l}{g}}. \tag{A.25}$$

The constant C has to be determined from experiment. The explicit solution of the problem via elliptic integrals shows that, for small pendulum motions, equation (A.25) is valid with $C = 2\pi$.

Newton's gravitational law. In 1619 Kepler discovered empirically that the motion of a planet satisfies the law

$$\frac{T^2}{a^3} = \text{const}$$

where T is the period of revolution, and a is the great semi-axis of the elliptic orbit. In order to guess Newton's gravitational law from this information, let us make the ansatz

$$m\ddot{\mathbf{x}} = C|\mathbf{x}|^\mu \mathbf{x}$$

for the motion $\mathbf{x} = \mathbf{x}(t)$ of the planet. Here, m is the mass of the planet, and C is a constant. We want to show that $\mu = -3$ is the only natural choice. To this end, consider the rescaled motion $\mathbf{y}(t) := \alpha\mathbf{x}(\beta t)$. Then

$$m\ddot{\mathbf{y}} = \beta^2\alpha^{-\mu}C|\mathbf{y}|^\mu \mathbf{y}.$$

We postulate that the equation of motion and the third Kepler law are independent of the rescaling. This means that $\beta^2\alpha^{-\mu} = 1$ and

$$(T\beta)^2/(\alpha a)^3 = T^2/a^3.$$

Hence $\mu = -3$. Summarizing, we obtain Newton's gravitational law

$$m\ddot{\mathbf{x}} = \frac{C}{|\mathbf{x}|^2} \cdot \frac{\mathbf{x}}{|\mathbf{x}|}.$$

The Kolmogorov law for energy dissipation in turbulent flows. It is a typical property of turbulent flow that there exist eddies of different diameters λ , where $\lambda_{\min} \leq \lambda \leq \lambda_{\max}$. One may think, for example, of clouds in the air or of nebulas in astronomy. One finds that the large eddies tend to break down into smaller eddies. This way, energy from large eddies flows to smaller eddies. Here, physicists assume that the energy of the smallest eddies with $\lambda = \lambda_{\min}$ is transformed into heat by friction (energy dissipation). Viscosity is of significance only for small eddies. We define

$$\varepsilon := \frac{\text{loss of energy by dissipation}}{\text{mass} \cdot \text{time}}.$$

This is the crucial physical quantity. Note that ε can be measured in experiments; it is equal to the produced heat. Using the method of dimensional analysis, Kolmogorov obtained the law

$$\varepsilon = \int_{\lambda_{\min}}^{\lambda_{\max}} s(\lambda) d\lambda$$

along with the spectral function

$$s(\lambda) := C \left(\frac{\lambda}{\lambda_{\min}} \right) \frac{\eta \varepsilon^{2/3}}{\varrho} \cdot \frac{1}{\lambda^{7/3}}.$$

Here, η and ϱ are viscosity and mass density, respectively. The function C is dimensionless. For values λ near λ_{\min} , the function C can be approximated by a constant. Therefore, physicists speak of Kolmogorov's 7/3-law. The proof can be found in Zeidler (1986), Vol. IV, p. 514.

It turns out that dimensional analysis represents a magic wand of physicists. In this setting, a minimum of hypotheses provides us a maximum of information.

A.6 The Similarity Principle in Physics

Rescaled SI units. Let us replace the SI units m, s, J, C, K with the rescaled units

$$m_*, s_*, J_*, C_*, K_*,$$

where $m_* = m_+ \cdot m$, $s_* = s_+ \cdot s, \dots$ with the real numbers m_+, s_+, \dots . Then, each physical quantity q can be represented as

$$q = q_* \cdot m_*^{\alpha} s_*^{\beta} J_*^{\gamma} C_*^{\mu} K_*^{\nu} = q_* \cdot [q].$$

The real number q_* is called the numerical value of q , and $[q]$ is called the dimension of q with respect to this system of units. In practice, one chooses m_*, s_*, \dots in such a way that the numerical values of the physical quantities are neither too large nor too small. For example, if we want to study thin layers, then it is convenient to use $m_* := 10^{-9} \text{m} = 1 \text{nm}$ (nanometer). In astronomy, one uses light years for measuring distances, and so on.

The role of small quantities in physics. It is impossible to speak of a small length L in physics. In fact, if

$$L = 1 \text{ meter},$$

then passing to a new length scale, we get

$$L = 10^{15} \text{ femtometer}.$$

Therefore, it makes sense to speak about smallness only for dimensionless quantities. For example, choose the radius r_E of earth and the radius r_p of a proton. Then the dimensionless ratio

$$\frac{r_p}{r_E} = 6 \cdot 10^{-21}$$

is a small quantity compared with 1.

The experience of physicists shows that two different theories are good approximations of each other if suitable dimensionless quantities are small. Let us consider two crucial examples.

- (i) Relativistic physics: Let v and c be the velocity of some particle and the velocity of light, respectively. If the dimensionless quotient

$$\frac{v}{c}$$

is sufficiently small, then the relativistic motion of the particle can be described approximately by Newton's classical mechanics. For example, the relativistic mass

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} = m_0 \left(1 - \frac{v^2}{2c^2} + o\left(\frac{v^2}{c^2}\right) \right), \quad \frac{v^2}{c^2} \rightarrow 0$$

is approximately equal to the rest mass m_0 if the quotient v/c is sufficiently small.

- (ii) Quantum mechanics: Let $S = E(t_1 - t_2)$ be the action for the motion of some particle with constant energy E during a fixed reasonable time interval $[t_1, t_2]$, say, one hour. If the dimensionless ratio

$$\frac{S}{\hbar}$$

is small, then the quantum motion of the particle can be approximately described by Newton's classical mechanics.

In (i) and (ii), corrections to classical mechanics can be obtained by perturbation theory if v/c and S/\hbar are small. These are the post-Newtonian approximation and the WKB approximation, respectively.

The fundamental similarity principle in physics. We postulate that

Physical processes are described by equations which are invariant under rescaling of units. Explicitly, we demand that the laws of physics can be written in such a way that, in a fixed system of units, they only depend on the dimensionless quotients

$$\frac{q}{[q]}, \quad \frac{r}{[r]}, \quad \dots$$

of all the physical quantities q, r, \dots

A special role is played by those physical quantities which are dimensionless in the SI system. We expect that such quantities are related to important physical effects. The experience of physicists confirms this. For example, the so-called fine structure constant

$$\alpha := \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{1}{137.04}$$

represents the most important dimensionless quantity that can be constructed from the universal constants. This constant measures the strength of the interaction between electrons, positrons, and photons in quantum electrodynamics. The smallness of α is responsible for the fact that perturbation theory can be successfully applied to quantum electrodynamics.

Example. Consider the Einstein relation

$$E = m_0c^2$$

between rest mass m_0 and rest energy E of a particle. In any rescaled SI system,

$$E = E_* \cdot J_*, \quad m_0 = (m_0)_* \cdot J_*s_*^2m_*^{-2}, \quad c = c_* \cdot m_*s_*^{-1}.$$

Hence $E_* = (m_0)_*c_*^2$. Moreover, $[E] = J_*$, and

$$[m_0][c]^2 = J_*s_*^2m_*^{-2} \cdot m_*^2s_*^{-2} = J_*.$$

This means that

$$\frac{E}{[E]} = \frac{m_0c^2}{[m_0][c]^2}.$$

Physicists frequently use such dimension tests in order to check the correctness of formulas.

Counterexample. Let x and t denote position and time, respectively. The equation

$$x = \sin t$$

is not allowed in the SI system, since it is not invariant under the rescaling $x \Rightarrow \alpha x$ and $t \Rightarrow \beta t$ for nonzero constants α and β . In contrast to this, the equation

$$\frac{x}{x_0} = \sin\left(\frac{t}{t_0}\right)$$

is admissible in any system of units if x and x_0 as well as t and t_0 possess the same dimensions.

Application to Reynolds numbers in turbulence. The motion of a viscous fluid in a 3-dimensional bounded domain \mathcal{G} is governed by the so-called Navier–Stokes equations¹⁶

$$\begin{aligned} \varrho \mathbf{v}_t - \nu \Delta_{\mathbf{x}} \mathbf{v} + \varrho(\mathbf{v} \nabla_{\mathbf{x}}) \mathbf{v} &= \mathbf{f} - \nabla_{\mathbf{x}} p \quad \text{on } \mathcal{G}, \\ \nabla_{\mathbf{x}} \mathbf{v} &= 0 \quad \text{on } \mathcal{G}, \\ \mathbf{v} &= 0 \quad \text{on } \partial \mathcal{G}. \end{aligned}$$

The symbols possess the following physical meaning: \mathbf{v} velocity vector, ϱ mass density, \mathbf{f} force density vector, p pressure, ν viscosity constant, \mathbf{x} position vector, and t time. Set

$$\mathbf{x} = \mathbf{X} \cdot \mathbf{m}_*, \quad t = T \cdot s_*, \quad \mathbf{v} = \mathbf{u} \cdot \mathbf{m}_* s_*^{-1}, \quad \varrho = \Omega \cdot \mathbf{J}_* s_*^2 \mathbf{m}_*^{-5}$$

and

$$\mathbf{f} = \mathbf{F} \cdot \mathbf{J}_* \mathbf{m}_*^{-4}, \quad p = P \cdot \mathbf{J}_* \mathbf{m}_*^{-3}, \quad \nu = N \cdot \mathbf{J}_* \mathbf{m}_*^{-3},$$

where the coefficients \mathbf{X}, T, \dots are dimensionless. Furthermore, let d and v denote the diameter of the domain \mathcal{G} and a typical velocity of the fluid, respectively. Naturally enough, we choose

$$\mathbf{m}_* := d, \quad s_* := dv^{-1}, \quad \mathbf{J}_* := \varrho v^2 d^{-3}.$$

This way, we obtain the rescaled dimensionless Navier–Stokes equations

$$\begin{aligned} \mathbf{u}_t - \text{Re}^{-1} \Delta_{\mathbf{X}} \mathbf{u} + (\mathbf{u} \nabla_{\mathbf{X}}) \mathbf{u} &= \mathbf{F} - \nabla_{\mathbf{X}} P \quad \text{on } \mathcal{H}, \\ \nabla_{\mathbf{X}} \mathbf{u} &= 0 \quad \text{on } \mathcal{H}, \\ \mathbf{u} &= 0 \quad \text{on } \partial \mathcal{H} \end{aligned}$$

with the dimensionless Reynolds number

$$\text{Re} := \frac{\varrho v d}{\nu}.$$

The rescaled domain \mathcal{H} is obtained from the original domain \mathcal{G} by replacing the points \mathbf{x} of \mathcal{G} by $d^{-1} \mathbf{x}$. Physical experiments show that if the Reynolds number Re is sufficiently large, then turbulence occurs.

The rescaled dimensionless Navier–Stokes equations reflect an important similarity principle in hydrodynamics. Explicitly, if two physical situations in different regions are governed by the same rescaled dimensionless Navier–Stokes equations, then the physics is the same up to suitable similarity transformations.

Discovery of errors in physical computations. Physicists use physical dimensions in order to detect errors in their computations. To explain this with a simple example, suppose that we arrive at the equation

$$p = c^3 m_0 \tag{A.26}$$

after finishing some computation. Here, we use the following notation: p momentum, m_0 particle mass, c velocity of light. We want to check this. In the SI system, we have the following dimensions:

$$[p] = \text{kg} \cdot \text{ms}^{-1}, \quad [m_0] = \text{kg}, \quad [c] = \text{ms}^{-1}.$$

Hence $[p] = [c] \cdot [m_0]$. It follows from (A.26) that $[p] = [c]^3 [m_0]$. This implies $[c]^2 = 1$, which is a contradiction. Consequently, our result (A.26) is wrong. The same argument can be used in the energetic system. However, we now have $[c] = 1$, which does not lead to any contradiction. In other words, the energetic system of units is too weak in order to detect that equation (A.26) is wrong, by checking physical dimensions.

¹⁶ Navier (1785–1836), Stokes (1819–1903).

Table A.3. Fundamental constants in nature

fundamental constant	SI units	natural SI units
velocity of light in a vacuum	$c = 2.998 \cdot 10^8 \text{ m/s}$	$c = \mathbf{m/s}$
Planck's action quantum	$h = 6.626 \cdot 10^{-34} \text{ Js}$ $\hbar = h/2\pi$	$\hbar = \mathbf{Js}$
gravitational constant	$G = 6.673 \cdot 10^{-11} \text{ m}^5/\text{Js}^4$	$G = \mathbf{m}^5/\mathbf{Js}^4$ $= l^2 c^3 / \hbar$
electric field constant	$\epsilon_0 = 8.854 \cdot 10^{-12} \text{ C}^2/\text{Jm}$	$\epsilon_0 = \mathbf{C}^2/\mathbf{Jm}$
magnetic field constant	$\mu_0 = 1/\epsilon_0 c^2$ $= 4\pi \cdot 10^{-7} \text{ Js}^2/\text{C}^2\text{m}$	$\mu_0 = \mathbf{Js}^2/\mathbf{C}^2\mathbf{m}$
Boltzmann constant	$k = 1.380 \cdot 10^{-23} \text{ J/K}$	$k = \mathbf{J/K}$
fine structure constant	$\alpha = e^2/4\pi\hbar c\epsilon_0$ (dimensionless)	$\alpha = 1/137.04$
charge of the proton	$e = 1.602 \cdot 10^{-19} \text{ C}$	$e = \sqrt{4\pi\alpha} \mathbf{C}$ $= 0.30 \mathbf{C}$
rest energy of the proton	$E_p = 1.5 \cdot 10^{-10} \text{ J}$ $= 0.938 \text{ GeV}$ (giga electron volt)	$E_p = 0.77 \cdot 10^{-19} \mathbf{J}$
rest mass of the proton	$m_p = 1.672 \cdot 10^{-27} \text{ kg}$ $= 0.938 \text{ GeV}/c^2$	$m_p = 0.77 \cdot 10^{-19} \mathbf{kg}$
Compton wave length of the proton $\lambda_p = h/m_p c$	$\lambda_p = 1.32 \cdot 10^{-15} \text{ m}$ $= 1.32 \text{ fm}$ (femtometer)	$\lambda_p = 0.83 \cdot 10^{20} \mathbf{m}$
rest energy of the electron	$E_e = 8.16 \cdot 10^{-14} \text{ J}$ $= 0.511 \text{ MeV}$ (mega electron volt)	$E_e = E_p/1838.1$

Table A.3. (continued)

rest mass of the electron	$m_e = 0.91 \cdot 10^{-30} \text{ kg}$	$m_e = m_p/1838.1$
Compton wave length of the electron $\lambda_e = \hbar/m_p c$	$\lambda_e = 2.43 \cdot 10^{-12} \text{ m}$	$\lambda_e = 1838.1 \lambda_p$
radius of the proton	$r_p = 1.3 \cdot 10^{-15} \text{ m}$ $= 1.3 \text{ fm (femtometer)}$	$r_p = 0.882 \cdot 10^{20} l$
fundamental constant	SI units	natural SI units
Bohr radius of the hydrogen atom	$r_B = 0.529 \cdot 10^{-10} \text{ m}$ $= 5.29 \text{ nm (nanometer)}$	$r_B = 40000 r_p$
Bohr magneton	$\mu_B = -e\hbar/2m_e$ $= -9.27 \cdot 10^{-24} \text{ mC/s}$	$\mu_B = -\mathbf{mC/s}$
magnetic moment of the electron	$\mu_e = (1 + \frac{\alpha}{2\pi} - \dots)\mu_B$	$\mu_e = 1.001\mu_B$
nuclear magneton	$\mu_n = e\hbar/2m_p$ $= 5.05 \cdot 10^{-27} \text{ mC/s}$	$\mu_B = 1836.1\mu_n$
magnetic moment of the proton	$\mu_p = 2.79\mu_n$	$\mu_p = 2.79\mu_n$

More precise values can be found in CODATA Bull. **63** (1986), and E. Cohen and B. Taylor, Review of Modern Physics **59**(4) (1986). A list of high-precision values can also be found in the Appendix to Zeidler, Oxford User’s Guide to Mathematics, Oxford University Press, 2004. In the following Table A.4, observe that the two quantities \mathbf{E} and $c\mathbf{B}$, possess the same physical dimension in the SI system. The same is true for $c\mathbf{D}$ and \mathbf{H} . Here, we use the notation:

- \mathbf{E} electric field vector,
- \mathbf{B} magnetic field vector,
- \mathbf{D} electric field intensity vector,
- \mathbf{H} magnetic field intensity vector.

In the literature, the terminology with respect to \mathbf{E} , \mathbf{B} , \mathbf{D} , \mathbf{H} is not uniform, for historical reasons. Since \mathbf{E} and \mathbf{B} generate the electromagnetic field tensor (see (14.51) on page 794), it follows from Einstein’s theory of special relativity that the vector fields \mathbf{E} and \mathbf{B} (resp. \mathbf{D} and \mathbf{H}) form a unit. The mean magnetic field of earth has the strength $\mathbf{B}_{\text{earth}} = 0.5 \text{ Gauss} = 0.5 \cdot 10^{-4} \text{ Tesla}$.

Table A.4. Units of physical quantities

Physical quantity	SI units $m^\alpha s^\beta J^\gamma C^\mu K^\nu$	natural SI units $\mathbf{m}^\alpha \mathbf{s}^\beta \mathbf{J}^\gamma \mathbf{C}^\mu \mathbf{K}^\nu$ $= l^A c^B \hbar^C \varepsilon_0^D k^E$
length	m (meter)	$\mathbf{m} = l$ (Planck length)
time	s (second)	$\mathbf{s} = l/c$ (Planck time)
energy, work	J (Joule)	$\mathbf{J} = \hbar c/l$ (Planck energy)
electric charge	C (Coulomb)	$\mathbf{C} = (c\hbar\varepsilon_0)^{\frac{1}{2}}$ (Planck charge)
temperature	K (Kelvin)	$\mathbf{K} = \hbar c/lk$ (Planck temperature)
mass	kg = J s ² /m ² (kilogram)	$\mathbf{J s}^2/\mathbf{m}^2 = \hbar/c l$ (Planck mass)
electric current strength	A = C/s (ampere)	$\mathbf{C/s} = c^{\frac{3}{2}} (\hbar\varepsilon_0)^{\frac{1}{2}}/l$
voltage	V = J/C (volt)	$\mathbf{J/C} = (c\hbar)^{\frac{1}{2}}/l\varepsilon_0^{\frac{1}{2}}$
action (energy × time)	J s	$\mathbf{J s} = \hbar$
momentum (mass × velocity)	J s/m	$\mathbf{J s/m} = \hbar/l$
power (energy/time)	W = J/s (Watt)	$\mathbf{J/s}$
force (energy/length)	N = J/m (Newton)	$\mathbf{J/m}$
frequency ν (number of oscillations/time)	1/s	$1/\mathbf{s}$
angular frequency $\omega = 2\pi\nu$	1/s	$1/\mathbf{s}$
pressure (force/area)	Pa = N/m ² = J/m ³	$\mathbf{J/m}^3$
area, cross section	m ²	$\mathbf{m}^2 = l^2$
volume	m ³	$\mathbf{m}^3 = l^3$

Table A.4. (continued)

Physical quantity	SI units $m^\alpha s^\beta J^\gamma C^\mu K^\nu$	natural SI units $m^\alpha s^\beta J^\gamma C^\mu K^\nu$ $= l^A c^B \hbar^C \varepsilon_0^D k^E$
velocity	m/s	m/s = c
acceleration	m/s^2	m/s² = c^2/l
mass density	$kg/m^3 = Js^2/m^5$	Js²/m⁵
electric charge density ρ (charge/volume)	C/m^3	C/m³
electric current density vector $\mathbf{j} = \rho\mathbf{v}$	C/m^2s	C/m²s
electric field vector \mathbf{E} (force/charge)	$N/C = V/m$ $= J/mC$	J/mC
magnetic field vector \mathbf{B}	$T = Vs/m^2$ $= Js/m^2C$ (Tesla)	Js/m²C
magnetic flow $\int \mathbf{B} d\mathbf{f}$	$Wb = Vs$ $= Js/C$ (Weber)	Js/C
electric field intensity vector \mathbf{D}	C/m^2	C/m²
magnetic field intensity vector \mathbf{H}	$A/m = C/sm$	C/sm
electric dipole moment	Cm	Cm
magnetic dipole moment	$Am^2 = m^2C/s$	m²C/s
polarization \mathbf{P} (electric dipole moment density), $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$	C/m^2	C/m²
magnetization \mathbf{M} (magnetic dipole moment density), $\mathbf{B} = \mu_0\mathbf{H} + \mathbf{M}$	C/ms	C/ms
scalar potential U ($\mathbf{E} = -\mathbf{grad} U - \mathbf{A}_t$)	$V = J/C$	J/C
vector potential \mathbf{A} ($\mathbf{B} = \mathbf{curl} \mathbf{A}$)	$Vs/m = Js/mC$	Js/mC
4-potential A^μ ($A^0 = U/c, \mathbf{A} = A^j \mathbf{e}_j$)	Js/mC	Js/mC

Table A.4. (continued)

Physical quantity	SI units $\text{m}^\alpha \text{s}^\beta \text{J}^\gamma \text{C}^\mu \text{K}^\nu$	natural SI units $\mathbf{m}^\alpha \mathbf{s}^\beta \mathbf{J}^\gamma \mathbf{C}^\mu \mathbf{K}^\nu$ $= l^A c^B \hbar^C \varepsilon_0^D k^E$
electromagnetic field tensor $F_{\mu\nu}$ ($F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$)	$\text{Vs}/\text{m}^2 = \text{Js}/\text{m}^2 \text{C}$	$\mathbf{Js}/\text{m}^2 \mathbf{C}$
electric 4-current j^μ ($j^0 = c\rho, \mathbf{j} = j^k \mathbf{e}_k$)	$\text{C}/\text{m}^2 \text{s}$	$\mathbf{C}/\text{m}^2 \mathbf{s}$
Schrödinger function ψ in N space dimensions (solution of the Schrödinger equation)	$\text{m}^{-\frac{N}{2}}$	$\mathbf{m}^{-\frac{N}{2}} = l^{-\frac{N}{2}}$
Dirac function ψ (solution of the Dirac equation, electron field, quark field, fermion fields)	$1/\text{ms}^{\frac{1}{2}}$	$\mathbf{1}/\mathbf{ms}^{\frac{1}{2}} = c^{\frac{1}{2}}/l^{\frac{3}{2}}$
Lagrangian L in classical mechanics action = $\int_{t_0}^{t_1} L(q, \dot{q}, t) dt$	J	$\mathbf{J} = \hbar c/l$
Lagrangian density \mathcal{L} in relativistic field theory action = $\int_{\mathbb{R}^4} \mathcal{L}(\psi, \partial\psi, x) d^4x$	Js/m^4	$\mathbf{Js}/\text{m}^4 = \hbar/l^4$
Hamiltonian H in classical mechanics, $H = p\dot{q} - L$	J	$\mathbf{J} = \hbar c/l$
Hamiltonian density $\mathcal{H} = \pi\dot{\psi} - \mathcal{L}$	Js/m^4	$\mathbf{Js}/\text{m}^4 = \hbar/l^4$
4-potential B_μ of the gluon field in QCD, ($iB_\mu \in SU(3)$)	Js/m	$\mathbf{Js}/\text{m} = \hbar/l$
field tensor $G_{\mu\nu}$ of the gluon field ($G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + ig_s[B_\mu, B_\nu]$)	Js/m^2	$\mathbf{Js}/\text{m}^2 = \hbar/l^2$

Epilogue

Mathematics is the gate and the key to the sciences.
Roger Bacon (1214–1294)

I love mathematics not only because it is applicable to technology but also because it is beautiful.

Rószsa Péter (1905–1977)

The perfection of mathematical beauty is such whatsoever is most beautiful is also found to be most useful and excellent.

D’Arcy Wentworth Thompson (1860–1948)

The observation which comes closest to an explanation for the mathematical concepts cropping up in physics which I know is Einstein’s statement that the only physical theories we are willing to accept are the beautiful ones.

Eugene Wigner (1902–1995)

A truly realistic mathematics should be conceived, in line with physics, as a branch of the theoretical construction of the one real world, and should adopt the same sober and cautious attitude toward hypothetic extensions of its foundations as is exhibited by physics.

Hermann Weyl (1885–1955)

The interplay between generality and individuality, deduction and construction, logic and imagination – this is the profound essence of live mathematics.

Any one or another of the aspects can be at the center of a given achievement. In a far-reaching development all of them will be involved. Generally speaking, such a development will start from the “concrete ground,” then discard ballast by abstraction and rise to the lofty layers of thin air where navigation and observations are easy; after this flight comes the crucial test of landing and reaching specific goals in the newly surveyed low plains of individual “reality.”

In brief, the flight into abstract generality must start from and return to the concrete and specific.¹⁷

Richard Courant (1888–1972)

¹⁷ Mathematics in the modern world, *Scientific American* **211**(3) (1964), 41–49 (reprinted with permission).

There are mathematicians who reject a binding of mathematics to physics, and who justify mathematical work solely by aesthetical satisfaction which, besides all the difficulty of the material, mathematics is able to offer. Such mathematicians are more likely to regard mathematics as a form of art than science, and this point of view of mathematical unselfishness can be characterized by the slogan “l’art pour l’art”.

On the other hand, there are physicists who regret that their science is so much related to mathematics. They fear a loss of intuition in the natural sciences. They consider the intimate relation with nature, the finding of ideas in nature itself, which was given to Goethe (1749–1832) in such a high degree, as being destroyed by mathematics, and their anger or sorrow is the more serious the more they are forced to realize the inevitability of mathematics.

Both points of view deserve serious consideration; because not only people with narrow minds have expressed such opinions. Yes, one can say that such a radical inclination to one side or the other, if not caused by a lack of talent, is sometimes evidence of a deeper perception of science, as if someone is interested in both sciences, but at the same time is satisfied with obvious connections between mathematics and physics. . .

Mathematics is an organ of knowledge and an infinite refinement of language. It grows from the usual language and world of intuitions as does a plant from the soil, and its roots are the numbers and simple geometrical intuitions. We do not know which kind of content mathematics (as the only adequate language) requires; we cannot imagine into what depths and distances this spiritual eye will lead us.¹⁸

Erich Kähler (1906–2000)

The most vitally characteristic fact about mathematics, in my opinion, is its quite peculiar relationship to the natural sciences, or more generally, to any science which interprets experience on a higher than purely descriptive level. . .

I think that this is a relatively good approximation to truth – which is much too complicated to allow anything but approximations – that mathematical ideas originate in *empirical facts*, although the genealogy is sometimes long and obscure. But, once they are so conceived, the subject begins to live a peculiar life of its own and is better compared to a creative one, governed by almost entirely aesthetic motivations, than to anything else and, in particular, to an empirical science.

But there is a grave danger that the subject will develop along the line of least resistance, that the stream, so far from its source, will separate into a multitude of insignificant tributaries, and that the discipline will become a disorganized mass of details and complexities. In other words, at a great distance from its empirical sources or after much *abstract inbreeding*, a mathematical object is in danger of degeneration. At the inception, the style is usually classical; when it shows signs of becoming *baroque*, then the danger signal is up. . .

Whenever this stage is reached, then the only remedy seems to be a rejuvenating return to the source: the re-injection of more or less directly *empirical ideas*. I am convinced that this is a necessary condition to con-

¹⁸ On the relations of mathematics to physics and astronomy (in German), *Jahresberichte der Deutschen Mathematiker-Vereinigung* **51** (1941), 52–63 (reprinted with permission).

serve the freshness and the vitality of the subject and that this will remain equally true in the future.¹⁹

John von Neumann (1903–1957)

I want to say a word about the communication between mathematicians and physicists.

It has been very bad in the past, and some of the blame is doubtless to be laid on the physicist's shoulders. We tend to be very vague, and we don't know what the problem is until we have already seen how to solve it. We drive mathematicians crazy when we try to explain what our problems are. When we write articles we don't do a good enough job of specifying how certain we are about our statements; we do not distinguish guesses from theorems.

On the other hand, since I have said a lot of nice things about mathematics, I have to say that the mathematicians carry an even greater burden of guilt for this communication problem, largely because of their elitism. They often have, it seems to me, as their ideal the savant who is understandable only to a few co-specialists and who writes articles that one has to spend years to try to fathom.

When physicists write articles, they generally start them with a paragraph saying, "Up until now, this has been thought to be the case. Now, so – and – so has pointed out this problem. In this article, we are going to try to suggest a resolution of this difficulty." On the other hand, I have seen books of mathematics, not just articles but books, in which the first sentence in the preface was, "Let H be a nilpotent subgroup of . . ." These books are written in what I would call a lapidary style. The idea seems to be that there should be no word in the book that is not absolutely necessary, that is inserted merely to help the reader to understand what is going on.

I think this is getting much better. I find it is wonderful how mathematicians these days are willing to explain their field to interested physicists. This situation is improving, partly because as Iz Singer mentioned, we realize now that in certain areas we have much more in common than we had thought, but I think a lot more has to be done. There is still too much mathematics written which is not only not understandable to experimental or theoretical physicists, but is not even understandable to mathematicians who are not the graduate students of the author.²⁰

Steven Weinberg (born 1933)

Relations between mathematics and physics vary with time. Right now, and for the past few years, harmony reigns and a honeymoon blossoms. However, I have seen other times, times of divorce and bitter battles, when the sister sciences declared each other as useless – or worse. The following exchange between a famous theoretical physicist and an equally famous mathematician might have been typical, some fifteen or twenty years ago: Says the physicist: "I have no use for mathematics. All the mathematics I ever need, I invent in one week."

¹⁹ The Mathematician. In: *The Works of the Mind*, Vol. 1, pp. 180–196. Edited by R. Heywood, University of Chicago Press, 1947 (reprinted with permission).

²⁰ Mathematics: The unifying thread in science: *Notices Amer. Math. Soc.* **33** (1986), 716–733 (reprinted with permission).

Answers the mathematician: “You must mean the seven days it took the Lord to create the world.”

A slightly more reliable document is found in the preface of the first edition of Hermann Weyl’s book on group theory and quantum mechanics from 1928. He writes: “I cannot abstain from playing the role of an (often unwelcome) intermediary in this drama between mathematics and physics, which fertilize each other in the dark, and deny and misconstrue one another when face to face.”

This dramatic situation, described here by one of the great masters in both sciences, is a result of recent times. At the time of Newton (1643–1727) disharmony between mathematics and physics seemed unthinkable and unnatural, since both were his brainchildren; and close symbiosis persisted through the whole of the eighteenth century. The rift arose around 1800 and was caused by the development of pure mathematics (represented by number theory) on the one hand, and of a new kind of physics, independent of mathematics, which developed out of chemistry, electricity and magnetism on the other. This rift was widened in Germany under the influence of Goethe (1749–1832) and his followers, Schelling (1775–1854) and Hegel (1770–1831) and their “Naturphilosophie”.

Our protagonists are Carl Friedrich Gauss (1777–1855), as the creator of modern number theory, and Michael Faraday (1791–1867) as the inventor of physics without mathematics (in the strict sense of the word).

It would be foolish, of course, to claim the nonexistence of number theory before Gauss. An amusing document may illustrate the historical development. Erich Hecke’s famous *Lectures on the Theory of Algebraic Numbers* has on its last page a “timetable”, which chronologically lists the names and dates of the great number theoreticians, starting with Euclid (300 B.C.) and ending with Hermann Minkowski (1864–1909). As a physicist, I am impressed to find so many familiar names in this Hall of Fame: Fermat (1601–1665), Euler (1707–1783), Lagrange (1736–1813), Legendre (1752–1833), Fourier (1768–1830), and Gauss. In fact, we cannot find a single great number theoretician before Gauss, whom we would not count among the great physicists, provided we disregard antiquity. Specialization starts after 1800 with names like Kummer (1810–1891), Galois (1811–1832), and Eisenstein (1823–1852); who were all under the great influence of Gauss’ *Disquisitiones arithmeticae* from 1801. In this specific sense, Gauss’ book marks the dividing line between mathematics as a universal science and mathematics as a union of special disciplines, and between the “géomètre” as a universal “savant” in the sense of the eighteenth century and the specialized “mathématicien” of modern times. As is typical for a man of transition, Gauss does not belong to either category, he was universal and specialized. The struggle raged within him – and made him suffer.

Res Jost (1918–1990)

*Mathematics and physics since 1800: discord and sympathy*²¹

By a particular prerogative, not only does each man advance day by day in the sciences, but all men together make continual progress as the universe ages. . . Thus, the entire body of mankind as a whole, over many centuries, must be considered as a single man, who lives forever and continues to learn.

Blaise Pascal (1623–1662)

²¹ In: R. Jost, *The Fairy Tale about the Ivory Tower, essays and lectures*. Edited by K. Hepp, W. Hunziker, and W. Kohn, Springer, Berlin, pp. 219–240 (reprinted with permission).

References

- Abdalla, E., Abdalla, M., Rothe, K. (2001), *Non-Perturbative Methods in Two-Dimensional Quantum Field Theory*, World Scientific, Singapore.¹
- Abraham, R., Marsden, J. (1978), *Foundations of Mechanics*, Addison-Wesley, Reading, Massachusetts.
- Abraham, R., Marsden, J., Ratiu, T. (1988), *Manifolds, Tensor Analysis, and Applications*, Springer, New York.
- Abramowitz, M., Stegun, I. (Eds.) (1984), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Wiley, New York, and National Bureau of Standards, Washington, DC.
- Adams, C. (1994), *The Knot Book*, Cambridge University Press, Cambridge, United Kingdom.
- Adams, F., Laughlin, G. (1997), A dying universe: the long-term fate and evolution of astrophysical objects, *Rev. Mod. Phys.* **69**, 337–372.
- Adams, F., Laughlin, G. (1999), *The Five Ages of the Universe: Inside the Physics of Eternity*, Simon and Schuster, New York.
- Adler, S. (2004), *Quantum Theory as an Emergent Phenomenon: the Statistical Mechanics of Matrix Models as the Precursor of Quantum Field Theory*, Cambridge University Press, Cambridge, United Kingdom.
- Adler, S. (2006), *Adventures in Theoretical Physics, Selected Papers with Commentaries*, World Scientific, Singapore.
- Agricola, I., Friedrich, T. (2002), *Global Analysis: Differential Forms in Analysis, Geometry and Physics*, Amer. Math. Soc., Providence, Rhode Island (translated from German into English).
- Aizenman, M. (1982), Geometric analysis of φ^4 fields and Ising models, *Commun. Math. Phys.* **86**, 1–48.
- Albert, C., Bleile, C., Fröhlich, J. (2008), Batalin–Vilkovisky integrals in finite dimensions, 35 pages. Internet: <http://arxiv.org/0812.0464>
- Albeverio, S. (1986), *Nonstandard Methods in Stochastic Analysis and Mathematical Physics*, Academic Press, New York.
- Albeverio, S. (1988), *Solvable Models in Quantum Mechanics*, Springer, New York.
- Albeverio, S., Jost, J., Paycha, S., Scarlatti, S. (1997), *A Mathematical Introduction to String Theory*, Cambridge University Press, Cambridge, United Kingdom.

¹ Hints for further reading can be found in Chap. 17. The author’s homepage contains a complete list of the references to Volumes I through VI.

Internet: <http://www.mis.mpg.de/>

We also refer to E. Zeidler (Ed.), *Oxford User’s Guide to Mathematics*, Oxford University Press, 2004, which contains a comprehensive list about the standard literature in mathematics.

- Albeverio, S., Mazzucchi, S. (2008), A survey on mathematical Feynman path integrals: construction, asymptotics, applications. In: B. Fauser, J. Tolksdorf, and E. Zeidler (Eds.) (2008), pp. 49–66.
- Albeverio, S., Høegh-Krohn, R. (2008), *Mathematical Theory of Feynman Path Integrals*, 2nd edition with S. Mazucchi, Springer, Berlin.
- Aleksandrov, M., Kontsevich, M., Schwarz, A., Zaboronsky, O. (1997), Geometry of the master equation, *Int. J. Mod. Phys.* **A12**, 1405–1430.
- Ali, S., Engliš, M. (2004), *Quantization methods: a guide for physicists and analysts*. Internet: <http://arxiv:math-ph/0405065>
- Alten, H., Naini, D., Folkerts, M., Schlosser, H., Schlote, K., Wußing, H. (2003), *4000 Jahre Algebra: Geschichte, Kulturen, Menschen (4000 Years of Algebra)*, Springer, Berlin (in German) (see also Scriba and Schreiber (2003), *5000 Jahre Geometrie*, and Wußing (2008), *6000 Jahre Mathematik*).
- Amann, H. (1990), *Ordinary Differential Equations: An Introduction to Nonlinear Analysis*, de Gruyter, Berlin (translated from German into English).
- Amann, H., Escher, J. (2004), *Analysis I–III*, Birkhäuser, Basel (translated from German into English).
- Amelino-Camelia, G., Kowalski-Glikman, J. (Eds.) (2005), *Planck-Scale Effects in Astrophysics and Cosmology*, Springer, Berlin.
- Anosov, D., Bolibruch, A. (1994), *The Riemann–Hilbert Problem*, Vieweg, Wiesbaden.
- Apostol, T. (1986), *Introduction to Analytic Number Theory*, Springer, New York.
- Apostol, T. (1990), *Modular Functions and Dirichlet Series in Number Theory*, 2nd edn., Springer, New York.
- Araki, H. (1999), *Mathematical Theory of Quantum Fields*, Oxford University Press, Oxford (translated from Japanese into English).
- Aref'eva, I., Slavnov, A., Faddeev, L. (1974), Generating functional for the S-matrix in gauge-invariant theories, *Theor. Math. Phys.* **21**, 1165–1172.
- Arnold, V. (1978), *Mathematical Theory of Classical Mechanics*, Springer, Berlin (translated from Russian into English).
- Arnold, V., Gusein-Zade, S., Varchenko, A. (1985), *Singularities of Differentiable Maps*, Vols. 1, 2, Birkhäuser, Basel (translated from Russian into English).
- Arnold, V. (2004), *Lectures on Partial Differential Equations*, Springer, Berlin (translated from Russian into English).
- Arnold, V., Faddeev, L., Filippov, V., Manin, Yu., Tikhomirov, V., Vershik, A., Bolibruch, A., Osipov, Yu., Sinai, Ya. (Eds.) (2006), *Mathematical Events of the Twentieth Century*, Springer, New York.
- Asch, J., Joye, A. (Eds.) (2006), *Mathematical Physics of Quantum Mechanics: Selected and Refereed Lectures from The State of the Art in the Mathematical Physics of Quantum Systems*, QMath9, held in Giens, France, in 2004, Springer, Berlin.
- Ashtekhar, A. (1991), *Lectures on Non-Perturbative Canonical Gravity*, World Scientific, Singapore.
- Ashtekhar, A., Lewandowski, J. (1997), Quantum theory of geometry I: Area operators, II: Volume operators, *Class. Quant. Grav.* **14**, A55–A81, *Adv. Theo. Math. Phys.* **1**, 388–429.
- Ashtekhar, A., Baez, J., Corichi, A., Krasnov, K. (1998), Quantum geometry and black hole entropy, *Phys. Rev. Letters* **80**(5), 904–907.
- Ashtekar, A., Bojowald, M., Lewandowski, J. (2003), Mathematical structure of loop quantum cosmology, *Adv. Theor. Math. Phys.* **7**, 233–268.
- Ashtekar, A., Lewandowski, J. (2004), Background independent quantum gravity: A status report, *Class. Quant. Grav.* **21**, R53–R152.

- Atiyah, M. (1970), Resolution of singularities and division of distributions, *Commun. Pure Appl. Math.* **23**, 145–150.
- Atiyah, M. (1979), *Geometry of Yang–Mills Fields*, Lezioni Fermiani, Accademia Nazionale dei Lincei Scuola Normale Superiore, Pisa, Italia.
- Atiyah, M., Hitchin, M. (1988), *The Geometry and Dynamics of Magnetic Monopoles*, Princeton University Press, Princeton, New Jersey.
- Atiyah, M. (1990a), The Work of Edward Witten, *Proc. Intern. Congr. Math. Kyoto 1990*, *Math. Soc. Japan* 1991. In: Atiyah and Iagolnitzer (Eds.) (2003), pp. 514–518.
- Atiyah, M. (1990b), *Geometry and Physics of Knots*, Cambridge University Press, Cambridge, United Kingdom.
- Atiyah, M. (2002), Mathematics in the 20th century, *Bull. London Math. Soc.* **34**, 1–15.
- Atiyah, M., Iagolnitzer, D. (Eds.) (2003), *Fields Medallists' Lectures*, World Scientific, Singapore.
- Atiyah, M. (2004), The impact of Thom's cobordism theory, *Bull. Amer. Math. Soc.* **41** (3), 337–340.
- Atiyah, M. (2005), *Collected Works*, Vols. 1–6, Cambridge University Press, Cambridge, United Kingdom.
- Awramik, M., Czakon, M. (2003a), Complete two loop electroweak contributions to the muon lifetime in the standard model, *Phys. Lett. B* **568**, 48–54.
- Awramik, M., Czakon, M. (2003b), Two loop electroweak corrections to the muon decay lifetime, *Nucl. Phys. Proc. Suppl.* **116**, 238–242.
- Awramik, M., Czakon, M., Freitas, A., Weiglin, G. (2004), Precise prediction for the W -boson mass in the standard model, *Phys. Rev. D* **69**, 53006–16.
Internet: <http://arxiv:hep-ph/0311148>
- Bandyopadhyay, P. (1996), *Geometry, Topology, and Quantization*, Kluwer, Dordrecht, 1996.
- Bailin, D., Love, A. (1996), *Introduction to Gauge Field Theory*, 2nd edn., Institute of Physics, Bristol.
- Bailin, D., Love, A. (1997), *Supersymmetric Gauge Field Theory and String Theory*, 2nd edn., Institute of Physics, Bristol.
- Baker, A. (2002), *Matrix Groups: An Introduction to Lie Group Theory*, Springer, New York.
- Bandyopadhyay, P. (1996), *Geometry, Topology, and Quantization*, Kluwer, Dordrecht.
- Banks, T. (2008), *Modern Quantum Field Theory: A Concise Introduction*, Cambridge University Press, Cambridge, United Kingdom.
- Barenblatt, G. (1996), *Scaling, Self-Similarity, and Intermediate Asymptotics*, Cambridge University Press, New York (translated from Russian into English).
- Barrow, J. (2007), *New Theories of Everything: The Quest for Ultimate Explanation*, Oxford University Press, New York.
- Barton, G. (1989), *Elements of Green's Functions and Propagation: Potentials, Diffusion, and Waves*, Clarendon Press, Oxford.
- Barut, A. (1967), *The Theory of the Scattering Matrix*, MacMillan, New York.
- Barut, A. (2002), *Quantum Theory, Groups, Fields, and Particles*, Springer, Berlin.
- Basdevant, J. (2000), *The Quantum-Mechanics Solver: How to Apply Quantum Theory to Modern Physics*, Springer, Berlin.
- Basdevant, J., Dalibard, J. (2002), *Quantum Mechanics*, Springer, Berlin.
- Batalin, I., Vilkovisky, G. (1981), Gauge algebra and quantization, *Phys. Lett.* **B102**, 27–31.

- Batalin, I., Vilkovisky, G. (1983), Quantization of gauge theories with linearly dependent generators, *Phys. Rev.* **D28**, 2567–2582.
- Batalin, I., Vilkovisky, G. (1984), Closure of the gauge algebra, generalized Lie equations and Feynman rules, *Nucl. Phys.* **B234**, 106–124.
- Batchelor, M. (1979), The structure of supermanifolds, *Trans. Amer. Math. Soc.* **253**, 329–338.
- Bateman, H. (Ed.) (1953), *Higher Transcendental Functions*, Vols. 1–3, MacGraw Hill, New York.
- Battle, G. (1999), *Wavelets and Renormalization*, World Scientific, Singapore.
- Baumgartel, H. and Wollenberg, M. (1983), *Mathematical Scattering Theory*, Birkhäuser, Basel.
- Baumgärtel, H. (1985), *Analytic Perturbation Theory for Matrices and Operators*, Birkhäuser, Boston.
- Baumgärtel, H., Wollenberg, M. (1992), *Causal Nets of Operator Algebras*, Akademie-Verlag, Berlin.
- Baumgärtel, H. (1995), *Operator-Algebraic Methods in Quantum Field Theory*, Akademie-Verlag, Berlin.
- Baxter, R. (1982), *Exactly Solved Models in Statistical Mechanics*, Academic Press, New York.
- Baym, G. (1969), *Lectures on Quantum Mechanics*, Benjamin, Menlo Park, California.
- Becker, K., Becker, M., Schwarz, J. (2006), *String Theory and M-Theory*, Cambridge University Press, Cambridge, United Kingdom.
- Belger, M., Schimming, R., Wünsch, V. (1997), A survey on Huygens' principle, *J. for Analysis and its Applications* **16**(1), 5–32.
- Bell, E. (1986), *Men of Mathematics: Biographies of the Greatest Mathematicians of all Times*, Simon, New York.
- Bellucci, S., Ferrara, S., Marrani, A. (2006), *Supersymmetric Mechanics*, Vol. 1: Supersymmetry, Noncommutativity and Matrix Models, Vol. 2: The Attractor Mechanism and Space Time Singularities, Springer, Berlin.
- Benfatto, G., Gallavotti, G. (1995), *Renormalization Group*, Princeton University Press, Princeton, New Jersey.
- Berezansky, Yu., Kondratiev, Yu. (1995), *Spectral Methods in Infinite-Dimensional Analysis*, Vols. 1, 2, Kluwer, Dordrecht (translated from Russian into English).
- Berezin, F. (1966), *The Method of Second Quantization*, Academic Press, New York (translated from Russian into English).
- Berezin, F. (1987), *Introduction to Superanalysis*, Reidel, Dordrecht (translated from Russian into English).
- Berezin, F., Shubin, M. (1991), *The Schrödinger Equation*, Kluwer, Dordrecht (translated from Russian into English).
- Berline, N., Getzler, E., Vergne, M. (1991), *Heat Kernels and Dirac Operators*, Springer, New York.
- Bertimann, R. (2000), *Anomalies in Quantum Field Theories*, Oxford University Press, Oxford.
- Bethe, H., Salpeter, E. (1957), *Quantum Mechanics of One-and Two-Electron Atoms*, Springer, Berlin.
- Bethe, H., Bacher, R., Livingstone, M. (1986), *Basic Bethe: Seminal Articles on Nuclear Physics 1936–37*, American Institute of Physics, New York.
- Bethuel, F., Brézis, H., Hélein, F. (1994), *Ginzburg–Landau Vortices*, Birkhäuser, Basel.
- Bigatti, D., Susskind, L. (1997) Review of matrix theory.
Internet: <http://arxiv.org/hep-th/9712072>

- Bjorken, J., Drell, S. (1964), *Relativistic Quantum Mechanics*, McGraw-Hill, New York.
- Bjorken, J., Drell, S. (1965), *Relativistic Quantum Fields*, McGraw-Hill, New York.
- Blasone, M., Jizba, P., Vitiello, G. (2009), *Quantum Field Theory and Its Macroscopic Manifestations: Boson Condensation, Ordered Patterns and Topological Defects*, World Scientific, Singapore.
- Blau, S., Visser, M., Wipf, A. (1988), Zeta functions and the Casimir energy, *Nucl. Phys. B* **310**, 163–180.
- Bloch, S., Esnault, H., Kreimer, D. (2006), Motives associated to graph polynomials, *Commun. Math. Phys.* **267**(1), 181–225.
Internet: <http://arxiv:math/0510011>
- Bocaletti, D., Pucacco, G. (1998), *Theory of Orbits, Vol 1: Integrable Systems and Non-Perturbative Methods, Vol. 2: Perturbative and Geometrical Methods*, Springer, Berlin.
- Bodanis, D. (2000), *$E = mc^2$: A Biography of the World's Most Famous Equation*, Walker, New York.
- Bogoliubov, N., Shirkov, D. (1956), Charge renormalization group in quantum field theory, *Nuovo Cimento* **3**, 845–863
- Bogoliubov, N., Parasiuk, O. (1957), Über die Multiplikation der Kausalfunktionen in der Quantentheorie der Felder (On the multiplication of propagators in quantum field theory), *Acta Math.* **97**, 227–326 (in German).
- Bogoliubov, N., Medvedev, B., Polivanov, M. (1958), *Problems of the Theory of Dispersion Relations*, Fizmatgiz, Moscow (in Russian).
- Bogoliubov, N. (1967), *Lectures on Quantum Statistics, Vols. 1, 2*, Gordon and Breach, New York (translated from Russian into English).
- Bogoliubov, N., Logunov, A., Todorov, I. (1975), *Introduction to Axiomatic Quantum Field Theory*, Benjamin, Reading, Massachusetts (translated from Russian into English).
- Bogoliubov, N., Shirkov, D. (1980), *Introduction to Quantum Field Theory*, 3rd edn., Wiley, New York (translated from Russian into English).
- Bogoliubov, N., Shirkov, D. (1983), *Quantum Fields*, Benjamin, Reading, Massachusetts (translated from Russian into English).
- Bogoliubov, N., Logunov, A., Orsak, A., Todorov, I. (1990), *General Principles of Quantum Field Theory*, Kluwer, Dordrecht (translated from Russian into English).²
- Bogoliubov, N. (1992), *Statistical Mechanics and the Theory of Dynamical Systems*. Amer. Math. Soc., Providence, Rhode Island (translated from Russian into English).
- Bohm, A. (1994), *Quantum Mechanics: Foundations and Applications*, 3rd ed., Springer, Berlin.
- Böhm, M., Denner, A., Joos, H. (2001), *Gauge Theories of the Strong and Electroweak Interaction*, 2nd edn., Teubner, Stuttgart.
- Bojowald, M. (2008a), Canonical gravity and effective theory. In: B. Fauser, J. Tolksdorf, and E. Zeidler (Eds) (2008), pp. 217–234.
- Bojowald, M. (2008b), Loop Quantum Cosmology, *Living Reviews* 11/4 (2008), Max Planck Institute for Gravitational Physics, Albert Einstein, Golm (Germany).
Internet: <http://relativity.livingreviews.org/Articles/Irr-2008-4>
- Bojowald, M. (2008c), Follow the bouncing universe, *Scientific American*, October 2008, pp. 44–51.

² 1200 references

- Bojowald, M. (2009), *Zurück vor den Urknall: die ganze Geschichte des Universums (Back before the Big Bang – the complete history of the universe)*, Fsciher, Frankfurt/Main (in German).
- Bordag, M. (Ed.) (1999), *The Casimir Effect 50 Years Later*, World Scientific, Singapore.
- Borel, A. (1998), *Twenty-Five Years with Nicolas Bourbaki, 1949–1973*, *Notices Amer. Math. Soc.* **45**(3), 373–380.
- Born, M. (1969), *Physics in My Generation*, Springer, New York (translated from German into English).
- Born, M., Wolf, E. (1970), *Principles of Optics*, 4th edn., Pergamon Press, New York.
- Born, M. (1977), *My Life: Recollection's of a Nobel Laureat*, Charles Scribner's Sons, New York (translated from German into English).
- Börner, G. (2003), *The Early Universe: Facts and Fiction*, 4th edn., Springer, Berlin.
- Bott, R., Tu, L. (1982), *Differential Forms in Algebraic Topology*, Springer, New York.
- Bott, R. (1994), *Collected Papers*, Vols. 1–4, Birkhäuser, Boston.
- Bouwmeester, D., Ekert, A., Zeilinger, A. (Eds.) (2002), *The Physics of Quantum Information: Quantum Cryptography, Quantum Teleportation, Quantum Computation*, Springer, Berlin.
- Bratelli, C., Robinson, D. (2002), *Operator Algebras and Quantum Statistical Mechanics*, Vols. 1, 2, 2nd edn., Springer, New York.
- Bredon, G. (1993), *Topology and Geometry*, Springer, New York.
- Brennan, R. (1997), *Heisenberg Probably Slept Here: The Lives, Times, and Ideas of the Great Physicists of the 20th Century*, Wiley, New York.
- Brennecke, F., Dütsch, M. (2008), *The quantum action principle in the framework of causal perturbation theory*. In: B. Fauser, J. Tolksdorf, and E. Zeidler (Eds.) (2008), pp. 177–196.
- Brezin, E. (1991), *The N Large Expansion in Quantum Field Theory and Statistical Physics: From Spin Systems to 2-Dimensional Gravity*, World Scientific, Singapore.
- Brézis, H., Browder, F. (1998), *Partial differential equations in the 20th century*, *Advances in Math.* **135**, 76–144.
- Brian, D. (2005), *Einstein – a Life*, Wiley-VCH, Weinheim, Germany.
- Bricmont, J., Gawędzki, K., Kupiainen, A. (1999), *Kolmogorov–Arnold–Moser (KAM) theorem and quantum field theory*, *Commun. Math. Phys.* **201** (3), 699–727.
- Brouder, C., Fauser, B., Frabetti, A., Oeckl, R. (2004), *Quantum field theory and Hopf algebra cohomology*, *J. Phys. A: Math. Gen.* **37** (2004), 5895–5927. Internet: [arxiv:hep-th/0311253](https://arxiv.org/abs/hep-th/0311253).
- Brouder, C. (2008), *The structure of Green functions in quantum field theory with a general state*. In: B. Fauser, J. Tolksdorf, and E. Zeidler (Eds.) (2008), pp. 1–176.
- Brown, L. (1992), *Quantum Field Theory*, Cambridge University Press, New York.
- Brown, L. (Ed.) (1993), *Renormalization: From Lorentz to Landau and Beyond*, Springer, New York.
- Brunetti, R., Fredenhagen, K., Köhler, M. (1996), *The microlocal spectrum condition and the Wick polynomials of free fields*, *Commun. Math. Phys.* **180**, 633–652.
- Brunetti, R., Fredenhagen, K. (2000), *Micro-local analysis and interacting quantum field theories: renormalization on physical backgrounds*, *Commun. Math. Phys.* **208**, 623–661.

- Brunetti, R., Fredenhagen, K., Verch, R. (2003), The generally covariant locality principle – a new paradigm for local quantum field theory, *Commun. Math. Phys.* **237**, 31–68.
- Brunetti, R., Dütsch, M., Fredenhagen, K. (2009), Perturbative algebraic quantum field theory and the renormalization groups, *Adv. Theor. Math. Phys.* **13**, 1–56.
- Buchholz, D. (1990), On quantum fields that generate local algebras, *J. Math. Phys.* **31**, 1839–1846.
- Buchholz, D., Porrmann, M., Stein, U. (1991), Dirac versus Wigner: towards a universal particle concept in local quantum field theory, *Phys. Lett.* **B267**, 377–381.
- Buchholz, D., Verch, R. (1995), Scaling algebras and renormalization group in algebraic quantum field theory, *Rev. Math. Phys.* **7**, 1195–2040.
- Buchholz, D., Iagolnitzer, D., Moschella, U. (Eds.) (2005), *Rigorous Quantum Field Theory: A Festschrift for Jacques Bros*, Birkhäuser, Basel.
- Bühler, W. (1981), *Gauss: A Bibliographical Study*, Springer, New York.
- Burgess, M. (2002), *Classical Covariant Fields*, Cambridge University Press, Cambridge, United Kingdom.
- Burgess, C., Moore, G. (2007), *The Standard Model: A Primer*, Cambridge University Press, Cambridge, United Kingdom.
- Buschhorn, G., Wess, J. (Eds.) (2004), *Fundamental Physics: Heisenberg and Beyond*, Springer, Berlin.
- Bytsenko, A., Cognola, G., Elizalde, E., Moretti, V., Zerbini, S. (2003), *Analytic Aspects of Quantum Fields*, World Scientific, Singapore.
- Calzetta, E., Hu, B. (2008), *Non-Equilibrium Quantum Field Theory*, Cambridge University Press, Cambridge, United Kingdom.
- Cao, H., Zhu, X. (2006), A complete proof of the Poincaré and geometrization conjectures – Application of the Hamilton–Perelman theory of Ricci flow, *Asian J. Math.* **10** (2).
- Cao, H., Yau, S., Zhu, X. (2006), *Structure of Three-dimensional Space: The Poincaré and Geometrization Conjectures*, International Press, Boston.
- Cameron, R. (1960), A family of integrals serving to connect the Wiener and Feynman integrals, *J. of Math. and Phys. Sci. of MIT* **39**, 126–140.
- Carmichael, H. (1995), *Statistical Methods in Quantum Optics: Master Equations and Fokker–Planck Equations*, Springer, New York.
- Carow-Watamura, U., Maeda, Y., Watamura, S. (Eds.) (2005), *Quantum Field Theory and Noncommutative Geometry*, Springer, Berlin.
- Cartier, P. (1995), An introduction to zeta functions. In: Waldschmidt et al. (Eds.) (1995), pp. 1–63.
- Cartier, P. (2000), *Mathemagics, A tribute to L. Euler and R. Feynman*, Séminaire Lotharingien **44**, 1–71.
- Cartier, P. (2001), A mad day’s work: from Grothendieck to Connes and Kontsevich. The evolution of concepts of space and symmetry. *Bull. Amer. Math. Soc.* **38**(4), 389–408.
- Cartier, P., Julia, B., Moussa, P., Vanhove, P. (Eds.) (2006), *Frontiers in Number Theory, Physics, and Geometry, Vols. 1, 2*, Springer, Berlin.
- Cartier, P. (2006), A primer of Hopf algebras. Preprint: Institute des Hautes Études, Bures-sur-Yvette (France) IHES/M/06/40.
Internet: <http://www.cartier@ihes.fr>
- Cartier, P., DeWitt-Morette, C. (2006), *Functional Integration: Action and Symmetries*, Cambridge University Press, Cambridge, United Kingdom.

- Cartier, P., Ebrahimi-Fard, K., Patras, F., Thibon, J.-Y. (Eds.) (2009), Algebraic and Combinatorial Structures in Quantum Field Theory, Conference in Cargese (Corsica, France) in April 2009.
<http://www.math.unice.fr/~patras/CargeseConference/index.html>
- Casimir, H. (1948), On the attraction between two perfectly conducting plates, Proc. Kon. Nederl. Akad. Wetensch. **51**, 793–795.
- Cattaneo, A., Felder, G. (2000), A path integral approach to the Kontsevich quantization formula, Commun. Math. Phys. **212**, 591–611.
- Chaichian, M., Nelipa, N. (1984), Introduction to Gauge Field Theories, Springer, Berlin.
- Chaichian, M., Demichev, A. (1996), Introduction to Quantum Groups, World Scientific, Singapore.
- Chaichian, M., Hagedorn, R. (1998), Symmetries in Quantum Mechanics: From Angular Momentum to Supersymmetry, Institute of Physics, Bristol.
- Chaichian, M., Demichev, A. (2001), Path Integrals in Physics. Vol. 1: Stochastic Processes and Quantum Mechanics; Vol. 2: Quantum Field Theory, Statistical Physics, and Other Modern Applications, Institute of Physics.
- Chang, K. (2005), Methods in Nonlinear Analysis, Springer, Berlin.
- Chang, S. (1990), Introduction to Quantum Field Theory, World Scientific, Singapore.
- Chern, S., Hirzebruch, F. (Eds.) (2001), Wolf Prize in Mathematics, Vols. 1, 2, World Scientific, Singapore.
- Chew, G. (1966), The Analytic S -Matrix: A Basis for Nuclear Democracy, Benjamin, New York.
- Choquet-Bruhat, Y., DeWitt-Morette, C., Dillard-Bleick, M. (1996), Analysis, Manifolds, and Physics. Vol. 1: Basics; Vol 2: 92 Applications, Elsevier, Amsterdam.
- Choquet-Bruhat, Y. (2008), General Relativity and the Einstein Equations, Oxford University Press, Oxford, United Kingdom.
- Chung, K., Zhao, Z. (1995), From Brownian Motion to Schrödinger's Equation, Springer, New York.
- Collins, J. (1984), Renormalization: An Introduction to Renormalization, the Renormalization Group, and the Operator-Product Expansion, Cambridge University Press, Cambridge, United Kingdom.
- Colombeau, J. (1984), New Generalized Functions and Multiplication of Distributions, North-Holland, Amsterdam.
- Connes, A., Gawędzki, K., Zinn-Justin, J. (Eds.) (1998), Quantum Symmetries, Les Houches 1995, Elsevier, Amsterdam.
- Connes, A. (1985), Noncommutative differential geometry, Publ. Math. IHES **62**, 257–360.
- Connes, A. (1988), The action functional in noncommutative geometry, Commun. Math. Phys. **117**, 673–683.
- Connes, A., Lott, J. (1990), Particle models and noncommutative geometry, Nucl. Phys. B (Proc. Suppl.) **18**, 29–47.
- Connes, A. (1994), Noncommutative Geometry, Academic Press, New York.
- Connes, A. (1995), Noncommutative geometry and reality, J. Math. Phys. **36**, 6194–6231.
- Connes, A., Kreimer, D. (1998), Hopf algebras, renormalization and noncommutative geometry, Commun. Math. Phys. **199**, 203–242.
- Connes, A., Kreimer, D. (2000a), Renormalization in quantum field theory and the Riemann–Hilbert problem I: The Hopf algebra structure of graphs and the main theorem, Commun. Math. Phys. **210**, 249–273.

- Connes, A., Kreimer, D. (2000b), Renormalization in quantum field theory and the Riemann–Hilbert problem II: The beta function, diffeomorphisms, and the renormalization group, *Commun. Math. Phys.* **216**, 215–241.
- Connes, A. (2000), *Noncommutative Geometry and the Riemann Zeta Function*, Frontiers and Perspectives, International Mathematical Union.
- Connes, A., Lichnerowicz, A., Schützenberger, M. (2001), *Triangle of Thoughts*, Amer. Math. Soc., Providence, Rhode Island.
- Connes, A. (2003), Symmétries galoisiennes et renormalisation. In: Duplantier and Rivasseau (2003), pp. 241–264.
- Connes, A., Marcolli, M. (2008), *Noncommutative Geometry, Quantum Fields, and Motives*, Amer. Math. Soc., Providence, Rhode Island.
Internet: <http://www.math.fsu.edu/~marcolli/bookjune4.pdf>
- Consani, C., Marcolli, M. (Eds.) (2006), *Noncommutative Geometry and Number Theory: Where Arithmetic meets Geometry and Physics*, Vieweg, Wiesbaden.
- Cottingham, W., Greenwood, D. (1998), *An Introduction to the Standard Model of Particle Physics*, Cambridge University Press, Cambridge, United Kingdom.
- Coughlan, G., Dood, J. (1991), *The Ideas of Particle Physics: An Introduction for Scientists*, 2nd edn., Cambridge University Press, Cambridge, United Kingdom.
- Courant, R. (1964), Mathematics in the modern world, *Scientific American* **211**(3), 41–49.
- Courant, R., Hilbert, D. (1989), *Methods of Mathematical Physics*, Vols. 1, 2, Wiley, New York (translated from German into English).
- Cox, D., Little, J., O’Shea, D. (1998), *Using Algebraic Geometry*, Springer, New York.
- Coxeter, H. (1995), *Projective Geometry*, 2nd edn., Springer, New York.
- Cycon, R., Froese, R., Kirsch, W., Simon, B. (1986), *Schrödinger Operators*, Springer, New York.
- Dardo, M. (2004), *Nobel Laureates and Twentieth-Century Physics*, Cambridge University Press, Cambridge, United Kingdom.
- Das, A. (2006), *Field Theory: A Path Integral Approach*, 2nd edn., World Scientific, Singapore.
- Das, A. (2008), *Lectures on Quantum Field Theory*, World Scientific, Singapore.
- Dautray, R., Lions, J. (1988), *Mathematical Analysis and Numerical Methods for Science and Technology*, Vols. 1–6, Springer, New York (translated from French into English).
- Dauxois, T., Peyrard, M. (2006), *Physics of Solitons*, Cambridge University Press, Cambridge, United Kingdom.
- Gennes, de P. (1979), *Scaling Concepts in Polymer Physics*, Cornell University Press, Ithaca.
- Gennes, de P., Prost, J. (1995), *The Physics of Liquid Crystals*, Clarendon Press, Oxford.
- Deligne, P., Etingof, P., Freed, D., Jeffrey, L., Kazhdan, D., Morgan, J., Morrison, D., Witten, E. (Eds.) (1999), *Lectures on Quantum Field Theory: A Course for Mathematicians Given at the Institute for Advanced Study in Princeton*, Vols. 1, 2, Amer. Math. Soc., Providence, Rhode Island.
- de Melo, W., van Strien, S. (1993), *One-Dimensional Dynamics*, Springer, Berlin.
- Derezinski, A., Gérard, C. (1997), *Scattering Theory of Classical and N -Particle Systems*, Springer, New York.
- DeSimone, A., Kohn, R., Müller, S., Otto, F. (2002), A reduced theory for thin-film micromagnetics, *Comm. Pure Appl. Math.* **55**, 1408–1460.
- De Vega, H. (2000), *Integrable Quantum Field Theories and Statistical models: Yang–Baxter and Kac–Moody Algebras*, World Scientific Singapore.

- Devlin, K. (2002), *The Millenium Problems: The Seven Greatest Unsolved Mathematical Puzzles of Our Time*, Basic Books, New York.
- DeWitt, B. (2003), *The Global Approach to Quantum Field Theory*, Vols. 1, 2, Clarendon Press, Oxford.
- Dierkes, U., Hildebrandt, S., Küster, A., Wohlrab, O. (1992), *Minmal Surfaces*, Vols. 1, 2, Springer, Berlin.
- Dieudonné, J. (1968), *Foundations of Modern Analysis*, Vols. 1–9, Academic Press, New York (translated from French into English). (German edition: *Grundzüge der modernen Analysis*, Vols. 1–9, Deutscher Verlag der Wissenschaften, Berlin 1972–1987.)
- Dieudonné, J. et al. (Eds.) (1978), *Abrégé d'histoire des mathématiques, 1700–1900*, Vols. 1, 2, Hermann, Paris (in French). (German edition: *Geschichte der Mathematik: ein Abriß 1700–1900*, Vieweg, Braunschweig, 1985.)
- Dieudonné, J. (1981), *History of Functional Analysis, 1900–1975*, North-Holland, Amsterdam.
- Dieudonné, J. (1982), *A Panorama of Pure Mathematics as Seen by Bourbaki*, Academic Press, New York (translated from French into English).
- Dieudonné, J. (1985), *History of Algebraic Geometry, 400 B.C.–1985 A.C.*, Chapman, New York.
- Dieudonné, J. (1989), *A History of Algebraic and Differential Topology, 1900–1960*, Birkhäuser, Boston.
- Di Francesco, P., Mathieu, P., Sénéchal, D. (1997), *Conformal Field Theory*, Springer, New York.
- Dimock, J. (1980), Algebras of local observables on a manifold, *Commun. Math. Phys.* **77**, 219–228.
- Dimock, J. (1982), Dirac quantum fields on a manifold, *Trans. Amer. Math. Soc.* **269**, 133–147.
- Dirac, P. (1930), *The Principles of Quantum Mechanics*, 4th edn. 1981, Clarendon Press, Oxford.
- Dirac, P. (1964), *Lectures on Quantum Mechanics*, Belfer Graduate School of Science, Yeshiva University, New York.
- Dirac, P. (1966), *Lectures on Quantum Field Theory*, Academic Press, New York.
- Dirac, P. (1970), *The Development of Quantum Mechanics*, Gordon and Breach, New York.
- Dirac, P. (1978), *Directions in Physics*, Wiley, New York.
- Dirac, P. (1995), *The Collected Works of P.A.M. Dirac*. Edited by R. Dalitz, Cambridge University Press, Cambridge, United Kingdom.
- Dirac, P. (1996), *General Theory of Relativity*, Princeton University Press, Princeton, New Jersey.
- Dodson, C., Parker, P. (1997), *A User's Guide to Algebraic Topology*, Kluwer, Dordrecht.
- Doetsch, G. (1956), *Handbuch der Laplace-Transformation (Handbook of the Laplace transformation)*, Vols. 1–3, Birkhäuser, Basel (in German).
- Dokshitzer, Yu., Khoze, V., Mueller, A., Troyan, S. (1991), *Basics of Perturbative Quantum Chromodynamics (QCD)*, Editions Frontières, Singapore.
- Dolzmann, G. (2003), *Variational Methods for Crystalline Microstructure: Analysis and Computation*, Springer, Berlin.
- Donaldson, S., Kronheimer, P. (1990), *The Geometry of Four-Manifolds*, Oxford University Press, Oxford.
- Donaldson, S. (1996), The Seiberg–Witten Equations and 4-Manifold Topology, *Bull. Amer. Math. Soc.* **33**, 45–70.
- Donaldson, S. (2002), *Floer Homology Groups*, Cambridge University Press, Cambridge, United Kingdom.

- Doplicher, S., Roberts, J. (1989), A new duality for compact groups, *Invent. Math.* **98**, 157–218.
- Doplicher, S., Roberts, J. (1990), Why there is a field algebra with compact gauge group describing the superselection structure in particle physics, *Commun. Math. Phys.* **131**, 51–107.
- Doplicher, S., Fredenhagen, K., Roberts, J. (1995), The structure of space-time at the Planck scale and quantum fields, *Commun. Math. Phys.* **172**, 187–220.
- Dorey, P. (1998), *Exact S-Matrices in Two-Dimensional Quantum Field Theory: An Introduction Through Affine Toda Models*, Cambridge University Press, Cambridge, United Kingdom.
- Dubrovin, B., Fomenko, A., Novikov, S. (1992), *Modern Geometry: Methods and Applications*, Vols. 1–3, Springer, New York (translated from Russian into English).
- Duistermaat, J., Kolk, J. (2000), *Lie Groups*, Springer, New York.
- Duistermaat, J., Heckmann, G. (1983), On the variation in the cohomology in the symplectic form of the reduced phase space, *Invent. Math.* **69** (1982), 259–268; **72** (1983), 153.
- Dunlap, R. (1997), *The Golden Ratio and Fibonacci Numbers*, World Scientific, Singapore.
- Duplantier, B., Rivasseau, V. (Eds.) (2003), *Vacuum Energy – Renormalization*, Poincaré Seminar 2002, Birkhäuser, Basel.
- Dütsch, M., Boas, F. (2002), The Master Ward identity, *Rev. Mod. Phys.* **14**, 977–1049.
- Dütsch, M., Rehren, K. (2003), Generalized free fields and AdS-CFT correspondence, *Ann. H. Poincaré* **4** (4), 613–635.
- Dütsch, D., Fredenhagen, K. (2003), The Master Ward identity and the generalized Schwinger–Dyson equation in classical field theory, *Commun. Math. Phys.* **243**, 275–314.
- Dütsch, M., Fredenhagen, K. (2004b), Causal perturbation theory in terms of retarded products, and a proof of the action Ward identity, *Rev. Math. Phys.* **16** (10), 1291–1348.
- Dyson, F. (1952), Divergence of perturbation theory in quantum electrodynamics, *Phys. Rev.* **85**, 631–632.
- Dyson, F. (1979), *Disturbing the Universe*, Harper and Row, New York.
- Dyson, F. (1992), *From Eros to Gaia*, Pantheon Books, New York.
- Dyson, F. (1993), *George Green and Physics*, *Phys. World* **6**, August 1993, 33–38.
- Dyson, F. (1996), *Selected Papers of Freeman Dyson with Commentaries*, Amer. Math. Soc., Providence, Rhode Island.
- Dyson, F. (1999a), *Origins of Life*, Cambridge University Press, Cambridge, United Kingdom.
- Dyson, F. (1999b), *The Sun, the Genome, and the Internet: Tools of Scientific Revolution*, Oxford University Press, New York.
- Ebrahimi-Fard, K., Guo, L., Kreimer, D. (2004), Spitzer’s identity and the algebraic Birkhoff decomposition in perturbative quantum field theory (pQFT), *J. Phys. A: Mathematical and General* **37**, 11037–11052.
- Ebrahimi-Fard, K., Kreimer, D. (2005), The Hopf algebra approach to Feynman diagram calculations. Topical Review. *J. Phys. A: Mathematical and General* **38**, R385–R407.
- Ebrahimi-Fard, E., Gracia-Bondia, J., Guo, L., Várilly, J. (2006), Combinatorics of renormalization as matrix calculus, *Phys. Lett.* **B632**, 552–558.

- Ebrahimi-Fard, K., Manchon, D., Patras, F. (2009), A noncommutative Bohnenlust–Spitzer identity for Rota–Baxter algebras solves Bogoliubov’s recursion (in renormalization theory), *J. of Noncommutative Geometry* **3**(2), 181–222.
- Economu, E. (2006), *Green’s Functions in Quantum Physics*, 3rd. edn., Springer, New York.
- Eden, R. et al. (1966), *The Analytic S-Matrix Theory*, Cambridge University Press, Cambridge, United Kingdom.
- Efetov, K. (1997), *Supersymmetry in Disorder and Chaos*, Cambridge University Press, Cambridge, United Kingdom.
- Egorov, Yu., Shubin, M. (1991), *Partial Differential Equations*, Vols. 1–4, Springer, New York (translated from Russian into English).
- Egorov, Yu., Shubin, M. (1998), *Foundations of the Classical Theory of Partial Differential Equations*, New York (translated from Russian into English).
- Egorov, Yu., Komech, A., Shubin, M. (1999), *Elements of the Modern Theory of Partial Differential Equations*, Springer, New York.
- Ehlers, J., Falco, E., Schneider, P. (1992), *Gravitational Lenses*, Springer, New York.
- Eichhorn, J. (2007), *Global Analysis on Open Manifolds*, Nova Science Publishers, New York.
- Einstein, A. (1949), *Albert Einstein als Philosoph und Naturforscher (Albert Einstein as philosopher and scientist)*. Edited by P. Schilpp, Kohlhammer Verlag, Stuttgart (in German).
- Einstein, A. (1982), *Ideas and Opinions*, Three Rivers Press, New York.
- Eisenbud, D. (1994), *Commutative Algebra with a View to Algebraic Geometry*, Springer, New York.
- Eisenbud, D., Harris, J. (2000), *The Geometry of Schemes*, Springer, New York.
- Eisenbud, D. (2005), *The Geometry of Syzygies: A Second Course in Commutative Algebra and Algebraic Geometry*, Springer, New York.
- Elizalde, E. (1995), *Ten Physical Applications of Spectral Zeta Functions*, Springer, Berlin.
- Emch, G. (1972), *Algebraic Methods in Statistical Physics and Quantum Field Theory*, Wiley, New York.
- Emch, G. (1984), *Mathematical and conceptual foundations of 20th century physics*, North-Holland, Amsterdam.
- Emch, G., Liu, C. (2002), *The Logic of Thermostatistical Physics*, Springer, New York.³
- Encyclopedia of Mathematical Sciences (1990), Vols. 1–142ff, Springer, Berlin, 1990ff (translated from Russian into English).
- Encyclopedia of Mathematical Physics (2006), Vols. 1–5. Edited by J. Francoise, G. Naber, and T. Tsun, Elsevier, Oxford.
- Engquist, B., Schmid, W. (Eds.) (2001), *Mathematics Unlimited – 2001 and Beyond*, Springer, New York.
- Epstein, H., Glaser, V. (1973), The role of locality in perturbation theory, *Ann. Inst. Poincaré* **A19**(3), 211–295.
- Erdélyi, A., Magnus, W., Oberhettinger, F., Tricomi, F. (Eds.) (1955), *Higher Transcendental Functions*, Vols. 1–3, McGraw-Hill, New York.
- Erdélyi, A. (1965), *Asymptotic Expansions*, Dover, New York.
- Eschrig, H. (2003), *The Fundamentals of Density Functional Theory*, 2nd edn., Teubner, Leipzig.
- Etingof, P. (2000), A note on dimensional regularization. In: P. Deligne et al. (Eds.) (2000), Vol. 1, pp. 597–607.

³ 1500 references

- Evans, C. (1998), *Partial Differential Equations*, Amer. Math. Soc., Providence, Rhode Island.
- Evans, E., Kawahigashi, Y. (1998), *Quantum Symmetries on Operator Algebras*, Clarendon Press, Oxford.
- Evans, G., Blackledge, J., Yardley, P. (2000), *Analytic Methods for Partial Differential Equations*, Springer, London.
- Faddeev, L., Slavnov, A. (1980), *Gauge Fields*, Benjamin, Reading, Massachusetts (translated from Russian into English).
- Faddeev, L. (1984), Integrable models in $1 + 1$ -dimensional quantum field theory. *Les Houches 1984, Session 43*, North-Holland Amsterdam, pp. 561–608.
- Faddeev, L., Takhtadzhian, L. (1987), *Hamiltonian Method in the Theory of Solitons*, Springer-Verlag, New York (translated from Russian into English).
- Faddeev, L. (1995), *40 Years in Mathematical Physics*, World Scientific, Singapore.
- Faddeev, L., Merkuriev, S. (1996), *Quantum Scattering Theory for Several Particle Systems*, Kluwer, Dordrecht (translated from Russian into English).
- Faddeev, L. (1999a), Elementary introduction to quantum field theory. In: P. Deligne et al. (Eds.) (1999), pp. 513–550.
- Faddeev, L. (1999b), Mathematical physics: what is it? In: *Proceedings of the Steklov Institute of Mathematics, Russia, Vol. 226, Papers dedicated to the 65th birthday of Academician Lyudvig Dimitrievich Faddeev*, pp. 1–4.
- Faddeev, L., Yakubovskii, O. (2009), *Lectures on Quantum Mechanics for Students of Mathematics*, Amer. Math. Soc., Providence, Rhode Island.
- Faltings, G. (1994), A proof of the Verlinde formula. *J. Alg. Geometry* **3**, 347–374.
- Faris, W. (1971), Perturbations and non-normalizable eigenvectors, *Helv. Phys. Acta* **44**, 930–936.
- Farkas, H., Kra, I. (1992), *Riemann Surfaces*, Springer, New York.
- Farkas, H., Kra, I. (2001), *Theta Constants, Riemann Surfaces and the Modular Group: An Introduction with Applications to Uniformization Theorems, Partition Identities and Combinatorial Number Theory*, Amer. Math. Soc., Providence, Rhode Island.
- Fausser, B., Stumpf, H. (1997), Positronium as an example of algebraic composite calculations, pp. 399–418. In: J. Keller and Z. Oziewicz (Eds.), *The Theory of the Electron, Mexico 1997*. Internet: <http://arxiv.org/hep-th/9510193>
- Fausser, B. (2001a), Clifford geometric quantization of inequivalent vacua, *Math. Meth. Appl. Sci.* **24**, 885–912. Internet: <http://arxiv.org/hep-th/9719947>
- Fausser, B. (2001b), On the Hopf algebraic origin of Wick normal-ordering. *J. Phys. A: Math. Gen.* **34**, 105–116. Internet: <http://arxiv.org/hep-th/0007032>
- Fausser, B., Tolksdorf, J., Zeidler, E. (Eds.) (2006), *Quantum Gravity: Mathematical Models and Experimental Bounds*, Birkhäuser, Basel.
- Fausser, B., Tolksdorf, J., Zeidler, E. (Eds.) (2008), *Quantum Field Theory – Competitive Methods*, Birkhäuser, Basel.
- Federbush, P. (1987), Quantum field theory in ninety minutes, *Bull. Amer. Math. Soc.* **17**(1), 93–103.
- Fedosov, B. (1996), *Deformation Quantization and Index Theory*, Akademie-Verlag, Berlin.
- Fefferman, C. (1983), The uncertainty principle, *Bull. Amer. Math. Soc.* **9**(2), 129–206.
- Fefferman, C. (1985), The atomic and molecular nature of matter, *Revista Mat. Iberoamer.* **1**(1), 1–44.
- Fefferman, C., de la Llave, R. (1986), Relativistic stability of matter, *Revista Mat. Iberoamer.* **2**, 119–213.

- Fefferman C., Seco, L. (1994), On the Dirac and Schwinger corrections to the ground-state energy of an atom, *Advances in Math.* **107**(1), 1–185.
- Feldman, J., Hurd, T., Rosen, L., Wright, J. (1988): *QED: A Proof of Renormalizability*, Springer, Berlin.
- Feldman, J., Trubowitz, E. (1992), Renormalization in classical mechanics and many body quantum field theory, *Jerusalem J. d'Analyse Mathématique* **52**, 213–247. Internet: <http://www.math.ubc.ca/~feldman/research.html>
- Feldman, J., Froese, R., Rosen, L. (1994), *Mathematical Quantum Theory: Field Theory and Many-Body Theory*, Amer. Math. Soc., Providence, Rhode Island.
- Feldman, J., Knörrer, H., Trubowitz, E. (2002), *Fermionic Functional Integrals and the Renormalization Group*, Amer. Math. Soc., Providence, Rhode Island.
- Feldman, J., Knörrer, H., Trubowitz, E. (2003), *A two-dimensional Fermi liquid*. Internet: <http://www.math.ubc.ca/~feldman/fl.html>
- Felsager, B. (1997), *Geometry, Particles, and Fields*, Springer, New York.
- Fernández, R., Fröhlich, J., Sokal, D. (1992), *Random Walks, Critical Phenomena, and Triviality in Quantum Field Theory*, Springer, Berlin.
- Ferris, T. (1991), *The World Treasury of Physics, Astronomy, and Mathematics*, Brown, Boston.
- Fetter, A., Walecka, J. (1971), *Quantum Theory of Many-Particle Systems*, McGraw-Hill, New York.
- Fewster, C. (2008), *Lectures on Quantum Field Theory in Curved Space-Time*, Department of Mathematics, University of York United Kingdom. Electronic address: cjf3yor.ac.uk
See also the Lecture Notes series of the Max Planck Institute for Mathematics in the Sciences, Leipzig. Internet: <http://www.mis.mpg.de/preprints>
- Feynman, R. (1961), *Quantum Electrodynamics*, Benjamin, Reading, Massachusetts.
- Feynman, F., Leighton, R., Sands, M. (1963), *The Feynman Lectures in Physics*, Addison-Wesley, Reading, Massachusetts.
- Feynman, R., Hibbs, R. (1965), *Quantum Mechanics and Path Integrals*, McGraw-Hill, New York.
- Feynman, R. (1966), *The Character of Physical Law*, MIT Press, Cambridge, Massachusetts.
- Feynman, R. (1985), *QED (Quantum Electrodynamics): The Strange Theory of Light and Matter*, Mautner Memorial Lectures, Princeton University Press, Princeton, New Jersey.
- Feynman, R., Leighton, R. (1985), *Surely You're Joking Mr. Feynman: Adventures of a Curious Character*, Norton, New York.
- Feynman, R. (1995), *Feynman Lectures on Gravitation*. Edited by B. Hatfield, Addison-Wesley, Boston.
- Feynman, R. (1998), *Statistical Mechanics: A Set of Lectures*, 14th edn., Addison Wesley, Reading, Massachusetts.
- Figuroa, H., Gracia-Bondia, J. (2005), Combinatorial Hopf algebras in quantum field theory I, *Rev. Math. Phys.* **17**, 881–975.
Internet: <http://arxiv:hep-th/0408145>
- Fikhtengol'ts, G. (1965), *The Fundamentals of Mathematical Analysis*, Vols. 1–3, Pergamon Press, Oxford (translated from Russian into English).
- Finster, F. (2006), *The Principle of the Fermionic Projector*, American Mathematical Society and International Press, Boston.
- Finster, F. (2008), From discrete space-time to Minkowski space: basic mechanisms, methods, and perspectives. In: B. Fauser, J. Tolksdorf, and E. Zeidler (Eds.) (2008), pp. 235–260.

- Finster, F., Kamran, N., Smoller, J., Yau, S. (2008), Linear waves in the Kerr geometry: a mathematical voyage to black hole physics.
Internet: <http://arxiv.org/0801.1423>
- Finster, F., Hainzl, C. (2008), Quantum oscillations prevent the Big Bang singularities in an Einstein–Dirac Cosmology. Internet: <http://arxiv.org/0809.1693>
- Flappan, E. (2000), When Topology Meets Chemistry: A Topological Look at Molecular Chirality, Cambridge University Press, Cambridge, United Kingdom.
- Fleischhack, C. (2006), Kinematical uniqueness of loop gravity, pp. 203–218. In: B. Fauser, J. Tolksdorf, and E. Zeidler (Eds.) (2006), pp. 203–218.
- Folland, G. (1984), Real Analysis; Modern Techniques and Their Applications, Wiley, New York.
- Folland, G. (1995a), Introduction to Partial Differential Equations, 2nd edn., Princeton University Press, Princeton, New Jersey.
- Folland, G. (1995b), A Course in Abstract Harmonic Analysis, CRC Press, Boca Raton, Florida.
- Folland, G. (2008), Quantum Field Theory: A Tourist Guide for Mathematicians, Amer. Math. Soc., Providence, Rhode Island.
- Folland G. (2009), Fourier Analysis and Its Applications, Amer. Math. Soc., Rhode Island.
- Ford, L. (1972), Automorphic Functions, 4th edn., Chelsea, New York.
- Fowler, A. (1997), Mathematical Models in the Applied Sciences, Cambridge University Press, Cambridge, United Kingdom.
- Franklin, E., Palchik, M. (1996), Conformal Quantum Field Theory in D Dimensions, Kluwer, Dordrecht.
- Frank, T. (2005), Nonlinear Fokker–Planck Equation, Springer, Berlin.
- Frankel, T. (1999), The Geometry of Physics, Cambridge University Press, Cambridge, United Kingdom.
- Frappat, L., Sciarrino, A., Sorba, P. (2000), Dictionary of Lie Algebras and Super Lie Algebras, Academic Press, New York.
- Fredenhagen, K., Rehren, K., Seiler, E. (2007), Quantum field theory: where we are, Lecture Notes in Physics **721** (2007), 61–87.
Internet: <http://arxiv.org/hep-th/0603155>
- Freed, D., Uhlenbeck, K. (1984), Instantons and Four-Manifolds, Springer, New York.
- Freed, D., Uhlenbeck, K. (1995), Geometry and Quantum Field Theory, Oxford University Press, New York.
- Freed, D. (1999), Five Lectures on Supersymmetry, Amer. Math. Soc., Providence, Rhode Island.
- Freed, D., Morrison, D., Singer, I. (2006), Quantum Field Theory, Supersymmetry, and Enumerative Geometry, Amer. Math. Soc., Providence, Rhode Island.
- Freidlin, M. (1985), Functional Integration and Partial Differential Equations, Princeton University Press, Princeton, New Jersey.
- Frenkel, I., Lepowski, J., Meurman, A. (1988), Vertex Operator Algebras and the Monster, Academic Press, New York.
- Frenkel, I., Ben-Zvi, D. (2001), Vertex Algebras and Algebraic Curves, American Mathematical Society, Providence, Rhode Island.
- Fried, H. (1972), Functional Methods and Models in Quantum Field Theory, MIT Press, Cambridge, Massachusetts.
- Friedrich, T. (Ed.) (1981), Self-Dual Riemannian Geometry and Instantons, Teubner, Leipzig.
- Friedrich, T. (2000), Dirac Operators in Riemannian Geometry, Amer. Math. Soc., Providence, Rhode Island (translated from German into English).

- Friedrichs, K. (1927), Eine invariante Formulierung des Newtonschen Gravitationsgesetzes und des Grenzübergangs vom Einsteinschen zum Newtonschen Gesetz (An invariant formulation of Newton's gravitational law and the passage from Einstein's theory of general relativity to Newton's classical theory), *Math. Ann.* **98**, 566–575.
- Friedrichs, K. (1953), *Mathematical Aspects of the Quantum Theory of Fields*, Interscience, New York.
- Friesecke, G., James, R., Müller, S. (2002), A theorem on geometric rigidity and the derivation of nonlinear plate theory from three-dimensional elasticity, *Comm. Pure Appl. Math.* **55**, 1461–1506.
- Fritzsche, H., Gell-Mann, M., Leutwyler, H. (1973), Advantages of the color octet gluon picture, *Phys. Lett.* **47B**, 365–368.
- Fritzsche, H. (1992), *Quarks*, Penguin, London (translated from German into English).
- Fröhlich, J., Spencer, T. (1984), A rigorous approach to Anderson localization, *Phys. Rev.* **103** (1984), 1–4, 9–25.
- Fröhlich, J. (1992), *Non-Perturbative Quantum Field Theory: Mathematical Aspects and Applications*, Selected Papers, World Scientific, Singapore.
- Fröhlich, J. (1993a), *Scaling and Self-Similarity in Physics: Renormalization in Statistical Physics*, Birkhäuser, Basel.
- Fröhlich, J., Kerler, T. (1993b), *Quantum Groups, Quantum Categories, and Quantum Field Theory*, Springer, Berlin.
- Fuchs, J. (1992), *Affine Lie Algebras and Quantum Groups: An Introduction with Applications in Conformal Field Theory*, Cambridge University Press, United Kingdom.
- Fuchs, J., Schweigert, C. (1997), *Symmetries, Lie Algebras, and Representations: A Graduate Course for Physicists*, Cambridge University Press, Cambridge, United Kingdom.
- Fujikawa, K., Suzuki, H. (2004), *Path Integrals and Quantum Anomalies*, Oxford University Press, Oxford.
- Fujita, T. (2007), *Symmetry and its Breaking in Quantum Field Theory*, Nova Science, New York.
- Fukugita, M., Yanagita, T. (2003), *Physics of Neutrinos and Applications to Astrophysics*, Springer, Berlin.
- Fulde, P. (1995), *Electron Correlations in Molecules and Solids*, 3rd edn., Springer, New York.
- Fulling, S. (1989), *Aspects of Quantum Field Theory in Curved Space-Time*, Cambridge, University Press, Cambridge, United Kingdom.
- Fulton, W., MacPherson, R. (1994), A compactification of configuration spaces, *Ann. of Math.* **139**, 183–225.
- Galindo, A., Pascual, P. (1990), *Quantum Mechanics*, Vols. 1, 2, Springer, Berlin (translated from Spanish into English).
- Gambini, R., Pulli, J. (2000), *Loops, Knots, Gauge Theories and Quantum Gravity*, Cambridge University Press, Cambridge, United Kingdom.
- Garbaczewski, P. (1985), *Classical and Quantum Field Theory of Exactly Soluble Nonlinear Systems*, World Scientific, Singapore.
- Gårding, L.: See Wightman and Gårding.
- Gasquet, C., Witomski, P. (2002), *Fourier Analysis and Applications*, Springer, New York.
- Gaudin, M. (1983), *La fonction d'onde de Bethe*, Masson, Paris (in French).
- Gauß, C. (1863/1933), *Werke* (Collected works with commentaries), Vols. 1–12, Göttingen, Germany.

- Ge, M., Bao-Heng Zhao (1989), *Introduction to Quantum Group and Integrable Massive Models of Quantum Field Theory*, World Scientific, Singapore.
- Gelfand, I., Shilov, G., Vilenkin, N. (1964), *Generalized Functions*, Vols. 1–5, Academic Press, New York (translated from Russian into English).
- Gelfand, I. (1989), *Collected Papers*, Vols. 1–3, Springer, New York.
- Gell-Mann, G., Low, F. (1951), Bound states in quantum field theory. *Phys. Rev.* **84**, 350–354.
- Gell-Mann, M., Low, F. (1954), Quantum electrodynamics at small distances, *Rev. Phys.* **95**(5), 1300–1317.
- Genz, H. (2004), *Nichts als das Nichts (The vacuum energy)*, Wiley–VCH, Weinheim, Germany (in German).
- Gerstenhaber, M. (1964), On the deformation of rings and algebras, *Ann. Math.* **79**, 59–103.
- Gerthsen, C. (2004), *Gerthsen Physik*. Edited by D. Meschede, 22nd edn., Springer, Berlin (in German).
- Giaquinta, M., Hildebrandt, S. (1995), *Calculus of Variations*, Vols. 1, 2, Springer, Berlin.
- Gilbarg, D., Trudinger, N. (1983), *Elliptic Partial Differential Equations of Second Order*, Springer, New York.
- Gilkey, P. (1995), *Invariance Theory, the Heat Equation, and the Atiyah–Singer Index Theorem*, 2nd edn., CRC Press, Boca Raton, Florida.
- Gilkey, P. (2003), *Asymptotic Formulae in Spectral Geometry*, Chapman, CRC Press, Boca Raton, Florida.
- Gilkey, P. (2008): The spectral geometry of Dirac and Laplace type. In: *Handbook of Global Analysis (2008)*, pp. 289–326.
- Gilmore, R. (2008), *Lie Groups, Physics, and Geometry: An Introduction for Physicists, Engineers, and Chemists*, Cambridge University Press, Cambridge, United Kingdom.
- Giulini, D., Kiefer, C., Lämmerzahl, C. (Eds.) (2003), *Quantum Gravity: From Theory to Experimental Search*, Springer, Berlin.
- Glimm, J., Jaffe, A. (1981), *Quantum Physics: A Functional Integral Point of View*, Springer, New York.
- Glimm, J., Jaffe, A. (1985), *Quantum Field Theory and Statistical Mechanics: Expositions*, Birkhäuser, Boston.
- Goldstein, H. (2002), *Classical Mechanics*, 3rd edn., Addison-Wesley, Reading Massachusetts.
- Golse, F., Saint-Raymond, L. (2001), The Navier–Stokes limit for the Boltzmann equation, *Comptes Rendus Acad. Sci. Paris, Ser. I, Math.* **333**, 897–902.
- Gomis, J., Paris, J., Samuel, S. (1995), Antibracket, antifields, and gauge-theory quantization, *Phys. Rept.* **259**, 1–145.
- Gottfried, K., Tung-Mow, Yan (2003), *Quantum Mechanics: Fundamentals*, 2nd edn., Springer, New York.
- Gottwald, S., Ilgands, H., Schlote, K. (1990), *Lexikon bedeutender Mathematiker (Biographies of important mathematicians)* (in German), H. Deutsch, Frankfurt/Main.⁴
- Gracia-Bondia, J., Várilly, J., Figueroa, H. (2001), *Elements of Noncommutative Geometry*, Birkhäuser, Boston.
- Gracia-Bondia, J. (2005), The Epstein–Glaser Approach to Quantum Field Theory, *Lecture Notes, AIP Conference Proceedings* **809**, American Institute of Physics, New York, pp. 24–43.

⁴ 1800 biographies

- Gradshteyn, I., Ryzhik, I. (1980), *Tables of Integrals, Series, and Products*, 4th edn., Academic Press, New York (translated from Russian into English).
- Graham, D., Brown, H., Harre, R. (1990), *Philosophical Foundations of Quantum Field Theory*, Clarendon Press, Oxford.
- Graham, N., Quandt, M., Weigel, H. (2009), *Spectral Methods in Quantum Field Theory*, Springer, Berlin.
- Gray, J. (2000a), *Linear Differential Equations and Group Theory: From Riemann to Poincaré*, Birkhäuser, Boston.
- Gray, J. (2000b), *The Hilbert Challenge: A Perspective on 20th Century Mathematics*, Oxford University Press, Oxford.
- Green, M., Schwarz, J., Witten, E. (1987), *Superstrings*, Vols. 1, 2, Cambridge University Press, Cambridge, United Kingdom.
- Greene, B. (1999), *The Elegant Universe: Supersymmetric Strings, Hidden Dimensions, and the Quest for the Ultimate Theory*, Norton, New York.
- Greiner, W. et al. (1996), *Course of Modern Theoretical Physics*, Vols. 1–13, Springer, New York (translated from German into English).
- Greiner, W., Schäfer, A. (1994), *Quantum Chromodynamics*, Springer, Berlin.
- Greiner, W., Müller, B. (1996), *Gauge Theory of Weak Interactions*, 2nd edn., Springer, New York.
- Greiner, W., Reinhardt, J. (1996a), *Quantum Electrodynamics*, 3rd edn., Springer, Berlin.
- Greiner, W., Reinhardt, J. (1996b), *Field Quantization*, Springer, Berlin.
- Greiner, W. (1997), *Relativistic Quantum Mechanics: Wave Equations*, Springer, Berlin.
- Gribbin, J. (1998), *Q is for Quantum: Particle Physics from A–Z*, Weidenfeld, London.
- Griffith, R. (1972), Rigorous results and theorems. In: C. Domb and M. Green (Eds.), *Phase Transitions and Critical Phenomena*, Academic Press, New York, 1972, pp. 9–108.
- Griffith, P., Harris, J. (1978), *Principles of Algebraic Geometry*, Wiley, New York.
- Grøn, Ø., Hervik, S. (2007), *Einstein’s Theory of General Relativity: with Modern Applications in Cosmology*, Springer, New York.
- Gromov, M. (1985), Pseudo-holomorphic curves in symplectic manifolds, *Invent. Math.* **82**, 307–347.
- Grosche, C., Steiner, F. (1998), *Handbook of Feynman Path Integrals*, Springer, New York.⁵
- Gross, D., Wilczek, F. (1973a), Ultraviolet behavior of non-Abelian gauge theories, *Phys. Letters* **30**(26), 1343–1346.
- Gross, D., Wilczek, F. (1973b), Asymptotically free gauge theories, *Phys. Rev. D* **9**(4), 980–993.
- Gross, F. (1993), *Relativistic Quantum Mechanics and Field Theory*, Wiley, New York.
- Grosse, H. (1988), *Models in Statistical Physics and Quantum Field Theory*, Springer, New York.
- Grosse, H., Wulkenhaar, R. (2005), Renormalisation of φ^4 -theory on noncommutative \mathbb{R}^4 in the matrix base, *Commun. Math. Phys.* **256**, 305–374.
- Grozin, A. (2007), *QED (Quantum Electrodynamics) and QCD (Quantum Chromodynamics): Practical Calculation and Renormalization of One- and Multi-Loop Feynman Diagrams*, World Scientific, Singapore.
- Grubb, G. (2009), *Distributions and Operators*, Springer, Berlin.

⁵ 950 references

- Guhr, T., Müller-Groeling, H., Weidenmüller, H. (1998), Random-Matrix Theories in Quantum Physics: Common Concepts, *Physics Reports* **299**, Sections 4–6.
- Guidry, M. (1991), *Gauge Field Theories: An Introduction with Applications*, Wiley, New York.
- Guillemin, V., Pollack, A. (1974), *Differential Topology*, Prentice Hall, Englewood Cliffs, New Jersey.
- Guillemin, V., Sternberg, S. (1989), *Geometric Asymptotics*, Amer. Math. Soc., Providence Rhode Island.
- Guillemin, V., Sternberg, S. (1990), *Symplectic Techniques in Physics*, Cambridge University Press, Cambridge, United Kingdom.
- Günter, N. (1934), *La théorie du potentiel et ses applications de la physique mathématique*, Paris. (German edition: *Potentialtheorie*, Teubner, Leipzig 1957.)
- Günther, P. (1988), *Huygens' Principle and Hyperbolic Differential Equations*, Academic Press, San Diego.
- Gustafson, S., Sigal, I. (2003), *Mathematical Concepts of Quantum Mechanics*, Springer, Berlin.
- Guth, A., Huang, K., Jaffe, A. (Eds.) (1983), *Asymptotic Realms of Physics, Essays in Honor of Francis Low*, MIT Press, Cambridge, Massachusetts.
- Haag, R., Kastler, D. (1964), An algebraic approach to quantum field theory, *J. Math. Phys.* **5**, 848–861.
- Haag, R., Lopuszanski, J., Sohnius, M. (1975), All possible generators of supersymmetries of the S -matrix, *Nucl. Phys.* **B88**, 257–274.
- Haag, R. (1996), *Local Quantum Physics: Fields, Particles, Algebras*, 2nd edn., Springer, New York.
- Hackbusch, W. (1985), *Multigrid Methods and Applications*, Springer, Berlin.
- Hackbusch, W. (1992), *Elliptic Differential Equations: Theory and Numerical Treatment*, Springer, New York.
- Hahn, Y., Zimmermann, W. (1968), An elementary proof of Dyson's power-counting theorem, *Commun. Math. Phys.* **10**, 330–342.
- Haken, H. (1976), *Quantum Field Theory of Solids*, North-Holland Amsterdam (translated from German into English). Second German edition: Teubner, Stuttgart, 1993.
- Haken, H. (1984), *Laser Theory*, 2nd edn. In: S. Flügge (Ed.), *Handbook of Physics*, Vol. XXV/2c, Springer, Berlin.
- Hall, B. (2003), *Lie Groups, Lie Algebras, and Representations: An Elementary Introduction*, Springer, New York.
- Halzen, F., Martin, A. (1984), *Quarks and Leptons*, Wiley, New York.
- Hamber, H. (2009), *Quantum Gravitation, The Feynman Path Integral Approach*, Springer, Berlin.
- Handbook of Differential Equations (2004), *Evolutionary Equations*, Vols. 1, 2. Edited by C. Dafermos et al., Elsevier, Boston.
- Handbook of Differential Equations (2004), *Stationary Partial Differential Equations*, Vols. 1, 2. Edited by M. Chipot and P. Quittner, Elsevier, Amsterdam.
- Handbook of Differential Equations (2005), *Ordinary Differential Equations*, Vols. 1, 2. Edited by A. Canada et al., Elsevier, Amsterdam.
- Handbook of Dynamical Systems (2002), Vols. 1A, 1B edited by B. Hasselblatt et al., Vol. 2 edited by B. Fiedler, Elsevier, Amsterdam.
- Handbook of Nonlinear Partial Differential Equations (2004). Edited by A. Polyanin and V. Zaitsev, Chapman and Hall, Boca Raton, Florida.
- Handbook of Mathematical Physics: See *Encyclopedia of Mathematical Physics* (2006), and *Modern Encyclopedia of Mathematical Physics* (2011).

- Handbook of Global Analysis (2008). Edited by D. Krupka and D. Saunders, Elsevier, Amsterdam.
- Harenberg Lexikon der Nobelpreisträger (2000) (Encyclopedia of Nobel Prize Laureates), Harenberg Lexikon Verlag, Dortmund, Germany (in German).
- Harer, J., Zagier, D. (1986), The Euler characteristic of the moduli space of curves, *Invent. Math.* **85**(3), 457–495.
- Harris, E. (1972), *A Pedestrian Approach to Quantum Field Theory*, Wiley, New York.
- Hartshorne, R. (1994), *Algebraic Geometry*, 3rd ed., Springer, New York.
- Hatcher, A. (2002), *Algebraic Topology*, Cambridge University Press, Cambridge, United Kingdom. Internet: <http://www.math.cornell.edu/~hatcher>
- Hatcher, A. (2005a), Spectral Sequences in Algebraic Topology. Internet: <http://www.math.cornell.edu/~hatcher>
- Hatcher, A. (2005b), *Vector Bundles and K-Theory*. Internet: <http://www.math.cornell.edu/~hatcher>
- Hatfield, B. (1992), *Quantum Field Theory of Point Particles and Strings*, Addison-Wesley, Redwood City, California.
- Hauser, H. (2003), The Hironaka theorem on resolution of singularities (or: a proof we always wanted to understand), *Bull. Amer. Math. Soc.* **40**(3), 323–403.
- Hausner, M., Schwartz, J. (1968), *Lie Groups and Lie Algebras* (lectures held at the Courant Institute, NYU), Gordon and Breach, New York.
- Havil, J. (2003), *Gamma: Exploring Euler’s Constant*, Princeton University Press, Princeton, New Jersey.
- Heath, H. (1981), *A History of Greek Mathematics*, Vols. 1, 2, Clarendon Press, Oxford, New York.
- Hein, W. (1990), *Introduction to Structure and Representation of the Classical Groups* (in German), Springer, Berlin.
- Heisenberg, W. (1925), Quantum-theoretical re-interpretation of kinematics and mechanical relations (in German), *Z. Physik* **33**, 879–893. (English translation: see van der Waerden (Ed.) (1968), pp. 261–276.)
- Heisenberg, W. (1927), Intuitiver Inhalt der quantentheoretischen Kinematik und Mechanik (The intuitive meaning of quantum-theoretical kinematics and mechanics), *Z. Phys.* **43**, 172–199.
- Heisenberg, W., Pauli, W. (1929), Zur Quantentheorie der Wellenfelder (On quantum field theory), *Z. Phys.* **56**, 1–61; **59**, 108–190 (in German).
- Heisenberg, W. (1930), *The Physical Principles of the Quantum Theory*, Chicago University Press, Chicago.
- Heisenberg, W. (1943), Die “beobachtbaren Größen” in der Theorie der Elementarteilchen (The observable quantities in particle physics), *Z. Phys.* **120**, 513–538. (1943), 673–702; **123** (1944), 93–112 (in German).
- Heisenberg, W. (1970), *Physics and Beyond: Encounters and Conversations*, Harper and Row, New York (translated from German into English).
- Heisenberg, W. (1984), *Gesammelte Abhandlungen* (Collected Papers). Edited by P. Dürr et al., Vols. 1–3, Springer, Berlin, New York.
- Heiss, D. (Ed.) (2002), *Fundamentals of Quantum Information: Quantum Computation, Communication, Decoherence and All That*, Springer, Berlin.
- Heitler, W. (1936), *The Theory of Radiation*, Clarendon Press, Oxford.
- Helffer, B. (2003), *Semiclassical Analysis*, World Scientific, Singapore.
- Hellagouarch, Y. (2002), *Invitation to the Mathematics of Fermat–Wiles*, Academic Press, New York.
- Henneaux, M., Teitelboim, C. (1993), *Quantization of Gauge Systems*, Princeton University Press, Princeton, New Jersey.

- Hepp, K. (1966), Proof of the Bogoliubov–Parasiuk theorem on renormalization, *Commun. Math. Phys.* **2**, 301–326.
- Hepp, K. (1969), *La théorie de la renormalisation*, Springer, Berlin.
- Hertling, C. (2002), *Frobenius Manifolds and Moduli Spaces for Singularities*, Cambridge University Press, Cambridge, United Kingdom.
- Hertling, C., Marcolli, M. (Eds.) (2004), *Frobenius Manifolds: Quantum Cohomology and Singularities*, Vieweg, Wiesbaden.
- Hida, T. (1970), *Stationary Stochastic Processes*, Princeton University Press, Princeton, New Jersey.
- Hida, T. (1980), *Brownian Motion*, Springer, New York.
- Hida, T., Potthoff, J., Streit, L. (1990), White noise analysis and applications. In: *Mathematics and Physics: Lectures on Recent Results*, Vol. 3, pp. 143–178, World Scientific, Singapore.
- Hilbert, D. (1900), Mathematical problems. Lecture delivered before the Second International Congress of Mathematicians at Paris 1900, *Bull. Amer. Math. Soc.* **8** (1902), 437–479.
- Hilbert, D. (1912), *Grundzüge einer allgemeinen Theorie der Integralgleichungen* (Foundations of the theory of integral equations), Teubner, Leipzig (in German). (Reprinted version: *Integralgleichungen und Gleichungen mit unendlich vielen Unbekannten*. Edited by A. Pietsch (with historical comments), Teubner, Leipzig, 1989.)
- Hilbert, D. (1932), *Gesammelte Werke* (Collected Works), Vols. 1–3, 12th edn., 1977, Springer, Berlin (in German).
- Hildebrandt, S., Tromba, T. (1985), *Mathematics and Optimal Form*, Scientific American Books, New York.
- Hirzebruch, F. (1966), *Topological Methods in Algebraic Geometry*, 3rd enlarged edn., Springer, New York. (translated from German into English).
- Hirzebruch, F. (1987), *Gesammelte Abhandlungen* (Collected Papers), Vols. 1, 2, Springer, Berlin.
- Hislop, P., Sigal, I. (1996), *Introduction to Spectral Theory With Applications to Schrödinger Operators*, Springer, New York.
- Hoddeson, L., Kolb, A., Westfall, C. (2009), *Fermilab: Physics, the Frontier and Megascience*, The University of Chicago Press, Chicago.
- Hollands, S., Ruan, W. (2002), The state space of perturbative quantum field theory in curved space-times, *Annales Henri Poincaré* **3**, 635–675.
Internet: <http://arxiv.org/gr-qc/0108032>
- Hollands, S., Wald, R. (2002), Existence of local covariant time-ordered products of quantum fields in curved spacetime, *Commun. Math. Phys.* **231**, 309–345.
Internet: <http://arxiv.org/gr-qc/0111108>
- Hollands, S., Wald, R. (2003), On the renormalization group in curved space-time, *Commun. Math. Phys.* **237**, 123–160.
Internet: <http://arxiv.org/gr-qc/0209029>
- Hollands, S., Wald, R. (2004), Quantum field theory is not merely quantum mechanics applied to low energy effective degrees of freedom, *Gen. Rel. Grav.* **36**, 2595–2603. Internet: [gr-qc/04055082](http://arxiv.org/gr-qc/04055082)
- Hollands, S., Wald, R. (2002), An alternative to inflation, *Gen. Rel. Grav.* **34**, 2043–2055. Internet: <http://arxiv.org/gr-qc/0205058>
- Hollands, S., Marolf, D. (2007), Asymptotic generators of fermionic charges and boundary conditions preserving supersymmetry, *Classical Quantum Gravity* **24** (9), 2301–2332.
- Hollands, S. (2007), The operator product expansion for perturbative quantum field theory in curved spacetime, *Commun. Math. Phys.* **273**, 1–36.
Internet: <http://arxiv.org/gr-qc/0605106>

- Hollands, S. (2008a), Renormalized Yang–Mills Fields in Curved Spacetime. *Rev. Math. Phys.* **20**, 1033–1172. Internet: <http://arxiv.org/0705.3340>
- Hollands, S. (2008b), Quantum field theory in terms of consistency conditions. I. General framework, 57 pages. Internet: <http://arxiv.org/0802.2198>
- Hollands, S., Wald, R. (2008), Quantum field theory in curved spacetime, the operator-product expansion, and dark energy, *Gen. Rel. Grav.* **40**, 2041–2051. Internet: <http://arxiv.org/0805.3419>
- Hollands, S., Wald, R. (2008), Axiomatic quantum field theory in curved spacetime, 44 pages. Internet: <http://arxiv.org/0803.2003>
- Hollik, W., Kraus, E., Roth, M., Rupp, C., Sibold, K., Stöckinger, D. (2002), Renormalization of the minimal supersymmetric standard model, *Nuclear Physics B* **639**, 3–65.
- Honerkamp, J., Römer, H. (1993), *Theoretical Physics: A Classical Approach*, Springer, Berlin (translated from German into English).
- Hörmander, L. (1979), The Weyl calculus for pseudo-differential operators, *Comm. Pure Appl. Math.* **32**, 359–443.
- Hörmander, L. (1983), *The Analysis of Linear Partial Differential Operators*, Vol. 1: Distribution Theory and Fourier Analysis, Vol. 2: Differential Operators with Constant Coefficients, Vol. 3: Pseudo-Differential Operators, Vol. 4: Fourier Integral Operators, Springer, New York.
- Hoste, J., Thistlewaite, M., Weeks, J. (1998), The first 1, 701, 936 knots, *Mathematical Intelligencer* **20**(4), 33–48.
- Hsiao, G., Wendland, W. (2008), *Boundary Integral Equations*, Springer, New York.
- Huang, K. (1987), *Statistical Physics*, Wiley, New York.
- Huang, K. (1992), *Quarks, Leptons, and Gauge Fields*, World Scientific, Singapore.
- Huang, K. (1998), *Quantum Field Theory: From Operators to Path Integrals*, Wiley, New York.
- Huebschmann, J., Stasheff, J. (2002), Formal solution of the (Batalin–Vilkovisky) master equation via homological perturbation theory and deformation theory, *Forum Mathematicum* **14** (2002), 847–868.
- Hughes, B. (1995), *Random Walks and Random Environments*, Vols. 1, 2, Clarendon Press, Oxford (e.g., percolation).
- Hurwitz, A., Courant, R. (1964), *Vorlesungen über allgemeine Funktionentheorie und elliptische Funktionen* (Lectures on complex function theory and elliptic functions), 4th edn., Springer, Berlin (in German).
- Hwa, R., Teplitz, V. (1966), *Homology and Feynman Diagrams*, Benjamin, Reading, Massachusetts.
- Iagolnitzer, D. (1993), *Scattering in Quantum Field Theory*, Princeton University Press, Princeton, New Jersey.
- I Calle, C. (2001), *Superstrings and Other Things: A Guide to Physics*, Institute of Physics Publishing, Bristol.
- Infeld, L. (1980), *Quest: An Autobiography*, 2nd edn., Chelsea, New York.
- International Congress on Mathematical Physics (ICMP) (2000ff), XIII: London 2000, XIV: Lisbon 2003, XV: Rio de Janeiro 2006, XVI: Prague 2009. The bibliographic material for the ‘Proceedings’ of the ICMP can be found on page 943.
- Itzykson, C., Zuber, J. (1980), *Quantum Field Theory*, MacGraw-Hill, New York.
- Itzykson, C., Drouffe, J. (1991), *Statistical Field Theory: From Brownian Motion to Renormalization and Lattice Gauge Theory*, Vols. I, II, Cambridge University Press, Cambridge, United Kingdom.

- Ivancevic, V., Ivancevic, T. (2007), *Differential Geometry: A Modern Introduction*, World Scientific, Singapore.
- Ivey, T., Landsberg, J. (2003), *Cartan for Beginners: Differential Geometry via Moving Frames and Exterior Differential Systems*, Amer. Math. Soc., Providence, Rhode Island.
- Jackson, J. (1995), *Classical Electrodynamics*, 2nd edn., Wiley, New York.
- Jacod, P., Protter, P. (2000), *Probability Essentials*, Springer, Berlin.
- Jaffe, A., Taubes, C. (1980), *Vortices and Monopoles: The Structure of Static Gauge Fields*, Birkhäuser, Boston.
- Jaffe, A., Witten, E. (2006), *Quantum Yang–Mills theory*. In: Carlson, J., Jaffe, A., Wiles, A. (Eds.), *The Millenium Prize Problems*, Amer. Math. Soc., Providence, Rhode Island, 2006, pp. 129–152.
- Jaffe, A. (2006), *The millennium grand challenge in mathematics*, *Notices Amer. Math. Soc.* **53**, 652–660.
- Jaffe, A. (2008), *Quantum Theory and Relativity*, *Contemporary Mathematics*, American Mathematical Society, Providence, Rhode Island, pp. 209–245.
- Jancewicz, B., Sobczyk, J. (Eds.) (1996), *From Field Theory to Quantum Groups*, World Scientific, Singapore.
- Jimbo, M. (1988), *Integrable Systems in Quantum Field Theory and Statistical Mechanics*, Academic Press, New York.
- Johnson, G., Lapidus, M. (2000), *The Feynman Integral and Feynman’s Operational Calculus*, Clarendon Press, Oxford.
- Jörgens, K., Rellich, F. (1976), *Eigenwertprobleme für gewöhnliche Differentialgleichungen (Eigenvalue problems for ordinary differential equations)*, Springer, Berlin (in German).
- Jost, J. (1994), *Differentialgeometrie und Minimalflächen (Differential geometry and minimal surfaces)*, Springer, Berlin (in German).
- Jost, J. (1997), *Compact Riemann Surfaces: An Introduction to Contemporary Mathematics*, Springer, Berlin.
- Jost, J., Xianqing Li-Jost (1998), *Calculus of Variations*, Cambridge University Press, Cambridge, United Kingdom.
- Jost, J. (2001), *The Bosonic String: A Mathematical Treatment*, International Press, Boston.
- Jost, J. (2002), *Partial Differential Equations*, Springer, New York.
- Jost, J. (2005a), *Postmodern Analysis*, 3rd edn., Springer, Berlin.
- Jost, J. (2005b), *Dynamical Systems: Examples of Complex Behavior*, Springer, Berlin.
- Jost, J. (2008), *Riemannian Geometry and Geometric Analysis*, 5th edn., Springer, Berlin.
- Jost, J. (2009), *Geometry and Physics*, Springer, Berlin.
- Jost, R. (1965), *The Generalized Theory of Quantum Fields*, Amer. Math. Soc., Providence, Rhode Island.
- Jost, R. (1995), *Das Märchen vom elfenbeinernen Turm: Reden und Aufsätze (The fairy tale about the ivory tower: essays and lectures in German and partly in English)*. Edited by K. Hepp, W. Hunziker, and W. Kohn, Springer, Berlin.
- Juncker, G. (1996), *Supersymmetric Methods in Quantum and Statistical Physics*, Springer, Berlin, New York.
- Junker, W., Schrohe, E. (2002), *Adiabatic vacuum states on general space-time manifolds: definition, construction, and physical properties*, *Ann. Henri Poincaré* **3**, 1113–1181.
- Kac, V. (1996), *Vertex Algebras for Beginners*, Amer. Math. Soc., Providence, Rhode Island.

- Kadanoff, L. (1966), Scaling laws for Ising models near critical temperature, *Physics* **2**, 263–272.
- Kadanoff, L. (2000), *Statistical Physics*, World Scientific, Singapore.
- Kadison, R., Ringrose, J. (1983), *Fundamentals of the Theory of Operator Algebras*, Vols. 1–4, Academic Press, New York.
- Kähler, E. (2003) *Mathematische Werke – Mathematical Works*. Edited by R. Berndt and Oswald Riemenschneider, de Gruyter, Berlin.
- Kaiser, D. (2005), *Drawing Theories Apart: The Dispersion of Feynman Diagrams in Postwar Physics*, The University of Chicago Press, Chicago.
- Kaku, M. (1999), *Quantum Field Theory: A Modern Introduction*, Oxford University Press, New York, 1999.
- Kaku, M. (2000), *Strings, Conformal Fields and M-Theory*, Springer, New York.
- Kalka, H., Soff, G. (1997), *Supersymmetrie*, Teubner, Stuttgart (in German).
- Kane, G. (2000), *Supersymmetry: Squarks, Photinos, and the Unveiling of the Ultimate Laws of Nature*, Perseus Publishing, Cambridge, Massachusetts.
- Kapusta, J. (1989), *Quantum Field Theory at Finite Temperature*, Cambridge University Press, Cambridge, United Kingdom.
- Kassel, C., Rosso, M., Turaev, V. (1997), *Quantum Groups and Knot Invariants*, Société Mathématique de France, Paris.
- Kastler, D. (Ed.) (1990), *The Algebraic Theory of Superselection Sectors*, World Scientific, Singapore.
- Kastler, D. (1992), Algebraic field theory: recollections and thoughts about the future, *Rev. Math. Phys.* **4**, Special Issue dedicated to Rudolf Haag, pp. 159–166.
- Kastler, D. (2003), Rudolf Haag – eighty years, *Commun. Math. Phys.* **237**, 3–6.
- Kato, T. (1966), *Perturbation Theory for Linear Operators*, Springer, Berlin.
- Kaufman, L. (2001), *Knots and Physics*, 3rd edn., World Scientific, Singapore.
- Ketov, S. (2000), *Quantum Non-Linear Sigma Models: From Quantum Field Theory to Supersymmetry, Conformal Field Theory, Black Holes, and Strings*, Springer, Berlin.
- Khrennikov, A. (1999), *Superanalysis*, Kluwer, Dordrecht.
- Kichenassamy, S. (1993), *Nonlinear Waves*, Pitman, London.
- Kichenassamy, S. (2007), *Fuchsian Reduction: Applications to Geometry, Cosmology, and Mathematical Physics*, Birkhäuser, Boston.
- Kiefer, C. (2004), *Quantum Gravity*, Oxford University Press, Oxford.
- Kirsten, K. (2002), *Spectral Functions in Mathematics and Physics*, Chapman, Boca Raton, Florida.
- Kittel, C. (1996), *Introduction to Solid State Physics*, 7th edn., Wiley, New York.
- Klauder, J., Sudarshan, E. (1968), *Fundamentals of Quantum Optics*, Benjamin, New York.
- Klauder, J., Daubchies, I. (1982), Quantum mechanical path integrals with Wiener measures for all polynomial Hamiltonians, *Phys. Rev. Lett.* **52** (1984), 1161–1164.
- Klauder, J., Skagerstam, B. (1985), *Coherent States: Applications in Physics and Mathematical Physics*, World Scientific, Singapore.
- Klauder, J. (1988), Quantization is geometry, after all, *Ann. Phys.* **188**, 120–141.
- Klauder, J. (1989), The regularized Feynman path integral. In: V. Syakanit et al. (Eds.), *Path Integrals from meV to MeV*, Bangkok, World Scientific, Singapore, 1989, pp. 48–59.
- Klauder, J., Onfri, E. (1989), Landau levels and geometric quantization, *Int. J. Mod. Phys.* **A4**, 3939–3949.
- Klauder, J. (2000), *Beyond Conventional Quantization*, Cambridge University Press, Cambridge, United Kingdom.

- Klein, F. (1884), *Vorlesungen über das Icosaeder und die Auflösung der Gleichungen vom fünften Grad*, Teubner, Leipzig. Reprinted 1993 by Birkhäuser and supplemented with commentaries by P. Slodowy. (English edition: *Lectures on the Icosahedron and the Solution of Fifth-Order Equations*, Dover, reprint 1956.)
- Klein, F. (1926), *Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert*, Vols. 1, 2, Springer-Verlag, Berlin. (English edition: *Development of Mathematics in the 19th Century*, with an appendix by R. Hermann, Math. Sci. Press, New York, 1979.)
- Klein, C., Richter, O. (2006), *Ernst Equation and Riemann Surfaces*, Springer, Berlin.
- Kleinert, H. (2004), *Path Integrals in Quantum Mechanics, Statistics, and Polymer Physics*, 3rd edn., World Scientific, River Edge, New York.
- Kleinert, H., Schulte-Frohlinde, V. (2001), *Critical Properties of φ^4 -Theories*, World Scientific, River Edge, New York.
- Kleinert, H. (2008), *Multivalued Fields: Condensed Matter, Electromagnetism, and Gravitation*, World Scientific, Singapore.
- Klein, C. and Richter, O. (2005), *Ernst Equation and Riemann Surfaces: Analytical and Numerical Methods*, Springer, Berlin.
- Klimyk, A., Schmüdgen, K. (1997), *Quantum Groups*, Springer, Berlin.
- Kline, M. (1972), *Mathematical Thought from Ancient to Modern Times*, Oxford University Press, New York.
- Knabner, P., Angermann, L. (2003), *Numerical Methods for Elliptic and Parabolic Partial Differential Equations*, Springer, New York (translated from German into English).
- Knapp, A. (1986), *Representation Theory of Semi-Simple Groups*, Princeton University Press, Princeton, New Jersey.
- Knapp, A. (2002), *Lie Groups Beyond an Introduction*, 2nd edn., Birkhäuser, Boston.
- Knörrer, H. (1996), *Geometrie*, Vieweg, Wiesbaden (in German).
- Kobayashi, S., Nomizu, K. (1963), *Foundations of Differential Geometry*, Vols. 1, 2, Wiley, New York.
- Kobzarev, I.: See Manin and Kobzarev.
- Koch, H. (1986), *Einführung in die klassische Mathematik (Introduction to classical mathematics)*, Akademie-Verlag, Berlin (in German).
- Komech, A. (1999), *Linear partial differential equations with constant coefficients*. In: Egorov, Komech, and Shubin (1999), pp. 121–255.
- Komech, A. (2005a), *Quantum Mechanics for Mathematicians*, Lecture Notes, Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany.
- Komech, A. (2005b), *On Global Attractors of Hamilton Nonlinear Wave Equations*. Lecture Notes, Max Planck Institute for Mathematics in the Sciences, Leipzig. Internet: <http://www.mis.mpg.de/preprints/ln/lecturenote-2405.pdf>
- Komech, A. (2007), *Book of Practical Differential Equations*. Lecture Notes, Max Planck Institute for Mathematics in the Sciences, Leipzig. Internet: <http://www.mis.mpg.de/preprints/ln/lecturenote-3307.pdf>
- Komech, A. (2007), *Lectures on Elliptic and Partial Differential Equations (pseudodifferential operator approach)*. Lecture Notes, Max Planck Institute for Mathematics in the Sciences, Leipzig. Internet: <http://www.mis.mpg.de/preprints-3207.pdf>
- Kontsevich, M. (2003), *Deformation quantization of Poisson manifolds*. *Lett. Math. Phys.* **66**(3), 157–216. Internet: <http://q-alg/9709040>

- Korevaar, J. (2004), *Tauberian Theory: A Century of Developments*, Springer, Berlin.
- Koshy, T. (2001), *Fibonacci and Lucas Numbers with Applications*, Wiley, New York.
- Kostrikin, A. (1989): See Manin and Kostrikin.
- Kosyakov, B. (2007), *Introduction to the Classical Theory of Particles and Fields*, Springer, Berlin.
- Kraft, F. (1999), *Vorstoß ins Unbekannte: Lexikon großer Naturwissenschaftler* (Encyclopedia of great scientists), Wiley, Weinheim, New York (in German).
- Kragh, H. (2000), *Quantum Generations: A History of Physics in the Twentieth Century*, Princeton University Press, Princeton, New Jersey.
- Krantz, S., Parks, H. (2002), *The Implicit Function Theorem: History, Theory, and Applications*, Birkhäuser, Boston.
- Kraus, E. (1998), Renormalization of the electroweak standard model to all orders, *Ann. Phys. (NY)* **262**, 155–259.
- Kreimer, D. (2000), *Knots and Feynman Diagrams*, Cambridge University Press, Cambridge, United Kingdom.
- Kriele, M. (2000), *Space-Time: Foundations of General Relativity and Differential Geometry*, Springer, Berlin.
- Kronheimer, P., Mirowka, T. (2007), *Monopoles and Three-Manifolds*, Cambridge University Press, Cambridge, United Kingdom.
- Kufner, A., John, O., Fučík, S. (1977), *Function Spaces*, Academia, Prague.
- Kugo, T. (1997), *Eichfeldtheorie* (Gauge field theory), Springer, Berlin (translated from Japanese into German).
- Kupradse, V. (1956), *Randwertaufgaben der Schwingungstheorie und Integralgleichungen* (Boundary-value problems for wave problems and integral equations), Deutscher Verlag der Wissenschaften, Berlin (translated from Russian into German).
- Lacki, J. et al. (Eds.) *Stueckelberg: An Unconventional Figure of Twentieth Century Physics. Selected Scientific Papers with Commentaries*, Birkhäuser, Boston, 2008.
- Lahiri, A., Pal, B. (2001), *A First Book of Quantum Field Theory*, Alpha Science International, Pangbourne, India.
- Landau, L., Lifshitz, E. (1982), *Course of Theoretical Physics*, Vol. 1: Mechanics, Vol. 2: The Classical Theory of Fields, Vol. 3: Quantum Mechanics, Vol. 4: Quantum Electrodynamics, Vol. 5: Statistical Physics, Part 1, Vol. 6: Fluid Mechanics, Vol. 7: Theory of Elasticity, Vol. 8: Electrodynamics of Continuous Media, Vol. 9: Statistical Physics, Part 2, Vol. 10: Physical Kinetics, Butterworth–Heinemann, Oxford (translated from Russian into English).
- Lang, S. (1972), *Introduction to Algebraic Geometry*, Addison-Wesley, Reading, Massachusetts.
- Lang, S. (1973), *Elliptic Functions*, Addison-Wesley, New York.
- Lang, S. (1983), *Abelian Varieties*, Springer, New York.
- Lang, S. (1986), *Algebraic Number Theory*, Springer, New York.
- Lang, S. (1993), *Real Analysis and Functional Analysis*, Springer, New York.
- Lang, S. (1995a), *Complex Analysis*, 4th edn., Springer, New York.
- Lang, S. (1995b), *Introduction to Algebraic and Abelian Functions*, 2nd edn., Springer, New York.
- Lang, S. (1995c), *Introduction to Diophantine Approximation*, Springer, Berlin.
- Lang, S. (1995d), *Introduction to Modular Forms*, 2nd edn., Springer, New York.
- Lang, S. (1999), *Fundamentals of Differential Geometry*, Springer, New York.
- Lang, S. (2002), *Algebra*, revised 3rd edn., Springer, New York.

- Laughlin, R. (2005), *A Different Universe: Reinventing Physics from the Bottom Down*, Basic Books, New York.
- Lawson, H., Michelsohn, M. (1994), *Spin Geometry*, 2nd edn., Princeton University Press, Princeton, New Jersey.
- Lax, P. (2002), *Functional Analysis*, Wiley, New York.
- Lax, P. (2003), Jürgen Moser, 1928–1999. In: *Ergodic Theory and Dynamical Systems* **22**, 1337–1342.
- Le Bellac, M. (1991), *Quantum and Statistical Field Theory*, Clarendon Press, Oxford (translated from French into English).
- Lieb, E. (1967), Exact solution of the problem of the entropy of two-dimensional ice, *Phys. Rev.* **18**, 692–694.
- Lieb, E., Mattis, D. (1968), *Mathematical Physics in One Dimension: Exactly Solvable Models of Interacting Particles* (a collection of reprints), Academic Press, New York.
- Lieb, E. (1982), Density functionals for Coulomb systems, *Intern. J. of Quantum Chemistry* **24**, 243–277.
- Lieb, E., Loss, M. (1997), *Analysis*, Amer. Math. Soc., Providence, Rhode Island.
- Lieb, E. (2002a), *The Stability of Matter: From Atoms to Stars*, *Selecta of Elliott Lieb*. Edited by W. Thirring, 3rd edn., Springer, New York.
- Lieb, E. (2002b), *Inequalities*, *Selecta of Elliott Lieb*. Edited by M. Loss, Springer, New York.
- Lieb, E., Seiringer, R. (2002c), Proof of Bose–Einstein condensation for dilute trapped gases, *Phys. Rev. Lett.* **88**, No. 170409, 1–4.
Internet: <http://www.arxiv.math-ph/0112032>
- Lieb, E., Yngvason, J. (1999), The physics and mathematics of the second law of thermodynamics, *Physics Reports* **310**(1), 1–96.
- Liebscher, D. (2005), *Cosmology*, Springer, Berlin.
- Longo, R., Roberts, E., Zsido, L. (Eds.) (1998), *Operator Algebras and Quantum Field Theory*, International Press, Boston.
- López Dávalos, A., Zanette, D. (1999), *Fundamentals of Electromagnetism*, Springer, Berlin.
- Lopuszanski, J. (1991), *An Introduction to Symmetry and Supersymmetry in Quantum Field Theory*, World Scientific, Singapore.
- Lörinczi, J., Hiroshima, F., Betz, V. (2009), *Feynman–Kac–Type Theorems and Gibbs Measures on Path Space: With Applications to Rigorous Quantum Field Theory*, De Gruyter, Berlin.
- Louis, J., de Wit, B. (1998), *Supersymmetry and dualities in various dimensions*, Lectures given at the Nato Advanced Study Institute on *Strings, Branes, and Dualities*, Cargese, Corsica (France), 1997.
Internet: <http://arxiv.hep-th/9801132>
- Lurié, D. (1968), *Particles and Fields*, Wiley, New York.
- Lüst, D., Theissen, S. (1989), *Lectures on String Theory*, Springer, Berlin.
- Macke, W. (1959), *Lehrbuch der Theoretischen Physik* (Textbook on theoretical physics), Vol. 1: Teilchen (particles), Vol. 2: Wellen (waves), Vol. 3 Quanten (quanta), Vol. 4: Elektromagnetische Felder (electromagnetic fields), Vol. 5: Thermodynamik und Statistik (thermodynamics and statistics), Vol. 6: Quanten und Relativität (quanta and relativity), Geest & Portig, Leipzig (in German).
- Madore, J. (1981), Geometric methods in classical field theory, *Physics Reports* **75**, 125–204.
- Madore, J. (1995), *An Introduction to Noncommutative Differential Geometry and Its Applications*, Cambridge University Press, Cambridge, United Kingdom.

- Madsen, I., Tornehave, J. (1997), *From Calculus to Cohomology: de Rham Cohomology and Characteristic Classes*, Cambridge University Press, Cambridge, United Kingdom.
- Maggiore, M. (2006), *A Modern Introduction to Quantum Field Theory*, Oxford University Press, Oxford.
- Maggiore, M. (2008), *Gravitational Waves*, Oxford University Press, Oxford.
- Magnus, W., Oberhettinger, F., Soni, R. (1966), *Formulae and Theorems for the Special Functions of Mathematical Physics*, 3rd edn., Berlin, Springer, Berlin.
- Majid, M. (1995), *Foundations of Quantum Group Theory*, Cambridge University Press, Cambridge, United Kingdom.
- Maldacena, J. (1998), The large N limit of superconformal field theories and supergravity, *Adv. Theor. Math. Phys.* **2**, 231–252.
- Mandel, L., Wolf, E. (1995), *Optical Coherence and Quantum Optics*, Cambridge University Press, Cambridge, United Kingdom.
- Manin, Yu. (1977), *A Course in Mathematical Logic*, Springer, New York (translated from Russian into English).
- Manin, Yu. (1978), Algebraic aspects of nonlinear differential equations, *J. Soviet Math.* **11**, 1–122.
- Manin, Yu. (1981), *Mathematics and Physics*, Birkhäuser, Boston.
- Manin, Yu. (1988), *Quantum Groups and Noncommutative Geometry*, CRM, Université de Montréal.
- Manin, Yu., Kobzarev, I. (1989), *Elementary Particles: Mathematics, Physics, and Philosophy*, Kluwer, Dordrecht (translated from Russian into English).
- Manin, Yu., Kostrikin, A. (1989), *Linear Algebra and Geometry*, Springer, New York (translated from Russian into English).
- Manin, Yu. (1989), Strings, *Math. Intelligencer* **11**(2), 59–65.
- Manin, Yu. (1991), *Topics in Noncommutative Geometry*, Princeton University Press.
- Manin, Yu., Zagier, D. (1996), Higher Weil–Peterson volumes of moduli spaces of stable n -pointed curves, *Commun. Math. Phys.* **181**(3), 763–787.
- Manin, Yu. (1997), *Gauge Fields and Complex Geometry*, Springer, Berlin.
- Manin, Yu. (1999), *Frobenius Manifolds, Quantum Cohomology, and Moduli Spaces*, Amer. Math. Soc., Providence, Rhode Island.
- Manin, Yu. (2007), *Mathematics as Metaphor: Selected Essays with a Foreword by Freeman Dyson*, Amer. Math. Soc., Providence, Rhode Island (translated from Russian into English).
- Manton, N., Sutcliffe, P. (2004), *Topological Solitons*, Cambridge University Press, Cambridge, United Kingdom.
- Manoukian, I. (1983), *Renormalization*, Academic Press, New York.
- Marathe, K., Martucci, G. (1992), *The Mathematical Foundations of Gauge Theories*, North-Holland, Amsterdam.
- Marathe, K., Martucci, G., Francaviglia, M. (1995), Gauge theory, geometry, and topology, *Seminario di Matematica dell' Università di Bari* **262**, 1–90.
- Marino, M., Labastida, J. (2007), Springer, New York.
- Marsden, J. (1972), *Basic Complex Analysis*, Freeman, New York.
- Martin, S. (1997), A supersymmetry primer. In: G. Kane (Ed.), *Perspectives on Supersymmetry II*, World Scientific Singapore, 1997, pp. 1–98.
- Martin, P., Rothen, F. (2002), *Many-Body Problems and Quantum Field Theory*, Springer, Berlin.
- Masujima, M. (2000), *Path Integral Quantization and Stochastic Quantization*, Springer, Berlin.

- Mathematical Physics (2000), Proceedings of the XIIIth International Congress on Mathematical Physics, London, United Kingdom, 2000. Edited by A. Fokal et al., Imperial College Press, River Edge, New York.
- Mathematical Physics (2003), Proceedings of the XIVth International Congress on Mathematical Physics, Lisbon, Portugal, 2003. Edited by J. Zambrini, World Scientific, Singapore.
- Mattis, D. (1993), *The Many-Body Problem: An Encyclopedia of Exactly Solved One-Dimensional Models*, World Scientific, Singapore.
- Mattuck, R. (1992), *A Guide to Feynman Diagrams in the Many-Body Problem*, Dover Publications.
- Maurin, K. (1968), *Generalized Eigenfunction Expansions and Unitary Representations of Topological Groups*, Polish Scientific Publishers, Warsaw.
- Maurin, K. (1972), *Methods of Hilbert Spaces*, Polish Scientific Publishers, Warsaw (translated from Polish into English).
- Maurin, K. (1976), *Analysis I, II*, Reidel, Boston (translated from Polish into English).
- Maurin, K. (1997), *The Riemann Legacy: Riemann's Ideas in Mathematics and Physics of the 20th Century*, Kluwer, Dordrecht.
- McComb, W. (2007), *Renormalization Methods: A Guide for Beginners*, Oxford University Press, Oxford.
- McCoy, B., Tai-Tsu Wu (1997), *The two-dimensional Ising Model*, Harvard University Press, Cambridge, Massachusetts.
- McDuff, D., Salamon, D. (1994), *J-holomorphic Curves and Quantum Cohomology*, Amer. Math. Soc., Providence, Rhode Island.
- McDuff, D., Salamon, D. (1998), *Introduction to Symplectic Topology*, 2nd edn., Clarendon Press, Oxford.
- Mehra, J., Milton, K. (2000), *Climbing the Mountain: The Scientific Biography of Julian Schwinger*, Oxford University Press, New York.
- Mehra, J., Rechenberg, H. (2002), *The Historical Development of Quantum Mechanics*, Vols. 1–6, Springer, New York.
- Mehta, M. (2004), *Random Matrices*, 3rd edn., Academic Press, New York.
- Melic, B., Passek-Kummericki, K., Trampetic, J., Schupp, P., Wohlgenannt, M. (2005a), The standard-model on noncommutative space-time: electroweak currents and Higgs sector. Internet: <http://hep-ph/0502249>
- Melic, B., Passek-Kummericki, K., Trampetic, J., Schupp, P., Wohlgenannt, M. (2005b), The standard-model on noncommutative space-time: strong interactions included. Internet: <http://hep-ph/0503064>
- Mielke, A. (2002), The Ginzburg–Landau equation in its role as a modulation equation. In: B. Fiedler (Ed.) (2002), *Handbook of Dynamical Systems*, Vol. 2, pp. 759–834.
- Milnor, J. (1963), *Morse Theory*, Princeton University Press, Princeton, New Jersey.
- Milnor, J. (1965), *Topology from the Differentiable Viewpoint*, Virginia University Press, Charlottesville.
- Milnor, J., Stasheff, J. (1974), *Characteristic Classes*, Princeton University Press, Princeton, New Jersey.
- Milnor, J. (2000), *Dynamics in One Complex Variable: Introductory Lectures*, 2nd edn., Vieweg, Wiesbaden.
- Milonni, P. (1994), *The Quantum Vacuum: An Introduction to Quantum Electrodynamics*, Academic Press, Boston.
- Milton, K. (2001), *The Casimir Effect: Physical Manifestations of Zero-Point Energy*, World Scientific, Singapore.

- Minlos, R. (2000), *Introduction to Mathematical Statistical Physics*, Amer. Math. Soc., Providence, Rhode Island.
- Misner, C., Thorne, K., Wheeler, A. (1973), *Gravitation*, Freeman, San Francisco.
- Mitra, A. (Ed.) (2000), *Quantum Field Theory: A 20th Century Profile*, Indian National Science Academy, Hindustan Book Agency, India.
- Modern Encyclopedia of Mathematical Physics (2011). Edited by I. Aref'eva and D. Sternheimer, Springer, Berlin (to appear).
- Moore, J. (1996), *Lectures on Seiberg–Witten Invariants*, Springer, Berlin.
- Morgan, J., Tian, G. (2007), *Ricci Flow and the Poincaré Conjecture*, Amer. Math. Soc., Providence, Rhode Island/Clay Mathematics Institute, Cambridge, Massachusetts.
- Morii, T., Lim, C., Mukherjee, S. (2004), *The Physics of the Standard Model and Beyond*, World Scientific, Singapore.
- Morse, P., Feshbach, H. (1953), *Methods of Theoretical Physics*, Vols. 1, 2, McGraw-Hill, New York.
- Mostepanenko, V., Trunov, N. (1997), *The Casimir Effect and its Applications*, Clarendon Press, Oxford.
- Moussardo, G. (2007), *Il Modello di Ising: Introduzione alla Teoria dei Campi e delle Transizioni di Fase (The Ising model: introduction to (quantum) field theory and phase transitions)*, Bollati Boringhieri, Italian. (in Italian).
- Müller-Kirsten, H., Wiedemann, A. (1987), *Supersymmetry: An Introduction with Conceptual Computational Details*, World Scientific, Singapore.
- Müller, S. (1999), *Variational Models for Microstructure and Phase Transitions*, In: F. Bethuel, S. Hildebrandt, and M. Struwe (Eds.) *Calculus of Variations and Geometric Evolution Problems*, Cetraro 1996, pp. 85–210, Springer, Berlin. <http://www.mis.mpg.de/preprints/ln/lecturenote-0298-abstr.html>
- Mumford, D. (1975), *Curves and their Jacobians*, The University of Michigan Press, Ann Arbor.
- Mumford, D. (1977), *Stability of projective varieties*, *L'Enseign. Math.* **23**, 39–110.
- Mumford, D. (1982), *Geometric Invariant Theory*, Springer, Berlin.
- Mumford, D. (1983), *Tata Lectures on Theta I, II*, Birkhäuser, Basel.
- Naber, G. (1980), *Topological Methods in Euclidean Spaces*, Cambridge University Press, Cambridge, United Kingdom.
- Naber, G. (1988), *Space-Time and Singularities*, Cambridge University Press, Cambridge, United Kingdom.
- Naber, G. (1992), *The Geometry of Minkowski Space-Time*, Springer, New York.
- Naber, G. (1997), *Topology, Geometry, and Gauge Fields*, Springer, New York.
- Nachtmann, O. (1990), *Elementary Particle Physics: Concepts and Phenomena*, Springer, Berlin (translated from German into English).
- Nagaosa, N. (2000), *Quantum Field Theory in Strongly Correlated Electronic Systems*, Springer, Berlin.
- Nahm, W. (1978), *Supersymmetries and their representations*, *Nucl. Phys.* **B135**, 149–166.
- Nahm, W., Craigie, N., Goddard, P. (Eds.) (1982), *Monopoles in Quantum Field Theory*, World Scientific, Singapore.
- Nahm, W. (1987), *Quantum field theories in one and two dimensions*, *Duke Math. J.* **54**, 579–613.
- Nahm, W., Randjbar-Daemi, S., Szegin, E. (1991), *Topological Methods in Quantum Field Theories*, World Scientific, Singapore.
- Nahm, W. (1996), *Conformally Invariant Quantum Field Theories in Two Dimensions*, World Scientific, Singapore.

- Nahm, W. (2000), Conformal field theory: a bridge over troubled waters. In: A. Mitra (Ed.) (2000), pp. 571–604.
- Nair, V. (2005), *Quantum Field Theory: A Modern Perspective*, Springer, New York.
- Nakahara, M. (1990), *Geometry, Topology and Physics*, Adam Hilger, Bristol.
- NASA home page, Internet: <http://www.nasa.gov/home/>
- Nash, C. (1978), *Relativistic Quantum Fields*, Academic Press, New York.
- Newman, D. (1980), Simple analytic proof of the prime number theorem, *American Math. Monthly* **87**, 693–696.
- Nielsen, M., Chuang, I. (2001), *Quantum Computation and Quantum Information*, Cambridge University Press, Cambridge, United Kingdom.
- Nirenberg, L. (2001), *Topics in Nonlinear Functional Analysis*, American Mathematical Society, Providence, Rhode Island.
- Noack, M., May, R. (2000), *Virus Dynamics: Mathematical Principles of Immunology and Virology*, Oxford University Press, Oxford.
- Nobel Prize Lectures (1954ff), Nobel Foundation, Stockholm.
- Nobel prize laureates. Internet: www.nobel.se/physics/laureats/index.html (see also Harenberg (2000) and Dardo (2004)).
- Nolting, W. (2002), *Grundkurs Theoretische Physik (Course in theoretical physics)*, Vol. 1: Klassische Mechanik, Vol. 2: Analytische Mechanik, Vol. 3: Elektrodynamik, Vol. 4: Spezielle Relativitätstheorie, Thermodynamik, Vol. 5/1 und Vol. 5/2 : Quantenmechanik, Vol. 6: Statistische Physik, Vol. 7: Vielteilchentheorie, Vieweg, Wiesbaden (in German).
- Novikov, S., Manakov, S., Pitaevskii, L., Zakharov, V. (1984), *Solitons: The Inverse Scattering Method*, Consultant Bureau, New York (translated from Russian into English).
- Novikov, S., Fomenko, A. (1987), *Basic Elements of Differential Geometry and Topology*, Kluwer, Dordrecht.
- Novikov, S. (1996), *Topology I: General Survey*, *Encyclopedia of Mathematical Sciences*, Vol. 12, Springer, Berlin (translated from Russian into English).
- Novikov, S., Taimanov, T. (2006), *Geometric Structures and Fields*, Amer. Math. Soc., Providence, Rhode Island.
- Novozhilov, Yu. (1973), *Introduction to Elementary Particle Physics*, Pergamon Press, Oxford (translated from Russian into English).
- Oberguggenberger, M. (1992), *Multiplication of Distributions and Applications to Partial Differential Equations*, Longman, Harlow, United Kingdom.
- Ocampo, H., Paycha, S., Vargas, A. (Eds.) (2005), *Geometric and Topological Methods for Quantum Theory*, Springer, Berlin.
- Olver, P. (1993), *Applications of Lie Groups to Differential Equations*, Springer, New York.
- Olver, P. (1995), *Equivalence, Invariants, and Symmetry*, Cambridge University Press, Cambridge, United Kingdom.
- Olver, P. (1999), *Classical Invariant Theory*, Cambridge University Press, Cambridge, United Kingdom.
- Omnès, R. (1994), *The Interpretation of Quantum Mechanics*, Princeton University Press, Princeton, New Jersey.
- Ooguri, H., Strominger, A., Vafa, C. (2004), Black hole attractors and the topological string, *Phys. Rev.* **D70**(3), 106007.
- Ortner, N. (1987), *Construction of Fundamental Solutions*, Lecture Notes, University of Innsbruck, Austria, Institute for Mathematics and Geometry.

- Ortner, N., Wagner, P. (1997), A survey on explicit representation formulas for fundamental solutions of partial differential operators, *Acta Applicandae Mathematicae* **47**, 101–124.
- Ortner, N., Wagner, P. (2008), *Distribution-Valued Analytic Functions: Theory and Applications*, Lecture Notes 37/2008, Max Planck Institute for Mathematics in the Sciences, Leipzig. Internet: <http://www.mis.de/preprints>
- Osterwalder, K., Schrader, R. (1973), Axioms for Euclidean Green's functions I, II, *Commun. Math. Phys.* **31** (1973), 83–112; **42** (1975), 281–305.
- Pais, A. (1982), *Subtle is the Lord: the Science and the Life of Albert Einstein*, Oxford University Press, Oxford.
- Pandey, J. (1996), *The Hilbert Transform of Schwartz Distributions and Applications*, Wiley, New York.
- Parisi, G. (1998), *Statistical Field Theory*, Perseus Publishing, Reading Massachusetts.
- Parker, L., Toms, D. (2009), *Quantum Field Theory in Curved Spacetime and Gravity*, Cambridge University Press, Cambridge, United Kingdom.
- Particle Data Group. Internet: <http://pdg.lbl.gov>
- Patras, F. (2001), *La pensée mathématique contemporaine*, (Philosophy of modern mathematics), Presses Universitaire de France, Paris (in French).
- Patterson, S. (1995), *An Introduction to the Theory of the Riemann Zeta Function*, Cambridge University Press, Cambridge, United Kingdom.
- Pauli, W. (1921), *Relativitätstheorie* (in German). In: *Enzyklopädie der Mathematischen Wissenschaften II*, 539–775, Teubner, Leipzig. New edition: Springer, Berlin 2000. (English enlarged edition: *Theory of Relativity*, Pergamon Press, London, 1958.)
- Pauli, W. (1933), *Die allgemeinen Prinzipien der Wellenmechanik*. In: *Handbuch der Physik*, Vol. XXIV, edited by H. Geiger and K. Scheel. Reprinted in: *Handbuch der Physik*, Vol. V, Part I, edited by S. Flügge, Springer, Berlin 1933, 1958 (classic survey article on quantum mechanics).
- Pauli, W. (1941), *Relativistic field theory of elementary particles*, *Rev. Modern Physics* **13**, 203–204 (classic survey article).
- Penrose, R. (2004), *The Road to Reality: A Complete Guide to the Laws of the Universe*, Jonathan Cape, London.
- Peres, A. (1993), *Concepts and Methods in Quantum Mechanics*, Kluwer, Dordrecht.
- Perkins, D. (1987), *Introduction to High Energy Physics*, Addison Wesley, Menlo Park, California.
- Peskin, M., Schroeder, D. (1995), *An Introduction to Quantum Field Theory*, Addison-Wesley, Reading, Massachusetts.
- Piguet, O., Sorella, S. (1995), *Algebraic Renormalization: Perturbative Renormalization, Symmetries, and Anomalies*. Springer, Berlin, 1995.
- Pike, E., Sarkar, S. (1995), *The Quantum Theory of Radiation*, Clarendon Press, Oxford.
- Planck, M. (1948), *Wissenschaftliche Selbstbiographie*, 5th edn., Barth, Leipzig, 1948/1970. (English edition: *Scientific Autobiography*, Philosophical Library, New York, 1949.)
- Planck, M. (2000), see: *Quantum Theory Centenary*.
- Planck, M. (2001), *Vorträge, Reden, Erinnerungen* (Lectures, speeches, and recollections). Edited by H. Roos, Springer, Berlin.
- Polchinski, J. (1984), *Renormalization and effective Lagrangians*, *Nucl. Phys. B* **231**, 269–295.

- Polchinski, J. (1998), *String Theory*, Vols. 1, 2, Cambridge University Press, Cambridge, United Kingdom.
- Poincaré Seminar 2002, *Vacuum Energy – Renormalization*. Edited by B. Duplantier and V. Rivasseau, Birkhäuser, Basel.
- Poincaré Seminar 2003, *Bose–Einstein Condensation/Entropy*. Edited by J. Dalibard, B. Duplantier, and V. Rivasseau, Birkhäuser, Basel.
- Poincaré Seminar 2004, *The Quantum Hall Effect*. Edited by B. Douçot, B. Duplantier, V. Rivasseau, and V. Pasquier, Birkhäuser, Basel.
- Poincaré Seminar 2004, *String Theory*.
Internet: La théorie des cordes. <http://www.bourbaphy.fr>
- Poincaré Seminar 2005, *Einstein 1905–2005*. Edited by T. Damour, O. Darrigol, B. Duplantier, and V. Rivasseau, Birkhäuser, Basel, 2006.
- Poincaré Seminar 2005, *Quantum Decoherence*. Edited by B. Duplantier, J. Raymond, and V. Rivasseau, Birkhäuser, Basel, 2006.
- Poincaré Seminar 2006, *Gravitation and Experience*.
Internet: Gravitation et expérience. <http://www.bourbaphy.fr>
- Poincaré Seminar 2007: *Quantum Spaces*. Edited by B. Duplantier and V. Rivasseau. Birkhäuser, Basel, 2007.
- Poincaré Seminar 2007: *Spin* (to appear).
- Polchinski, J. (1998), *String Theory*, Vols. 1, 2, Cambridge University Press, Cambridge, United Kingdom.
- Politzer, D. (1973), Reliable perturbative results for strong interactions? *Phys. Rev.* **30**(2), 1346–1083.
- Polyakov, A. (1987), *Gauge Fields and Strings*, Harwood, Chur, Switzerland.
- Pontryagin, L. (1965), *Foundations of Combinatorial Topology*, 5th edn., Rochester, New York.
- Pontryagin, L. (1966), *Topological Groups*, 2nd edn., Gordon and Breach.
- Prudnikov, A. Brychkov, Yu., Manichev, O. (1986), *Integrals and Series*, Vols. 1–5, Gordon and Breach, New York (translated from Russian into English).
- Quantum Theory Centenary (2000), *Proceedings of the Symposia Quantum Theory Centenary*, December 2000 in Berlin, *Ann. Physik (Leipzig)* **9** (2000), 11/12; **10** (2001), 1/2.
- Radovanović, V. (2007), *Problem Book in Quantum Field Theory*, Springer, Berlin.
- Radzikowski, M. (1996), Micro-local approach to the Hadamard condition in quantum field theory on curved space-time, *Commun. Math. Phys.* **179**, 529–553.
- Ramis, J. (1993), *Séries divergentes et théories asymptotiques* (in French), Société Mathématique de France, Paris.
- Rammer, J. (2007), *Quantum Field Theory of Non-Equilibrium States*, Cambridge University Press, Cambridge, United Kingdom.
- Rammal, R., Toulouse, G., Virasoro, M. (1986), Ultrametricity for physicists, *Rev. Mod. Phys.* **58** (1986), 765–788.
- Ramond, P. (1990), *Field Theory: A Modern Primer*, Addison-Wesley, Reading, Massachusetts.
- Randall, L., Sundrum, R. (1999), Large mass hierarchy from a small extra dimension, *Phys. Rev. Lett.* **83**, 4690–4693.
- Randall, L. (2005), *Warped Passages: Unravelling the Mysteries of the Universe's Hidden Dimensions*, Ecco, New York.
- Reed, M., Simon, B. (1972), *Methods of Modern Mathematical Physics*, Vol. 1: Functional Analysis, Vol. 2: Fourier Analysis and Self-Adjointness, Vol. 3: Scattering Theory, Vol. 4: Analysis of Operators (Perturbation Theory), Academic Press, New York, 1972/79.

- Regis, E. (1989), Who Got Einstein's Office? Eccentricity and Genius at the Institute for Advanced Study in Princeton, Addison-Wesley, Reading, Massachusetts.
- Rehren, K. (2000), Algebraic holography, *Ann. Institute Henri Poincaré* **1**, 607–623.
- Rehren, K. (2002), QFT (Quantum Field Theory) Lectures on AdS/CFT (Lectures on the AdS/CFT correspondence between string theory or some other theory including quantum gravity on bulk Anti-de Sitter spaces (AdS) and supersymmetric Yang-Mills theory on its conformal boundary (CFT–conformal field theory). Internet: <http://arxiv.org/hep-th/0411086>
- Reid, C. (1970), *Hilbert*, Springer, New York.
- Reid, C. (1976), *Courant: the Life of an Improbable Mathematician*, Springer, New York.
- Remmert, R. (1991), *Theory of Complex Functions*, Springer (translated from German into English).
- Remmert, R. (1998), *Classical Topics in Complex Function Theory*, Springer, New York (translated from German into English).
- Rendall, A. (1994), The Newtonian limit for asymptotically flat solutions of the Vlasov–Einstein system, *Commun. Math. Phys.* **163**, 89–112.
- Rendall, A. (1998), *Lectures on Nonlinear Hyperbolic Differential Equations* (in German), Max Planck Institute Albert Einstein for Gravitational Physics, Golm/Potsdam, Germany.
Internet: <http://www.aei-potsdam.mpg.de/rendall/vorlesung1.html>
- Renn, J. (Ed.) (2005a), *Einstein's Annalen Papers: The Complete Collection 1901–1922*, Wiley-VCH, Weinheim, Germany.
- Renn, J. (Ed.) (2005b), *Albert Einstein – Engineer of the Universe: 100 Authors for Einstein – Essays*, Wiley-VCH, Weinheim, Germany.
- Rieffel, M. (1993), 'Deformation Quantization for Actions of \mathbb{R}^d ', *Mem. Amer. Math. Soc.* **106**.
- Riemann, B. (1990), *Collected Mathematical Works* (with commentaries), Springer, New York, and Teubner, Leipzig.
- Riesz, F., Nagy, B. (1975), *Functional Analysis*, Frederick Ungar, New York (translated from French into English).
- Rivasseau, V. (1991), *From Perturbative to Constructive Renormalization*, Princeton University Press, Princeton, New Jersey.
- Rivers, R. (1990), *Path Integral Methods in Quantum Field Theory*, Cambridge University Press, Cambridge, United Kingdom.
- Roepstorff, G. (1996), *Path Integral Approach to Quantum Physics*, Springer, New York.
- Römer, H. (2005), *Theoretical Optics: An Introduction*, Wiley-VHC, New York (translated from German into English).
- Rogers, L., Williams, D. (2000), *Diffusion Markov Processes and Martingales Vols. 1–2*, Cambridge University Press, Cambridge, United Kingdom.
- Rohlf, R. (1994), *Modern Physics from α to Z^0* , Wiley, New York.
- Roman, P. (1969), *Introduction to Quantum Field Theory*, Wiley, New York.
- Rosner, J. (2003), Resource letter SM-1: The standard model and beyond, *Am. J. Phys.* **71**, 302–318. Internet: <http://arxiv:hep-ph/0206176>
- Rothe, H. (2005), *Lattice Gauge Theories*, 3rd edn., World Scientific Singapore.
- Rovelli, C. (2004), *Quantum Gravity*, Cambridge University Press, Cambridge, United Kingdom.
- Rozanov, Yu. (1969), *Introductory Probability Theory*, Prentice-Hall, Englewood Cliffs, New Jersey (translated from Russian into English).
- Ruelle, D. (1969), *Statistical Mechanics: Rigorous Results*, Benjamin, New York.

- Ryder, L. (1999), *Quantum Field Theory*, Cambridge University Press, United Kingdom.
- Sachdev, S. (1999), *Quantum Phase Transitions*, Cambridge University Press, Cambridge, United Kingdom.
- Sachs, I., Sen, S., Sexton, J. (2006), *Elements of Statistical Mechanics: With an Introduction to Quantum Field Theory and Numerical Simulation*, Cambridge University Press, Cambridge, United Kingdom.
- Sakurai, J. (1967), *Advanced Quantum Mechanics*, Reading, Massachusetts.
- Sakurai, J., Tuan, S. (1994), *Modern Quantum Mechanics*, Benjamin, New York.
- Salam, A., (Ed.) (1968), *From a Life of Physics. Evening Lectures at the International Center for Theoretical Physics, Trieste, Italy, with outstanding contributions by Abdus Salam, Hans Bethe, Paul Dirac, Werner Heisenberg, Eugene Wigner, Oscar Klein, and Eugen Lifshitz*, International Atomic Energy Agency, Vienna, Austria.
- Salmhofer, M. (1999), *Renormalization: An Introduction*, Springer, Berlin.
- Sauvigny, F. (2004), *Partielle Differentialgleichungen der Geometrie und der Physik (Partial differential equations in geometry and physics)*, Vol. 1: Grundlagen und Integraldarstellungen (basics and integral representations of the solutions), Vol. 2: Funktionalanalytische Lösungsmethoden (functional analytic methods), Springer, Berlin (in German).
- Scharf, G. (1994), *From Electrostatics to Optics*, Springer, Berlin.
- Scharf, G. (1995), *Finite Quantum Electrodynamics: the Causal Approach*, 2nd edn., Springer, New York.
- Scharf, G. (2001), *Quantum Gauge Theories: A True Ghost Story*, Wiley, New York.
- Schechter, M. (1982), *Operator Methods in Quantum Mechanics*, North-Holland, Amsterdam.
- Schechter, M. (1986), *Spectra of Partial Differential Operators*, North-Holland, Amsterdam.
- Scheck, F. (2000), *Theoretical Physics*, Vol. 1: Mechanics: From Newton's Law to Deterministic Chaos, 4th edn., Vol. 2: Quantum Physics, Vol. 3: Classical Field Theory: From Electrodynamics to Gauge Theories, Vol. 4: Quantized Fields: From Symmetries to Quantum Electrodynamics, Vol. 5: Theory of Heat: From the Fundamental Laws in Thermodynamics to Quantum Statistics (to appear), Springer, Berlin. (Vols. 3ff are written in German; the English translations of these volumes are in preparation.)
- Scheck, F., Wend, W., Upmeyer, H. (Eds.) (2002), *Noncommutative Geometry and the Standard Model of Elementary Particle Physics*, Springer, Berlin.
- Scheck, F. (Ed.) (2002), *Theory of Renormalization and Regularization*, Workshop Hesselberg (Germany), 2002.
Internet: <http://www.thep.physik.uni.mainz.de/~scheck/Hessbg02.html>
- Schlichenmaier, M. (2008), *An Introduction to Riemann Surfaces, Algebraic Curves, and Moduli Spaces*, enlarged second edition, Springer, New York.
- Schmidt, A. (2008), *Towards a q -deformed supersymmetric theory*. In: B. Fauser, J. Tolksdorf, and E. Zeidler (Eds.) (2008), pp. 238–302.
- Schmüser, P. (1995), *Feynman Graphs and Gauge Theories for Experimental Physicists (in German)*, 2nd edn., Springer, Berlin.
- Schulmann, L. (1981), *Techniques and Applications of Path Integration*, Wiley, New York.
- Schuster, P. (1994), *Deterministic Chaos: An Introduction*, Physik-Verlag, Weinheim, Germany.

- Schutz, B. (1980), *Geometrical Methods of Mathematical Physics*, Cambridge University Press, Cambridge, United Kingdom.
- Schutz, B. (2003), *Gravity from the Ground Up*, Cambridge University Press, Cambridge, United Kingdom.
- Schwabl, F. (2000), *Quantum Mechanics*, 3rd edn., Springer, Berlin.
- Schwabl, F. (2002), *Statistical Mechanics*, Springer, Berlin (translated from German into English).
- Schwabl, F. (2003), *Advanced Quantum Mechanics*, 3rd edn., Springer, Berlin (translated from German into English).
- Schwartz, L. (1965), *Méthodes mathématiques pour les sciences physiques*, Hermann, Paris (in French).
- Schwartz, L. (1978), *Théorie des distributions*, 3rd edn., Vols. 1, 2, Hermann, Paris (in French).
- Schwarz, A. (1993), *Quantum Field Theory and Topology*, Springer, Berlin (translated from Russian into English).
- Schwarz, A. (1994), *Topology for Physicists*, Springer, Berlin (translated from Russian into English).
- Schwarz, J. (1985), *Superstrings: The First 15 Years of Superstring Theory*, Vols. 1, 2, World Scientific, Singapore. (Ed.)
- Schwarz, M. (1993), *Morse Homology*, Birkhäuser, Basel.
- Schweber, S. (1961), *An Introduction to Relativistic Quantum Field Theory*, Harper and Row, New York.
- Schweber, S. (1994), *QED (Quantum Electrodynamics) and the Men Who Made It: Dyson, Feynman, Schwinger, and Tomonaga*, Princeton University Press, Princeton, New Jersey.⁶
- Schweber, S. (2000), *In the Shadow of the Bomb: Bethe, Oppenheimer, and the Moral Responsibility of the Scientist*, Princeton University Press, Princeton, New Jersey.
- Schweber, S. (2008), *Einstein and Oppenheimer: the Meaning of Genius*, Harvard University Press, Cambridge, Massachusetts.
- Schwinger, J. (Ed.) (1958), *Quantum Electrodynamics: 34 Selected Articles*, Dover, New York.
- Schwinger, J. (1970), *Particles, Sources, and Fields*, Vols. 1–3, Addison-Wesley, Reading, Massachusetts.
- Schwinger, J. (1979), *Selected Papers*. Edited by M. Flato et al., Reidel, Dordrecht.
- Schwinger, J. (1998), *Classical Electrodynamics*, Perseus Books, Reading, Massachusetts.
- Schwinger, J. (2001), *Quantum Mechanics*, Springer, New York.
- Scriba, C., Schreiber, P. (2003), *5000 Jahre Geometrie: Geschichte, Kulturen, Menschen (5000 years of geometry)*, Springer, Berlin (in German) (see also Alten et al. (2003), *4000 years of algebra*, and Wußing (2008), *6000 years of mathematics*).
- Seiberg, N., Witten, E. (1994a), *Electric-magnetic duality, monopole condensation, and confinement in $N = 2$ supersymmetric Yang–Mills theory*, *Nuclear Phys. B* **426**, 19–52.
- Seiberg, N., Witten, E. (1994b), *Monopoles, duality and chiral symmetry breaking in $N = 2$ supersymmetric QCD*, *Nucl. Physics B* **431**, 485–550.
- Seiden, A. (2005), *Particle Physics: A Comprehensive Introduction*, Addison-Wesley, San Francisco.
- Seiler, E., Sibold, K. (Eds.) (2008), *Quantum Field Theory and Beyond: Essays in Honor of Wolfhart Zimmermann*, World Scientific, Singapore.

⁶ 1300 references

- Séminaire Poincaré 2002ff, Institut Henri Poincaré, Paris. See also Poincaré Seminar 2002ff. Internet: <http://www.juissieux.fr.poincare>
- Sen Hu (1999), Witten's Lectures on Three-Dimensional Topological Quantum Field Theory, World Scientific, Singapore.
- Shafarevich, I. (1990), Algebra I: Basic Notions of Algebra, Encyclopedia of Mathematical Sciences, Vol. 11, Springer, Berlin (translated from Russian into English).
- Shafarevich, I. (1994), Basic Algebraic Geometry, Vols. 1, 2, 2nd edn., Springer, Berlin (translated from Russian into English).
- Shiryayev, A. (1996), Probability, 6th edn., Springer, New York (translated from Russian into English).
- Shohat, J., Tamarkin, J. (1950), The Problem of Moments, American Mathematical Society, New York.
- Shore, S. (2003), The Tapestry of Modern Astrophysics, Wiley, New York.
- Shuir, Ya. (2005), Magnetic Monopoles, Springer, Berlin.
- Sigal, I. (1983), Scattering Theory for Many-Body Quantum Mechanical Systems: Rigorous Results, Springer, Berlin.
- Sibold, K. (2001), Theorie der Elementarteilchen (Theory of elementary particles), Teubner, Stuttgart (in German).
- Siegel, C., Moser, J. (1971), Lectures on Celestial Mechanics, Springer, Berlin.
- Simon, B. (1971), Quantum Mechanics for Hamiltonians Defined on Quadratic Forms, Princeton University Press, Princeton, New Jersey.
- Simon, B. (1974), The $P(\varphi)_2$ -Euclidean Quantum Field Theory, Princeton University Press, Princeton, New Jersey.
- Simon, B. (1979), Functional Integration and Quantum Physics, Academic Press, New York.
- Simon, B. (1984), Fifteen problems in mathematical physics. In: W. Jäger (Ed.), Perspectives in Mathematics, pp. 423–454, Birkhäuser, Basel.
- Simon, B. (1993), The Statistical Theory of Lattice Gases, Princeton University Press, Princeton, New Jersey.
- Simon, B. (1996), Representations of Finite and Compact Groups, Amer. Math. Soc., Providence, Rhode Island.
- Sinai, Ya. (1982), Theory of Phase Transitions: Rigorous Results, Pergamon Press, Oxford (translated from Russian into English).
- Singh, S. (1997), Fermat's Last Theorem: The Story of a Riddle that Confounded the World's Greatest Minds for 358 Years, Fourth Estate, London.
- Skolbelytyn, D. (1967), Quantum Field Theory and Hydrodynamics, Academic Press, New York.
- Smirnov, V. (1964), A Course of Higher Mathematics, Vols. 1–5, Pergamon Press, New York (translated from Russian into English).
- Smirnov, V. (2004), Evaluating Feynman Integrals, Springer, Berlin.
- Smoller, J. (1994), Shock Waves and Reaction-Diffusion Equations, 2nd edn., Springer, New York.
- Sommerfeld, A. (1949), Lectures on Theoretical Physics, Vol. 1: Mechanics, Vol. 2: Mechanics of Continua, Vol. 3: Electrodynamics, Vol. 4: Thermodynamics, Vol. 5: Quantum Mechanics, Vol. 6: Partial Differential Equations in Mathematical Physics, Academic Press, New York (translated from German into English).
- Spanier, E. (1989), Algebraic Topology, 2nd edn., Springer, New York.
- Srednicki, M. (2007), Quantum Field Theory, Cambridge University Press, Cambridge, United Kingdom.
- Stein, E., Shakarchi, R. (2003), Princeton Lectures in Analysis. I: Fourier Analysis, II: Complex Analysis, III: Measure Theory, IV: Selected Topics, Princeton University Press, Princeton, New Jersey.

- Steinmann, O. (1971), *Perturbation Expansions in Axiomatic Field Theory*, Springer, Berlin.
- Steinmann, O. (2000), *Perturbative Quantum Electrodynamics and Axiomatic Field Theory*, Springer, Berlin.
- Stephani, H. (1989), *Differential Equations: Their Solution Using Symmetries*, Cambridge University Press, Cambridge, United Kingdom (translated from German into English).
- Stephani, H., Kramer, D., MacCallum, M., Hoenselaers, C., Herlt, E. (2003), *Exact Solutions of Einstein's Field Equations*, 2nd edn., Cambridge University Press, Cambridge, United Kingdom (translated from German into English).
- Stone, M. (2000), *The Physics of Quantum Fields*, Springer, New York.
- Straumann, N. (2002), *Ein Grundkurs über nicht-relativistische Quantentheorie (Quantum mechanics: a basic course of non-relativistic quantum theory)*, Springer, Berlin (in German).
- Straumann, N. (2004), *General Relativity with Applications to Astrophysics*, Springer, New York.
- Straumann, N. (2005), *Relativistische Quantentheorie: Eine Einführung in die Quantenfeldtheorie (Relativistic quantum theory: an introduction to quantum field theory)*, Springer, Berlin (in German).
- Streater, R., Wightman, A. (1968), *PCT, Spin, Statistics, and All That*, 2nd edn., Benjamin, New York.
- Strocchi, F. (2005), *An Introduction to the Mathematical Structure of Quantum Mechanics: A Short Course for Mathematicians*, Lecture Notes, Scuola Normale Superiore, Pisa (Italy), World Scientific, Singapore.
- Strocchi, F. (1993), *Selected Topics on the General Properties of Quantum Field Theory*, World Scientific, Singapore.
- Stroke, H. (Ed.) (1995), *The Physical Review: The First Hundred Years – A Selection of Seminal Papers and Commentaries*, American Institute of Physics, New York.⁷
- Strominger, A., Vafa, C. (1996), Microscopic origin of the Bekenstein–Hawking entropy of black holes, *Phys. Lett.* **B379**, 99–104.
- Stueckelberg, E., Rivier, D. (1950), Causalité et structure de la matrice S , *Helv. Phys. Acta* **23**, 215–222.
- Stueckelberg, E., Petermann, A. (1953), La normalisation des constantes dans la théorie des quanta, *Helv. Phys. Acta* **26**, 499–520.
- Stueckelberg, E. (2008), *Stueckelberg: An Unconventional Figure of Twentieth Century Physics. Selected Scientific Papers with Commentaries*. Edited by J. Lacki, H. Ruegg, and G. Wanders, Birkhäuser, Boston.
- Stumpf, H., Borne, T., Lochak, G. (2001), *Nonperturbative Quantum Field Theory and the Structure of Matter*, Kluwer, Dordrecht.
- Sudberry, A. (1986), *Quantum Mechanics and the Particles of Nature*, Cambridge University Press, Cambridge, United Kingdom.
- Sulem, C., Sulem, P. (1999), *Nonlinear Schrödinger Equations: Self-Focusing and Wave Collapse*, Springer, New York.
- Sweedler, W. (1969), *Hopf Algebras*, Benjamin, New York.
- Szabo, R. (2000), *Equivariant Cohomology and Localization of Path Integrals*, Springer, Berlin.
- Szabo, R. (2004), *An Introduction to String Theory and D-Brane Dynamics*, Imperial College Press, London.

⁷ 14 survey articles on general developments, 200 fundamental articles, and 800 additional articles on CD

- Tai-Kai Ng (2009), *Introduction to Classical and Quantum Field Theory*, Wiley, New York.
- Takhtajan, L. (2008), *Quantum Mechanics for Mathematicians*, Amer. Math. Soc., Providence, Rhode Island.
- Tao, T. (2009), Why are solitons stable? *Bull. Amer. Math. Soc.* **46**(1), 1–34.
- Taschner, R. (2005), *Der Zahlen gigantische Schatten: Mathematik im Zeichen der Zeit* (The role of mathematics in the history of human culture), Vieweg, Wiesbaden (in German).
- Taylor, J. (1972), *Scattering Theory*, Wiley, New York.
- Taylor, M. (1997), *Partial Differential Equations*, Vols. 1–3, Springer, New York.
- Teller, P. (1997), *An Interpretive Introduction to Quantum Field Theory*, Princeton University Press, Princeton, New Jersey.
- Teschl, G. (2005), *Mathematical Methods in Quantum Mechanics: with Applications to Schrödinger Operators*, Lectures held at the University of Vienna, Austria. Internet: <http://www.mat.univie.ac.at/~gerald/ftpbook-Schroe/>
- Thiemann, T. (2007), *Modern Canonical Quantum General Relativity*, Cambridge University Press, Cambridge, United Kingdom.⁸
- 't Hooft, G, Veltman, M (1973), *Diagrammar*, CERN, Report 73/9. Internet: <http://doc.cern.ch/yellowrep/1973/1973-009/p1.pdf>
- Thirring, W. (1997), *Classical Mathematical Physics: Dynamical Systems and Fields*, 3rd edn., Springer, New York (translated from German into English).
- Thirring, W. (2002), *Quantum Mathematical Physics: Atoms, Molecules, and Large Systems*, 2nd edn., Springer, New York (translated from German into English).
- Thomas, C. (2004), *Representations of Finite and Lie Groups*, Imperial College Press, London.
- Thorne, K. (1993), *Black Holes & Time Warps: Einstein's Outrageous Legacy*, Norton, New York.
- Thouless, D. (1961), *The Quantum Mechanics of Many-Body Systems*, Academic Press, New York.
- Thouless, D. (Ed.) (1998), *Topological Quantum Numbers in Non-Relativistic Physics*, World Scientific, Singapore (collection of 40 important articles on superfluidity, quantum Hall effect, phase transitions, etc.)
- Tian Yu Cao (1998), *Conceptual Developments of 20th Century Field Theories*, Cambridge University Press, Cambridge, United Kingdom.
- Tian Yu Cao (Ed.) (1999), *Conceptual Foundations of Quantum Field Theory* (with contributions made by leading physicists), Cambridge University Press, Cambridge, United Kingdom.
- Ticciati, R. (1999), *Quantum Field Theory for Mathematicians*, Cambridge University Press, Cambridge, United Kingdom.
- Tilley, D., Tilley, J. (1995), *Superfluidity and Superconductivity*, Institute of Physics, Bristol.
- Tipler, P. (1999), *Physics for Scientists and Engineers*, 4th edn., Freeman, New York.
- Titchmarsh, E. (1946), *Eigenfunction Expansions Associated with Second-Order Differential Equations*, Vols. 1, 2, Clarendon Press, Oxford.
- Titchmarsh, E. (1967), *Introduction to the Theory of Fourier Integrals*, Clarendon Press, Oxford.
- Titchmarsh, E., Heath-Brown, D. (1986), *The Theory of the Riemann Zeta-Function* (first edition, 1930), Cambridge University Press, Cambridge, United Kingdom.

⁸ 900 references

- Toda, M. (1989), *Nonlinear Waves and Solitons*, Kluwer, Dordrecht.
- Todorov, I. (1971), *Analytic Properties of Feynman Diagrams in Quantum Field Theory*, Pergamon Press, London.
- Tolksdorf, J. (2007), On the square of first order differential operators of Dirac type and the Einstein–Hilbert action, *J. Geometry and Physics* **57**, 1999–2013.
- Tolksdorf, J., Thumstädter, T. (2007), Dirac-type gauge theories and the mass of the Higgs boson, *J. Phys. A: Math. Theor.* **40**, 9691–9716.
- Tomonaga, S. (1971), *Scientific Papers*, Vols. 1, 2. Edited by T. Miyazima, Tokyo, Japan.
- Topping, P. (2006), *Lectures on the Ricci Flow*, Cambridge University Press, Cambridge, United Kingdom.
- Triebel, H. (1987), *Analysis and Mathematical Physics*, Kluwer, Dordrecht (translated from German into English).
- Triebel, H. (1989), *Higher Analysis*, Barth, Leipzig (translated from German into English).
- Triebel, H. (1992), *Theory of Function Spaces*, Birkhäuser, Basel.
- Tromba, A. (1996), *Teichmüller Theory in Riemannian Geometry*, Birkhäuser, Basel.
- Tsvetik, A. (2003), *Quantum Field Theory in Condensed Matter Physics*, 2nd edn., Cambridge University Press, Cambridge, United Kingdom.
- Turaev, V. (1994), *Quantum Invariants of Knots and 3-Manifolds*, de Gruyter, Berlin.
- van Baal, P. (2000), *A Course in Quantum Field Theory*,
Internet: <http://rulgm4.leidenuniv.nl/van-baal/FTcourse.html>
- van Suijlekom, W. (2008), Renormalization of gauge fields using Hopf algebras. In: B. Fauser, J. Tolksdorf, and E. Zeidler (Eds.) (2008), pp. 137–154.
- van der Waerden, B. (1930), *Moderne Algebra* (in German), Vols. 1, 2, 8th edn. 1993, Springer, New York (English edition: Frederick Ungar, New York 1975).
- van der Waerden, B. (1932), *Group Theory and Quantum Mechanics*, Springer, New York 1974 (translated from the 1932 German edition into English).
- van der Waerden, B. (1939), *Introduction to Algebraic Geometry*, 2nd edn. 1973, Springer, Berlin (in German).
- van der Waerden, B. (Ed.) (1968), *Sources of Quantum Mechanics 1917–1926*, Dover, New York.
- van der Waerden, B. (1984), *A History of Algebra: From al-Khwarizmi to Emmy Noether*, Springer, New York.
- Varadarajan, V. (2004), *Supersymmetry for Mathematicians*, Courant Lecture Notes, Amer. Math. Soc., Providence, Rhode Island.
- Varadarajan, V. (2006), *Euler through Time: A New Look at Old Themes*, Amer. Math. Soc., Providence, Rhode Island.
- Varadarajan, V. (2007), *Geometry of Quantum Theory*, 2nd edn., Springer, New York.
- Vassilievich, D. (2003), Heat Kernel Expansion: User’s Manual, *Physics Reports* **388**, 279–360.
- Velo, G.: See Wightman and Velo.
- Veltman, M. (1995), *Diagrammatica: the Path to Feynman Diagrams*, Cambridge University Press, Cambridge, United Kingdom.
- Veltman, M. (2003), *Facts and Mysteries in Elementary Particle Physics*, World Scientific, Singapore.
- Verch, R. (2004), The current status of quantum fields in curved space time. Lecture held on the occasion of the 125th anniversary of Einstein’s birth. Deutsche

- Physikalische Gesellschaft, Ulm, 2004. Preprint of the Max Planck Institute for Mathematics in the Sciences, Leipzig.
 Internet: <http://www.mis.mpg.de/preprints/>
- Vilenkin, N., Klimyk, A. (1991), *Special Functions and Representations of Lie Groups*, Vols. 1–4, Kluwer, Dordrecht (translated from Russian into English).
- Villani, C. (2002), A review of mathematical topics in collisional kinetic theory. In: S. Friedlander and D. Serre (Eds.), *Handbook of Fluid Dynamics*, Vol. 1, pp. 71–306, Elsevier, Boston, 2002.
- Villani, C. (2003), *Topics in Optimal Transportation*, Amer. Math. Soc., Providence, Rhode Island.
- Vladimirov, V. (1966), *Methods of the Theory of Many Complex Variables*, MIT Press, Cambridge, Massachusetts (translated from Russian into English).
- Vladimirov, V. (1971), *Equations of Mathematical Physics*, Marcel Dekker, New York (translated from Russian into English).
- Voisin, C. (2002), *Hodge Theory and Complex Algebraic Theory*, Vols. 1, 2, Cambridge University Press, Cambridge, United Kingdom.
- von Neumann, J. (1947), The mathematician. In: *The Works of the Mind*, Vol. 1, pp. 180–196. Edited by R. Heywood, University of Chicago Press.
- von Neumann, J. (1955), *Mathematical Foundations of Quantum Mechanics*, Princeton University Press, New Jersey (translated from the 1932 German edition into English).
- von Neumann, J. (1961), *Collected Papers*, Vols. 1–5, Pergamon Press, New York.
- Wachter, H. (2008), Towards a q -deformed quantum field theory. In: B. Fauser, J. Tolksdorf, and E. Zeidler (Eds.) (2008), pp. 261–281.
- Wagner, P. (2009), A new constructive proof of the Malgrange–Ehrenpreis theorem, *Amer. Math. Monthly* **116**, 457–462.
- Wald, R. (1984), *General Relativity*, University of Chicago Press, Chicago.
- Wald, R. (1992), *Space, Time, and Gravity: The Theory of the Big Bang and Black Holes*, University of Chicago Press, Chicago.
- Wald, R. (1994), *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*, The University of Chicago Press, Chicago.
- Waldmann, S. (2007), *Poisson-Geometrie und Deformationsquantisierung (Poisson geometry and deformation quantization)*, Springer, Berlin (in German).
- Waldmann, S. (2008), Noncommutative field theories from a deformation point of view. In: B. Fauser, J. Tolksdorf, and E. Zeidler (Eds.) (2008), pp. 117–136.
- Waldschmidt, M., Moussa, P., Luck, L., Itzykson, C. (Eds.) (1995), *From Number Theory to Physics*, 2nd edn., Springer, New York.
- Wasow, W. (1965), *Asymptotic Expansions for Ordinary Differential Operators*, Interscience, New York.
- Weil, A. (1974), *Basic Number Theory*, Springer, Berlin.
- Weinberg, S. (1960), High energy behavior in quantum field theory, *Phys. Rev.* **118**, 838–849.
- Weinberg, S. (1972), *Gravitation and Cosmology*, Wiley, New York.
- Weinberg, S. (1977), *The First Three Minutes: A Modern View of the Origin of the Universe*, Basic Books, New York.
- Weinberg, S. (1983), *The Discovery of Subatomic Particles*, Scientific American Library, New York.
- Weinberg, S. (1992), *Dreams of a Final Theory*, Pantheon Books, New York.
- Weinberg, S. (1995), *Quantum Field Theory*, Vols. 1–3, Cambridge University Press, Cambridge, United Kingdom.
- Weinberg, S. (2008), *Cosmology*, Oxford University Press, Oxford.

- Wentzel, G. (1949), *Quantum Theory of Wave Fields*, Interscience, New York (translated from German into English).
- Wess, J., Zumino, B. (1974), Supergauge transformations in four dimensions, *Nucl. Phys. B* **70**, 39–50.
- Wess, J., Zumino, B. (1977), Superspace formulation of supergravity, *Phys. Lett* **66B**, 361–364.
- Wess, J., Bagger, J. (1991), *Supersymmetry and Supergravity*, 2nd edn., Princeton University Press, Princeton, New Jersey.
- Wess, J. (2000), Noncommutative spaces and quantum groups, *Fortschritte der Physik* **48** (13), 233–240.
- Wess, J. (2005), Gauge theories on noncommutative spacetime treated by the Seiberg-Witten method. In: U. Carow-Watamura et al. (eds.) (2005), 179–192.
- Wess, J. (2005), Gauge theories on noncommutative spacetime treated by the Seiberg-Witten method. In: U. Carow-Watamura et al. (Eds.) (2005), pp. 179–192.
- Weyl, H. (1910), On ordinary differential equations with singularities (in German), *Math. Ann.* **68**, 220–269.
- Weyl, H. (1911), On the asymptotic distribution of eigenvalues (in German), *Göttinger Nachr.* 1911, 110–117.
- Weyl, H. (1913), *Die Idee der Riemannschen Fläche* (in German). Teubner, Leipzig. New edition with commentaries supervised by R. Remmert, Teubner, Leipzig, 1997. (English edition: *The Concept of a Riemann Surface*, Addison Wesley, Reading, Massachusetts, 1955.)
- Weyl, H. (1918), *Raum, Zeit, Materie* (in German), 8th edn. 1993, Springer, Berlin. (English edition: *Space, Time, Matter*, Dover, New York, 1953.)
- Weyl, H., Peter, F. (1927), On the completeness of the irreducible representations of compact continuous groups (in German), *Math. Ann.* **97**, 737–755.
- Weyl, H. (1929), *Gruppentheorie und Quantenmechanik* (in German), Springer, Berlin. (English edition: *Theory of Groups and Quantum Mechanics*, Dover, New York, 1931.)
- Weyl, H. (1938), *The Classical Groups: Their Invariants and Representations*, Princeton University Press, Princeton, New Jersey (8th edn. 1973).
- Weyl, H. (1940), The method of orthogonal projection in potential theory, *Duke Math. J.* **7**, 414–444.
- Weyl, H. (1943), On Hodge's theory of harmonic integrals, *Ann. of Math.* **44**, 1–6.
- Weyl, H. (1944), David Hilbert and its mathematical work, *Bull. Amer. Math. Soc.* **50**, 612–654.
- Weyl, H. (1949), *Philosophy of Mathematics and Natural Sciences*, Princeton University Press, Princeton, New Jersey.
- Weyl, H. (1952), *Symmetry*, Princeton University Press, Princeton, New Jersey.
- Weyl, H. (1968), *Gesammelte Werke* (Collected Works), Vols. 1–4, Springer, New York.
- Whittaker, E., Watson, G. (1979), *A Course of Modern Analysis*, Cambridge University Press and MacMillan Company, London, 1944; American Mathematical Society, Providence, Rhode Island, 1979.
- Widder, D. (1944), *The Laplace Transform*, Princeton University Press, Princeton, New Jersey.
- Wigner, E. (1939), On unitary representations of the inhomogeneous Lorentz group, *Ann. of Math.* **40**, 149–204.
- Wigner, E. (1959), *Group Theory and its Applications to the Quantum Mechanics of Atomic Spectra*, Academic Press, New York. (German edition: Springer, Berlin, 1931.)

- Wigner, E. (1993), *Collected Works*, Vols. 1ff. Edited by J. Mehra and A. Wightman, Springer, New York.
- Wigner, E. (1995), *Philosophical Reflections and Syntheses*. Annotated by G. Emch. Edited by J. Mehra and A. Wightman, Springer, New York.
- Wightman, A., Gårding, L. (1954a): Representations of the anticommutation relations, *Proc. Natl. Acad. Sci. U.S.A.* **40**, 617–621.
- Wightman, A., Gårding, L. (1954b): Representations of the commutation relations, *Proc. Natl. Acad. Sci. U.S.A.* **40**, 622–625.
- Wightman, A. (1956), Quantum field theories in terms of vacuum expectation values, *Phys. Rev.* **101**, 860–866.
- Wightman, A., Hall, D. (1957), A theorem on invariant analytic functions with applications to relativistic quantum field theory, *Danske Vid. Selsk. Mat.-Fys. Medd.* **31**, 1–41.
- Wightman, A. (1960), Quantum field theory and analytic functions of several complex variables, *J. Indian Math. Soc.* **24**, 625–677.
- Wightman, A. (1962), On the localizability of quantum mechanical systems, *Rev. Mod. Phys.* **34**, 845–872.
- Wightman, A., Gårding, L. (1964), Fields as operator-valued distributions in relativistic quantum theory, *Arkiv för Fysik* **28**, 129–189.
- Wightman, A., Streater, R. (1968), *PCT, Spin, Statistics, and All That*, 2nd edn., Benjamin, New York.
- Wightman, A., Velo, G. (Eds.) (1973), *Constructive Quantum Field Theory*, Springer, Berlin.
- Wightman, A., Velo, G. (Eds.) (1976), *Renormalization Theory*, Reidel, Dordrecht.
- Wightman, A. (1976): Orientation on renormalization. In: A. Wightman and G. Velo (Eds.) (1976), pp. 1–24.
- Wightman, A., Velo, G. (Eds.) (1980), *Rigorous Atomic and Molecular Physics*, Plenum Press, New York.
- Wilson, K., Kogut, J. (1974), The renormalization group and the ε -expansion, *Physics Reports* **12C**, 75–199.
- Witten, E. (1981), A new proof of the positive energy theorem, *Commun. Math. Phys.* **80**, 381–396.
- Witten, E. (1982), Supersymmetry and Morse theory, *J. Diff. Geom.* **17**, 661–692.
- Witten, E. (1988a), Geometry and quantum field theory, *Proc. AMS Centennial Symposium*, 1988. In: Atiyah and Iagolnitzer (Eds.) (2003), pp. 523–535.
- Witten, E. (1988b), Topological quantum field theory, *Commun. Math. Phys.* **117**, 353–386.
- Witten, E. (1988c), Topological sigma models, *Commun. Math. Phys.* **118**, 411–449.
- Witten, E. (1989), Quantum field theory and the Jones polynomials, *Commun. Math. Phys.* **212**, 359–399.
- Witten, E. (1990), On the structure of the topological phase of two-dimensional gravity, *Nucl. Phys. B* **340** (1990), 281–332.
- Witten, E. (1991), Two-dimensional gravity and intersection theory on moduli spaces, *Surv. Diff. Geometry* **1** (1991), 243–310.
- Witten, E. (1992), Geometry and quantum field theory, *AMS Centennial Symposium 1988*, Amer. Math. Soc. 1992. Reprinted in Atiyah and Iagolnitzer (2003), pp. 523–535.
- Witten, E. (1994), Monopoles and four-manifolds, *Math. Research Letters* **1**, 769–796.
- Witten, E. (1996), Reflections on the fate of space-time, *Physics Today* **49**, April 1996, 24–30.

- Witten, E. (1997), Duality, space-time, and quantum mechanics, *Physics Today* **50**, May 1997, 28–33.
- Witten, E. (1998a), Magic, mystery and matrix (Gibbs Lecture), *Notices Amer. Math. Soc.* **45**, 1124–1129.
- Witten, E. (1998b), New perspectives in the quest for unification.
Internet: [arxiv:hep-th/9812208](http://arxiv.org/hep-th/9812208)
- Witten, E. (1998), Anti-de Sitter space and holography, *Adv. Theor. Math. Phys.* **2**, 253–291.
- Witten, E. (1999a), Lectures on perturbative quantum field theory. In: P. Deligne et al (Eds.) (1999), Vol. 1, 421–509.
- Witten, E. (1999b), Lectures on dynamical aspects of quantum field theories. In: P. Deligne et al (Eds.) (1999), Vol. 2, 1119–1424.
- Witten, E. (1999c), Lectures on Three-Dimensional Topological Quantum Field Theory. See Sen Hu (1999).
- Witten, E. (2003), Physical law and the quest for mathematical understanding, *Bull. Amer. Math. Soc.* **40**, 21–30.
- Witten, E. (2004), Perturbative gauge theory as a string theory in twistor space, *Commun. Math. Phys.* **252**, 189–209.
- Witten, E., Kapustin, A. (2006), Electric-magnetic duality and the geometric Langlands program, 225 pages.
Internet: <http://arxiv.org/hep-th/0604151>
- Witten, E. (2007), Three-dimensional gravity revisited, 82 pages.
Internet: <http://arxiv.org/hep-th/0706.3359>
- Witten, E., Gukov, S (2008), Branes and quantization, 70 pages.
Internet: <http://arxiv.org/hep-th/0809.0305>
- Woodhouse, N. (1997), *Geometric Quantization*, 3rd edn., Oxford University Press, New York.
- Woodhouse, N. (2003), *Special Relativity*, Springer, New York.
- Wulkenhaar, R. (2005), Lectures on Euclidean quantum field theory and commutative and noncommutative spaces. In: Ocampo, Paycha, and Vargas (Eds.) (2005), pp. 59–100.
- Wußing, H. (Ed.) (1983), *Geschichte der Naturwissenschaften (History of the natural sciences)*, Edition Leipzig (in German).
- Wußing, H. (2008), *6000 Jahre Mathematik: Eine kulturgeschichtliche Zeitreise (6000 years of mathematics: a cultural journey through time)*, Vols. I, II, Springer, Heidelberg (in German).
- Xiaoping Xu (1998), *Introduction to Vertex Operator Superalgebras and Their Modules*, Kluwer, Dordrecht.
- Yandell, B. (2001), *The Honors Class: Hilbert’s Problems and Their Solvers*, Peters, Natick, Massachusetts.
- Yang, Y. (2001), *Solitons in Field Theory and Nonlinear Analysis*, Springer, New York.
- Yeh, J. (1973), *Stochastic Processes and the Wiener Integral*, Marcel Dekker, New York.
- Yosida, K. (1960), *Lectures on Differential-and Integral Equations*, Interscience, New York (translated from Japanese into English).
- Yosida, K. (1995), *Functional Analysis*, 7th edn., Springer, New York.
- Zagier, D. (1981), *Zetafunktionen und quadratische Zahlkörper: Eine Einführung in die höhere Zahlentheorie (Zeta functions and quadratic number fields: an introduction to advanced number theory)*, Springer, Berlin (in German).

- Zagier, D. (1990), The Bloch–Wigner–Ramakrishnan polylogarithm function, *Math. Ann.* **286**, 613–624.
- Zagier, D. (1995), Introduction to modular forms. In: Waldschmidt et al. (Eds.) (1995), pp. 238–291.
- Zagier, D. (1996), Newman’s short proof of the prime number theorem, *Amer. Math. Monthly* **104**, 705–708.
- Zagier, D. (2001), Leçon inaugurale, Jeudi 17 Mai 2001, Collège de France, Paris.
- Zee, A. (1982), *Unity of Forces in the Universe*, World Scientific, Singapore.
- Zee, A. (1999), *Fearful Symmetry: The Search for Beauty in Modern Physics*, Princeton University Press, New Jersey.
- Zee, A. (2001), *Einstein’s Universe: Gravity at Work and Play*, Oxford University Press, New York.
- Zee, A. (2002), *Swallowing Clouds*, University of Washington Press, Seattle, Washington.
- Zee, A. (2003), *Quantum Field Theory in a Nutshell*, Princeton University Press, Princeton, New Jersey.
- Zeidler, E. (1986), *Nonlinear Functional Analysis and Its Applications*. Vol. I: Fixed-Point Theory, 3rd edn. 1998; Vol. IIA: Linear Monotone Operators, 2nd edn. 1997; Vol. IIB: Nonlinear Monotone Operators; Vol. III: Variational Methods and Optimization; Vol. IV: Applications to Mathematical Physics, 2nd edn. 1995, Springer, New York.
- Zeidler, E. (1995a), *Applied Functional Analysis, Vol 1: Applications to Mathematical Physics*, 2nd edn. 1997, Applied Mathematical Sciences, AMS 108, Springer, New York.
- Zeidler, E. (1995b), *Applied Functional Analysis, Vol. 2: Main Principles and Their Applications*, Applied Mathematical Sciences, AMS 109, Springer, New York.
- Zeidler, E. (Ed.) (2002a), *Teubner-Taschenbuch der Mathematik (in German)*, Vol. 1 (English edition: see Zeidler (2004)); Vol. 2 edited by G. Grosche, D. Ziegler, V. Ziegler, and E. Zeidler (English edition in preparation), Teubner, Stuttgart/Leipzig.
- Zeidler, E. (2002b), *Mathematics – the Cosmic Eye of Humanity*.
Internet: <http://www.mis.mpg.de/>
- Zeidler, E. (Ed.) (2004), *Oxford User’s Guide to Mathematics*, Oxford University Press, Oxford (translated from German into English).
- Zeidler, E. (2008), Gedanken zur Zukunft der Mathematik (Reflections on the future of mathematics). In: H. Wußing (2008), Vol. II, last chapter, pp. 553–586 (in German).
- Zimmermann, W. (1969), Convergence of Bogoliubov’s method of renormalization in momentum space, *Commun. Math. Phys.* **15**, 208–234.
- Zinn-Justin, J. (2004), *Quantum Field Theory and Critical Phenomena*, 4th edn., Clarendon Press, Oxford.
- Zinn-Justin, J. (2007), *Phase Transitions and Renormalization Group*, Oxford University Press, New York.
- Zorich, V. (2003), *Analysis I, II*, Springer, New York (translated from Russian into English).
- Zwiebach, B. (2004), *A First Course in String Theory*, Cambridge University Press, Cambridge, United Kingdom.

List of Symbols

$f(x) := x^2$ (definition of f)
 $f(x) \simeq g(x)$, $x \rightarrow a$ (asymptotic equality); this means
 $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = 1$, 949
 $f(x) = o(g(x))$, $x \rightarrow a$ (Landau symbol); this means
 $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = 0$, 949
 $f(x) = O(g(x))$, $x \rightarrow a$, 949
 $f(x) \sim \sum_{n=0}^{\infty} a_n x^n$ (asymptotic expansion), 308, 863

$\text{sgn}(a)$ (sign of the real number a), 949
 $[a, b]$, $]a, b[$, $]a, b]$ (intervals), 949
 $\sum_{n=-\infty}^{\infty} b_n$, 215
 δ_{ij} (Kronecker symbol), 949
 $\delta_{11} := 1$, $\delta_{12} := 0$
 $\delta^{ij} = \delta_{ij} = \delta_i^j$, 949
 $\delta_{\mathbf{pq}}$, 672
 ε_{ij} (skew-symmetric symbol)
 $\varepsilon_{12} = -\varepsilon_{21} = 1$, $\varepsilon_{11} = \varepsilon_{22} = 0$, 337

x, y, z (right-handed Cartesian coordinates)
 $\mathbf{i}, \mathbf{j}, \mathbf{k}$ (right-handed orthonormal basis)
 $\mathbf{x} := x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ (position vector)
 $\|\mathbf{x}\|$ (length (norm) of the vector \mathbf{x})

t (time)
 $x^1 := x, x^2 := y, x^3 := z, x^0 := ct$
 (space-time point in Minkowski space), 949
 $\mu = 0, 1, 2, 3$ (indices for space-time variables in Minkowski space), 949
 $j = 1, 2, 3$ (indices for spatial variables in Minkowski space), 949
 $\eta_{\mu\nu}$ (Minkowski symbol), $\eta_{00} := 1$,
 $\eta_{11} := -1$, $\eta_{01} := 0$, 949
 $\eta^{\mu\nu} = \eta_{\mu\nu} = \eta_{\nu}^{\mu}$, 949
 $\epsilon^{\alpha\beta\gamma\delta}$, $\epsilon_{\alpha\beta\gamma\delta}$ (skew-symmetric symbol)
 $\epsilon_{0123} := 1$, $\epsilon_{1023} := -1$, 950

$a_{\mu}b^{\mu} := \sum_{\mu=0}^3 a_{\mu}b^{\mu}$ (Einstein's convention in Minkowski space), 950
 $\zeta(s)$ (zeta function), 280
 B_n (Bernoulli number), 280

$\theta(t)$ (Heaviside function), 92, 579
 $\delta(t)$ (Dirac delta function), 593
 δ, δ_x (Dirac delta distribution), 612
 δ_{μ} (Dirac delta function with respect to the measure μ), 605
 $\delta_{\Delta t}(t)$ (standard discrete Dirac delta function), 582
 $\delta_{\Delta^4 x}, \delta_{\mathcal{C}(L)}, \delta_{\mathcal{G}(N)}, \delta_T, \delta_{\text{dis}}$ (discrete Dirac delta functions), 443, 672
 $\mathcal{C}(N)$ (cube in position space), 671
 $\mathcal{G}(N)$ (grid in momentum space), 671
 $\Delta^3 p$, 672
 \mathcal{V} (normalization volume), 671
 $\delta(x^2 - a^2)$ (special distribution), 598
 $\mathcal{P}(\frac{1}{x})$ (special distribution), 621
 $\frac{1}{x \pm 0_{\pm i}}$ (special distribution), 623
 $\mathcal{P}(\ln|x|)$ (special distribution), 738

I , id (identity operator)
 $x \in U$ (the point x is an element of U)
 $U \subseteq V$ (U is a subset of V)
 $U \subset V$ (U is a proper subset of V), 947
 $U \cup V$ (the union of two given sets U and V)
 $U \cap V$ (the intersection of two given sets U and V)
 $U \setminus V$ (the difference of two sets U and V , i.e., the set of elements of U not belonging to V)
 ∂U (boundary of the set U)
 $\text{int}(U)$ (interior of U)
 $\text{cl}(U) \equiv U \cup \partial U$ (closure of U), 545
 \emptyset (empty set)
 $\{x : x \text{ has the property } \mathcal{P}\}$ (the set of all things which have the property \mathcal{P})

- $f : X \rightarrow Y$ (map), 947
 $\text{im}(f)$ (image of the map f), 947
 $\text{dom}(f)$ (domain of f), 947
 $f^{-1} : Y \rightarrow X$ (inverse map), 947
 $f(U)$ (image of the set U), 947
 $f^{-1}(V)$ (pre-image of the set V), 947
- $z = x + yi$ (complex number)
 $\Re(z) := x$ (real part of z)
 $\Im(z) := y$ (imaginary part of z)
 $|z|$ (modulus of z), 211
 $\arg(z)$ (principal argument of z),
 $-\pi < \arg(z) \leq \pi$, 211
 $\arg_*(z)$ (argument of z), 211
 $z^\dagger := x - yi$ (conjugate complex number), 338
 $\ln z$ (logarithmic function), 222
 $\text{res}_z(f)$ (residue of the function f at the point z), 215
- \mathbb{R} (set of real numbers)
 \mathbb{C} (set of complex numbers)
 $\overline{\mathbb{C}}$ (closed complex plane), 219
 $\mathbb{C}_>$ (open upper half-plane), 665
 \mathbb{C}_\geq (closed upper-half plane)
 $\mathbb{C}_<$ (open lower half-plane), 665
 $\mathbb{K} = \mathbb{R}, \mathbb{C}$ (set of real or complex numbers)
 \mathbb{Z} (set of integers, $0, \pm 1, \pm 2, \dots$)
 \mathbb{N} (set of natural numbers, $0, 1, 2, \dots$)
 \mathbb{Q} (set of rational numbers)
 $\mathbb{R}^N, \mathbb{C}^N, \mathbb{K}^N$ ($N = 1, 2, \dots$), 330
 \mathbb{M}^4 (Minkowski space), 771
- \mathbb{R}^\times (set of nonzero real numbers)
 \mathbb{N}^\times (set of nonzero natural numbers, $1, 2, \dots$)
 \mathbb{C}^\times (set of nonzero complex numbers)
 \mathbb{K}^\times (set of nonzero numbers in \mathbb{K})
 \mathbb{R}_{\geq} (set of nonnegative real numbers, $x \geq 0$)
 $\mathbb{R}_{>}$ (set of positive real numbers, $x > 0$)
 \mathbb{R}_{\leq} (set of non-positive real numbers, $x \leq 0$)
 $\mathbb{R}_{<}$ (set of negative real numbers, $x < 0$)
 \mathbb{R}_+ (additive semigroup of nonnegative real numbers)
 \mathbb{R}_+^\times (multiplicative group of positive real numbers)
- \mathbb{B}^2 (closed unit disc)
 $\text{int}(\mathbb{B}^2)$ (open unit disc)
- $\mathbb{S}^1 \equiv \partial\mathbb{B}^2$ (unit circle)
 \mathbb{B}^3 (closed 3-dimensional unit ball)
 $\text{int}(\mathbb{B}^3)$ (open 3-dimensional unit ball)
 $\mathbb{S}^2 \equiv \partial\mathbb{B}^3$ (2-dimensional unit sphere)
 \mathbb{B}^n (closed n -dimensional unit ball), 270
 $\mathbb{S}^n \equiv \partial\mathbb{B}^{n+1}$ (n -dimensional unit sphere)
- \bar{A} (mean value), 353
 ΔA (mean fluctuation), 353
- $\dim X$ (dimension of the linear space X), 332
 $\text{span } S$ (linear hull of the set S), 331
 $\langle x|y \rangle$ (inner product), 338
 $\langle \varphi|, |\psi \rangle$ (Dirac calculus), 361
 $\|\varphi\|$ (norm), 338, 368
 $L(X, Y)$ (space of linear operators), 334
 X^d (dual space), 334
 A^\dagger (adjoint operator), 359
 A^d (dual operator), 359
 A^{-1} (inverse operator), 947
 A^c (conjugate complex operator); this means $(A^\dagger)^d$
 A^\ddagger (adjoint matrix), 343
 A^d (dual or transposed matrix), 343
 A^c (conjugate complex matrix), 343
 A^{-1} (inverse matrix), 947
 $[A, B]_- := AB - BA$, 56
 $[A, B]_+ := AB + BA$,
 $\text{tr}(A)$ (trace), 343, 365
 $\det(A)$ (determinant), 335
 e^A (exponential function), 347
 $\ln A$ (logarithmic function), 348
 $\sigma(A)$ (spectrum), 367
 $\varrho(A) = \mathbb{C} \setminus \sigma(A)$ (resolvent set), 367
- $GL(X), SL(X), U(X), SU(X)$
 (Lie groups), 343
 $U(1), U(n), SU(n), O(n), SO(n),$
 $GL(n, \mathbb{R}), SL(n, \mathbb{R}), GL(n, \mathbb{C}),$
 $SL(n, \mathbb{C})$ (matrix Lie groups), 343
 $gl(X), sl(X), u(X), su(X)$
 (Lie algebras), 344
 $u(n), su(n), o(n), so(n), gl(n, \mathbb{R}),$
 $sl(n, \mathbb{R}), gl(n, \mathbb{C}), sl(n, \mathbb{C})$
 (matrix Lie algebras), 345
 $T_x M$ (tangent space), 350
- $\lim_{n \rightarrow \infty} \varphi_n = \varphi$ (limit), 339
 $f(+0)$, 95
 $\text{curl } \mathbf{E}$ (curl of the vector field \mathbf{E}), 172
 $\text{div } \mathbf{E}$ (divergence of \mathbf{E}), 172

- $\text{grad } U$ (gradient of the scalar field U), 172
 ∂ (vector differential operator), 172
 $\Delta = -\partial^2$ (Laplacian), 544
 \square (wave operator), 797
 $f^*\omega$ (pull-back), 257
- $\dot{\psi}(t) \equiv \frac{d\psi(t)}{dt}$ (time derivative)
 $f'(x) \equiv \frac{df(x)}{dx}$ (derivative)
 $\partial_\mu f \equiv \frac{\partial f}{\partial x^\mu}$ (partial derivative)
 $\partial^\alpha f$ (partial derivative of the function f of order $|\alpha|$), 538
 $\partial^\alpha F$ (partial derivative of the distribution F), 613
 $\alpha = (\alpha_1, \dots, \alpha_N)$ (multi-index), 538
 $|\alpha| = |\alpha_1| + \dots + |\alpha_N|$ (order of α), 538
 $\alpha! = \alpha_1! \alpha_2! \dots \alpha_N!$ (factorial)
 ∇_α (covariant derivative), 793
 $\delta F(\psi; h)$ (variation of the functional F at the point ψ in direction of h), 398
 $F'(\psi) \equiv \frac{\delta F(\psi)}{\delta \psi}$ (functional derivative of F at the point ψ), 398
 $\frac{\delta Z(J)}{\delta J(x)}, \frac{\delta F(\psi)}{\delta \psi(x)}$ (local functional derivative at the point x), 405, 444, 752, 763
- $\int f(x)dx$ (Lebesgue integral), 531
 $\int f(x)d\mu(x)$ (measure integral), 531
 $\int f(\lambda)dE_\lambda$ (Hilbert–von Neumann spectral integral), 37, 371
 $PV \int_{-\infty}^{\infty} f(x)dx$ (principal value), 90, 621
 $\int F(q)d\mu(q), \int F(\varphi)\mathcal{D}\varphi$ (functional integral), 418, 444, 755
 $\Pi_{\Delta t}(q_{\text{in}}, q_{\text{out}})$ (space of curves), 422
- $\text{zero}(f)$, 611
 $\text{supp}(f)$ (support of the function f), 611
 $\text{supp}(F)$ (support of the distribution F), 613
 $\text{supp}(\mu)$ (support of the measure μ), 605
 $\text{sing supp}(F)$ (singular support of the distribution F), 707
 $\text{Char}(L)$ (characteristic set of the differential operator L), 713
 $WF(G)$ (wave front set of the distribution G), 712
- $\mathcal{F}g$ (Fourier transform of the function g), 537
 FG (Fourier transform of the distribu-
- tion G), 620
 $\mathcal{F}_M g$ (Fourier–Minkowski transform of the function g), 774
 $\mathcal{L}g$ (Laplace transform of g), 94
 $f * g$ (convolution of two functions), 95, 536
 $F * G$ (convolution of two distributions), 619
 $f \otimes g$ (tensor product of two functions), 619
 $F \otimes G$ (tensor product of two distributions), 619
- $C[a, b]$ (space of continuous functions), 368
 $C^1[a, b]$ (space of continuously differentiable functions), 552
 $C^\infty(\Omega)$ (space of smooth functions), 545
 $C^\infty(\bar{\Omega})$, 545
 $C_0^\infty(\Omega) \equiv \mathcal{D}(\Omega)$, 545
 $C^\alpha(\Omega), C^{k, \alpha}(\Omega)$ (Hölder spaces), 556
 $C^{0,1}(\Omega), C^{k,1}(\Omega)$ (Lipschitz spaces), 556
- $L_2(\Omega)$ (Lebesgue space), 533
 $L_2(-\pi, \pi)$, 535
 $L_{\text{loc}}(\mathbb{R}^N)$, 612
 l_2 (classical Hilbert space), 536
 $L_2(\mathcal{M})$, 443 (discrete Lebesgue space)
- $W_2^1(\Omega), W_2^1(\Omega)$ (Sobolev spaces), 559
 $W_2^k(\Omega)$, 559
 $W_2^{1/2}(\Omega)$ (fractional Sobolev space), 559
- $\mathcal{D}(\Omega) \equiv C_0^\infty(\Omega)$ (space of smooth test functions with compact support), 545
 $\mathcal{S}(\mathbb{R}^N)$ (space of rapidly decreasing test functions), 539
 $\mathcal{E}(\mathbb{R}^N) \equiv C^\infty(\mathbb{R}^N)$ (space of smooth test functions), 617
 $\mathcal{D}'(\mathbb{R}^N)$ (space of distributions), 611
 $\mathcal{S}'(\mathbb{R}^N)$ (space of tempered distributions), 618
 $\mathcal{E}'(\mathbb{R}^N)$ (space of distributions with compact support), 617
- $\gamma^0, \gamma^1, \gamma^2, \gamma^3$ (Dirac–Pauli matrices), 791
 $\sigma^0, \sigma^1, \sigma^2, \sigma^3$ (Pauli matrices), 791
 $\bar{\psi} \equiv \psi^\dagger \gamma^0$, 793

- $\partial, \cancel{\partial}$ (Feynman's slash symbols), 794
 c (velocity of light in a vacuum), 965
 h (Planck's quantum of action),
 $\hbar \equiv h/2\pi$, 965
 k (Boltzmann constant), 965
 G (gravitational constant), 965
 ϵ_0 (electric field constant of a vacuum),
965
 μ_0 (magnetic field constant of a
vacuum), 965
 e (electric charge of the proton), 965
 $-e$ (electron charge)
 m_e (electron mass), 965
 α (fine structure constant), 965
 λ_C (Compton wave length), 144
 $\tilde{\lambda}_C \equiv \lambda_C/2\pi$ (reduced Compton wave-
length), 144

 m (meter), 950
 s (second), 950
 J (Joule), 950
 C (Coulomb), 950
 K (Kelvin), 950
 $\mathbf{m}, \mathbf{s}, \mathbf{J}, \mathbf{C}, \mathbf{K}$ (Planck units), 953
 eV (electron volt), 953
 MeV (mega electron volt), 953
 GeV (giga electron volt), 953

 $\hat{\rho}$ (statistical operator), 760
 ϱ (density operator), 760
 $\beta \equiv 1/kT$, 760

 $P(t, s)$ (propagator), 385
 $P^+(t, s)$ (retarded propagator), 386
 $P^-(t, s)$ (advanced propagator), 386
 $S[\varphi]$ (action of the field φ), 754
 $S(s, t)$ (S -matrix on the time interval
 $[t, s]$), 392

 $S(T)$ (S -matrix on $[-\frac{T}{2}, \frac{T}{2}]$), 824
 $|0\rangle, \Phi_0$ (ground state of a free quantum
field)
 $|0_{\text{int}}\rangle, \Phi_{\text{int}}$ (ground state of a quantum
field under interactions)
 T (chronological operator), 390, 746
 $: AB :$ (normal product), 824

 $Z(J), C_n, C_n, G_n$ (discrete model of a
quantum field), 446
 $Z(J, \varphi), Z_{\text{free}}(J, \varphi)$, 450
 S_n , 452
 φ_{mean} , 459
 Z_{red} , 461
 V_n , 462
 $C_{n, \text{free}}$, 468

 $Z(J)$ (full generating functional), 751
 $Z_{\text{free}}(J)$ (free generating functional), 751
 C_n (full n -point correlation function),
746
 $C_{n, \text{free}}$ (free n -point correlation func-
tion), 745
 G_n (full n -point Green's function), 746
 $G_{n, \text{free}}$ (free n -point Green's function),
745

 $\frac{1}{p^2 - m_0^2 + 0_+ i}$ (special distribution), 782
 \mathcal{G}_{F, m_0} (Feynman propagator for mesons),
777
 \mathbf{G}_{F, m_0} (Feynman propagator distribu-
tion for mesons), 780
 $\mathbf{G}_F \equiv \mathbf{G}_{F, m_0=0}$ (Feynman propagator
distribution for the wave equation)
 $D_F^{\alpha\beta} \equiv -\eta^{\alpha\beta} \mathbf{G}_F$ (Feynman propagator
distribution for photons), 802
 $\mathbf{S}_F \equiv (i\gamma^\alpha \partial_\alpha + m_e) \mathbf{G}_{F, m_e}$ (Feynman
propagator distribution for elec-
trons), 802

Index

- Abdera, 102
- Abel, 222, 288, 691
 - prize in mathematics, 75
- Abelian
 - function, 551
 - group, 343
 - integral, 221, 551
 - regularization, 691
 - theorem, 288
- Abrikosov, 70
- absolute time, 25
- action, 22, 30, 31, 110, 404, 409, 411, 447, 463, 493, 692, 754, 768, 776, 795, 806, 818
- actual information, 944
- addition theorem, 212
- additive group, 344
- adiabatic
 - limit, 623, 687
 - regularization, 691
- adjoint
 - matrix, 342, 359
 - operator, 358
- Adler–Bell–Jackiw anomaly, 207
- advanced
 - fundamental solution, 715
 - propagator, 386, 585
- age of the universe, 82, 115
- Aharonov, 73
- Ahlfors, 71, 74
- AKSZ (Aleksandrov, Kontsevich, Schwarz, Zaboronsky), 906
 - master equation, 906
- Alferov, 71
- Alfvén, 70
- algebra, 334
- algebraic
 - Feynman integral, 636
 - software systems, 946
 - integral, 221
 - quantum field theory, 868, 921
 - renormalization, 860
- almost
 - all, 533
 - everywhere, 532
- alpha rays, 131
- amplitude, 84
- analytic
 - continuation, 220, 226
 - operator function, 369
 - S -matrix theory, 221, 226
- analyticity, 211
- Anderson, 69, 133, 186
- angle-preserving map, 212
- angular
 - frequency, 26, 84
 - momentum, 147
 - quantum number, 182
- anharmonic oscillator, 63
 - renormalization, 628
- annihilation operator, 51, 55, 820
- anomalous magnetic moment of the electron, 4
- anomaly, 906, 934
- anti-quark, 135
- anticolor charge, 158
- antidistribution, 682
- antiduality map, 681
- antifield, 906
- antighost, 879, 885, 892, 903
- antilinear, 682
- antineutron, 133
- antiparticle, 132, 133, 157
- antisymmetric, 335
- Apéry, 280
- arc length, 250
- Archimedes, 529
- Archimedean ordering, 399
- arcwise connected, 241
- Ariadne’s thread, VIII
 - in quantum field theory, 328

- in scattering theory (see also Vol. II), 328
- Arnold, 75, 499, 653
- Artin, 60, 67
- Ashtekar program, 744
- asymptotic
 - expansion, 308, 864
 - freedom, 203
 - freedom of quarks, 137
- asymptotically free model in quantum field theory, 871
- Atiyah, 3, 71, 75, 259, 650, 925
- Atiyah–Singer index theorem, 259, 894, 928
- atom, 99
- atomic
 - model, 152
 - number, 117, 152
- axiomatic quantum field theory, 868, 922

- background radiation, 114
- backward light cone, 716
- Bacon, 971
- Baker, 71
- Baker–Campbell–Hausdorff formula, 455, 497, 510
- Balmer, 122
 - series, 122
- Banach
 - fixed-point theorem, 368
 - space, 368
- Bardeen, 70, 577
- bare parameters, 770
- barn, 119, 130
- baryon, 135, 158
 - number, 156, 159
- basic laws in physics, 952
- basis, 332
- Basov, 69, 128
- Batalin–Vilkovisky quantization, 905, 933
- Bednorz, 70
- Belevanin, 939
- Bellman, 724
- Bénard cell, 184
- Bequerel, 69, 130
- Bernays, 67
- Bernoulli
 - Jakob, 20, 107, 280, 311, 549
 - Johann, 549
 - number, 108, 280, 311
 - polynomial, 314, 320
- Berry, 73
- Besov space, 562
- Bessel function, 716
- beta function
 - and renormalization group, 200
 - of Euler, 292
- Bethe, 69, 131, 741
 - amplitude, 60
- Bethe–Salpeter equation, 60
- Betti, 253
 - number, 253, 898
- Bianchi identity, 811
- bicharacteristic curves, 715
 - and light rays, 724
- bifurcation, 184, 505
 - and renormalization, 633
 - equation, 508, 634
 - of a flow, 184
 - theorem, 505
 - theory, 504
- biholomorphic, 213, 553
- bijjective, 948
- bilinear functional, 335
- Binnet, 289
- Biot, 254
- Birch and Swinnerton–Dyer conjecture, 79
- Bjorken scaling, 203
- black
 - body, 104
 - radiation, 104
 - hole, 143, 145
- Blumenthal, 67, 543
- Bochner theorem, 226
- Böhm, 101
- Bogoliubov, 577, 651, 855, 856
 - formula, 859
- Bohr, 61, 70, 122
 - model of the hydrogen atom, 123
- Boltzmann, 100, 284
 - constant k , 142, 281, 760, 952
 - statistics, 108
- Bólyai, 21
- Bolzano, 230
 - existence principle, 231
- Bombielli, 217
- Bombieri, 72
- Borchers, 72, 937
- Borel, 530
- Born, 29, 33, 37, 48, 63, 64, 67, 70
 - approximation, 41
- Bose, 285
- Bose–Einstein

- condensation, 149, 687, 791
- statistics, 149, 285
- boson, 147, 517
- Bott, 75
- bottomness, 156
- bound state, 527
 - of a quantum field, 60
- boundary operator, 894
- bounded
 - operator, 368
 - set, 368
- Bourgain, 72
- BPHZ (Bogoliubov, Parasiuk, Hepp, Zimmermann), 856
 - renormalization, 855
- bra symbol, 361
- Bragg, 69
- braking radiation (bremsstrahlung), 858
- brane, 230
- Brattain, 70
- Breit–Wigner lifetime, 144
- Broglie, 65
- Brout, 74
- Brouwer, 232
- Browder, 558
- Brown, 513
- Brownian motion, 397, 657, 663
- BRST (Becchi, Rouet, Stora, Tyutin), 892
 - quantization, 399
 - symmetry, 892
 - transformation, 904, 905
- Brunetti, 623

- Calabi–Yau manifold, 936
- Calderon, 74
- Callan–Symanzik equation, 505
- Cambridge school, 547
- canonical
 - commutation relation, 54
 - equation, 47
 - transformation, 394
- Carathéodory, 68, 724
- Carleman, 22
- Carleson, 74, 75
- Cartan
 - Élie, 187, 724, 893, 894, 896
 - Henri, 74, 400
- Cartier, 856
- Casimir, 301
 - effect, 301
 - force, 301
- Catalan, 311
 - constant, 316
- category theory, 14
- Cauchy, 215, 359, 527, 653
 - characteristic system, 724
 - integral formula, 215
 - problem in general relativity, 917
 - residue method, 381, 735
 - residue theorem, 216
 - sequence, 339
- causal
 - convolution, 95
 - correlation function, 355, 426
- causality, 379
 - and analyticity, 93, 697
- Cavalieri, 577
- Cavendish laboratory, 100, 102
- cavity radiation, 105
- Cayley, 267, 365
- cell decomposition, 244
- central limit theorem, 431, 696
- CERN (European Organization for Nuclear Research at Geneva, Switzerland), 134, 138
- Chadwick, 69, 100, 102, 130
- Chamberlain, 70, 133
- Chandrasekhar, 69
- chaotic motion of asteroids, 290
- character, 938
- characteristic
 - curves, 724
 - equation, 367, 721
 - set of a differential operator, 713
 - surfaces and wave fronts, 721
 - system, 413
- charge conjugation, 174
- charm, 156
- chart map, 237
- Chebyshev, 696
- chemical potential, 281
- Cherenkov, 69
- Chern, 74, 251
 - class, 251
 - number, 249
- Chern–Simons theory, 266, 813
- Chew, 578
- chiral matrix, 792
- chirality, 147
- chronological operator, 44, 384
- Chu, 70
- CKM (Cabibbo, Kobayashi, Maskawa) mixing matrix, 161
- Clay Mathematics Institute (CMI), 78

- closed, 238, 341
 - complex plane, 219
 - Jordan curve, 242
 - upper half-plane, 665
- COBE (Cosmic Background Explorer)
 - sky maps, 115
- cobordism theory (see Vol. IV), 236
- coboundary, 895
 - operator, 894
- cocycle, 123, 895
- coercive, 571
- Cohen, 71
- cohomology, 123, 399, 400, 549, 893
 - and physical states, 895
 - functor, 14, 275
 - group, 900
 - of geometric objects, 898
 - of Lie groups, 902
 - ring, 14
- color
 - charge, 162
 - of quarks, 162, 882
- combinatorics and renormalization, 931
- commutation relation, 48
- commutative group, 343
- compact
 - subset, 611
 - support, 545
 - topological space, 241
- compactification, 219
- compactness, lack of, 572
- complete
 - measure, 530
 - orthonormal system, 357, 534
- completeness, 339, 368
 - relation, 358, 534
- complex
 - curve, 237
 - energy, 379
 - number, 211
 - plane, 211
- complexity, 282
- component, 242
- Compton, 69, 114
 - scattering (see also Vol. II), 5, 749
 - wave length, 124, 136, 144, 957
 - reduced, 136, 144
- condensation of a gas, 685
- conditional probability, 40
- conformal, 213, 237
 - field theory, 172, 929, 939
 - group, 77, 139
 - quantum field theory, 939
- conformally equivalent, 554
- conjugate complex
 - matrix, 342
 - number, 948
- connected
 - correlation function, 470, 753
 - Feynman graph, 470
- connectedness, 241
- connection, 186, 885
- Connes, 72, 862, 929
- Conrey, 298
- conservation
 - laws for quantum numbers, 158
 - of energy, 31
- constitutive law, 699
- constrained variational problem, 490
- constraining force, 493
- continuous, 238
 - spectrum, 527
- continuum
 - φ^4 -model, 463, 775
 - limit, 852
- contractible, 241
- contraction, 827
- convergence, 339, 368
- convolution
 - causal, 96
 - of distributions, 619
 - of functions, 536, 539
- Cook, 133
 - problem, 79
- Cooper, 70, 577
 - pair, 577
- coordinates, 332, 356
 - in gauge theory, 886
- Cornell, 70, 687
- correlation
 - coefficient, 354, 761
 - function, 57, 446, 449, 485, 888
 - n -point, 745
 - free, 471, 745
 - full, 471, 745, 784
- cosmic strings, 140
- cosmology, 916
- costate, 351, 352, 599, 683
- cotangent bundle, 708, 709
- Coulomb
 - field, 700
 - force, 115, 700
- counterterm, 511, 636, 852, 853, 860
- coupling constant, 35, 409, 755, 884
- Courant, 67, 971
 - Institute, 67

- covariance principle, 744
- covariant derivative, 185, 794
- covering
 - group, 268
 - space, 238
- CP (Charge Conjugation, Parity) violation, 180
- CPT (Charge Conjugation, Parity, Time Reversal) symmetry principle, 173
- creation operator, 51, 55, 820
- Crick, 71
- critical
 - action, 806
 - point, 410, 807
 - regular, 252
- Cronin, 70, 180
- cross section, 116, 119, 788, 841, 960
 - differential, 841
 - total, 841
- crossing
 - point, 265
 - symmetry, 179
- cumulant, 752, 753
- Curie
 - Marie, 69, 131
 - Pierre, 131
- Curl, 70, 247
- curl of a vector field, 173
- current density vector, 34
- curvature, 185, 885
 - in modern physics, 185
- cycle, 894
- cyclic vector, 871
- cyclotomic field, 936

- Dalton, 100
- damped
 - oscillation, 379
 - wave, 91
- Davies, 131
- Davis, 74
- de Broglie, 70, 114, 144, 693
- de Gennes, 70, 73
- de Giorgi, 74, 191
- de la Vallée-Poussin, 295
- de Rham cohomology, 399, 899
- Debye, 69
- Dedekind, 311
 - eta function, 286
- deformation
 - invariance, 214
 - quantization, 931
- retract, 241
- degenerate quantum state, 181
- Dehmelt, 70
- Delbrück, 71
- Deligne, 72, 75
- Democritus, 100, 102
- Denner, 101
- dense set, 545, 679
- density
 - functional method, 155
 - matrix, 283
 - of a functional, 405
 - operator, 760
- DESY (Deutsches Elektronensynchrotron), 135
- determinant, 335, 337
 - trick for Grassmann variables, 520, 891
- Diesenhofer, 71
- Dieudonné, 21, 546, 579
- diffeomorphism, 236, 237
- differential
 - cross section, 116, 119, 841
 - form, 518, 549, 896
 - pull-back, 256
 - geometry and modern physics, 186, 251, 885
 - topology, 236, 898
- diffraction of light, 726
- diffusion, 657
- dilute gas, 790
- dimension, 332
- dimensional
 - analysis, 960
 - regularization, 638, 853, 855
- dimensionless physical quantities, 962
- Diophantus, 18
- dipole moment, 700
- Dirac, 29, 60, 62, 70, 130, 186, 285, 327, 357, 361, 527, 579, 702, 850
 - calculus, 90, 261, 284, 361, 534, 599, 677
 - completeness relation, 358, 534
 - delta distribution, 612
 - delta function, 98, 592, 677
 - see also discrete and truncated Dirac delta function, 817
 - equation, 152, 812, 956
 - interaction picture, 43, 396
 - magic formula, 803
 - measure, 531
 - notation, 682
 - substitution trick, 362, 601

- Dirac–Fermi statistics, 285
- Dirac–Pauli matrices, 792, 882
- Dirichlet, 60, 259, 293, 299, 311, 544, 669, 688, 718
 - L -function, 298
 - function, 718
 - integral, 669, 690, 717, 736
 - principle, 542, 547, 558
 - problem, 542, 544, 568
 - series, 263, 311
- discrete
 - φ^4 -model, 466
 - Dirac delta function, 443, 582, 672, 677, 817
 - Fourier transform, 534, 672
 - functional derivative, 444
 - integral, 443
 - Laplace transform, 287
 - model in quantum field theory, 462
 - symmetries, 796
- discretization, 465
 - lattice approximation in quantum field theory, 817
- disorder, 282
- dispersion, 699
 - relation, 26, 84, 93, 704
- Disquisitiones arithmeticae, 298, 974
- distribution, 611
 - tempered, 617
- divergence of a vector field, 173
- Doetsch, 91
- domain of holomorphy, 226
- Donaldson, 72
- Douglas, 71
- Drinfeld, 72
- dual
 - basis, 518
 - matrix, 342
 - operator, 359, 364
 - space, 352, 369
- duality, 352
 - between light rays and wave fronts, 723
 - between strong and weak interaction, 705
 - in physics, 692
 - map, 681
- Duhamel, 387, 610
 - principle, 387, 610
- Duistermaat, 705
- Dynkin formula, 511
- Dyson, 1, 2, 5, 27–29, 73, 578, 741, 863
 - magic S -matrix formula, 392, 824
 - no-go argument, 862
 - series, 44, 390, 825
- Dyson–Schwinger equation, 455, 789
- early universe, 145
- east coast convention, 950
- effective
 - coupling constant, 770
 - electron charge, 196, 770
 - electron mass, 770
 - fine structure constant, 198
 - quantities in physics, 849
 - quantum action, 462, 486, 753
- Ehresmann, 251
- eigendistribution, 677
- eigenstate, 353
- eigenvalue, 353
- eikonal, 724
 - equation, 723
- Eilenberg, 74
- Einstein, 22, 24, 26, 29, 61, 67, 102, 113, 143, 172, 251, 327, 397
 - light particle hypothesis, 113
 - summation convention, 886, 949
 - theory of
 - general relativity, 113
 - special relativity, 112
- Eisenstein, 974
- electric
 - charge, 156, 158
 - energy, 544
 - field, 698
 - field constant, 721, 849
 - of a vacuum ϵ_0 , 698, 952
 - intensity, 698, 701
 - potential, 700
 - susceptibility, 699
- electromagnetic
 - field, 102
 - force, 130
 - wave, 25, 87, 720
- electron
 - lepton number, 156
 - spin, 267
 - volt, 959
- electroweak force, 130, 135
- elliptic
 - curve, 19
 - differential operator, 713
 - function, 19, 244
 - integral, 244
- encyclopedias, 942
- energetic

- Fourier transform, 606
- system of units, 958
- energy
 - density of the early universe, 145
 - of a relativistic particle, 25
 - operator, 374, 606, 678, 820
 - production on sun, 131
- energy-frequency relation, 143
- energy-mass relation, 143
- energy-time uncertainty, 144
- enthalpy, 761
- entire function, 212, 512
- entropy, 108, 168, 169, 282, 761
- Epstein, 311
 - zeta function, 304, 314
- Epstein–Glaser approach, 751, 856
- equation of motion, 793
- equator, 250
- Erdős, 74
- Erlanger program, 365
- Esaki, 70
- essential map, 272
- essentially self-adjoint operator, 679
- Euclid, 293, 569
- Euclidean
 - Fourier transform, 540
 - inner product, 338
 - trick, 661, 669
- Euler, 28, 30, 259, 279, 285, 294, 359, 513, 549, 974
 - beta function, 292, 640, 939
 - characteristic, 244, 245, 247–249, 259, 898, 899, 901
 - constant, 197, 639
 - exponential formula, 84
 - numbers, 313
 - partition function, 285
 - polyhedra formula, 246
- Euler–Lagrange equation, 31, 404, 410, 448, 549, 550, 754, 808, 809
- Euler–Maclaurin summation formula, 321
- evolution of the universe, 285
- exponential
 - function, 212, 347, 369, 370
 - matrix function, 347
- extended
 - quantum action functional, 489, 784, 805
 - response model, 481
- extension strategy in mathematics, 625
- Faddeev–Popov
 - determinant, 892
 - ghost formula, 891
- Faddeev–Popov–De Witt ghost approach, 888
- Faltings, 72
- Faraday, 102, 974
- fast oscillating integral, 717
- Faust, VII
- Fefferman, 72
- Feigenbaum, 73
- Fejér theorem, 628
- Feldman, 850
- femto, 951
- Fenn, 71
- Fermat, 18, 577, 723
 - last theorem, 18
- Fermi, 69, 131, 285
 - liquid, 920
- fermi (unit of length), 129
- Fermi–Dirac statistics, 149, 150
- fermion, 147, 517
- Fert, 71, 74
- Feynman, 4, 27–29, 70, 329, 354, 397, 513, 569, 741, 863
 - algebraic integral, 636, 946
 - dagger symbol, 794
 - gauge, 797
 - integration trick, 645
 - magic formula, 755
 - rules, 834, 837, 846
 - transition amplitude, 39
- Feynman diagram, 5, 41, 463, 470, 750, 757, 775, 831, 834, 837, 838, 845
 - equivalent, 838, 839
- Feynman path integral, 32, 418, 419, 654, 726
 - discrete, 418
 - in string theory, 939
 - main trouble, 663
- Feynman propagator, 32, 57, 98, 421, 468, 579, 778, 781, 802, 803
 - n -point, 745
 - distribution, 779
 - for electrons, 803
 - for mesons, 781
 - for photons, 802
 - formula, 419
 - kernel, 423, 590, 608, 662
- Feynman–Kac formula, 657
- fiber, 271, 709
 - bundle, 885
- Fibonacci, 288
 - number, 289

- fibration, 271
- Fields medal in mathematics, 71
- fine structure constant, 4, 121, 963
- finite
 - measure, 530
 - measure integral, 418
 - part of a divergent integral, 621
- Finnegans Wake, 100
- first law of
 - progress in theoretical physics, 81
 - thermodynamics, 168
- Fischer, 533
- Fischer–Riesz theorem, 533
- Fisher, 73
- Fitch, 70, 180
- five ages of the universe, 82
- flow, 201
- fluctuation, 34
 - of energy, 38
- Fock, 60
 - space, 51, 56
 - state, 51
- force
 - advanced, 584
 - retarded, 584
- form factor, 748
- formally self-adjoint, 525, 679
- forward light cone, 716
- four-manifold, 927
- Fourier, 28, 88, 259, 285, 374, 527, 535, 974
 - coefficient, 356, 357
 - integral operator, 731
 - method, 259
 - quantization, 49, 55, 819
 - series, 357, 535
- Fourier transform, 88, 89, 537, 538
 - and Dirac calculus, 601
 - discrete, 466, 672
 - energetic, 606
 - generalized, 680, 681
 - terminology of the, 540
- Fourier–Laplace transform, 542, 663, 703
- Fourier–Minkowski transform, 464, 542, 774
- Fowler, 69
- Fréchet, 397
- fractional Sobolev space, 559
- Franck, 67, 69
- Fraunhofer, 726
- Fredenhagen, 623
- Fredholm, 29, 543
- free
 - correlation function, 745
 - energy, 761
 - enthalpy, 761
 - Green’s function, 745, 822
 - quantum field, 745, 819
- Freedman, 72
- frequency, 83
- Fresnel, 669, 718, 726
 - integral, 669, 718, 736
- Frey curve, 18
- Friedman
 - Herbert, 73
 - Jerome, 70
- Friedrichs, 67, 562
 - extension, 562
- Fritzsche, 81
- fugacity, 282
- Fukui, 70
- full
 - correlation function, 471, 745, 784
 - generating functional, 751
 - Green’s function, 746, 769, 847
 - quantum field, 745, 750, 859
- fullerene, 247
- functional, 351
 - calculus, 397
 - derivative, 58, 397, 402, 594, 597, 752
 - discrete, 444
 - local, 594
 - partial, 403
 - integral, 32, 57, 418, 654, 752
 - discrete, 418
 - generating, 789
 - global quantum action principle, 789
 - mnemonic beauty, 789
 - see also Feynman path integral, 789
- functions of observables, 359
- functor, 14
- fundamental
 - constants in nature, 952, 964
 - interactions in nature, 129
 - particle, 133
 - solution, 580, 648
 - theorem of
 - algebra, 217
 - calculus, 213, 547
- Gabor, 69
- Galilei transformation, 112
- Galois, 12, 936, 974
 - functor, 14

- group (motivic), 862
- gamma
 - convergence, 191
 - function, 292, 639
- Gamow, 131
- Gårding, 150, 175, 921
- Gårding–Wightman axioms, 868
- Gâteaux, 397
- gauge
 - boson, 135, 883
 - boson propagator, 888
 - condition, 796
 - field tensor
 - of gauge bosons, 883
 - rescaled, 884, 887
 - field theory, 186, 187, 251, 879
 - basic ideas, 882
 - invariance principle, 797, 881
 - lattice theory, 135
 - Lie algebra, 883
 - Lie group, 883
 - potential, 796, 883
 - rescaled, 884
 - transformation, 796, 884
 - of the Schrödinger–Maxwell equation, 176
- Gauss, 10, 20, 21, 30, 60, 217, 254, 293, 294, 298, 542, 544, 546, 548, 936, 974
 - fundamental theorem of algebra, 217
 - integral, 430, 521
 - infinite-dimensional, 660
 - main formula, 431
 - integral theorem, 548
 - method of least squares, 354, 534
 - principle of critical constraint, 493
 - prize, 73
 - probability distribution, 431, 696
 - system of units, 957
- Gauss–Bonnet theorem, 248
- Gauss–Bonnet–Chern theorem, 249
- Gauss–Grassmann integral, 521
- Gelfand, 74, 527
 - triplet, 580, 677
- Gelfand–Kostyuchenko spectral theorem, 681
- Gell-Mann, 70, 81, 100, 137, 156, 182, 691, 767
- Gell-Mann–Low (GL) reduction formula, 429, 847
- general
 - linear group, 343
 - relativity, 251
- generalized
 - function, 611
 - state, 599, 683
- generating
 - function, 58, 280, 285, 287
 - functional, 751, 785, 787, 805
 - integral, 58, 789, 806
- generations of elementary particles, 132
- genus, 243, 247
- geometric optics, 723
- geometrization of physics, 327
- Geyer, XI
- ghost, 879, 885, 892, 895, 903
 - field, 892
- Giacconi, 73
- Giaever, 70
- Giaquinta, 571
- Gibbs, 284
 - potential, 282
- giga, 951
 - electronvolt (GeV), 959
- Ginzburg, 70, 73
- Ginzburg–Landau equation, 813
- GL (Gell-Mann–Low) reduction formula, 429, 847
- Glashow, 3, 70, 81, 137
- Glauber, 70
- Glimm, 189, 872, 922
- global
 - properties of the universe, 229
 - quantum action principle, 748
- gluon, 132, 883
 - field tensor, 882
 - potential, 882
- Goeppert-Mayer, 70
- Goethe, VII, VIII, 20, 60, 974
- Göttingen, 60
 - tragedy, 67
- golden ratio, 290
- Goldhaber, 73
- Goudsmit, 150
- Gowers, 72
- Grünberg, 71, 74
- gradient of a vector field, 173
- Grand Unified Theories (GUT), 207
- Grassmann, 517
 - calculus, 517, 519
 - manifold, 276
 - product, 517
 - variable, 519, 805, 892
- gravitation, 102
- gravitational
 - constant G , 952
 - law, 961

- lens, 711
- wave, 138
- gravitino, 140
- graviton, 135, 138, 139
- Green, 29, 544, 546, 548, 577, 583
 - integral formula, 548
 - operator, 376, 500
 - regularized, 376
- Green’s function, 32, 548, 579, 585, 648
 - n -point, 745, 769, 848
 - 2-point, 57, 426
 - 4-point, 57
 - advanced, 585
 - free, 745, 822
 - full, 746, 769, 847
 - history of, 577
 - in quantum field theory, 57
 - prototype, 583
 - renormalized, 636
 - retarded, 98, 585
- Griess, 936
- Griffith, 75
- Gromov, 74, 75
- Gross, 70, 166, 203
- Grothendieck, 71
- ground state (vacuum), 55, 426, 819
 - energy of
 - the electromagnetic quantum field, 301
 - the harmonic oscillator, 142
- group, 342
 - epimorphism, 342
 - equation, 416
 - isomorphism, 342
 - morphism, 342
 - simple, 936
 - velocity, 84
- Gudermann, 551
- GUT (Grand Unified Theories), 207

- Hänsch, 70
- Haag, 868, 922
 - theorem on quantum fields, 751
- Haag–Kastler axioms, 869
- Haag–Ruelle theory for the S -matrix, 872
- Haar, 667
- Haar measure, 667
- Hackbusch, XI, 570
- Hadamard, 295, 556, 621
 - regularization of integrals, 621
 - state, 744
- hadron, 135

- Hahn, 69, 73
- Hall, 70
- Halmos, 741
- Halperin, 74
- Hamilton, 29, 30, 267, 724
- Hamilton–Jacobi differential equation, 726
- Hamiltonian, 374
 - approach to quantum field theory, 47
- Hammurabi, 569
- Har Gobind Khorana, 71
- Hardy, 285, 288, 569
- Hardy–Littlewood theorem, 689
- Hardy–Ramanujan theorem, 286
- harmonic
 - analysis, 533
 - map, 554, 813
 - oscillator, 23, 45, 96, 409
 - wave, 83
- Hausdorff, 237
- Hawking, 73
 - temperature of a black hole, 145
- heat conduction equation, 589
- heat kernel, 259, 589, 590, 609, 928
 - global, 263
 - method, 263
- Heaviside, 579
 - function, 92, 303, 579, 663
 - adiabatic regularization, 690
 - system of units, 957
- Hecke, 285, 974
 - algebra, 287
 - operator, 287
- Hegel, 974
- Heisenberg, 23, 29, 34, 46, 47, 60, 62, 63, 70, 111, 122, 130, 142, 155, 523
 - algebra, 936
 - particle picture, 42
 - philosophical principle, 47
 - picture, 395
 - S -matrix, 38
 - uncertainty inequality, 525
- Heisenberg–Born–Jordan commutation relation, 42, 48, 64
- helicity, 147
- Helmholtz, 24, 254, 726
 - equation, 723, 727
 - potential, 727
- Hepp, 855, 856
- Herring, 73
- Hershey, 71
- Hertz
 - dipole, 722

- Gustav, 69
- Heinrich, 25
- Hess, 69, 131
- Hessian, 720
- Hewish, 69
- Higgs, 74
 - mechanism, 183
 - particle, 140, 183, 888
- high-energy limit, 684, 852
- highlights
 - in the sciences, 69
 - of physics in the 20th century, 943
- Hilbert, 17, 21, 29, 67, 68, 337, 370, 527, 542, 544, 551, 724
 - action, 813
 - Paris lecture, 17
 - problems, 17
 - space morphism, 340
 - spectral
 - family, 38, 370
 - integral, 371
 - transform, 93, 666
- Hilbert space, 337, 339, 527
 - approach, 35
 - isomorphism, 340
 - rigged, 580
 - separable, 680
- Hildebrandt, XI, 544, 571
- Hilton, 68
- hints for further reading, 225, 226, 242, 300, 306, 533, 544, 549, 567, 568, 570, 580, 647, 653, 662, 702, 705, 791, 856, 864, 873, 877, 906, 909
- hints for quick reading, 30
- Hironaka, 71, 650, 653
 - theorem on the resolution of singularities, 650
- Hironaka–Atiyah–Bernstein–Gelfand (HABG) theorem, 653
- Hirsh, 73
- Hirzebruch, XI, 74, 925
- historical remarks, 21, 60, 69, 71, 99, 106, 129, 137, 150, 186, 202, 246, 284, 527, 741, 743, 768, 815, 850, 856, 859, 860, 862, 892, 939
- history of quantum mechanics, 60, 63
- Hodge
 - conjecture, 79
 - homology, 399
 - theory, 569, 926
- Hölder
 - continuous, 556
 - Ernst, 724
- Otto, 557
 - space, 556
- Hörmander, 71, 74, 649
- Hoffmann
 - Karl-Heinz, XI
 - Roald, 70
- Hofstadter, 70
- Holley, 71
- Holmgren, 543
- holomorphic, 211, 212, 221
 - extension, 220
 - function of several variables, 225
- homeomorphism, 238
- homological algebra, 399, 894
- homology, 399
 - functor, 14, 275
 - group, 14
- homotopic, 240
- homotopically
 - equivalent, 240
 - trivial map, 240
- homotopy
 - class, 273
 - functor, 275
 - group $\pi_k(X)$, 273
- Hopf, 270
 - $U(1)$ -bundle, 270
 - algebra, 861
 - fibration, 270
 - map, 270
- Hubble, 115
 - law, 145
- Huber, 71
- Hulse, 69, 138
- Hurd, 850
- Hurwitz, 24, 311
 - zeta function, 314
- Huygens, 723
 - duality, 723
 - principle, 621, 727
- hydrogen atom, 181, 527
 - see also Vol. III (functional analytic approach), 122
- Hylleraas, 570
- hypercharge, 156
- hyperelliptic integral, 551
- icosahedron, 247
- iff (if and only if), 94
- image, 947
- imaginary part, 211, 948
- implicit function theorem, 508
- index, 260

- of a stationary point, 247
- of an operator, 260
- picture, 772
- indistinguishability principle, 149
- induced topology, 238
- inertial system, 111
- infinitely large number, 399
- infinitesimal
 - rotation, 411, 412
 - transformation, 351, 416, 893
- infinitesimally small number, 399
- infinitesimals, 398
- information, 169, 282
- infrared
 - catastrophe, 230
 - limit, 852
- injective, 948
- inner
 - energy, 761
 - product, 338
- instanton, 927
- Institute for Advanced Study in Princeton, 67
- integral, 530
 - equation, 388
 - on Riemann surfaces, 222
- integration
 - by parts, 546
 - over orbit spaces, 888
 - tricks, 643
- interaction
 - four fundamental forces in nature, 129
 - picture, 43, 396, 751
 - Haag’s theorem, 751
 - see also gauge field theory, 884
- International Congress of Mathematicians (ICM), 71
- International Congress on Mathematical Physics (ICMP), 943
- interplay between mathematics and physics, 924
- intersection number, 232
- introductory literature on quantum field theory, 909
- invariant theory, 365
- inverse
 - Laplace transform, 374
 - map, 948
- inversion with respect to the unit sphere, 564
- irreducible vertex function, 753
- irreversible, 167, 181
- isolated pole, 512
- isometric operator, 340
- isomorphic Hilbert spaces, 340
- isomorphism, 332, 345
- isospin, 155
 - number, 156
- isotope, 152
- Itô, 73, 74
- iterative method, 368
- Ivanenko, 114, 130

- Jacobi, 19, 30, 259, 311, 551, 724
 - inverse problem, 551
- Jacobian, 275
- Jaffe, 20, 78, 872, 922
- Janke, XI
- Jensen, 70
- John, 215
- Joliot, 69
- Joliot-Curie, 69
- Jones, 72
 - polynomial, 266
- Joos, 101
- Jordan
 - Camille, 242
 - curve, 242
 - theorem, 242
 - Pascual, 29, 49, 64
- Jordan–Wigner bracket, 56
- Jorgenson, 259
- Josephson, 70
- Jost
 - Jürgen, XI, 191
 - Res, 175, 225, 974
- Joule, 23
- Joyce, 100

- Kähler
 - geometry, 926
- Kac–Moody algebra, 936
- Kadanoff, 73
- Kähler, 972
 - manifold, 72, 277, 936
- KAM (Kolmogorov, Arnold, Moser), 653
 - theory, 290, 499, 653
- Kammerlingh-Onnes, 70
- Kant, VII
- Kapusta, 759
- Kastler, 821, 868, 922
- Keller
 - Gottfried, XI
 - Joseph, 75

- Kelvin, 542, 547, 719
 - transformation, 564
- Kendall, 70
- Kepler, 1, 182, 347, 961
- kernel theorem, 683
- ket symbol, 361
- Ketterle, 70, 687
- kick force, 586
- Killing form, 887
- Kirby, 71
- Kirchhoff, 24, 105, 726
- Kirchhoff–Green representation
 - formula, 727
- Klein
 - Felix, VII, 19, 60, 243, 247, 365, 552
 - Oscar, 118
- Klein–Gordon equation, 463, 714, 755, 808, 810, 816, 867
- Klein–Nishina formula, 118, 121
- Kleinberg, 73
- Kleppner, 74
- Klima, 741
- Kline, 246, 547
- KMS (Kubo, Martin, Schwinger), 744
 - state, 744
- knot
 - classification, 265
 - theory, 267
- Kobayashi, 70
- Kodaira, 71, 74
- Koebe, 19
- Kohn, 70, 155, 571
- Kolmogorov, 74, 499, 530, 653
 - law in turbulence, 961
- Kontsevich, 72, 254, 906
- Koshiha, 74
- Kostyuchenko, 527
- Kramers, 61
- Kramers–Kronig dispersion relation, 704
- Kreimer, 861
 - Hopf algebra, 743, 861
- Krein, 74
- Kroemer, 71
- Kronecker, 257
 - integral, 257
 - symbol, 56, 356, 949
 - generalized, 594
- Kroto, 70, 247
- Kummer, 974
- Kusch, 70
- Lafourge, 72
- Lagrange, 28, 385, 548, 549, 653
- Lagrangian, 30
 - and the principle of critical action, 30
 - approach to physics, 47
 - density, 754, 776, 795, 799, 807
 - multiplier, 490, 799, 879
- Lamb, 69
- Landau
 - Edmund, 67, 948
 - Lev, 60, 70, 285
 - symbol, 948
- Landau–Ginzburg potential, 182
- Lang, 259
- Langlands, 75
 - program, 926
- Laplace, 91, 254, 259, 285, 374, 544, 557
 - transform, 91, 94, 288, 292
 - discrete, 285, 287, 291
- Laplace transform, 379
 - discrete, 287
- Laplacian, 260, 544, 557, 561, 956
- laser, 128
- lattice, 671
 - approximation, 817
 - gauge theory, 206, 578
- Laue, 69
- Laughlin, 70
- Lawrence, 69
- laws of progress in theoretical physics, 81
- Lax, 75
- Le Verrier, 113
- least-squares method, 354
- Lebesgue, 528, 529
 - integral, 532
 - measure, 532
- Lebowitz, 192
- Lederman, 70, 73
- Lee, 3, 70
- left-handed neutrino, 147
- left-invariant vector field, 903
- Legendre, 294, 974
 - transformation, 461, 486
- Leggett, 70, 74
- Lehmann, 441, 767
- Leibniz, 1, 103, 254, 336, 397, 398, 547, 577, 578
- Lenard, 1, 26, 69, 114
- lepton, 132
 - number, 156, 158
- Leray, 74, 232, 400
- Leucippus, 100

- Lewis, 26, 114
- Lewy, 74
- LHC (Large Hadron Collider), 141
- l'Huilier, 247
- Libchaber, 73
- Lichtenstein, 67
- Lie, 24, 29, 60, 199, 347, 411, 724
 - algebra, 24, 344
 - $so(3)$, $su(2)$, 269, 345
 - $u(X)$, $su(X)$, $gl(X)$, $sl(X)$, 345
 - basis of, 885
 - isomorphism, 345
 - morphism, 344
 - structure constants of, 885, 886
 - bracket, 48, 56, 344
 - product, 269, 344
 - functor, 14
 - group, 349
 - $SO(3)$, $U(n)$, $SU(n)$, $Spin(3)$, 269, 344
 - $U(X)$, $SU(X)$, $GL(X)$, $SL(X)$, 344
 - basic ideas, 201
 - isomorphism, 350
 - morphism, 350
 - one-parameter, 201, 416
 - linearization principle, 350
 - subalgebra, 345
 - theory for differential equations, 201
- Lieb, 1
- lifetime, 91, 380, 382
 - of a black hole, 145
 - of elementary particles, 136
- light
 - cone, 715
 - particle (photon), 26
 - ray, 725
 - wave, 87
- LIGO (Laser Interferometer Gravitational-Wave Observatory), 139
- limits in physics, 684
- linear
 - functional, 334, 351
 - hull, 331
 - isomorphism, 332
 - material, 699
 - morphism, 332
 - operator, 332
 - response and causality, 703
 - response theory, 703
 - space, 331
 - subspace, 331
- link, 265
- linking number, 254
 - and magnetic fields, 253
- Lions, 72
- Liouville, 527
 - theorem, 220
- Lippmann, 29
- Lippmann–Schwinger integral equation, 40, 726
- Lipschitz, 556
 - continuous, 556
 - space, 556
- Lipschitz-continuous boundary, 548
- LISA (Laser Interferometer Space Antenna), 139
- Listing, 254
- Littlewood, 288
- local
 - degree of homogeneity, 624
 - functional derivative, 594, 752
 - properties of the universe, 229
 - symmetry, 176
- local-global principle, 220
- locality, 871
- locally
 - holomorphic, 212, 221
 - holomorphic at ∞ , 219
- logarithmic
 - function, 222
 - matrix function, 348
- Lojasewicz, 649
- loop, 242
 - cosmology, 917
 - gravity, 916
- Lorentz, 69
 - boost, 869
 - condition, 797
 - transformation, 112
- Lovasz, 75
- Low, 691, 767
- low-energy limit, 852
- lower
 - half-plane, 665
 - semicontinuous, 571
- LSZ (Lehmann, Symanzik, Zimmermann), 441, 767
 - axiom, 788
 - reduction formula, 446, 451, 485, 748, 767, 769, 786
- Luria, 71
- Lyapunov–Schmidt method, 634
- Maclaurin, 311
- macrocosmos, 229

- magic
 - Dyson S -matrix formula, 824
 - Dyson series for the propagator, 390
 - Faddeev–Popov formula, 890
 - Feynman formula, 419, 764
 - formulas for the Green’s operator, 374
 - Gell-Mann–Low formula, 429, 847
 - LSZ reduction formula, 451, 485, 748, 767, 769, 786
 - quantum action formula, 450
 - quantum action reduction formula, 748, 767, 769, 784, 805
 - survey on magic formulas, 328, 767
 - trace formula, 758
 - Wick formula, 427
 - zeta function formula, 436
- magnetic
 - field, 698
 - field constant, 721, 849
 - of a vacuum μ_0 , 698, 952
 - intensity, 698, 701
 - moment, 152
 - anomalous, 150
 - of the electron, 4
 - of the myon, 6
 - monopole, 701, 702
 - quantum number, 182
 - susceptibility, 699
- magnetic monopole, 927
- magnetism, 152
- magnetization, 698, 701, 968
- Maiman, 73
- majorant criterion, 496, 531
- Mandelbrot, 73
- manifold, 236, 237
 - complex, 237
 - oriented, 237
 - with boundary, 548
- Manin, 929, 1002
- Mann, 180
- mapping degree, 230
 - and electric fields, 255
- Marathe, 13, 254, 264
- Marczewski, 67
- Margulis, 72, 75
- Maslov, 705
 - index, 433
- mass
 - density, 594
 - hyperboloid, 465
 - of a relativistic particle, 25
 - shell, 465, 637, 715
- Masukawa, 70
- mathematical physics, 13
- matrix
 - algebra, 342
 - calculus, 336, 347, 348
 - elements, 357
 - of an operator, 336
 - group, 342
 - mechanics, 64, 65
 - rules, 342
- Maupertius, 30
- Maurin, 67, 569, 925
- maximum principle, 261
- Maxwell, 25, 102, 254, 284
 - equations, 173, 721, 811, 955
 - for material media, 698
- McMullen, 72
- mean
 - energy, 38
 - field approximation, 459, 753
 - fluctuation, 352, 761
 - inner energy, 761
 - lifetime, 379, 380
 - particle number, 761
 - value, 34, 352, 530, 761
- mean-square convergence, 533
- measurable function, 530
- measure, 530
 - integral, 418, 530, 605
 - zero, 533
- measurement of an observable, 356, 760
- mega, 951
 - electron volt (MeV), 960
- Mellin, 292, 311
 - transform, 263, 292, 307, 668
 - generalized, 307
 - normalized, 292
- Mendeleev, 152
- meridian, 250
- meromorphic, 215
- meson, 135, 158
 - model, 463, 775, 867
- messenger particle, 132, 133
- method of
 - least squares, 354, 534
 - orthogonal projection, 566, 569
 - quantum fluctuations, 658
 - second quantization, 52
 - stationary phase, 32, 432, 437, 717
- metric tensor, 250
- Meyer, 152
- Michel, 71
- Michelson, 25, 69

- micro, 951
- microcosmos, 229
- microlocal analysis, 705
- microstructure, 191
- Mikusiński calculus, 290
- millennium prize problems, 78
- milli, 951
- millibarn, 130
- Millikan, 26, 69, 114
- Mills, 183, 187, 251
- Milnor, 71, 74
- minimal surface, 813
- Minkowski, 24, 771, 974
 - metric, 770, 950
 - space, 771
 - symbol, 771, 949
- Minkowskian versus Euclidean model, 866
- mirror symmetry, 705, 926, 927, 936
- Mittag-Leffler theorem, 512
- models
 - exactly soluble, 918
 - general relativity, 920
 - matrix models, 920
 - quantum field theory, 914
 - random matrices, 920
 - solitons, 914
 - statistical physics, 914
- modular
 - curve, 19
 - form, 286, 323
 - function, 18
- moduli space, 15, 225, 554, 932, 935, 939
- modulus, 211, 948
- Möbius, 243
- Möbbaauer, 70
 - effect, 115
- moment
 - of a probability distribution, 58
 - problem, 753
 - trick, 434
- momentum, 35, 147
 - operator, 677
 - on the real line, 33
- monomorphism, 345
- monster group, 936
- monstrous moonshine module, 936
- Montesquieu, 732
- moon landing, 491
- Mori, 72
- morphism, 332, 345
- Morse, 253
 - index, 252
 - theorem, 252
 - theory, 252
- Moser, XI, 75, 499, 653
- motivic Galois group, 862
- Müller
 - Karl, 70
 - Stefan, XI, 191
- multi-grid method, 570
- multi-index, 538
- multilinear functional, 334, 335
- multiplicity, 506
- Mumford, 72, 75
- muon, 132
 - lepton number, 156
- Nambu, 70, 73, 939
- nano, 951
- Napier, 182, 347
- NASA, 82
- natural
 - number, 948
 - SI units, 953, 966
- Navier, 964
- Navier-Stokes equations, 79, 964
- Ne’eman, 100
- Néel, 70
- negative
 - energy, 380
 - real number, 948
- neighborhood, 238
 - open, 238
- net of local operator algebras, 744
- Neumann
 - Carl, 543
 - John von (see von Neumann), 21
- neutrino, 130–132
 - mass, 147
 - oscillations, 147
- neutron, 102
- Nevanlinna prize in computer sciences, 72
- Newman, 295
 - adiabatic theorem, 295, 688
- Newton, 28, 102, 103, 129, 397, 398, 547, 577, 961
 - equation of motion, 35
 - polygon, 652
 - potential, 557
- Nirenberg
 - Louis, 705
 - Marshall, 71
- Nishina, 118

- Nobel prize
 - in chemistry, 69
 - in physics, 69
- Noether, 22, 29, 60, 67, 894
 - theorem (see also Vol. II), 22, 31
- non-degenerate ground state (vacuum), 426
- non-positive, 948
- non-relativistic approximation, 693
- non-resonance, 376, 500
- non-standard analysis, 399
- noncommutative geometry, 141, 876
 - and the Standard Model in particle physics, 929
- nonnegative, 948
- norm, 339, 368
- normal product, 824, 826
 - principle, 826
- normalization volume, 671
- normalized state, 352
- normed space, 368
- notation, 947
- Novikov, 71, 75
- Nozieres, 73
- nucleon, 131
- null Lagrangian, 799, 809
- number theory and physics, 930

- observable, 38, 352
- observer, 356
- Occhialini, 73
- Okunkov, 72
- one-parameter Lie group, 201, 416
- one-to-one, 948
- Onsager, 71
- open, 341
 - neighborhood, 238
 - set, 238
 - upper half-plane, 665
- operator
 - algebra, 334
 - approach, 749, 815
 - calculus, 37
 - function, 359, 369
- Oppenheimer, 60
- orbifold, 938
- orbit, 271
 - space, 271
- oriented manifold, 237
- orthogonal matrix, 343
- orthogonality, 356, 566
 - relation, 671
- orthonormal
 - basis, 339
 - system, 357
 - complete, 357
- orthonormality condition
 - continuous, 695
 - discrete, 358
- oscillating integral, 669, 717
- Osterwalder, 868
- Osterwalder–Schrader axioms, 868
- Ostrogradski, 548
- Ostwald’s classic library, 218

- paired normal product, 827
- Paley–Wiener–Schwartz theorem, 664
- panorama of
 - literature, 909
 - mathematics, 15
- parallel
 - of latitude, 249
 - transport, 186, 251
- Parasiuk, 855, 856
- parity, 156, 161
 - transformation, 174
 - violation, 167
- Parseval
 - de Chénes, 358
 - equation, 358, 361, 534, 537, 538
- partial
 - derivative, 538
 - functional derivative, 403, 405, 752
- particle
 - density, 34
 - stream, 34
- partition
 - function, 15, 108, 281, 285, 759
 - functional, 759
- Pascal, 974
- path integral, 32, 57, 418, 654, 934
- Paul, 70
- Pauli, 64, 70, 130, 131
 - exclusion principle, 149, 150, 153, 163, 178
 - matrices, 792, 887
 - spin-statistics principle, 149, 150
- Pauli–Villars regularization, 640, 855
- Pauling, 70
- pendulum, 960
- Penrose, 73
- Penzias, 69, 115
- Perelman, 72
- period, 83
- periodic table of chemical elements, 152
- Perl, 70, 73

- Perrin, 69
- perturbation theory, 45, 182, 392, 749, 756
 - software systems, 946
- peta, 951
- Péter, 971
- phase
 - shift, 84
 - transition, 685
 - in the early universe, 687
 - velocity, 84
- Phillips, 70
- phonon, 914
- photoelectric effect, 25, 26
- photon, 26, 114, 132, 135
- physical
 - mathematics, 13, 75, 264
 - states, 895
- physics, basic laws in, 952
- Picard–Lefschetz theorem, 647
- pico, 951
- picture
 - Feynman path integral, 31
 - Feynman propagator kernel, 50
 - Heisenberg particle, 42, 48
 - Schrödinger wave, 35, 49
 - von Neumann operator, 37
- piecewise smooth boundary, 548
- Piguet, 860
- Planck, 22, 26, 70, 100, 103, 285, 741, 952
 - charge, 953
 - constant, 23, 110
 - energy, 953, 959
 - function and number theory, 107
 - length, 953
 - mass, 953
 - quantization rule, 23
 - quantum of action h , 23, 110, 952
 - radiation law, 100, 104
 - reduced quantum of action, $\hbar = h/2\pi$, 952
 - scale, 950
 - system of units, 950, 954
 - temperature, 953
 - time, 953
- plane wave, 721
- Plato, 246
- Platonic solids, 246
- Poincaré, 19, 21, 29, 229, 543, 549, 726, 894
 - conjecture, 79
 - group, 869
 - hairy ball theorem, 248
 - lemma, 399, 897
- Poincaré conjecture, 928
- Poincaré–Friedrichs inequality, 573
- Poincaré–Hopf index, 248
- Poincaré–Hopf theorem, 247
- Poincaré–Lindstedt series, 866
- Poincaré–Stokes integral theorem, 549
- Poisson, 28, 311, 544, 557
 - approach, 48
 - bracket, 47, 48
 - equation, 543, 544, 557, 701, 713
 - summation formula, 312
- polarization, 87, 698, 701, 968
- Polchinski equation, 515, 875
- pole, 215, 512
- Politzer, 70, 203
- Polyakov, 939
- Pontryagin, 724
- Pople, 70, 155
- position operator, 677
 - on the real line, 33
- positive real number, 948
- post-Newtonian approximation, 963
- potential, 35, 896
- Pound, 115
- Powell, 69
- power series expansion, 211
- power-counting theorem, 641
- pre-Hilbert space, 339
- pre-image, 947
- pressure, 761
- Prigogine, 71
- prime number, 293
 - theorem, 293, 294
- principal
 - argument, 211
 - axis theorem, 359
 - branch, 222
 - part of the square root, 85
 - symbol, 713
 - value, 90, 621, 704
- principle of
 - critical action, 30, 404, 411, 447, 463, 754, 776, 795, 806
 - summary, 806
 - under constraints, 492
 - critical constraint, 493
 - general relativity, 113
 - indistinguishable particles, 150, 163
 - least action, 409, 411
 - special relativity, 112
 - the right index picture, 772

- probability, 370
 - distribution function, 371
- Prochorov, 69, 128
- propagation of singularities, 712
- propagator, 385, 421, 579, 585
 - advanced, 585
 - equation, 386
 - retarded, 585
- proton rest energy, 959
- pseudo-differential operator, 731
- pseudo-holomorphic curves, 926
- pseudo-limit, 12
- pseudo-resolvent, 379, 632
- Puiseux expansion, 652
- punctured open neighborhood, 215
- Pythagoras, 17, 569
- Pythagorean theorem, 569

- QA (quantum action reduction formula), 446, 767, 769
- QA axiom, 786
- QCD (quantum chromodynamics), 135, 882
- QED (quantum electrodynamics; see also Vol. II), 791, 795, 813, 848, 881, 888
- quadrupole moment, 700
- quantization, 32
 - Batalin–Vilkovisky, 905, 933
 - in a nutshell, 26
 - of phase space, 840
 - of Poisson structures, 933
 - second, 52
- quantum
 - chemistry, 155
 - computer, 128
 - fluctuation, 32, 658, 794
 - gravity, 141, 916, 946
 - group, 928
 - information, 128
 - of action, 100, 110, 142
 - of action (reduced), 142
 - particle, 29
 - state, 271
 - statistics, 285
 - symmetry, 906
- quantum action
 - axiom, 785, 805
 - functional, 784, 805
 - extended, 489, 784, 805
 - principle
 - global, 449, 756, 789
 - local, 455, 789
 - prototype, 434
 - reduction formula, 446, 748, 767, 769, 785
- quantum chromodynamics, 578, 882
- quantum electrodynamics (see also QED), 791
- quantum field, 52
 - Bogoliubov’s formula, 859
 - classical, 860
 - creation and annihilation operators (see also Vol. II), 55
 - free, 745
 - full, 745, 750
 - generalized, 860
 - trouble with interacting quantum fields, 750
- quantum field theory
 - algebraic, 868
 - Haag–Kastler approach, 744, 868, 921
 - Hadamard states, 744
 - Kubo–Martin–Schwinger (KMS) states, 744
 - survey on quantum gravity, 912
 - Tomita–Takesaki theory for von Neumann algebras, 744
 - Ariadne’s thread, 328
 - as a low-energy approximation of string theory, 744
 - Ashtekar program, 744
 - at finite temperature, 759
 - axiomatic approach, 868
 - Epstein–Glaser, 751, 856
 - Gårding–Wightman, 868, 921
 - Glimm–Jaffe, 872, 922
 - Haag–Kastler, 921
 - Osterwalder–Schrader, 868
 - Segal, 921
 - Wightman, 868, 921
 - basic formulas, 741
 - (QA) and (LSZ), 767
 - Dyson’s S -matrix formula, 824
 - magic formulas (see also magic), 328, 767
 - basic strategies, 741, 815
 - Dyson’s operator approach, 815
 - Feynman’s functional integral approach, 57, 755, 806
 - Schwinger’s response approach, 767
- Batalin–Vilkovisky quantization, 905
- Becchi–Rouet–Stora–Tyutin (BRST) symmetry, 892
- conformal, 939

- constructive, 872
- Faddeev–Popov ghosts, 890
- Faddeev–Popov–De Witt ghost approach, 888
- fascination of, 4
- Haag theorem, 751
- in a nutshell, 27
- interplay between physics and mathematics, 924
- key formula
 - for the cross section of scattering processes, 841
 - for the transition probability, 839
- lattice approximation, 817
- method of
 - Fourier quantization (see also Vol. II), 55
 - Heisenberg–Pauli canonical quantization, 52, 762
 - Lehmann–Symanzik–Zimmermann (LSZ), 767
 - moments and correlation functions (Green’s functions), 744
 - quantum action (QA), 767
 - second quantization, 52
- model
 - asymptotically free, 871
 - continuum, 775
 - discrete, 462
 - exactly soluble, 918
 - trivial, 871
- panorama of literature, 909
- paradox of, 2
- perturbation theory and
 - Feynman diagrams (see also Vol. II), 749
 - Feynman rules (see also Vol. II), 845
- recent developments, 911, 916, 917, 923
- references
 - actual information, 944
 - introductory, 909
 - rigorous approach, 921
 - standard, 920
- renormalization (see also Vol. II), 850
- revolution of physics, 22
- rigorous
 - finite-dimensional approach, 327
 - perspectives, 864
 - rigorous approaches, 921
 - soluble models, 914
 - topological methods, 266, 927
- quantum number, 145, 156
 - of leptons, 157
 - of quarks, 157
- quark, 132
 - confinement, 136, 137, 206, 207
 - hypothesis, 182
- quark-gluon field, 883
- quasi-crystal, 290
- quaternion, 267
- Quillen, 72
- rabbit problem, 288
- Rademacher, 285, 557
 - theorem, 557
- radiation law, 104
- Radzikowski, 707
- Raman, 69
- Ramanujan, 285
- Ramsey, 70
- random matrices, 920
- randomness of quantum processes, 371
- ray, 276
- Rayleigh, 100, 547, 726
- Rayleigh–Jeans radiation law, 105
- Razborov, 73
- real part, 211, 948
- recent developments in quantum field theory, 911, 912, 916, 917, 923
- red shift, 145
- reduced
 - correlation function, 752
 - Planck’s quantum of action, $\hbar = h/2\pi$, 952
- reduction formulas, 748
- references (see also hints for further reading), 975
- refractive index, 721
- regular solution, 492
- regularity condition, 506
- regularization of integrals, 513
- regularized Green’s operator, 376, 502
- regularizing term, 29, 502, 511, 514, 621
- Reines, 70
- relativistic electron, 812
- Rellich theorem, 509
- Remmert, 215, 863
- renormalizability, 856
- renormalization, 5, 377, 499, 511, 621, 627, 757, 770, 786, 848, 850, 858, 864
 - algebraic, 860
 - and bifurcation, 633
 - and Hopf algebras, 862
 - and tempered distributions, 625, 859

- basic ideas, 198, 627, 628, 770, 852
- of the anharmonic oscillator, 628
- see also Vol. II, 770
- renormalization group, 199, 505, 851, 874
 - basic ideas, 198
 - differential equation, 200
 - equation, 199, 636
- renormalized
 - electron charge, 196
 - electron mass, 770
 - Green’s function, 636
 - integral, 627
- Repka, 115
- residue
 - method, 215, 643, 735
 - theorem, 216
- resolvent, 360, 367
 - set, 367
- resonance, 91, 376, 500, 628
 - condition, 376
- response
 - and causality, 703
 - approach, 748, 767
 - rigorous, 440
 - equation, 448
 - function, 96, 98, 448, 483, 488, 769
 - for electrons, 802
 - for gauge bosons, 887
 - for mesons, 777
 - for photons, 801
- rest mass, 25
- retarded
 - fundamental solution, 715
 - propagator, 374, 375, 380, 386, 390, 421, 585
- retract, 241
- reversible, 167
- Reynolds number, 964
- Ricci flow, 928
- Richter, 70
- Riemann, 10, 19, 60, 222, 229, 259, 293, 551
 - conjecture, 79
 - curvature tensor, 250
 - moduli space, 225
 - and string theory, 225
 - sphere, 219
 - surface, 19, 222, 237, 553, 932
 - zeta function, 279, 293, 323
- Riemann–Hilbert problem, 665, 681, 862
- Riemann–Roch–Hirzebruch theorem, 894
- Riemannian geometry of the sphere, 249
- Riesz
 - Fryges, 22, 533
 - Marcel, 716
 - representation theorem for functionals, 358
- rigged Hilbert space, 35, 580, 677
- Ritt theorem, 863
- Ritz, 122
 - method in quantum chemistry, 155
- Rivasseau, 851
- Roberval, 577
- Robinson, 399
- Röntgen, 69, 130
- Rosanes, 64
- Rosen, 850
- Rossi, 73
- Roth, 71
- Rubbia, 70, 138, 184
- Rudolph, XI
- running (renormalized)
 - coupling constant, 198, 204
 - prototype, 502
 - fine structure constant, 198, 204
- Rutherford, 69, 100, 102, 122, 131
- Rydberg, 122
- Rydberg–Ritz energy formula, 124
- Ryle, 69
- saddle, 248
- Salam, 3, 60, 70, 81, 137
 - criterion, 641, 850
- Salmhofer, XI
- Sato, 75
- Savart, 254
- scattering
 - cross section, 841
 - function, 446, 787
 - modified, 452, 787
 - functional, 451, 787
 - matrix (S -matrix), 38, 372, 393, 748, 830
 - generalized, 856
 - state, 527
 - theory (see also Vol. II), 830
- Schauder, 232, 562
 - theory, 562
- Schechter, 1
- Schelling, 974
- Scherk, 939

- Schmidt, 22
- Schoenflies, 164
- Schrader, 868
- Schrieffer, 70, 577
- Schrödinger, 29, 37, 62, 65, 70, 130, 527
- equation, 36, 394, 810, 955
 - stationary, 36
 - operator picture, 394
 - quantization, 36, 755
 - wave picture, 35
- Schrödinger–Maxwell equation, 174, 176
- Schwartz
- kernel theorem, 683
 - Laurent, 71, 327, 527, 577–579
 - Melvin, 70
 - space $S(\mathbb{R}^N)$, 538
- Schwarz
- Albert, 906
 - Amandus, 543
 - inequality, 340
 - John, 939
- Schwarzschild radius, 143, 144
- Schweber, 741, 863
- Schwinger, 4, 28, 29, 66, 70, 285, 376, 397, 741, 767, 863
- function, 867
 - integration trick, 646, 780
- second law of
- progress in theoretical physics, 81
 - thermodynamics, 168
- second quantization, 52
- secular equation, 367
- Segal, 921
- Segré, 70, 133
- Seiberg–Witten equation, 206, 813
- Selberg, 71, 74
- self-adjoint operator, 359, 679
- self-similarity, 199
- semicontinuous, 571
- separable Hilbert space, 680
- separated, 238
- sequentially
- closed, 341
 - continuous, 539
- Serre, 71, 75
- set
- arcwise connected, 241
 - bounded, 368
 - closed, 238
 - compact, 241
 - neighborhood of a point, 238
 - open, 238
 - neighborhood of a point, 238
 - simply connected, 242
- sharp state, 353
- sheaf cohomology, 400, 932
- shell structure of atoms, 152
- Shimura–Taniyama–Weil conjecture, 19
- shock wave, 616
- Shockley, 70
- Shore, 73
- short-wave asymptotics for light, 720
- SI system of units, 950
- rescaled, 962
 - tables, 966
- Sibold, XI
- Siegel, 74
- sigma
- additivity (σ -additivity), 530
 - algebra (σ -algebra), 530
- similarity principle in physics, 962
- simple group, 936
- simply connected, 242
- Sinai, 75
- Singer, 75, 259
- singular
- limits in physics, 691
 - support, 707
- sink, 248
- skein relation, 266
- skew-adjoint, 345
- skew-symmetric, 346
- SLAC (Stanford Linear Accelerator Center, California), 137
- Smale, 71, 75, 236
- small divisor, 629
- Smalley, 70, 247
- S -matrix (see scattering matrix), 787
- smooth, 236
- boundary, 548
 - function, 31, 523
- Sobolev, 544
- embedding theorem, 560
 - space, 559
- software systems in perturbation theory, 946
- Sokhotski formula, 666
- solid forward light cone, 668
- solitons, 920
- solitons in mathematics, physics, and molecular biology, 702
- Solvay Conference, 62
- Sommerfeld, 122, 726
- radiation condition, 727
- Sorella, 860

- source, 248
 - term, 376
 - trick, 751
- space reflection, 174
- special
 - functions, 666
 - linear group, 343
 - unitary group, 343
- specific volume, 685
- spectral
 - family, 370, 371
 - geometry, 262, 928
 - theorem, 680, 681
 - theory in functional analysis (see Vol. II), 371
- spectrum, 360, 367
- Sperber, 196
- spherical
 - coordinates, 249
 - wave, 722
- spin, 146, 147
 - operator, 163
 - quantum number, 156, 270
- spin geometry, 928
- spin-orbit coupling, 154
- splitting of spectral lines, 181
- spontaneous
 - emission, 126
 - symmetry breaking, 182
- square-integrable functions, 580
- stability of matter, 1
- stable manifold, 208
- Standard Model in particle physics
 - elementary introduction, 913
 - emergence of the, 70
 - history, 137
 - minimal supersymmetric, 140, 861
 - renormalization, 861
 - resource letter, 81
 - see also Vols. III–VI, 129
- standing wave, 86
- state, 271, 351, 352, 599
 - generalized, 599
 - of an elementary particle, 163
- state of the art in
 - gravitation and cosmology, 946
 - quantum field theory, 945
- state, generalized, 683
- stationary
 - phase, 432, 437, 717
 - Schrödinger equation, 375
- statistical
 - operator, 760
 - potential, 282
- Stein, 75
- Steinberger, 70
- Steinmann, 625
 - extension, 625
 - renormalization theorem, 625
- step function, 530
- stereographic projection, 219
- Stern, 69
- Stern–Gerlach effect, 150
- Stevin, 288
- stimulated
 - absorption, 126
 - emission, 126
- stochastic process, 663
- Störmer, 70
- Stokes, 547, 548, 964
 - integral theorem, 549
 - on manifolds, 549
- Stone, 22, 29
- Stone–von Neumann theorem, 821
- strangeness, 156
- string theory, 1, 2, 77, 139, 224, 293, 744, 813, 925
 - history of, 939
 - references to, 911
- strong
 - correlation, 355
 - force, 129, 133
 - hypercharge, 156
 - isospin, 156
- structure constants of a Lie algebra, 886
- Sturm, 527
- sub-velocity of light, 25
- subgroup, 342
- submanifold (see Vol. III), 265
- substitution trick, 362
- Sudan, 73
- suggested reading (see also hints for further reading), 909
- sum rule for spins, 149
- summation convention, 771, 949
- superconductivity, 577
- supernova, 131
- supersymmetric Standard Model in particle physics, 140, 861
- supersymmetry, 140, 813, 934
- support, 611
 - of a distribution, 613
 - of a measure, 605
 - singular, 707
- surjective, 948

- Sylvester, 365
 Symanzik, 441, 767
 symbol of a differential operator, 713
 symmetric, 335
 symmetry, 164–180
 - and special functions, 666
 - breaking, 180, 934
 - in atomic spectra, 154
 - factor of a Feynman diagram, 834
 symplectic, 47, 708
 - topology, 926
 system of units
 - energetic system, 958
 - Gauss, 957
 - Heaviside, 957
 - natural SI units, 953, 966
 - Planck, 954
 - SI (Système International), 83, 950
 - tables, 966
 Tacoma Narrows Bridge, VIII
 Tamm, 114
 Tanaka, 71
 tangent bundle, 251
 Tannoudji, 70
 Tao, 72
 Tarjan, 72
 Tate, 75
 tau lepton number, 156
 Tauber, 288
 - theorem, 288, 689
 tauon, 132
 Taylor
 - Joseph, 69, 73, 138
 - Richard, 70
 Taylor–Slavnov identity, 855
T-duality, 705
 Teichmüller space, 14, 15
 Telegdi, 73
 tempered distribution, 617
 tensor product of distributions, 618
 tera, 951
 Tesla, 966
 test function, 611
 theorema egregium, 11, 250
 theory of
 - general relativity, 113
 - probability, 530
 - special relativity, 111
 thermal Green’s function, 577
 thermodynamic limit, 685
 third law of progress in theoretical physics, 81
 Thirring, 1
 Thom, 71, 229, 236
 Thompson, 75
 Thompson, D’Arcy, 971
 Thompson, John, 71, 74
 Thomson
 - George, 693
 - Joseph John, 69, 100, 102, 114, 119, 130
 - series, 938
 t’Hooft, 70, 73, 137
 Thouless, 73
 Thurston, 72
 time
 - period, 83
 - reversal, 174
 time-ordered
 - contraction, 829
 - paired normal product, 829
 - product, 424, 829
 time-ordering, 390
 Ting, 70
 Tits, 74, 75
 Tomonaga, 4, 28, 70, 741, 863
 Tonelli, 544, 571
 topness, 156
 topological
 - charge, 216
 - invariant, 241
 - methods in quantum field theory, 927
 - quantum field theory, 254, 266, 926
 - quantum number, 216, 243
 - space, 238
 - arcwise connected, 241
 - compact, 241
 - separated, 238
 - simply connected, 242
 topology, 229
 total
 - cross section, 841
 - pairing, 828
 Townes, 69, 128
 trace, 342, 365
 - formula, 758
 transformation theory, 327, 357
 transition
 - amplitude, 39, 353, 757, 825
 - maps, 236
 - probability, 40, 353, 788, 825
 transport equation, 723
 triangle inequality, 368
 Triebel–Lizorkin space, 562
 trivial

- linear space, 332
- model in quantum field theory, 871
- Trotter product formula, 509, 656
- trouble
 - with divergent perturbation series, 863
 - with interacting quantum fields, 750
 - with scale changes, 189
- truncated
 - damped wave, 91
 - Dirac delta function, 817
 - lattice in momentum space, 672
- Tsu, 70
- tube, 226
- tunnelling of α -particles, 131
- turbulence, 961
 - problem, 79
- Tycho Brahe, 182

- Uehling, 196
- Uhlenbeck, 73, 150
- Uhlmann, XI, 60
- ultrafilter, 399
- ultraviolet limit, 852
- uncertainty
 - inequality, 34, 144, 525
 - relation, 62
- uniformization theorem, 19, 223, 554
- unit
 - ball, 270
 - sphere, 270
 - surface measure, 563
- unitarity of the S -matrix, 372, 892
- unitary
 - equivalence, 340, 360
 - group, 343
 - matrix, 343
 - operator, 340
- universe
 - global structure, 229
 - local structure, 229
- unknot, 265
- unphysical states, 895
- unstable manifold, 208
- upper half-plane, 286, 665

- vacuum (ground state), 55, 183, 819
 - energy, 302
 - expectation value, 427
 - polarization, 197
 - state, 426
- Valiant, 73
- van der Meer, 70, 138
- van der Waerden, 60
- van Dyck, 243
- Vandermonde, 254
- vanishing measure, 533
- Varadhan, 75
- variation of the parameter, 385
- variational
 - lemma, 405
 - complex, 546
 - real, 545
 - problem, 549
- vector calculus, 173
- velocity of light c , 698, 952
- Veltman, 5, 70, 81, 137
- Veneziano, 293, 939
 - model, 939
- Verch, XI
- vertex
 - algebra, 938
 - distribution, 782
 - function, 446, 462, 486, 753
 - functional, 461
- vertex algebra, 938
- vibrating string, 807
- Vilenkin, 529
- Virasoro algebra, 936
- virtual particle, 59, 838, 846
- virus dynamics, 291
- Voevodsky, 72
- Volterra, 388, 397
 - differential calculus, 752
 - integral equation, 43, 388
- volume
 - form, 250
 - potential, 557
- von Klitzing, 70
- von Neumann, 21, 29, 35, 37, 38, 60, 67, 68, 283, 285, 370, 374, 527, 537, 538, 569, 821, 973
 - spectral theorem, 680
- von Waltershausen, 10

- Ward–Takehashi identity, 855
- warning to the reader, 741, 778, 786
- Watson, 71
- wave, 83
 - equation, 807
 - front, 721, 725
 - equation, 721
 - set, 708, 710, 712
 - length, 84
 - of matter waves, 144
 - mechanics, 65

- number, 84
- operator, 463
- packet, 84
- vector, 708
- weak
 - convergence, 572
 - correlation, 355
 - force, 130
 - gauge bosons W^\pm, Z^0 , 132
 - hypercharge, 156
 - isospin, 156
- weakly lower semicontinuous, 572
- Weber, 254
- wedge product, 517
- Weierstrass, 19, 512, 542, 551, 653
 - counterexample, 551
 - product theorem, 512
- Weil, 74
- Weinberg, 3, 70, 81, 100, 137, 203, 641, 973
 - power-counting theorem, 641
- Weinberg–Salam theory, 578
- Weisskopf, 73
- Wentzel, 66
- Werner, 72
- Wess, XI, 935
- Wess–Zumino model, 813
- west coast convention, 950
- Weyl, X, 3, 24, 67, 68, 259, 365, 523, 527, 544, 569, 971, 974
 - asymptotics of the spectrum, 262
 - lemma, 614
- Wheeler, 39, 73, 230
- Whitney, 74
- Wick
 - moment trick, 58, 434
 - rotation, 427, 591, 638, 645
 - theorem
 - first, 827
 - main, 826
 - second, 828
- Wick theorem
 - main, 830
- Widgerson, 73
- Wieman, 70, 687
- Wien, 69
 - radiation law, 104
- Wiener, 21, 29, 397
 - integral, 657, 658, 663
 - measure, 657
- Wightman, 150, 175, 641, 651, 707, 850, 868, 921
 - axioms, 868
 - functional, 860
- Wigner, 49, 70, 165, 527, 971
- Wilczek, 70, 203
- Wiles, 18, 72, 75, 78
- Wilson
 - Charles, 69
 - Kenneth, 70, 73, 192
 - loop, 578
 - Robert, 69, 115
- winding number, 216
- Witten, 13, 72, 75, 76, 78, 102, 940
 - functor, 14
 - important papers, 925
- WKB (Wentzel, Kramers, Brillouin), 433
 - approximation, 963
 - method, 433
- WMAP (Wilkinson Microwave Anisotropy Probe), 82, 115
- Wolf prize
 - in mathematics, 74
 - in physics, 73
- wormhole, 230
- Wright, 850
- Wu, 73
- Wüthrich, 71
- Yang, 3, 70, 183, 187, 251
- Yang–Lee condensation, 685
- Yang–Mills
 - action, 885
 - equation, 813
- Yau, 72
- Yoccoz, 72
- Yukawa, 69
 - meson, 145, 714
 - potential, 727
- Zagier, XI, 18, 295, 306, 307
- Zamolodchikov, 939
- Zariski, 74
- Zeeman, 69, 182
 - effect, 181
- Zelmanov, 72
- zero
 - measure, 532
 - set, 530, 532
- zeta function, 262, 293, 314, 436, 661
 - determinant formula, 263, 661
 - of a compact manifold, 262
 - regularization, 303, 660
 - trick, 436
- Zimmermann, 441, 767, 855, 856

– forest formula, 743, 855, 862

Zumino, 935

Zustandssumme (partition function),
759

Zweig, 100, 137