Passive components used in power converters

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Summary
In power converters, passive components play an important role, and have in general specific nature and properties. The goal of this tutorial is to give an overview, first on inductive components for power conversion, and second on dedicated power capacitors. In a third part, new components—supercapacitors—will be presented. Generally, inductors for power applications must be custom designed. In this tutorial, the most important effects encountered when realising inductive components will be presented in the first part, without entering into the detailed design of such components. For that purpose, the referenced documents that have served as a base for this tutorial must be consulted [1], [2], and mainly [3]. The second part of this tutorial (Capacitors used in power electronics) is dedicated to power capacitors. Unlike inductors, capacitors cannot be specifically designed, but must be selected from a manufacturer's list of components. Here, the documentation corresponds to a subset of Ref. [4] that has been translated by Dr. Martin Veenstra. The third part of the tutorial (Supercapacitors and applications) presents supercapacitors, new components that have very high energy density and high power density. Modelling and design rules for several applications are presented. This part of the document uses as a base the study made by Dr. Philippe Barrade [5]. Finally, it must be noted that, even with a correct selection or design of passive elements, there can be parasitic effects caused by interactions between components of the same or different nature. As an example, by designing filters combining several passives like inductors and capacitors, the primary specification may be modified by the interaction of parasitics, typically a mutual coupling between the parasitic inductances of neighbouring capacitors. A good description of such effects can be found in Ref. [6].

PART I

Inductors used in power electronics and applications

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Abstract
This tutorial gives an overview of inductive components for power conversion, and of the common effects that have to be considered in their design. Generally, inductors for power applications must be custom designed, because they are solicited in many different ways according to the given application. In this tutorial, the most important effects encountered

while making inductive components will be presented, whether they result from the typical properties of magnetic materials and cores, or from the effect of time-varying currents and related effects of the magnetic field generated on the winding itself. The phenomena occurring in magnetic material and wires will be described, and their thermal consequences (losses) will be discussed.

1 Overview, typical applications

Inductors can be found in most power electronics applications and in converters; they can be classified thus:

- AC applications
- DC applications
- Filtering
- Smoothing (limiting di/dt)
- Components of resonance circuits

For each application or function of these components, a specific design is required, taking the best advantage of the properties of the materials, mainly conductive materials like copper and magnetic core materials. In the conductive materials, the limiting factors are of a thermal nature because of the dissipative losses, and in the magnetic materials they have to do with linearity, because of saturation effects.

Generally, two main types of inductors are considered:

- Air inductors
- Inductors with magnetic core

These two types can be described by the following parameters.

Solenoid (air)

The main parameter of the inductor shown in Fig. 1 is its inductance L, defined as:

$$L = N^2 \mu A / l \qquad (1)$$

with A the area of coil; l the length; N the number of turns; and μ the permeability.

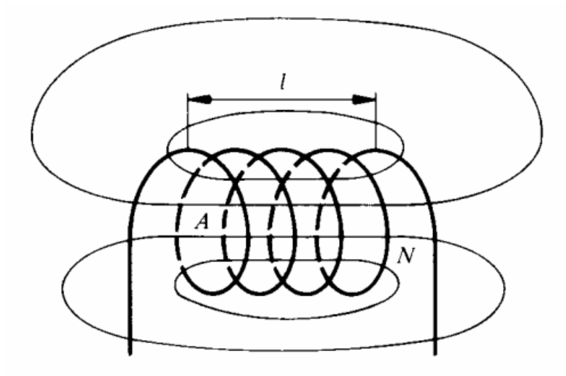

Fig. 1: Solenoid

Toroid (core)

The main parameter of the inductor shown in Fig. 2 is its inductance L, defined as:

$$L = N^2 \mu A / \pi d_\mathrm{m} \tag{2}$$

with A the section of core; d_m the mean diameter of the toroid; N the number of turns; and μ the permeability.

The permeance of a magnetic circuit is defined as the reciprocal of its reluctance:

$$\mathcal{P} = \frac{1}{\mathcal{R}} \ .$$

$\boldsymbol{l = \pi d_\mathrm{m}}$

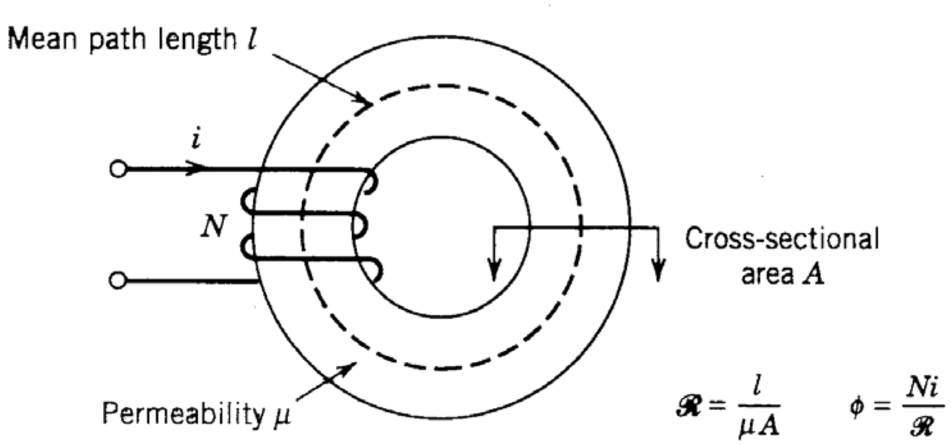

Fig. 2: Toroidal inductance

2 Main parameters of inductors and equivalent scheme

The main parameters of an inductor can be represented schematically by an equivalent circuit as shown in Fig. 3.

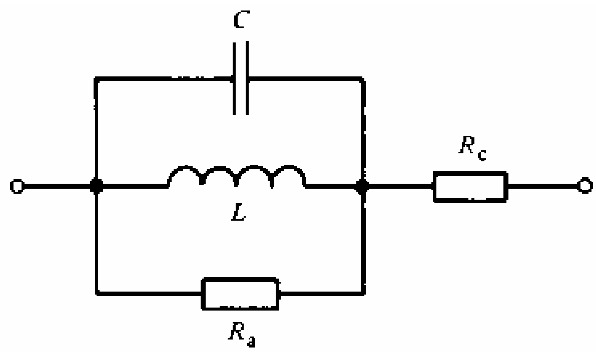

Fig. 3: Equivalent scheme

The main parameters are defined as:

- inductance
- quality factor

– capacity
– rated current.

For the equivalent scheme:

– L: main inductance
– R_a: losses related to ac current component
– R_c: resistance of winding
– C: capacity of winding.

Relationships to the equivalent scheme:

when: $\omega^2 LC << 1$ $\qquad \underline{Z} \cong R' + j\omega L'$

with $\qquad R' = R_c + R_a/(1+Q_a^2)$

and $\qquad Q_a = R_a/\omega L$

if $Q_a^2 >> 1$ $\qquad L' \cong L$

$$R' \cong R_c + \omega^2 L^2/R_a\ .$$

Factor of losses and quality factor:

when $Q_a^2 >> 1$ $\qquad \tan\delta = R'/\omega L' \cong R_c/\omega L + \omega L/R_a$

$$\tan\delta = \tan\delta_c + \tan\delta_a$$

$$Q = 1/\tan\delta = \omega L'/R'\ .$$

These factors are important for resonant circuits.

3 Magnetic materials and cores

Two main classes of materials are used to make inductors and transformers.

Iron-based materials

– Alloys of iron with small amounts of chrome and silicium exhibiting high electrical conductivity and high saturation limit (near 1.8 T).
– Two types of losses are found in iron alloy materials: hysteresis losses and eddy-current losses.
– Powdered iron cores (small iron particles isolated from each other) are better suited for higher frequencies, because of their greater resistivity and smaller eddy current losses.
– Amorphous alloys of iron with other transition metals like cobalt and nickel in combination with boron, silicon, and other glass-forming elements also offer interesting properties for inductor and transformer applications. Alloy compositions containing cobalt such as METGLAS 2705M appear particularly suitable for high-frequency applications: saturation induction around 0.76 T at room temperature, and 0.65 T at 150ºC.

Ferrites

The second class of materials used for cores are ferrites which are basically oxide mixtures of iron and other magnetic elements. They show a high electrical resistivity, but rather low saturation flux density, typically 0.3 T. Ferrites have only hysteresis losses, but no significant eddy current losses because of their high electrical resistivity. Ferrites are the material of choice for cores that operate at high frequencies (greater than 10 kHz), because of low eddy current loss.

4 Losses in magnetic components

4.1 Hysteresis losses

The area between the B–H curve of Fig. 4 represents the work done on the material by the applied field, which causes so-called hysteresis losses. The work (energy) dissipated in the material and the heat generated by the dissipation raise the temperature of the material. The hysteresis losses in the core material increase with the increase in ac flux density, B_{ac}, and in operating or switching frequency f. The general form of the losses per unit volume (specific losses) $P_{m,sp}$ is given by:

$$P_{m,sp} = kf^{a}(B_{ac})^{d} , \qquad (3)$$

where k, a, d, are constants depending on the material.

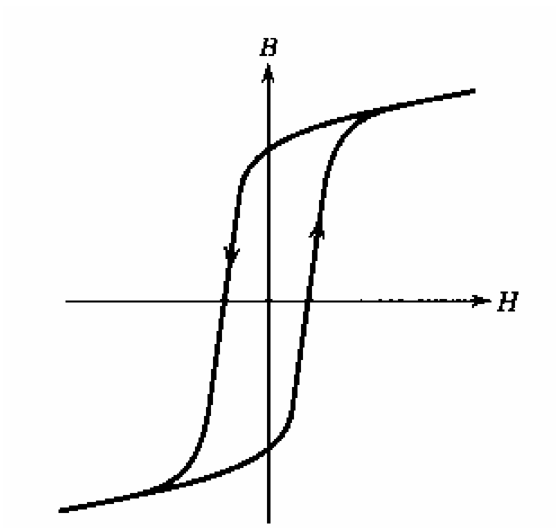

Fig. 4: Hysteresis curve

The expression for hysteresis loss applies over a limited range of frequency and flux density, with the range of validity depending on the material. The flux density B_{ac} is the peak value of the ac waveform shown in Fig. 5(a), if the flux density averages zero over time.

$$B_{ac} = \hat{B} .$$

When the flux density has a DC component as shown in Fig. 5(b), B_{avg}, then the appropriate value to use in Eq. (3) is

$$B_{ac} = \hat{B} - B_{avg} .$$

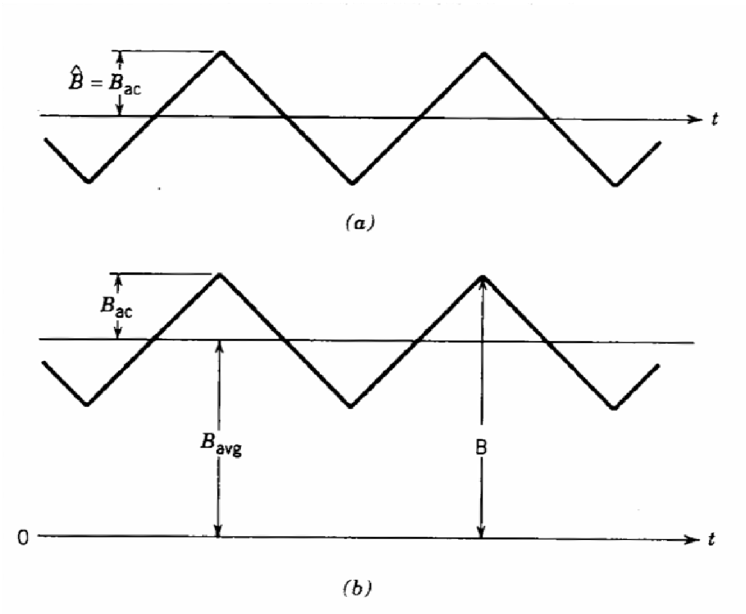

Fig. 5: Waveforms of the flux density

Core manufacturers provide detailed information about core loss, usually in the form of graphs of specific loss $P_{m,sp}$ as a function of flux density, and with the operating frequency as a parameter. An example is given in Fig. 6.

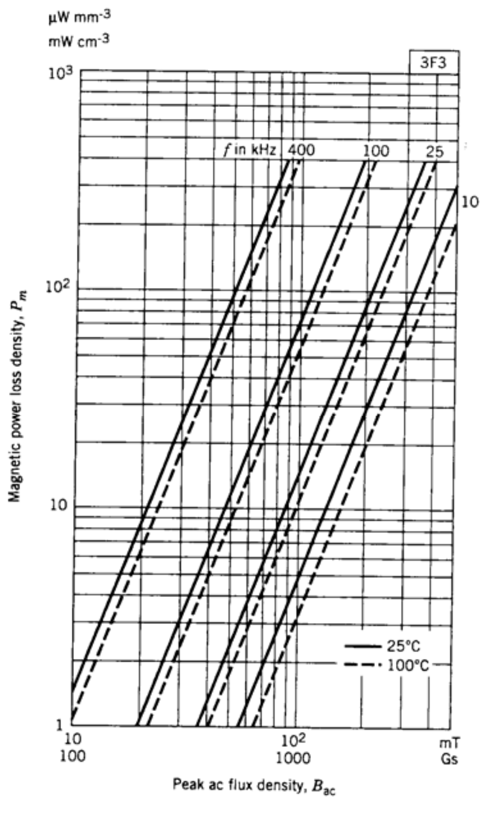

Fig. 6: Magnetic losses depending on flux density and operating frequency

Two examples of specific losses are given below.

Ferrite material 3F3

$$P_{\mathrm{m,sp}} = 1.5\times10^{-6} f^{1.3} (B_{\mathrm{ac}})^{2.5}$$

where $P_{\mathrm{m,sp}}$ is in mW/cm^3 with f in kHz and B_{ac} in mT.

METGLAS

$$P_{\mathrm{m,sp}} = 3.2\times10^{-6} f^{1.8} (B_{\mathrm{ac}})^{2} .$$

For 100 kHz and 100 mT: $P_{\mathrm{m,sp}}$ = 127 mW/cm^3.

$P_{\mathrm{m,sp}}$ depends finally on how efficiently the dissipated heat is removed. $P_{\mathrm{m,sp}}$ is even smaller because of the presence of eddy current loss.

4.2 Skin effect limitations (in core)

If a conducting material is used, a circulation of currents appears when the magnetic field is time-varying (eddy currents) (Fig. 7).

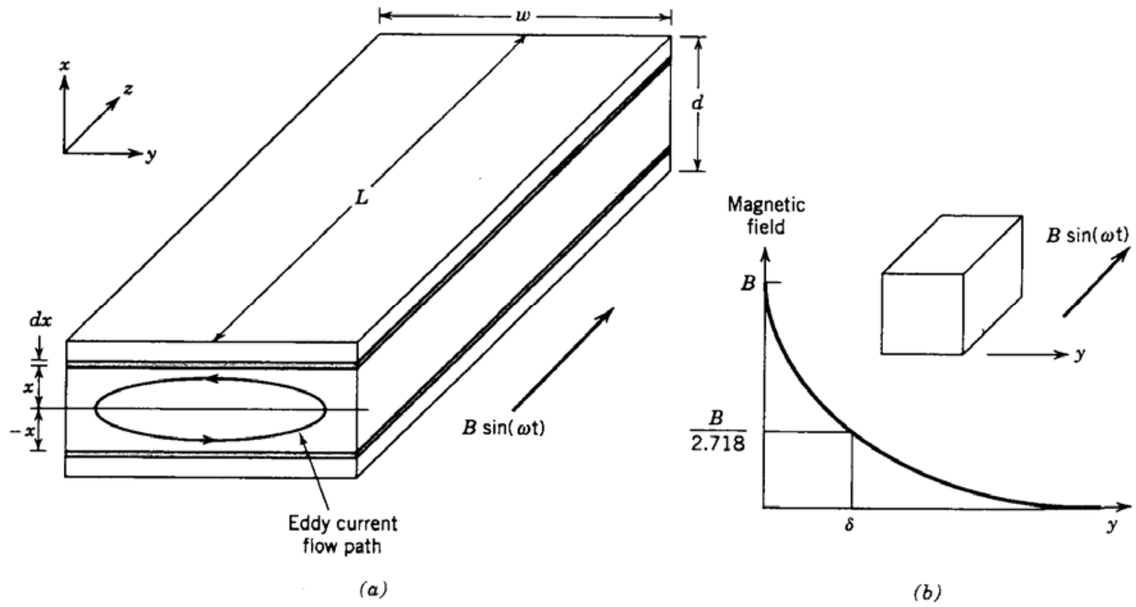

Fig. 7: Eddy currents generated in a thin transformer lamination by applied time-varying field, (a), and (b), decay of magnetic field versus depth into the interior of a thick bar of magnetic material.

The magnetic field in the core decays exponentially with distance into the core:

$$B(y) = B_0 e^{-y/\delta} .$$

The skin depth δ is defined as

$$\delta = \sqrt{2/\omega\mu\sigma}$$

where $\omega = 2\pi f$, μ is the permeability, and σ is the conductivity.

The magnetic cores for inductors and transformers made of conducting magnetic materials consist of many thin laminations as shown in Fig. 8.

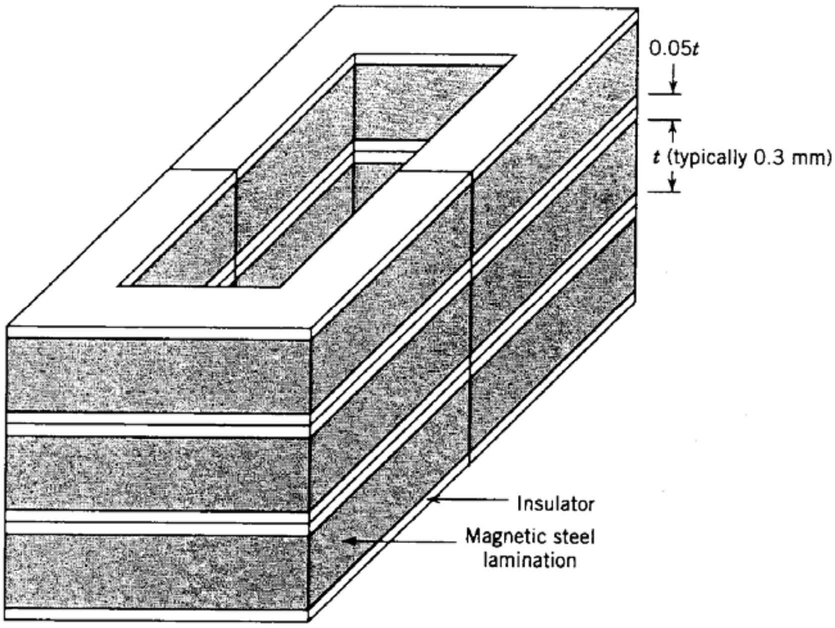

Fig. 8: Laminations of a magnetic core for a transformer or an inductor

To illustrate this effect, an example of typical skin depth is considered:

– material with high permeability:
– skin depth ~ 1 mm at 60 Hz
– choice of thin laminations isolated from each other
– stacking factor (0.9…0.95).

For materials with increased resistivity, there is an increase of the skin depth, but the magnetic properties are reduced.

For 50/60 Hz transformers, a reasonable compromise would be

– alloy of 97% iron and 3% silicon, and a lamination of 0.3 mm.

4.3 Eddy current loss in laminated cores

The eddy currents generated in the conductive core dissipate power, generically known as eddy current loss, which raises the core's temperature. Considering the magnetic conductor shown in Fig. 8, which is immersed in a uniform time-varying magnetic field having a flux density $B(t) = B\sin(\omega t)$, it is assumed that the thickness d is less than the skin depth δ so that the induced eddy currents do not reduce the magnetic field in the interior of the material.

The specific eddy current loss, $P_{\mathrm{ec,sp}}$, which correspond to the loss per unit volume can be calculated by:

$$P_{\mathrm{ec,sp}} = \frac{d^2\omega^2 B^2}{24\rho_{\mathrm{core}}}$$

where d is the thickness of the lamination and $d < \delta$ the skin depth, and

$$B(t) = B\sin(\omega t) \ .$$

5 Core shapes and optimum dimensions

Magnetic cores are available in a wide variety of shapes and sizes to suit the given application. This is particularly true for ferrite cores, which are available as toroids, pot cores with an air-gap, and in U-, E-, and I-shapes. Laminated material is available as tape-wound toroids and C-cores. A double E-core is shown in Fig. 9(a) as an example.

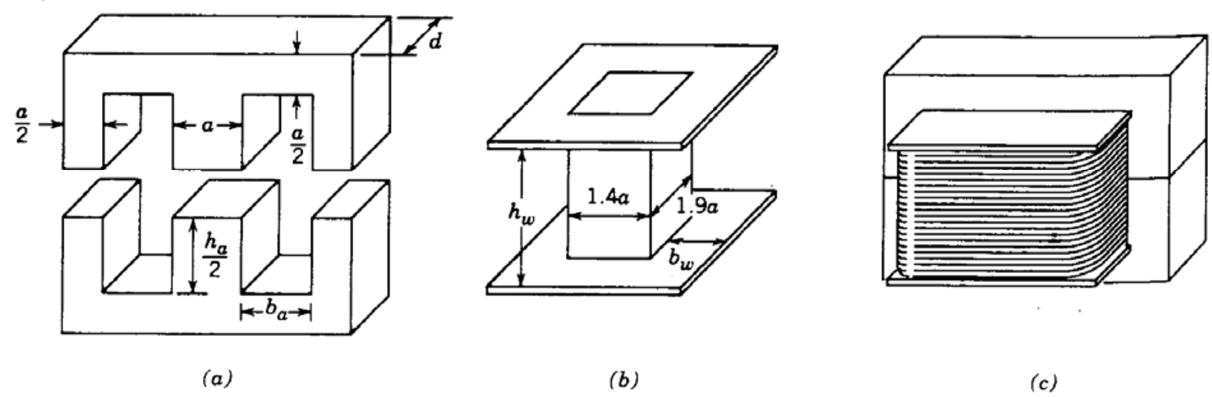

Fig. 9: Magnetic cores with coil former

A bobbin or a coil former is provided with most cores, and the effective cross-sectional area $A_w = h_w b_w$ available for the copper windings on the bobbin is given, as shown in Fig. 9(b).

An example of a nearly optimal geometry is given by the following parameters:

$$b_a = a, \quad d = 1.5a, \quad h_a = 2.5a, \quad b_w = 0.7a, \quad h_w = 2a .$$

6 Windings

6.1 Copper windings

The conductor windings of inductances or transformers are made from copper because of its high conductivity. In addition it is easy to bend.

Single round wires are normally used, but under specific conditions where skin effect would lead to dissipation problems, so-called Litz-wire with strands of a few hundred microns diameter can also be used.

Figure 10 shows the different cross-sections for a double E-core and bobbin assembly. The ratio of the total copper area to the winding window area is defined as the copper fill factor k_{Cu} and is given by

$$k_{Cu} = \frac{NA_{Cu}}{A_w};$$

typical values range from 0.3 (Litz) to 0.5...0.6 for round conductors.

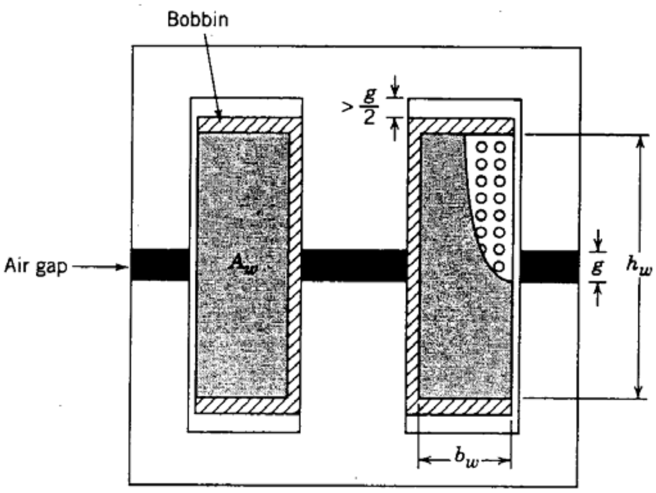

Fig. 10: Definition of the parameters of a core, and of a bobbin: geometric considerations

The (specific) power dissipated per unit of copper volume can be calculated by:

$$P_{\mathrm{Cu,sp}} = \rho_{\mathrm{Cu}} (J_{\mathrm{rms}})^2 \quad \text{where} \quad J_{\mathrm{rms}} = I_{\mathrm{rms}}/A_{\mathrm{Cu}}$$

or by

$$P_{\mathrm{w,sp}} = k_{\mathrm{Cu}} \rho_{\mathrm{Cu}} (J_{\mathrm{rms}})^2$$

as power dissipated per unit of winding volume.

6.2 Skin effect in copper windings

The skin effect in the copper conductors of the windings occurs in the same manner as in the conductive magnetic core. When considering the single copper conductor shown in Fig. 11(a), which is carrying a time-varying current $i(t)$, a magnetic field $H(t)$ is generated. This time-varying field gives rise to the eddy currents represented in Fig. 11(b), circulating in a direction opposite to that of the conductor current in the interior of the winding, and thus tending to shield the interior of the conductor from the applied current. The result is a non-linear current density in the conductor as shown in Fig. 11(c).

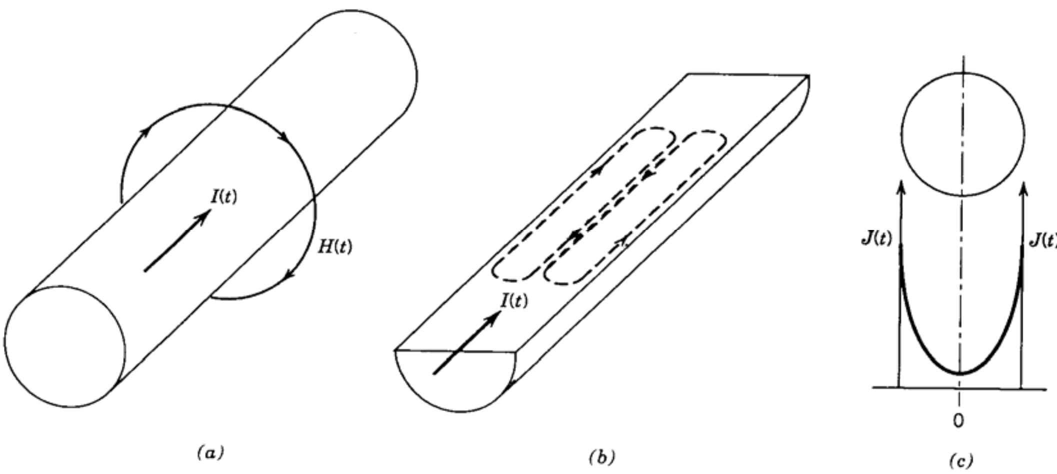

Fig. 11: Current density in a conductor, skin effect

The current density decays exponentially, the skin depth is called δ.

Table 1 gives the value of the skin depth in copper at 10°C for several different frequencies.

Table 1: Skin depth in copper at 100^{o}C for different frequencies

Frequency	50 Hz	5 kHz	20 kHz	500 kHz
δ	10.6 mm	1.06 mm	0.53 mm	0.106 mm

7 Thermal considerations

A temperature increase of the core and of the windings degrades their performance in several respects. Two main effects can be mentioned:

- the resistivity of the copper winding is increased, thus the conduction losses increase with temperature;
- the value of the saturation flux density decreases as temperature increases.

Thus, in practice, it is important to keep the core and winding temperature under 100–125°C.

A classical thermal model can be used to calculate the temperature as a function of the dissipated power.

The following parameters are used:

thermal resistance from surface to ambient $R_{\theta sa}$,

thermal resistance related to radiation $R_{\theta rad}$,

thermal resistance related to convection $R_{\theta conv}$,

$$R_{\theta sa} = \frac{k_1}{a^2}$$

with k_1 constant.

The thermal model is given by the relation

$$\Delta T = R_{\theta sa}\left(P_{core} + P_{w}\right),$$

where the core and the winding dissipation are calculated from their specific values multiplied by their respective volumes:

$$P_{core} = P_{c,sp}V_{c}$$

$$P_{w} = P_{w,sp}V_{w} \ .$$

On account of the geometry, the specific power dissipation P_{sp} is generally given by the relation

$$P_{sp} = \frac{k_3}{a}$$

and the current density J_{rms} corresponds to

$$J_{rms} = \frac{k_5}{\sqrt{k_{Cu}a}} \ .$$

An example of these curves is given in Fig. 12.

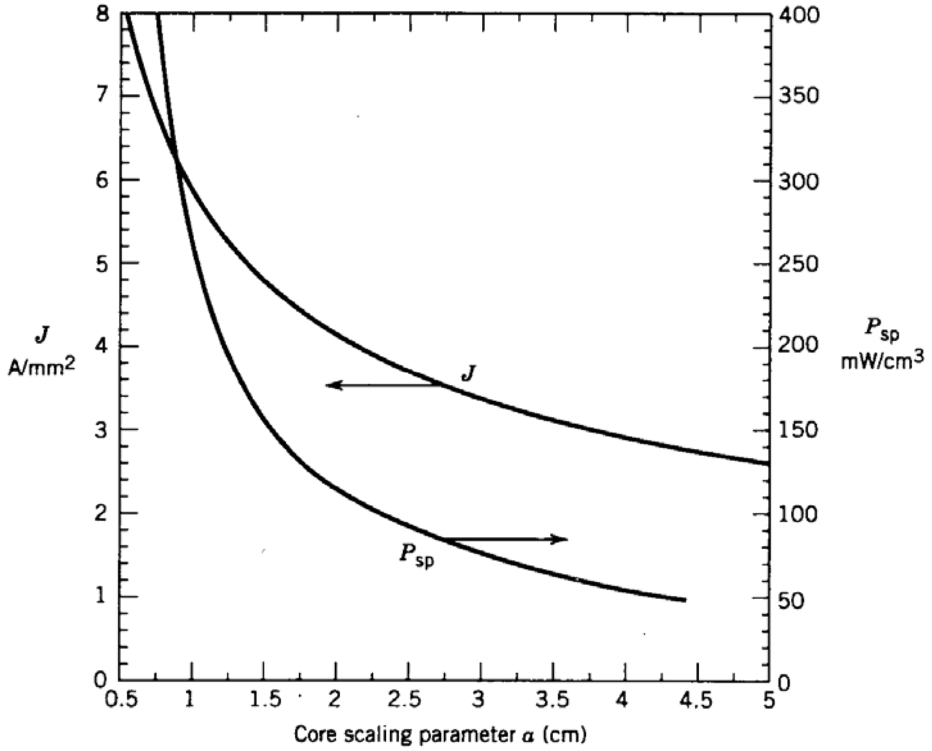

Fig. 12: Maximum current density and specific power dissipation P_{sp} as a function of the scaling parameter a of a double-E core

References

[1] P. Robert, Matériaux de l'électrotechnique, in *Traité d'électricité*, Vol II, PPUR, ISBN 2-88074-042-8.

[2] M. Jufer and F. de Coulon, Introduction à l'électrotechnique, in *Traité d'électricité*, Vol I, PPUR, ISBN 2-88074-042-8.

[3] T. Undeland, N. Mohan and P. Robbins, *Power Electronics, Converters, Applications and Design,* Wiley, ISBN 0-471-58408-8.

[4] G. Mouriès, Condensateurs utilisés en électronique de puissance, in *Techniques de l'Ingénieur, Traité de Génie Electrique* (1995).

[5] P. Barrade and A. Rufer, Current capability and power density of supercapacitors: considerations on energy efficiency, EPE 2003: European Conference on Power Electronics and Applications, 2–4 September 2003, Toulouse, France.

[6] S. Wang, R. Chen, F. C. Lee, and J. D. van Wyk, Improved passive filter configurations for high-frequency conducted EMI in power electronics, EPE 2005, 11th European Conference on Power Electronics and Applications, 11–14 September 2005, Dresden, Germany.

PART II

Capacitors used in power electronics

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Abstract
Capacitors in power-electronic circuits operate under conditions quite opposite to those prevalent in industrial networks. In Section 1, we see the distinctive features of this application. In Section 2, we study the technologies and the materials used and some applications. The use and specifications of such capacitors are developed in Section 3.

1 Distinctive features

1.1 Introduction and equivalent circuit

Capacitors used in electronics to increase the power factor, start single-phase asynchronous motors, etc., operate under the following conditions:

- almost sinusoidal waveforms at industrial frequencies (50 Hz or 60 Hz);
- no significant constant voltage on their terminals.

Finally, their parasitic series inductance is not specified as it can be quite high without serious consequences for these applications.

Such conditions are quite opposite to those applying to capacitors employed in power electronic circuits. Indeed, in these circuits:

- the currents are not sinusoidal, the residual harmonics can easily exceed 60%; very frequently, these currents are pulse-like under various forms, with current variations (di/dt) easily exceeding 10 A/µs; the fundamental frequencies are often very high (1–50 kHz);
- a high permanent constant voltage is generally superimposed on the alternating or pulse-like component;
- the parasitic series inductance and resistance must be as small as possible.

The equivalent circuit of a real capacitor can take various forms. We shall use the one of Fig.1(a), with the following notations:

- C: ideal capacitor,
- L_s: series inductance,
- R_s: series resistance,
- R_p: equivalent parallel resistance used to define the dielectric losses,
- R_f: leakage resistance,
- R_{eq}: equivalent series resistance [Fig. 1(b)] used to define the total capacitor losses at a given frequency.

The leakage resistance is generally very large and its influence can be neglected; the self-discharge time constants ($R_f C$) are often bigger than 1000 s.

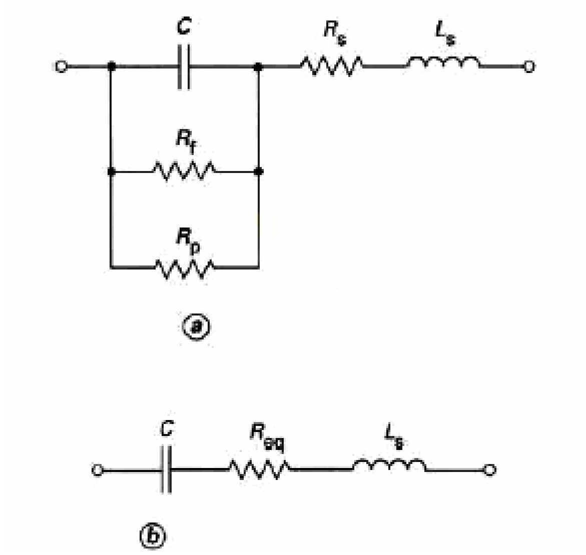


Fig. 1: Power capacitor. a) Equivalent circuit. b) Simplified equivalent circuit.

1.2 Constraints

The constraints applied to power-electronic capacitors are very diverse. Each type of usage requires a study and a particular technology; that is why classical capacitors for industrial networks are seldom suitable.

Choosing a capacitor implies solving three main problems linked to the dielectric, the pulse-like currents, and the thermal environment.

- ***Dielectric problem:*** The ageing of dielectrics depends on the voltage waveform (continuous, alternating, or both superimposed), its frequency and its harmonics. The temperature and overvoltage stress are determining parameters.
- ***Problems linked to pulse-like currents:*** The peak currents, if of high amplitude, subject the terminals and internal connections to forces which can result in rupture or breakdown. Because of their structure (Section 2.3.2), capacitors with metallized electrodes are sensitive to this parameter. The manufacturers specify maximum values for the terms dv/dt or, more convenicutly, for I^2t.
- ***Thermal problem:*** This is the most important problem, because it determines the component reliability. The heating calculations are delicate and require a lot of experience. Unfortunately, the dielectrics used in capacitors are quite limited in temperature tolerance (generally θ = 85ºC, against 150ºC to 200ºC for those used in transformers or motors). Their lifetime is an exponential function of this parameter (for example, between 70ºC and 85ºC the lifetime can be divided by 10; for more details, see Ref. [1]).

It is essential to know the thermal characteristics of the capacitor, its heat-exchange rate with the cooling fluid, and, in particular, the thermal impedance between the hottest point of the dielectric and the case.

1.3 Limitations

Power-electronics systems create strong currents whose frequency is high and rich in harmonics (Section 1.1); this complicates the evaluation of losses. We use the notations of the equivalent circuit in Fig. 1(a).

- The ohmic losses in the connections and the electrodes (R_s) depend on the frequency (skin effect).
- The dielectric losses (R_p) are the product of the reactive power (proportional to the squared amplitude of the electric-field variations and to their frequency) and the tangent of the loss angle:

$$\tan\delta = C\omega/R_p\ .$$

The latter depends, according to the nature of the dielectric, on the voltage, the frequency, and the temperature.

Finally, one must add the electromagnetic losses due to the induced currents in the metal case, which often requires using non-magnetic metals such as aluminium.

Figure 2 illustrates, for sinusoidal operation, the various capacitor limitations as a function of the frequency. Three zones can be distinguished.

- In zone A (low frequencies), the limitation comes from the maximum voltage, which must not be exceeded. The reactive power Q increases with the frequency:

$$Q = U^2 C\omega$$

with U the r.m.s. value of the capacitor's terminal voltage, and $\omega = 2\pi f$ its angular frequency. We write f_1 to designate the frequency at which the capacitor dissipates its maximum active power P_{max} at the maximum voltage U_{max}; the reactive power is also at its maximum value Q_{max}.

- Zone B corresponds to intermediate frequencies; the maximum allowable losses are attained.
- In zone C (high frequencies), the limitation is set by the maximum allowable current (I being the r.m.s. value of the current traversing the capacitor); this current decreases with the frequency, owing to the skin effect in the connections and to the electromagnetic losses induced in the case. The maximum current is attained at the frequency f_2.

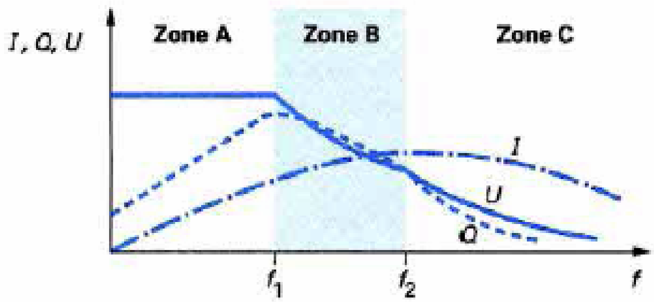


Fig. 2: Capacitor limitations (voltage U, current I, reactive power Q) for sinusoidal operation as a function of the frequency f

1.4 Series inductance

The series inductance L_s generates a transient voltage drop $L_s \cdot \mathrm{d}i/\mathrm{d}t$, which can be large compared to the operating voltage of the capacitor.

A typical component of 20 µF, for example, has a series inductance of about 0.1 µH. If the current variation is di/dt = 50 A/µs, quite a common value, the transient voltage over L_s attains 5 V, whereas the peak value of the capacitor terminal voltage must not exceed 100 V (this is the case for a low-power, moderate-voltage converter). This same capacitor has, moreover, a proper series-resonant frequency of 100 kHz.

Thus one cannot consider such a component as being an ideal capacitor, and its impedance as a function of the frequency takes the form indicated in Fig. 3. The minimum corresponds to the series resonance at angular frequency ω_0, such that:

$$L_s C \omega_0^2 = 1 \, .$$

This series inductance can be bothersome, even in stationary operation, if ω_0 is close to the angular frequency of some higher-rank harmonics of the fundamental. This can occur, particularly in high-frequency resonant converters (above 5–10 kHz). In practice, it is recommended not to use a capacitor at a frequency above 1/5th of its resonance frequency.

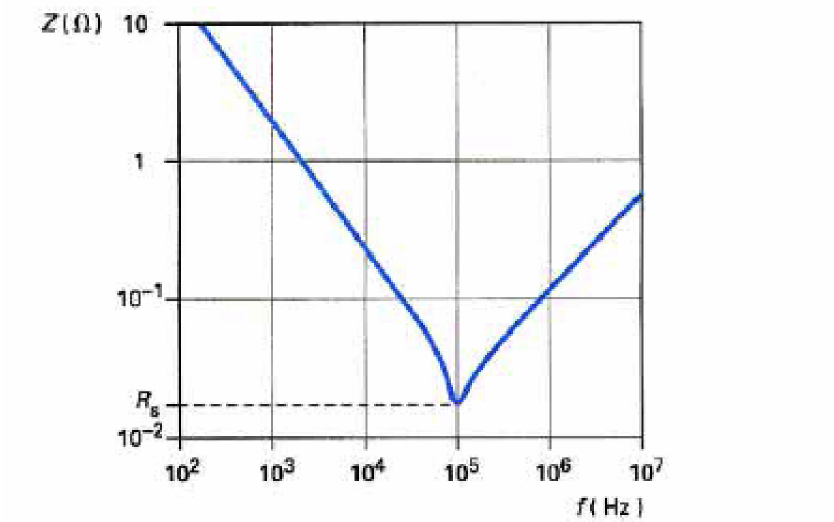


Fig. 3: Impedance *Z* of a real capacitor as a function of the frequency *f*

These parasitic phenomena have been known for a long time. It was usual to surround paper capacitors, which present a big residual inductance, with silver-plated mica capacitors of about 100 times smaller capacitance, to ensure an efficient decoupling at the highest frequencies (above several MHz). A similar arrangement is used nowadays in so-called compound capacitors.

1.5 Conclusion

The constraints prevalent in power electronics require capacitor technologies suited to each application. Large currents at high frequency and the temperature limits of existing dielectrics require components with very low losses and low thermal impedance. The orders of magnitude are generally as follows:

- series resistance R_s = 0.1–10 m
- series inductance L_s = 5–400 nH
- dielectric losses (loss-angle tangent) $\tan\delta = 2 \cdot 10^{-4}$ to $100 \cdot 10^{-4}$
- thermal impedance Z_{th} = 0.5–20 K/W

Capacitors for industrial networks are only a special case; they are designed to operate under sinusoidal conditions at 50 Hz or 60 Hz, and do not have the characteristics required by power electronics.

2 Technologies used

2.1 Technology families

For power-electronics applications, three types of capacitors are used:

– electrolytic aluminium capacitors;

– ceramic capacitors;

– film capacitors (paper, plastic, dry, or impregnated).

Electrolytic capacitors (Section 2.4) are only used for filtering continuous voltages. They have been used in decoupling power supplies (models with a small resistance in series) for a long time. Nowadays, their increased lifetime means that they can be used in medium-power equipment (P > 10 kW; U < 1000 V); applications to high power (P > 100 kW) are tested up to 3500 V.

Ceramic capacitors are traditionally used at high frequencies (f > 1 MHz) for resonance or decoupling at high voltages (U > 3000 V), for example in radio emitters. A new generation of ceramic multi-layered capacitors covers low continuous voltages (U < 500 V) at the present time. Their current characteristics (series resistance and inductance) are exceptional. Their high capacitance in a small volume makes them suitable for output filtering of high-frequency decoupling power supplies (200 kHz < f < 1 MHz). Unfortunately, their still considerable cost restricts their use to very sophisticated systems (military applications).

Film capacitors are generally made by simultaneously winding dielectric ribbons (paper or plastic film) and metallic ribbons, the latter constituting the electrodes (Section 2.3.2). One can impregnate these elements; the dielectric is thus the association of the film and the impregnation.

As one can vary the nature and the dimensions of each constitutent component, it is thus possible to create a large palette of characteristics adapted to the various constraints in power-electronics applications. These capacitors are the best adapted and the most used. We shall develop all the possibilities of this technology in detail.

2.2 Dielectrics

The dielectrics used are common electrical engineering materials, but they are thinner and their quality carefully selected (Table 1). Table 2 gives the application domain of the various dielectrics.

Table 1: Characteristics of the dielectrics used

dielectric	relative permittivity ε_r	$\tan\delta$ at 1 kHz and 25 °C (10^{-4})	flash point (°C)	fire point (°C)	dielectric strength (kV/mm)	volumetric mass (kg/m^3)	temperature coefficient (10^{-6}/K)
paper (cellulose fibre)	6.6					1200	
polypropylene (PP)	2.2	2			600	900	−200
polyester (*Mylar*)	3.2	50			500	1400	+1200
polyethylene naphtalate (PEN)	3.0	39			250	1360	+65
mineral oil	2.3	10	150	165	60	860	−1400
castor oil	4.6	100	250	305	60	900	−800
vegetable oil (e.g. rape)	3.0	10	330	370	> 40	910	−1000
phenylxylylethane (PXE)	2.7	2	150	160	60	988	−2000
mono/dibenzyltoluene (M/DBT) (*Jarylec*)	2.7	2	144	154	60	1000	−1850
wax	2.6	2	245	290	60	900	−2000
silicone	2.8	2	305	360	60	900	−3300

Table 2: Application domains of the dielectrics used

dielectric	applications	key advantages	observations	relative permittivity ε_r	$\tan\delta$ at 1 kHz and 25 °C (10^{-4})
paper + mineral oil	• DC filtering • energy storage	• high $\theta \leq 85$ °C	• considerable volume	≈ 4	
paper + castor oil	• DC filtering • energy storage • very high energy banks	• smaller volume than mineral oil • cost	• $\theta \leq 60$ °C	5.7	≈ 60
metallised paper + wax	• DC filtering	• very small volume • self healable	• $\theta \leq 70$ °C	4.6	150
paper and polyester + oil	• DC filtering • discharges	• small volume • high $\theta \leq 85$ °C	• protection by internal fuses possible	≈ 4	≈ 50
paper and PP + oil	• commutation • fast discharges • IPF • DC filtering	• best loss/volume ratio at high power	• protection by internal fuses possible	≈ 3	≈ 15
polyester and armatures + M/DBT	• DC filtering	• small volume	• $\theta \leq 70$ °C • protection by internal fuses	3.2	70
metallised polyester	• filtering • LV commutation	• high $\theta \leq 85$ °C • very low voltage	• high losses	3.2	60
rough PP + M/DBT	• commutation • snubbers • AC filtering • induction heating	• very low losses		2.4	≈ 2
PP + paper + M/DBT	• commutation • snubbers	• low losses		3.2	10
mica + mineral oil	• decoupling • HF resonance	• very low HF losses • high stability		7.0	2
metallised PP	• LV filtering • commutation • snubbers	• high $\theta \leq 85$ °C • dry dielectric • self healable	• limited dv/dt due to shoopage	2.2	5
segmented metallised PP	• DC filtering • energy storage	• high $\theta \leq 85$ °C • small volume • self protected	• limited dv/dt due to shoopage	2.2	50
segmented metallised rough PP + mineral oil	• DC filtering	• small mass • small volume • self protected • ecologic	• $\theta \leq 70$ °C	2.3	100
segmented metallised PP + gas	• DC filtering	• small mass • small volume	• $\theta \leq 70$ °C	2.2	70
metallised PP and armatures	• HF resonance • induction heating	• very low losses • very high current	• can replace mica	2.2	2

HF high frequency
IPF impulse former (association of inductors and capacitors)
LV low voltage
snubber semiconductor-commutation aid (*RC* protection circuit)
θ temperature

2.3 Capacitor fabrication

2.3.1 Objectives

Research on capacitors has two objectives:

- minimize the volume;
- increase the reliability over the lifetime required by the application.

The reference value is the ***volumetric energy***, ratio of the stored energy at nominal voltage and the volume *V*:

$$W_{\mathrm{V}} = \frac{1}{2}\frac{CU^2}{V}.$$

It is usual to express it in joules per litre.

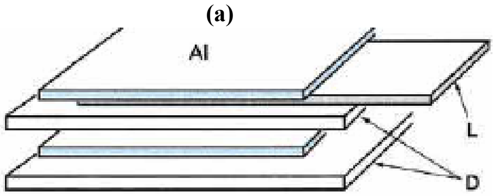
Another significant quantity is the ***voltage gradient***, equal to the ratio of the applied voltage to the dielectric thickness (electric field in the case of infinite plane electrodes), expressed in volt per micrometre.

The volumetric energy is proportional to the dielectric permittivity and to the squared voltage gradient. The permittivity being a physical quantity set by Nature, the manufacturer's know-how will aim to push the dielectrics to always higher voltage gradients as a function of the desired lifetimes. For example, a metallized polypropylene filter capacitor had a volumetric energy of 50 J/l (voltage gradient 100 V/µm) in 1985; in 1994, with this same material, one could obtain 140 J/l (voltage gradient 165 V/µm) thanks to the improvements in manufacturing.

2.3.2 Technological configurations

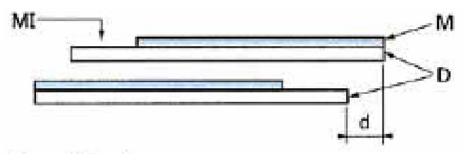
The different technologies used to realize capacitors are schematized in Fig. 4 and the applications are detailed in Table 3.

(a)



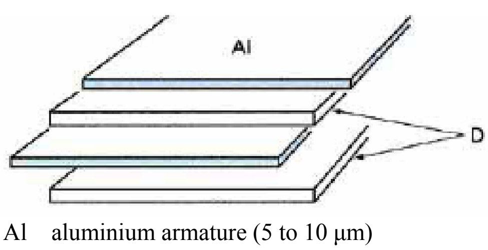
Al aluminium armature (5 to 10 µm)
L copper strip
D dielectrics

(c)



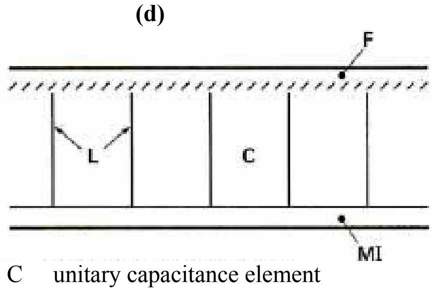
D dielectrics
d margin for shoopage bonding
M metal (10 to 50 nm)
MI insulating margin

(b)



Al aluminium armature (5 to 10 µm)
D dielectrics

(d)



C unitary capacitance element
F region of fused connection
L line uncoated with metal
MI insulating margin

Fig. 4: Different capacitor technologies. a) Capacitor with armatures; connection by strips. b) Capacitor with armatures sticking out. c) Metallized capacitor. d) Segmented metallized film capacitor.

Table 3: Capacitor technologies

technology	main applications	presentation
armatures–strip outputs (Figure 4a), flat windings	• HV filtering (3 to 100 kV) • HV energy storage (3 to 100 kV)	parallelepipedic steel cases or cylindric PP cases
armatures sticking out (Figure 4b), flat windings	• very-high-current commutation • filtering • energy storage • laser discharges	parallelepipedic steel or aluminium cases
armatures sticking out (Figure 4b), cylindric windings	• IPF • HF discharges • snubbers	cylindric aluminium, polypropylene or glass cases; steel cases for IPF
shooped metallised paper (Figure 4c), single or composite dielectric	• filtering • discharges	steel cases
wound shooped metallised film (PP or polyester), segmented or not, dry or impregnated (Figure 4d)	• commutation • DC filtering • AC filtering • snubbers • laser discharges	parallelepipedic or cylindric aluminium, steel, stainless steel or plastic cases
HF high frequency HV high voltage IPF impulse former (association of inductors and capacitors for radars) snubber semiconductor-commutation aid (*RC* protection circuit)		

Capacitors with armatures: In this type of capacitor [Fig. 4(a)], the electrodes consist of an aluminium ribbon of 5 to 10 µm thickness. The most common connection system consists of inserting thin copper strips while winding the element. These strips are thereafter easily connected to the component terminals. It is also very simple to make series–parallel associations between the elements and the two terminals can come out at the same side.

The series inductance (characteristic winding inductance) is inherently high. Nevertheless, some configurations can allow large capacitors to be made (volume bigger than 20 l) with inductances of the order of some tens of nanohenrys. On the other hand, the rather small strip dimension does not allow currents with a r.m.s. value higher than 0.2 A/µF.

Capacitors with armatures sticking out: This other system [Fig. 4(b)] has been created especially for high currents. In fact, it allows soldering of all winding turns in parallel. The series resistance is practically negligible.

The parasitic inductance is due to the loop formed by the element and the capacitor terminals. Moreover, the excellent thermal conductivity of aluminium minimizes the thermal impedance.

Metallized capacitors: In this capacitor [Fig. 4(c)], the electrodes are directly deposited on a dielectric by evaporation under vacuum. These metal layers (aluminium or zinc) have a thickness of 10 to 50 nm, which is quite thin compared to the film thickness of about two micrometres. The external contact of this type of electrode is made by shoopage, a process that consists of spraying a molten metal (generally zinc) that clings to the films after solidifying. This contact is destroyed if one exceeds the current limits (r.m.s. or pulse-like) set by the manufacturer. Beside the volume gained, the electrodes being thin, the main interest of metallized films is their ***self-healing property***.

Under normal operating conditions, when a local dielectric breakdown occurs, the dissipated energy blasts the metallized layer around the fault, thus isolating it. This property cleans the dielectrics of their inevitable weak points and permits using them with ***optimized voltage gradients***. Today, to increase the latter, one uses films with so-called reinforced-edge metallization. The thickness is defined as a function of the metal used and of the dielectric-film thickness, in order to make self-healing easier; at the edge, the metallization is thicker by about 2 mm to ensure good contact with the shoopage.

Segmented or crenellated metallized film capacitors: Some techniques, generally called segmentation or crenellation [Fig. 4(d)], divide the metallized layer into small capacitive elements connected together by means of fuses. This improves the self-healing by energy limitation in case of a breakdown. The fuses increase the reliability, particularly in case of abnormal overvoltages.

Since 1990, advances in ***segmentation techniques*** have contributed to an increase in the operating voltage gradients of metallized films.

- Originally, the segmentation was done by ***electro-erosion***; this method resulted in gross demetallized lines, and only metallized paper resisted this treatment.
- Since the end of the 1970s, ***lasers*** have been used to segment the plastic dielectrics with a good definition (0.1 mm). Today, lasers have an excellent definition (0.01 mm).
- A new method has appeared, called offset because of its similarity with printing techniques.

As opposed to lasers, which segment the metallized film by demetallization, the offset method prints the segmentation pattern on the film with oil, which prevents metal deposition during metallization under vacuum. The difficulty with this method is to control the oil traces. The definition is still inferior to that of the laser method.

With alternating voltage, the film metallization (segmented or not) can corrode electrochemically, as a result of corona effects; if one exceeds the set limits (voltage, temperature), this leads to considerable loss of capacitance.

Generally, with both AC and DC, the ageing of metallized capacitors results in a loss of capacitance. Thus the life expectancy also depends on the capacitance drift limits that the equipment can tolerate (for example, for filter capacitors, one can accept 20%).

2.4 Electrolytic capacitors

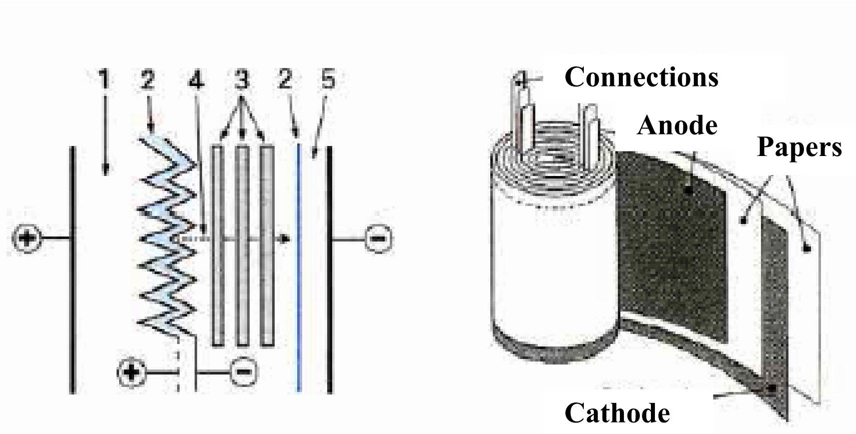
Electrolytic capacitors use alumina as a dielectric, whose characteristics are excellent:

- relative permittivity: $\varepsilon_r = 8.5$;
- dielectric strength: 850 V/µm.

If one could metallize and wind alumina in the same way as polypropylene film, one would obtain capacitors with a volumetric energy of 30 000 J/l, 50 times smaller than high-energy capacitors (paper–castor oil). Unfortunately, this marvellous dielectrics requires a paper-supported electrolyte and a thick cathode (Fig. 5), which reduces the volumetric energy to about 400 J/l. Potentially, the electrolytic capacitor can progress towards higher energies. The electrolyte brings a second drawback: its resistance is high and generates current-limiting losses.

With modern electrolytes, filtering capacitors rated at 350–600 V are used in equipment with power higher than 10 kW. As an example, Figs. 6 and 7 show the characteristics of electrolytic capacitors used in power electronics. These curves reflect, in fact, the electrolyte properties:

- conductivity as a function of the temperature (Fig. 6);
- dielectric losses due to the electrolyte resistance [Fig. 7(c) and 7(d)];
- ageing acceleration due to the effect of temperature [Fig. 7(a)].



1. **anode: aluminium foil**
2. **dielectrics: alumina**
3. **electrolyte impregnated separation paper**
4. **ionic conduction by electrolyte**
5. **cathode: aluminium foil**

Fig. 5: Structure of an aluminium electrolytic capacitor

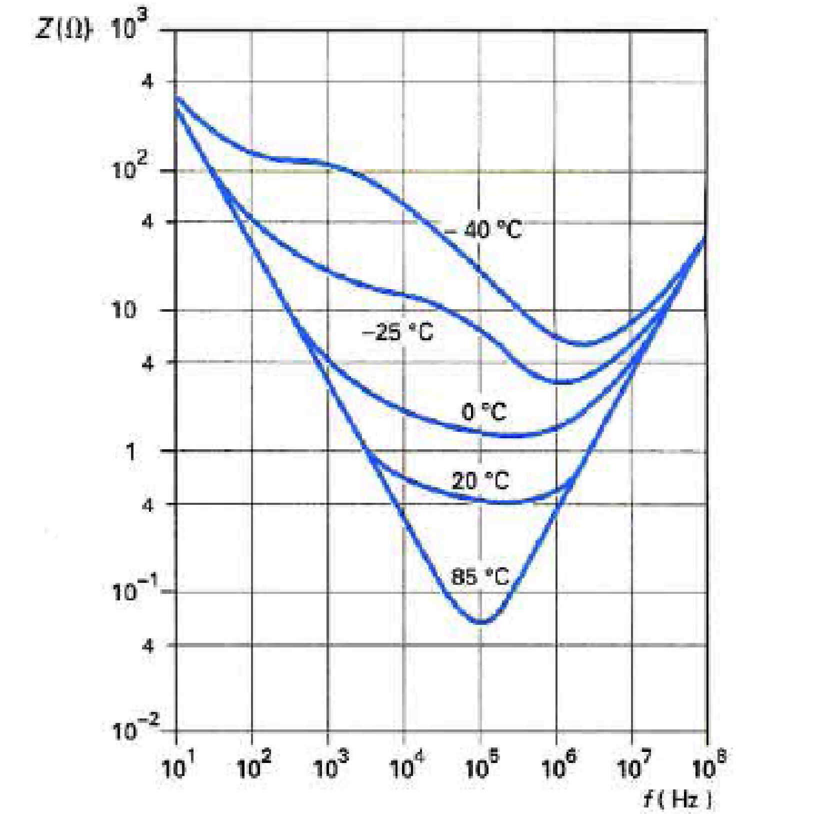
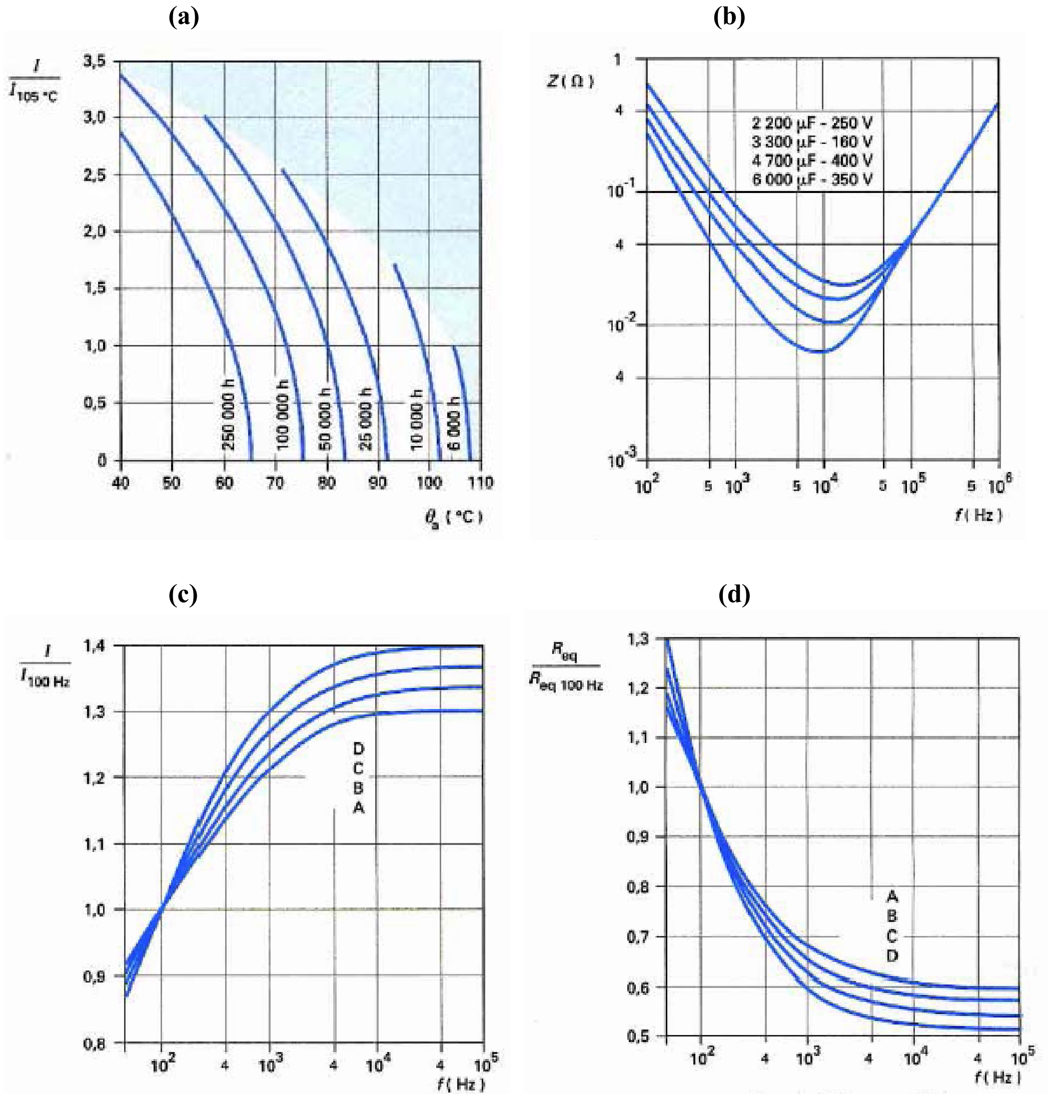


Fig. 6: Impedance variation of an electrolytic capacitor as a function of frequency and temperature



A: *d* = 76.9 mm	**A: *d* = 91.0 mm or 76.9 mm**
B: *d* = 64.3 mm	**B: *d* = 64.3 mm**
C: *d* = 51.6 mm or 91.0 mm	**C: *d* = 51.6 mm**
D: *d* = 35.7 mm	**D: *d* = 35.7 mm**

Fig. 7: Characteristics of aluminium-foil electrolytic capacitors. a) Lifetime as a function of ambient temperature and admissible r.m.s. current. b) Impedance for different capacitor values. c) Alternating-current limit for different capacitor diameters. d) Equivalent series resistance for different capacitor diameters.

The curves of Figs. 6 and 7(b) also show the effect of the series inductance, due to the capacitor winding, on the impedance. Battery assemblies, series–parallel (Fig. 8), have been made up to 3500 V. Generally, the application's current determines the number of parallel elements and its voltage the number of series elements. In spite of balancing resistors, one must slightly reduce the battery operating voltage with respect to a simple multiplication of the number of elements and their nominal voltage, to take into account imbalance, mainly due to temperature differences between the elements.

However, electrolytic capacitors do to some extent withstand overvoltages (1.2 U_n), as opposed to film capacitors (2 U_n) with a plastic or paper dielectric, which require control and protection electronics to avoid dimensioning the series connection according to phenomena whose duration is smaller than 1% of the operating time. The current rating can be multiplied by 2.5 for some models which can be fitted with air or water coolers (for example, Fig. 9).

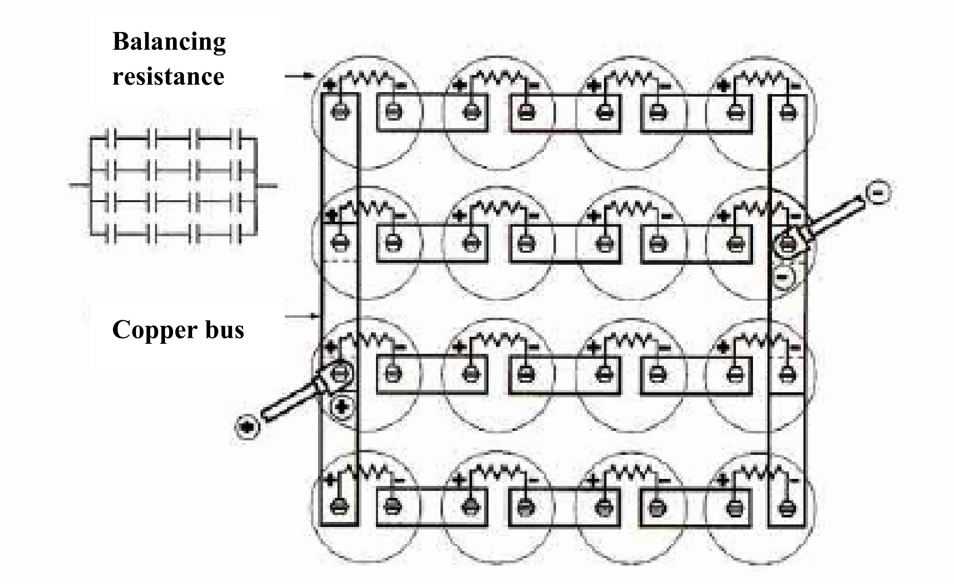


Fig. 8: Battery assembly of electrolytic capacitors

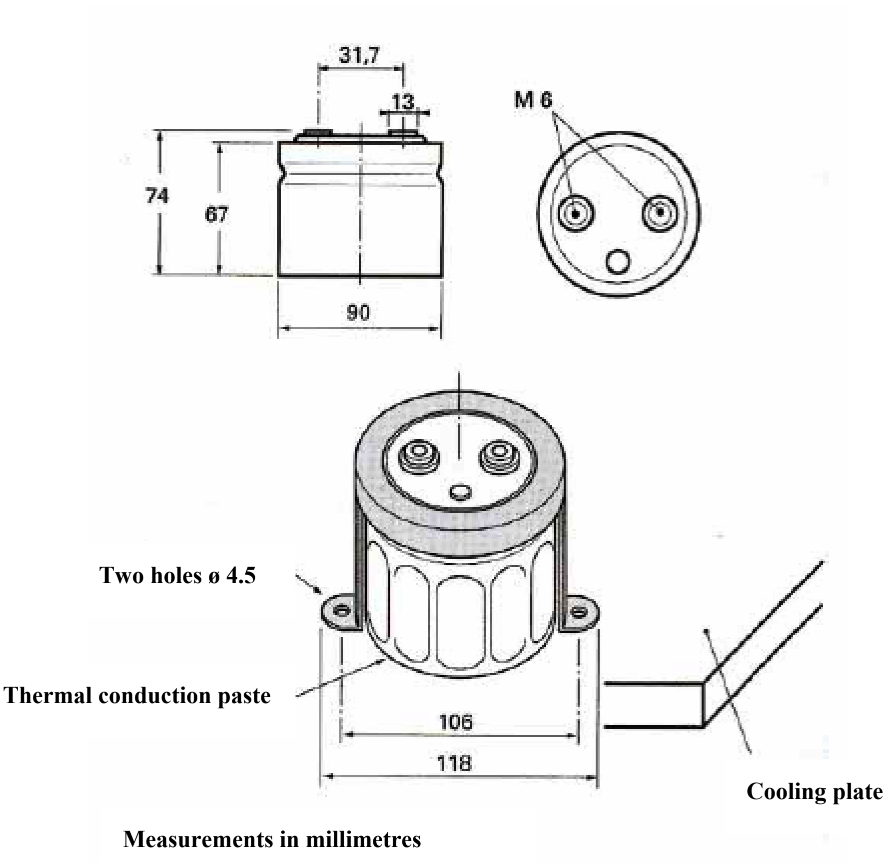


Fig. 9: Mounting of an electrolytic capacitor on a cooling plate

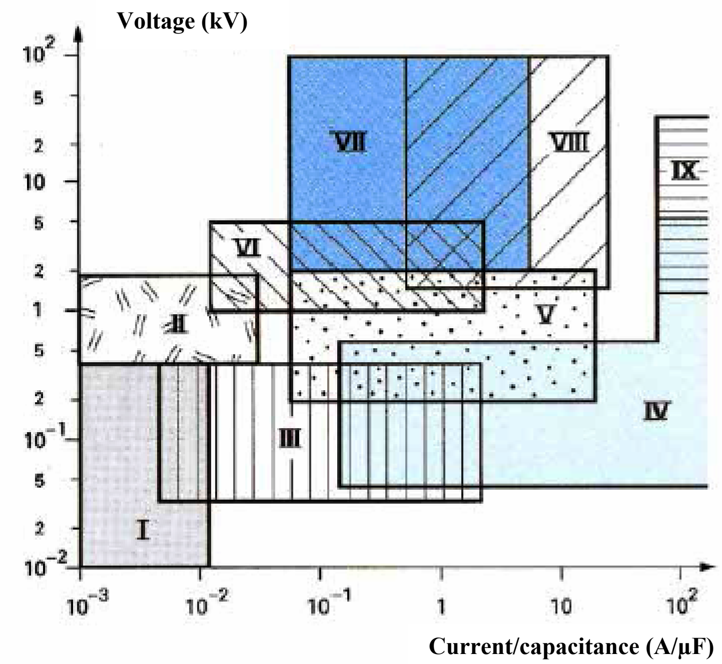
3 Applications and specifications

3.1 Introduction

It is difficult and expensive to manufacture capacitors meeting all the requirements for power-electronic capacitors. As a consequence, manufacturers have been compelled to develop components adapted to each application. Basically, there are two large families of capacitors:

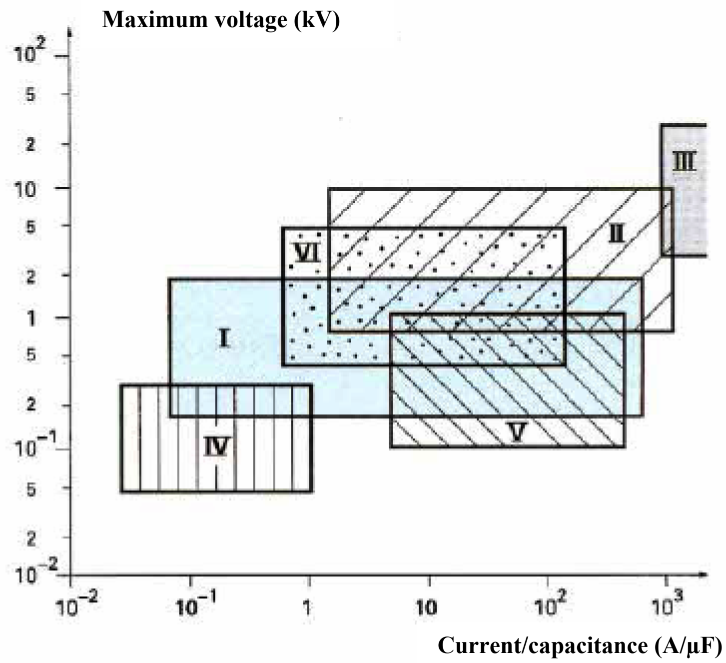
- those whose ***operating voltage*** is ***continuous and unipolar*****:** filtering (Section 3.2), decoupling (Section 3.3), energy storage (Section 3.7);
- those whose ***operating voltage*** is ***alternating:*** harmonic filtering (Section 3.2), commutation (Section 3.4), resonance (Section 3.5), commutation aid or semiconductor protection (Section 3.6).

Figures 10 and 11 show the different technologies, with their voltage application domain and their r.m.s. current-to-capacitance ratio. For thermal and power-dimensioning calculations, it is best to consult the manufacturers with detailed specifications including voltage and current waveforms, environmental conditions (mechanical and thermal), as well as life expectancy. However, the manufacturers' catalogues give indications for reducing the problem. One has to remember as we have seen before, that the losses are of two natures: dielectric and galvanic; the latter are the ohmic losses in the connections.



I electrolytic capacitor
II electrolytic capacitor in series–parallel
III polyester film, dry metallized capacitor
IV dry multilayer ceramic capacitor
V dry metallized polypropylene capacitor
VI polypropylene, metallized impregnated capacitor and metallized impregnated paper capacitor
VII impregnated paper capacitor with armature
VIII paper and film, impregnated capacitor
IX dry HV ceramic capacitor

Fig. 10: Capacitor technologies for continuous voltage



I	**dry metallized polypropylene capacitor**
II	**polypropylene, impregnated film**
III	**ceramic capacitor, HV, HF**
IV	**dry metallized polyester capacitor**
V	**dry polypropylene capacitor with armature**
VI	**MKV type capacitor**

Fig. 11: Capacitor technologies for alternating voltage

Capacitors for ***continuous voltage*** or ***energy storage*** with a low discharge recurrence (a few Hz) are subjected to low reactive powers and the dielectric losses are not dominant; one can thus tolerate dielectrics with rather high losses (paper, type-II ceramic, electrolyte systems, etc.). The series resistance and the r.m.s. current are the essential heating factors.

On the other hand, for ***alternating voltage***, both loss components are important. Table 4 gives r.m.s. values for currents, and reactive and loss powers for some usual waveforms.

In Section 2, Tables 2 and 3 specify the application domains of the dielectrics and the technologies used, respectively.

3.2 Filter capacitors for rectifiers at industrial frequencies

These capacitors are used in low-pass filters. One can compare the oscillating voltage to a sinusoid of frequency pf, p being the pulse number of the circuit used (most often two for single-phase, three or six for three-phase systems) and f the network frequency. These capacitors only have to withstand unipolar voltages, the peak value of the oscillating voltage always remaining smaller than the average rectified voltage.

The ***main constraint*** for these capacitors is the continuous voltage, equal to the average rectified voltage, taking the presence of the capacitor into account. To this voltage, one must add the peak value of the oscillating voltage, the sum of both defining the nominal operating voltage U_n of the capacitor used.

Table 4: Calculation of the current r.m.s. value, the reactive power and the loss power for a metallized polypropylene capacitor for waveforms often seen in commutations

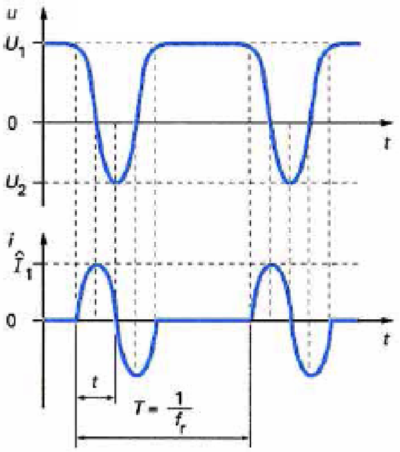
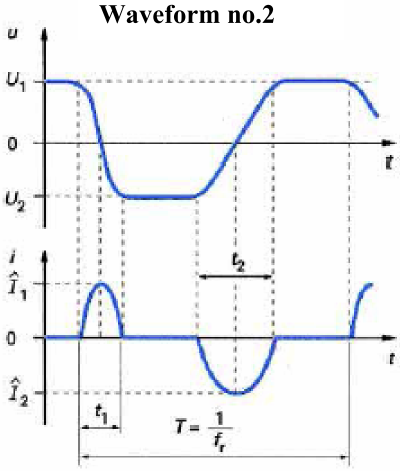
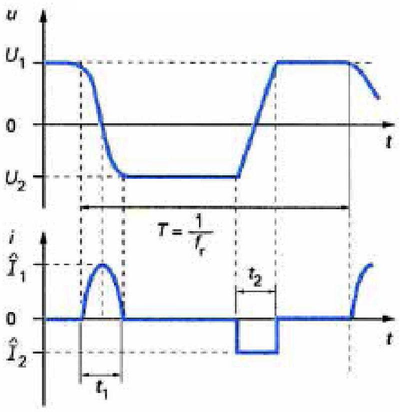
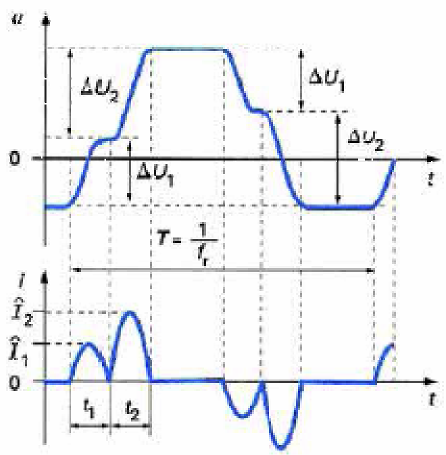
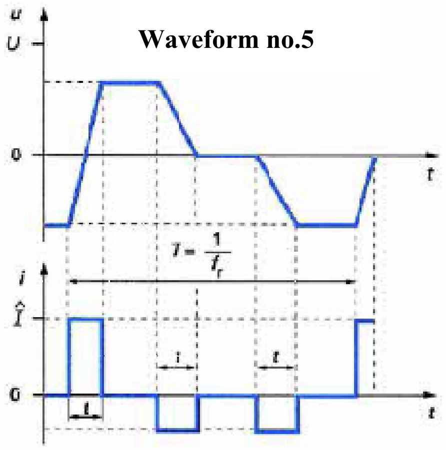
Waveform no.1 	$\hat{I} = \frac{\pi}{2} C \left(\frac{U_1 + U_2}{t} \right)$ $I_{rms} = \frac{\pi}{2} C (U_1 + U_2) \sqrt{\frac{f_r}{t}}$ $Q = \frac{\pi}{4} C (U_1 + U_2)^2 f_r$ $f = \frac{1}{2t}$ $P = Q (2 + kf)\, 10^{-4}$
Waveform no.2 	$\hat{I}_1 = \frac{\pi}{2} C \left(\frac{U_1 + U_2}{t_1} \right)$ $\hat{I}_2 = \frac{\pi}{2} C \left(\frac{U_1 + U_2}{t_2} \right)$ $I_{rms} = \frac{\pi}{2} C (U_1 + U_2) \sqrt{\frac{f_r}{2t_1} + \frac{f_r}{2t_2}}$ $Q = \frac{\pi}{4} C (U_1 + U_2)^2 f_r$ $f_1 = \frac{1}{2t_1}$ $f_2 = \frac{1}{2t_2}$ $P = \frac{Q}{2} \left[(2 + kf_1)\, 10^{-4} + (2 + kf_2)\, 10^{-4} \right]$
Waveform no.3 	$\hat{I}_1 = \frac{\pi}{2} C \left(\frac{U_1 + U_2}{t_1} \right)$ $\hat{I}_2 = C \left(\frac{U_1 + U_2}{t_2} \right)$ $I_{rms} = C (U_1 + U_2) \sqrt{\frac{\pi^2 f_r}{8t_1} + \frac{f_r}{t_2}}$ $Q = \frac{\pi}{4} C (U_1 + U_2)^2 f_r$ $f_1 = \frac{1}{2t_1}$ $f_2 = \frac{1}{2t_2}$ $P = \frac{Q}{2} \left[(2 + kf_1)\, 10^{-4} + (2 + kf_2)\, 10^{-4} \right]$
$\tan\delta = \tan\delta$ dielectric $+ CR_s \cdot 2\pi f$ $\tan\delta$ dielectric $= 2 \cdot 10^{-4}$ for polypropylene $CR_s \cdot 2\pi = k \cdot 10^{-4}$ galvanic component $\tan\delta = (2 + kf)\, 10^{-4}$	f sinusoidal frequency f_r recurrent waveform frequency

Table 4 (continued): Calculation of the current r.m.s. value, the reactive power and the loss power for a metallized polypropylene capacitor for waveforms often seen in commutations

Waveform no.4  	$\hat{I}_1 = \frac{\pi}{2} C \frac{\Delta U_1}{t_1}$ $\hat{I}_2 = \frac{\pi}{2} C \frac{\Delta U_2}{t_2}$ $I_{rms} = \frac{\pi}{2} C \sqrt{\left(\frac{\Delta U_1^2}{t_1} + \frac{\Delta U_2^2}{t_2}\right) f_r}$ $Q = \frac{\pi}{4} C \left(\Delta U_1 + \Delta U_2\right)^2 f_r$ $f_1 = \frac{1}{2t_1}$ $f_2 = \frac{1}{2t_2}$ $P = \frac{\pi}{4} C f_r \left[\Delta U_1^2 (2 + kf_1)\, 10^{-4} + \right.$ $+ \Delta U_2^2 (2 + kf_2)\, 10^{-4} +$ $\left. + 2\Delta U_1 \Delta U_2 (2)\, 10^{-4}\right]$
Waveform no.5  	$\hat{I} = C \frac{2U}{t}$ $I_{rms} = CU \sqrt{\frac{6 f_r}{t}}$ $Q = \frac{3\pi}{4} C U^2 f_r$ $f = \frac{1}{2t}$ $P = Q\,(2 + kf)\, 10^{-4}$
$\tan\delta = \tan\delta \text{ dielectric} + CR_s \cdot 2\pi f$ $\tan\delta \text{ dielectric} = 2 \cdot 10^{-4}$ for polypropylene $CR_s \cdot 2\pi = k \cdot 10^{-4}$ galvanic component $\tan\delta = (2 + kf)\, 10^{-4}$	f sinusoidal frequency f_r recurrent waveform frequency

The ***second constraint*** concerns the r.m.s. value of the current crossing the capacitor, proportional to *f* and $U_\sim$, this last quantity representing the r.m.s. value of the oscillating voltage. For a given current, crossing a given capacitor, the product $U_\sim f$ should be constant, but, due to various phenomena linked to the skin effect, the dissipating power, etc., $U_\sim$ decreases more slowly than expected as the frequency increases. This fact is illustrated by the curve in Fig. 12; it corresponds to an oil-impregnated paper capacitor; one notes that, in this figure, $U_\sim$ is defined as a function of the nominal operating voltage U_n. However, certain manufacturers prefer to specify the maximum allowable current directly.

The ***series inductance*** must not be taken into account in such an application, its effects being negligible for capacitors of present-day technology, used for rectifier filtering operating at a power supply with a frequency smaller than or equal to 400 Hz. On the other hand, one must examine the temperature influence and take into consideration the operating-voltage reduction as a function of the temperature increase.

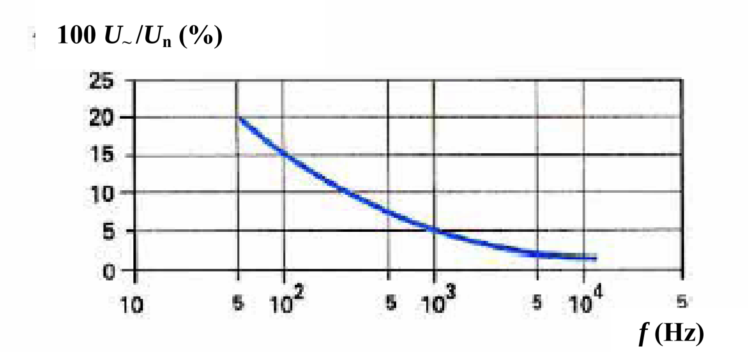


Fig. 12: Alternating voltage $U_{\sim}/U_n$ applicable to an oil-impregnated paper capacitor as a function of the frequency f

If the capacitor constitutes an element of both the rectifier output filter and the input-filter capacitor of a chopper or an inverter, one must take into account the fast commutations required by the latter. The capacitors thus suffer much more severe constraints and the previous conclusions are no longer valid. One must, in this case, use capacitors aimed at decoupling (Section 3.3), and this holds also if sizeable components of high frequencies exist in the voltage to be filtered.

3.3 Decoupling capacitors

The role of these capacitors is quite similar to that of the preceding ones: they constitute links of theoretically zero impedance in circuits where occasionally super-imposed continuous and alternating components can be found. But, as opposed to filter capacitors, the peak value of the alternating component can be larger than the continuous voltage. Thus the resulting capacitor terminal voltage might be inverted.

Among the ***principal uses*** of these capacitors, one can mention:

- the input and output filters of decoupled power supplies;
- the input filters of voltage-source converters;
- the decoupling of parasitic supply-cable inductances and batteries (autonomous supplies).

One could classify in the same category capacitors aimed at improving the waveform of converters with forced commutations. But these capacitors rather belong to the class of resonant components, which will be studied in Section 3.5. In the case of input filters, the functions of converter decoupling and rectifier filtering are often assured by the same capacitors. One speaks of ***filter capacitors.***

The variable components applied to decoupling capacitors, in general, are not mainly sinusoidal such as the ones met at the terminals of the filter capacitors examined in Section 3.2. On the contrary, the high dv/dt values (in the order of 500 to 1000 V/µs) give rise to very considerable peak current values that one must take into account.

Thus, these capacitors must be designed to meet constraints very similar to those withstood by commutation capacitors (Section 3.4).

Since the advent of IGBTs (Insulated-Gate Bipolar Transistors), the parasitic inductance of the decoupling circuit becomes critical; these transistors have very rapid commutation characteristics (high di/dt) which generate destructive overvoltages proportional to the circuit inductance. The cabling of capacitors with low series inductance is performed by parallel plates; this results in global filter-circuit inductances smaller than 50 nH. Figures 13 and 14 show typical examples of converter input-filter capacitors.

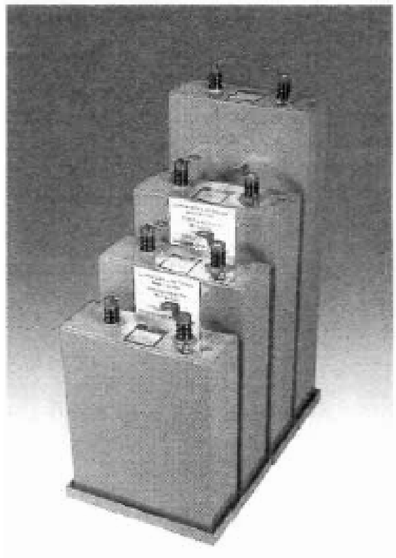


Fig. 13: Filter capacitor for the TGV Atlantique (2000 μF, 1800 V). Evolution from metallized wax-impregnated paper (125 × 340 × 787 mm^3, 49 kg) to segmented metallized rape-oil impregnated polypropylene film, fourth generation (125 × 340 × 430 mm^3, 21 kg).

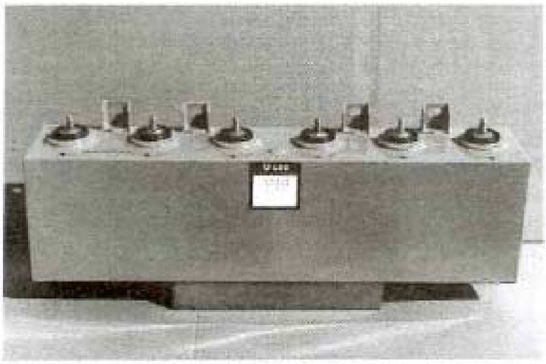


Fig. 14: Filter capacitor for an IGBT traction converter (tramway). Segmented metallized rape-oil impregnated polypropylene technology. The 3 element with flat terminals give this capacitor a series inductance < 30 nH (3150 μF, 1000 V, 690 × 140 × 185 mm^3).

3.4 Commutation capacitors

These capacitors are intended to deliver current pulses necessary to block thyristors. It is these that withstand the most severe constraints, because of complex applied waveforms (Table 4).

Apart from dimension, reliability and cost considerations, the ambient temperature constitutes an important constraint that limits the performances of a given capacitor type, independently of the electrical constraints. The latter depend a lot on applied voltage and current waveforms, which are defined in each case. In a general manner, the constraints are either of a dielectric nature, that means linked to the voltage, or due to the ohmic losses, that means linked to the current.

3.4.1 Dielectric constraints

Applied continuous voltage: independently from an average voltage different from zero, the applied voltages can be unipolar.

Voltage r.m.s. value: it must be defined in each case.

Voltage peak value: it must always remain smaller than the nominal voltage U_n; a security margin must be kept to account for eventual non-repetitive voltage peaks (start-up, load variations, etc.).

Voltage variation rate, or d*v*/d*t*, sometimes incorrectly called ***voltage gradient***: the dielectric losses increase significantly with high values of this quantity; what plays a role is not the duration of the voltage levels, but that of the transitions; it is the same for all hysteresis phenomena, whether magnetic, electrical or mechanical.

For each capacitor type, the manufacturers give an acceptable maximum d*v*/d*t* value. But it is acceptable to exceed this value if the peak voltage remains smaller than U_n, and the sum of the dielectric losses due to d*v*/d*t* and to U_{max} remains constant.

In practice, a simple rule leads to an increase of the considered capacitor d*v*/d*t* value by the U_n/U_{max} ratio (U_{max} being the maximum voltage).

The nominal value U_n of the chosen capacitor may have to be derated as a function of the ambient temperature of about 1%/K above 85ºC, provided the case itself can withstand the over-heating.

3.4.2 Constraints due to ohmic losses and to the frequency

The essential quantity is, of course, the ***r.m.s. value*** I_{rms} of the current traversing the capacitor. But, in the calculation, one has to account for the derating induced by the temperature and the frequency.

For the temperature, one can adopt the curve of Fig. 15. The frequency has a more complex impact, because this variable acts both on the dielectric losses and the skin effect, quantities which are themselves functions of the temperature.

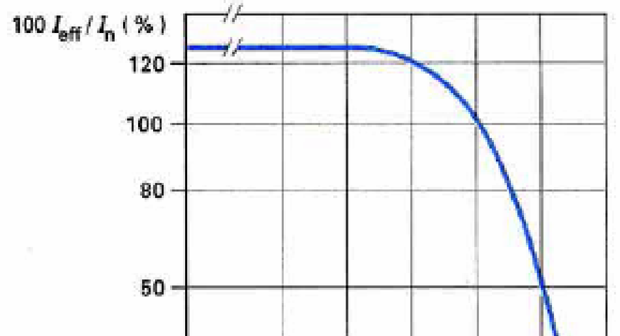


Fig. 15: Current density I_{rms}/I_n of a polypropylene capacitor as a function of the ambient temperature θ

In any case, the current r.m.s. value must never exceed the security limits given by the manufacturer. This calculation is easily done if the waveform can be obtained with an oscilloscope during the tests.

To the current r.m.s. value, one must add another important characteristic: its ***peak value***. If dealing with impulses, one must take their duration into account.

Finally, knowing the ***reactive power*** allows one to obtain an estimate of the loss power, using the tan δ value specified by the manufacturer in the data sheets.

3.5 Resonance capacitors

As their name indicates, these capacitors are used to tune series or parallel resonant circuits used in industrial medium-frequency systems (resonant converters, tuning circuits for HF furnaces, etc.). The operating frequencies are thus comprised between several hundred hertz and several hundred kilohertz.

The capacitors used in ferro-resonant controllers, either for tuning, or for filtering (see also Ref. [2], belong to this category).

As opposed to components met in the previous sections, where the capacitance is not critical, the capacitors studied here must have relatively tight tolerances. One often requires $\Delta C/C \leq 2\%$. This can lead to the exclusion of certain dielectrics.

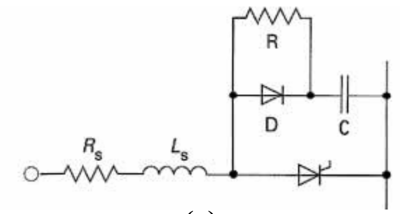
As opposed to previously described capacitors, the components used here operate, in principle, under a pure alternating voltage, without a superimposed continuous component.

The currents that traverse these capacitors are, according to their use, quite regular; the only constraints to take into account are the voltage peak value which must remain smaller than the maximum allowable voltage and the r.m.s. current, depending on the dielectric losses and on the ohmic losses, i.e., the series resistance.

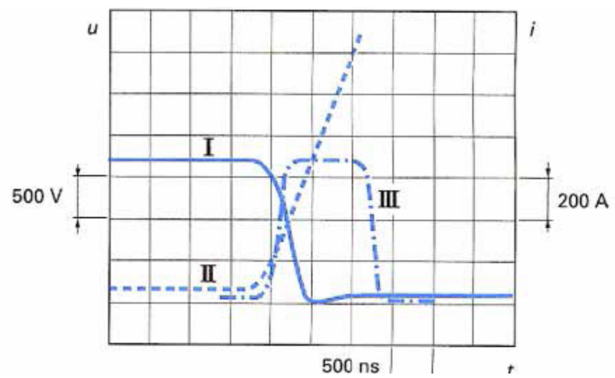
3.6 Capacitors for semiconductor commutation assistance

In order to minimize semiconductor commutation losses and to keep them from exceeding their d*v*/d*t* limits, one uses RCD networks (resistor, capacitor, diode; Fig. 16). The capacitor temporarily absorbs the load current at the switch opening; it thus withstands strong pulsed currents; the series inductance L_s must be minimum.

In the case of GTO thyristors, the parasitic inductance of the RCD circuit is very critical (< 100 nH), which led to the development of capacitors with a very low specific inductance for this application (< 10 nH).



(a)



I **current in the thyristor GTO(d*i*/d*t* = 1600 A/µs)**
II **voltage across the capacitor**
III **current in the capacitor**

(b)

Fig. 16: Semiconductor switch. a) Usual construction. b) Voltage and current waveforms.

3.7 Energy-storage capacitors

The aim is to accumulate a maximum of energy in a minimum of volume and to discharge this energy in very short times, thus with very high currents.

The typical applications are lasers, lightning-wave or nuclear-electromagnetic-pulse simulators, etc. Figure 17 shows an example of such a capacitor.

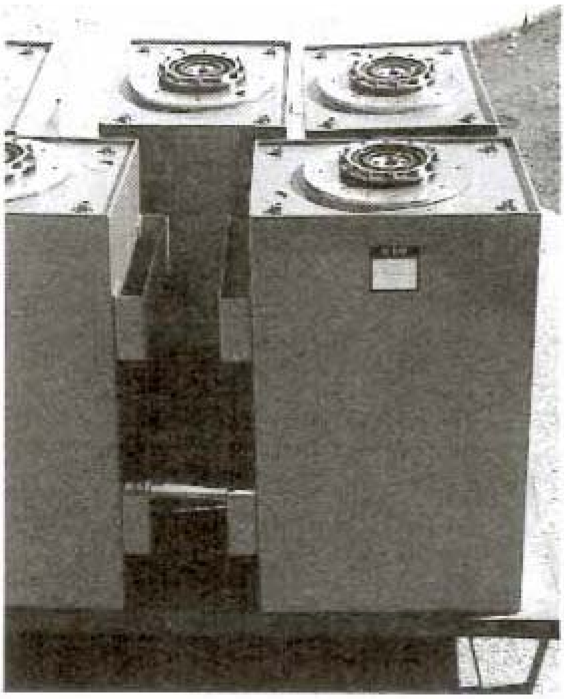


Fig. 17: Example of an energy-storage capacitor (50 kJ, 10 kV, peak current 60 kA, volumetric energy 600 J/l). Castor-oil-impregnated paper technology.

Generally, one uses the dielectrics at their maximum strength; the lifetime is reduced (for example, for telemetric lasers, it corresponds to 500 000 charge–discharge cycles).

Pulse formers are a special case. They consist of an association of LC cells to generate current pulses of rectangular shape and a given duration (10 μs to 1 ms). The main application is the power supply of radar tubes.

References

[1] C. Menguy, Mesure des caractéristiques des matériaux isolants solides, Techniques de l'Ingénieur, traité Génie électrique, D 2 310, Aug. 1997.

[2] J. Bonfante and F. Gallin, Régulateurs statiques de tension alternative, Techniques de l'Ingénieur, traité Génie électrique, D 3 500, Mar. 1990.

PART III

Supercapacitors and applications

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Abstract
New components called supercapacitors are presented from the point of view of the related new field of applications, namely, energy storage. The principle and properties are briefly presented, as well as a sizing method on the basis of energy requirements. In this field, energy considerations lead to efficiency calculations for constant current and constant power charge and discharge. Finally, the power availability of a supercapacitive tank will be discussed for a given energetic efficiency.

1 Introduction

Supercapacitors are components for energy storage, used where both energy and power density are needed. Even if their energy density is ten times lower than the energy density of batteries, supercapacitors offer new alternatives for applications where energy storage is needed. The application fields of supercapacitors can be divided into two families. In the first one, supercapacitors are used as main energy sources. The constraints placed on supercapacitors are due to energy and power requirements.

The second application field for supercapacitors is as energy buffers, where these components are used as complementary storage devices to minimize power constraints on an energy source [1–4]. The aim here is to compensate the power variations of a load in order to draw power from the main energy source at a rate as constant as possible. In such applications, energy requirements are not so critical for the design of the supercapacitive tank, but there is the need to match power requirements. Regarding the power density of supercapacitors as given by manufacturers, they are well suited for such applications as load levelling in elevators, hybrid vehicles, compensation of weak distribution networks, etc.

Whatever the application, the sizing of a supercapacitive tank depends on two criteria, which are linked and have to be satisfied simultaneously: energy and power requirements. Energy requirements define mainly the volume and the size of the supercapacitive tank, as the amount of energy to be stored defines the number of supercapacitors. However, even if the energy and the power provided to or absorbed by a source are linked by differentiation or integration operations, the sizing of a supercapacitive tank would be incomplete if energy requirements only were considered. The power constraints applied to these components have to be compatible with their power availability [5]. The main difficulty is then to define the power availability of supercapacitors, versus the energy these components are able to store.

The link between the stored energy and the power availability of supercapacitors is their energy efficiency, which defines internal losses within the components during their charge and discharge. The aim of this paper is to define the energy efficiency of supercapacitors during their discharge. The target is to be able to correctly size a supercapacitive tank with respect to its energy and power requirements. In other words, the target is to be able to store a given amount of energy, with a pre-defined power availability.

2 Principle of supercapacitors

2.1 Structure

Supercapacitors are electrochemical double layer capacitors. Their internal structure is shown in Fig. 1. In these components, energy is stored by charge transfer at the boundary between electrodes and electrolyte. The amount of stored energy is a function of the:

- electrode surface,
- size of the ions,
- electrolyte decomposition voltage.

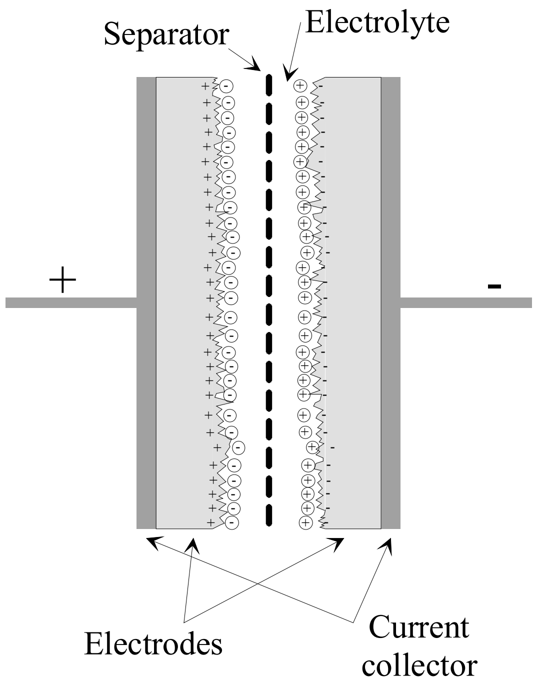

Fig. 1: Internal structure of a supercapacitor

2.2. Technology

The electrodes are made of activated carbon on foil:

- large surface area
- define the energy density

Current collectors:

- high conduction elements

Membrane:

- prevents electrical contact between the electrodes
- can be crossed by the charged ions

Electrolyte:

- supplies and conducts ions
- dissociation voltage of organic electrolytes less than 3 V
- organic electrolytes have a low ionic conductivity (reduced power capability).

3 Sizing of a supercapacitive tank

3.1 Sizing of a supercapacitive tank with energy requirements

3.1.1 Requirements

The first step in the sizing of a supercapacitive tank is to identify the energy and the power requirements. These two specifications define the required number of supercapacitors, but do not have the same constraints. The energy profile defines the amount of energy that the supercapacitive tank has to store during the charge and to provide during the discharge. The required number of supercapacitors is directly linked to the energy profile.

The power profile defines the constraints applied to the supercapacitive tank. As the supercapacitors have a limited power density, this last parameter has to be taken into account and compared with the power that the supercapacitive tank has to manage. If this power is not compatible with the power density of the components, then the number of supercapacitors has to be increased in order to limit the charge and discharge power per supercapacitor.

As an example, we can consider an application where a supercapacitive tank has to be able to deliver 50 kJ (13.9 Wh). This energy is called usable energy and represents the energy requirements. Once the energy requirements are known, the power requirements have to be identified. The stored energy can be consumed in ten seconds, or in a few minutes; this does not lead to the same power range. For the example we are considering, the supercapacitive tank has to deliver 50 kJ in five seconds. Given the 50 kJ stored energy, the constant discharge power is 10 kW. The discharge time (5 s) thus defines the power rating of the supercapacitive tank. These requirements are summarized in Table 1.

Table 1: Energy and power requirements

Usable energy	Time allowed for discharge
50 kJ	5 s

3.2 Main design principle using energy requirements

Supercapacitors can be sized simply on the basis of the energy requirements only, considering them as ideal components, with no internal losses and without any relaxation phenomena [6]. The main equations that define the properties of supercapacitors are

$$i_c = C\frac{du_c}{dt} \text{ and } W = \frac{1}{2}Cu_c^2, \tag{1}$$

where i_c is the current through the supercapacitor, and u_c is the voltage across it. The parameter C is its capacitance, and W is the stored energy. This energy is a function of the voltage across the component. The voltage across a supercapacitor has to be limited to a maximum value U_M (typically 2.5 V), in order to avoid any electrochemical reactions in the component that could shorten its lifetime. Under this condition, the stored energy is at its maximum W_M when the voltage is at its maximum value U_M.

The voltage across the component has then to be decreased from its maximum allowable value U_M to 0 V in order to use the total amount of stored energy. This is not possible because the current provided by the supercapacitor would then be infinite. For this reason, the minimum voltage when discharging the component has a lower limit. All the energy stored will not be used. It is necessary to define the parameter *d*, that is the ratio between the minimum allowed voltage U_m once the discharge has ended, and the maximum voltage U_M that defines a full charge of the component. The parameter *d* is expressed in per cent and is called **voltage discharge ratio**:

$$d = \frac{U_m}{U_M} 100 \,. \tag{2}$$

The usable energy W_u of a supercapacitor is then only a part of the total stored energy W_M:

$$W_n = \frac{1}{2} C U_M^2 - \frac{1}{2} C U_m^2 = \underbrace{\frac{1}{2} C U_M^2}_{W_m} \left[1 - \left(\frac{d}{100} \right)^2 \right] \,. \tag{3}$$

For a supercapacitive tank that contains *N* supercapacitors, the usable energy W_{bu} is the product of the usable energy of each supercapacitor by the number *N*. Assuming that the supercapacitors are identical, then the usable energy of the tank is defined by the equation:

$$W_{bu} = N W_u \,. \tag{4}$$

Using Eqs. (3) and (4), it is then possible to identify the number of supercapacitors needed for a supercapacitive tank that has to manage a usable energy W_{bu}:

$$N = \frac{2 W_{bu}}{C U_M^2 \left[1 - \left(\frac{d}{100} \right)^2 \right]} \,. \tag{5}$$

Regarding the requirements defined in Table 1, we propose in Table 2 various possible sizings of a supercapacitive tank, depending on the voltage discharge ratio *d*. It has to be emphasized that this sizing is achieved with energy requirements only. The total stored energy of the tank W_{bM} is higher than the usable energy, because of the limitation of the voltage variation of the supercapacitive tank. It is much higher than the chosen voltage discharge ratio *d*.

Table 2: Sizing of the supercapacitive tank taking into account energy requirements only

N	*d* (%)	Volume (l)	Weight (kg)	W_{bM} (kJ)
9	50	3.78	4.72	73.12
10	60	4.20	5.25	81.25
13	70	5.46	6.82	105.62

$W_{bu} = 50$ kJ (13.9 Wh), $C = 2600$ F, $U_M = 2.5$ V.

3.3 Energy efficiency of a supercapacitor

3.3.1 Power electronics interfaces

It is now well established that supercapacitors cannot be directly connected to their load, because of their voltage variation during charge and discharge [Eq. (1)]. Most of the applications require a constant voltage level. This is why power converters are needed to interface the supercapacitive tank with its load, as presented in Fig. 2(a).

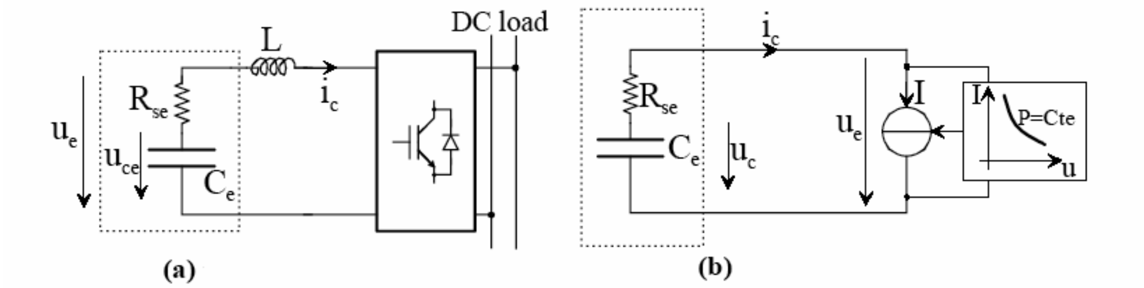

Fig. 2: Power electronics interfaces for the discharge of supercapacitors. a) Power electronics interface. b) Constant power discharge.

Another aspect of the use of power converters is the control of the discharge current. This also controls, according to Eq. (1), the voltage variations across the supercapacitors. It controls the energy provided by the supercapacitive tank to its load. Regarding the controlled current discharge, two main modes can be considered: discharge at constant current, and discharge at constant power. For this last discharge mode, presented in Fig. 2(b), the current taken from the supercapacitive tank is adjusted to maintain the product $u_e.I$ constant. As the supercapacitive tank is discharged, the voltage u_e decreases. The current I must then be increased.

The topologies presented in Fig. 2 take into account the series resistor R_{se} of the supercapacitive tank. As a consequence, for a discharge current i_c, the voltage u_e across the supercapacitive tank is lower than the voltage u_{ce} across the equivalent ideal capacitor C_e. Assuming that all the supercapacitors of the storage tank are connected in series, the equivalent capacitance C_e, the equivalent resistor R_{se}, and the equivalent maximum voltage across the ideal capacitor C_e are given by Eq. (6), [7]:

$$C_e = \frac{C}{N} \quad R_{se} = NR_s \quad u_{ce} = Nu_c \tag{6}$$

where C is the capacitance, R_s is the series resistor, and u_c is the voltage of one single supercapacitor. N is the number of series connected components, assuming that all the supercapacitors are connected in series and have identical parameters. The energy and power constraints are then the same for all the components.

For the constant current discharge, the current i_c is set constant and is positive according to the convention indicated in Fig. 2. For the constant power discharge, the current i_c is set to correspond to the equation:

$$i_c = \frac{P}{u_e} \tag{7}$$

where P is the constant power that the supercapacitive tank has to provide. The supercapacitors are assumed to be initially charged, with the maximum voltage $u_{ce} = U_{Me} = NU_M$. The voltage at the end of the discharge is defined by the voltage discharge ratio d, as shown in Eq. (2).

3.3.2 Constant current discharge

The voltage variation across the ideal equivalent capacitor C_e (Fig. 2) is defined with the relation:

$$u_{ce} = U_{Me} - \frac{1}{C_e} i_c t \Rightarrow u_c = U_M - \frac{1}{C} i_c t \,. \tag{8}$$

The time for discharging the supercapacitive tank from $U_{ce} = U_{Me}$ to $U_{ce} = U_{Med}/100$ is

$$T_{ch} = C_e \frac{U_{Me}}{i_c} \frac{100 - d}{100} = C \frac{U_M}{i_c} \frac{100 - d}{100} . \tag{9}$$

The energy that is then provided by the supercapacitive tank during its discharge is the usable energy W_{bu} defined by Eq. (4). During the discharge of the supercapacitor, energy is dissipated into the equivalent series resistor R_{se}:

$$W_r = \int_0^t R_{se} i_c^2 dt = R_{se} i_c^2 t = N R_s i_c^2 t . \tag{10}$$

Regarding the duration T_{ch} of the discharge in Eq. (9), the energy dissipated into the series resistors of the tank is

$$W_r = R_{se} C_e U_{Me} i_c \frac{100 - d}{100} = N R_s C U_M i_c \frac{100 - d}{100} . \tag{11}$$

The total energy injected into the power converter in Fig. 2 is the difference between the energy W_{bu} provided by the supercapacitive tank and the energy lost in the series resistors R_s:

$$W_T = W_{bu} - W_r . \tag{12}$$

The energy efficiency of the discharge with a constant current is the ratio between W_T and the energy W_{bu} provided by the supercapacitive tank. It can be expressed as a function of the discharge current or as a function of the discharge time:

$$\eta_i = \frac{W_T}{W_u} = 1 - 2R_s \frac{i_c}{U_M} \frac{100 - d}{100} = 1 - 2R_s C \frac{1}{T_{ch}} \frac{100 - d}{100 + d} . \tag{13}$$

3.3.3 Constant power discharge

The main equation that can be obtained from Fig. 2 and Eq. (6) is

$$u_e = u_{ce} - R_{se} i_c \Rightarrow N R_s i_c^2 - N u_c i_c + P = 0 . \tag{14}$$

From this equation, the current during the discharge with a constant power can be expressed as

$$i_c = \frac{1}{2R_s} u_c \pm \frac{1}{2R_s} \sqrt{\Delta} \quad \text{with} \quad \Delta = u_c^2 - 4 \frac{R_s P}{N} . \tag{15}$$

The two possible solutions for i_c are real if Δ is positive. This is the case with the condition:

$$u_c \geq 2 \sqrt{\frac{R_s P}{N}} . \tag{16}$$

The variation range of the voltage u_c across one single capacitor has to be limited in order to satisfy Eq. (16). This is a new limitation on the minimum voltage that can be obtained during the discharge of the component. This is derived from the definition of the voltage discharge ration d, taking into account the condition (16). Then, the minimum voltage discharge ratio is

$$d_{min} = \frac{u_{cm}}{U_M} 100 = 200 \frac{\sqrt{\frac{R_s P}{N}}}{U_M} . \tag{17}$$

As a consequence, the maximum value $W_{bu\ max}$ of the usable energy of the supercapacitive tank as defined in Eq. (4) is limited by this new condition:

$$W_{\mathrm{bu\,max}} = \underbrace{N\frac{1}{2}CU_{\mathrm{M}}^{2}}_{W_{\mathrm{M}}}\left(1-4\frac{R_{\mathrm{s}}P}{NU_{\mathrm{M}}^{2}}\right). \tag{18}$$

The usable energy of a supercapacitor during a discharge with a constant power is limited by the discharge power P. The choice of a voltage discharge ratio cannot be arbitrary here. It is limited by the condition (17). It can be seen that the discharge with a constant power is impossible for the condition $d_{\min} > 100\%$. Considering Eq. (18), this leads to another limiting condition:

$$P_{\max} \frac{1}{4}\frac{NU_{\mathrm{M}}^{2}}{R_{\mathrm{s}}}. \tag{19}$$

These specific limits are particular to the discharge with a constant power. Once they are defined, it must be decided which of the two possible solutions defined in Eq. (15) for the current i_{c} has to be chosen. This can be achieved by the calculation of the derivative of Eq. (14). The analysis of this new equation leads to the following condition:

$$i_{\mathrm{c}} < \frac{u_{\mathrm{c}}}{2R_{\mathrm{s}}}. \tag{20}$$

As a consequence, the only solution to be considered for the expression of the current i_{c} is

$$i_{\mathrm{c}} = \frac{1}{2R_{\mathrm{s}}}u_{\mathrm{c}} - \frac{1}{2R_{\mathrm{s}}}\sqrt{u_{\mathrm{c}}^{2} - 4\frac{R_{\mathrm{s}}P}{N}}. \tag{21}$$

From Eqs. (14) and (21), the total voltage u across one of the supercapacitors connected in series can be expressed as

$$u = \frac{1}{2}u_{\mathrm{c}} + \frac{1}{2}\sqrt{u_{\mathrm{c}}^{2} - 4\frac{R_{\mathrm{s}}P}{N}}. \tag{22}$$

Once the links between the voltages u, u_{c} and I are known, it is possible to identify the energy provided by the current source. Two methods can be used. The first one is the easier. As the discharge power is constant, the expression of the energy W_{T} provided by the current source is

$$W_{\mathrm{T}} = PT_{\mathrm{ch}}, \tag{23}$$

where T_{ch} is the time needed for discharging the supercapacitors from their maximum voltage U_{M} to the minimum voltage level defined by the voltage discharge ratio in Eq. (2). Another method, more complex, leads to a more complex expression of the energy W_{T}. First of all, the relation between the voltages u and u_{c} has to be found:

$$u_{c} = \frac{\frac{P}{N}}{i_{\mathrm{c}}} \text{ with } i_{\mathrm{c}} = \frac{u_{c}-u}{R_{\mathrm{s}}} \Rightarrow u_{\mathrm{c}} = \frac{u^{2} + \frac{R_{\mathrm{s}}P}{N}}{u}. \tag{24}$$

The derivative of this last equation gives a new expression of the current i_{c}:

$$\frac{du_{\mathrm{c}}}{dt} = \frac{1}{u^{2}}\frac{du}{dt}\left(u^{2} - \frac{R_{\mathrm{s}}P}{N}\right) \Rightarrow i_{\mathrm{c}} = C\frac{1}{u^{2}}\frac{du}{dt}\left(\frac{R_{\mathrm{s}}P}{N} - u_{\mathrm{c}}\right). \tag{25}$$

Then, the energy provided to the current source can be found by integrating the product of the voltage u by the current i_{c}, from $t = 0$ s to T_{ch}:

$$W_{\mathrm{T}} = \int_0^{T_{\mathrm{ch}}} u i_{\mathrm{c}} \mathrm{d}t = C\int_0^{T_{\mathrm{c}}} \frac{1}{u}\frac{\mathrm{d}u}{\mathrm{d}t}\left(\frac{R_{\mathrm{s}}P}{N} - u_{\mathrm{c}}\right) dt\,. \tag{26}$$

Then W_{T} can be expressed as follows:

$$W_{\mathrm{T}} = \frac{1}{2}C\left(2\frac{R_{\mathrm{s}}P}{N}\mathrm{In}\frac{u_{\{t=T_{\mathrm{ch}}\}}}{u_{\{t=0\}}} - u^2_{\{t=T_{\mathrm{ch}}\}} + u^2_{\{t=0\}}\right) \text{ with } \begin{cases} u_{\{t=0\}} = \dfrac{U_{\mathrm{M}}}{2} + \dfrac{1}{2}\sqrt{U^2_{\mathrm{M}} - 4\dfrac{R_{\mathrm{s}}P}{N}} \\ u_{\{t=T_{\mathrm{ch}}\}} = \dfrac{U_{\mathrm{M}}}{2}\dfrac{d}{100}\dfrac{1}{2}\sqrt{\left(U_{\mathrm{M}}\dfrac{d}{100}\right)^2 - 4\dfrac{R_{\mathrm{s}}P}{N}} \end{cases}. \tag{27}$$

Thanks to Eqs. (23) and (27), the time for the discharge of the supercapacitor from U_{M} to $U_{\mathrm{M}}d/100$ is easy to define:

$$T_{\mathrm{ch}}\frac{1}{2}\frac{C}{P}\left(2\frac{R_{\mathrm{s}}P}{N}\mathrm{In}\frac{u_{\{t=T_{\mathrm{ch}}\}}}{u_{\{t=0\}}} - u^2_{\{t=T_{\mathrm{ch}}\}} + u^2_{\{t=0\}}\right) \text{ with } \begin{cases} u_{\{t=0\}} = \dfrac{U_{\mathrm{M}}}{2} + \dfrac{1}{2}\sqrt{U^2_{\mathrm{M}} - 4\dfrac{R_{\mathrm{s}}P}{N}} \\ u_{\{t=T_{\mathrm{ch}}\}} = \dfrac{U_{\mathrm{M}}}{2}\dfrac{d}{100}\dfrac{1}{2}\sqrt{\left(U_{\mathrm{M}}\dfrac{d}{100}\right)^2 - 4\dfrac{R_{\mathrm{s}}P}{N}} \end{cases}.$$

The energy that is then provided by the supercapacitive tank during its discharge is the usable energy W_{bu} defined by Eq. (4).

The energy efficiency of the discharge at constant power can be expressed with Eqs. (27) and (3) as the ratio of the energy W_{T} injected into the power converter to the usable energy W_{u} provided by one supercapacitor of the tank:

$$\eta_{\mathrm{P}} = \frac{W_{\mathrm{T}}}{W_{\mathrm{u}}} = \frac{2\dfrac{R_{\mathrm{s}}P}{N}\mathrm{In}\dfrac{u_{\{t=T_{\mathrm{ch}}\}}}{u_{\{t=0\}}} - u^2_{\{t=T_{\mathrm{ch}}\}} + u^2_{\{t=0\}}}{U^2_M\left[1-\left(\dfrac{d}{100}\right)^2\right]} \text{ with } \begin{cases} u_{\{t=0\}} = \dfrac{U_{\mathrm{M}}}{2} + \dfrac{1}{2}\sqrt{U^2_{\mathrm{M}} - 4\dfrac{R_{\mathrm{s}}P}{N}} \\ u_{\{t=T_{\mathrm{ch}}\}} = \dfrac{U_{\mathrm{M}}}{2}\dfrac{d}{100}\dfrac{1}{2}\sqrt{\left(U_{\mathrm{M}}\dfrac{d}{100}\right)^2 - 4\dfrac{R_{\mathrm{s}}P}{N}} \end{cases}.$$

The different equations that have been defined are summarized in Fig. 3 for a 2600 F/0.7 mΩ/2.5 V supercapacitor ($N = 1$).

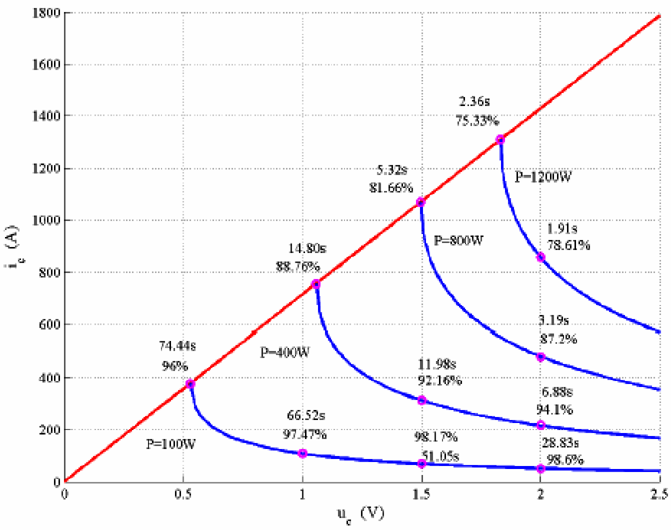

Fig. 3: Discharge of a supercapacitor at constant power

The discharge current i_c is plotted versus the voltage u_c across the ideal capacitor C, for various constant discharge powers. As the voltage decreases from 2.5 V, the current i_c is increased to maintain the power P constant. As defined by Eq. (17), the discharge is limited to a minimum voltage ratio d_{min}. As a consequence, the discharge has to be stopped in order to respect Eqs. (17) and (19). The usable energy will then be reduced as much as the discharge power is high. For a high level of discharge power, the discharge is impossible ($P > 2.32$ kW in this example). On each curve related to a constant power discharge are indicated the time for the discharge together with the efficiency from Eqs. (28) and (29). The efficiency is affected most when the discharge power is high.

The definitions of the energy efficiency of supercapacitors during their discharge can be used for the sizing of supercapacitive tanks. They will be designed to reach at least 90% energy efficiency, according to the specifications defined in Table 1 (50 kJ in 5 s).

3.4 Discharge at constant current

The first step is to define which voltage discharge ratio d matches a 90% energy efficiency for the five seconds allowed for the discharge. This is obtained thanks to Eq. (9) that gives $d = 75.84\%$ for a 2600 F/0.7 mΩ/2.5 V supercapacitor. Knowing d, Eq. (13) gives the maximum possible discharge current i_c to match a 90% energy efficiency. The discharge current has then to be set at 314 A. The total number of supercapacitors can be identified thanks to Eq. (5), where the usable energy stored in the supercapacitive tank must amount to 50 kJ taking the energy efficiency of the tank into consideration.

4 Energy efficiency and power availability

4.1 Energy efficiency and sizing of a supercapacitive tank

$$N = \frac{1}{\eta_i} \frac{2W_{bu}}{CU_M^2\left[1-\left(\frac{d}{100}\right)^2\right]}. \tag{30}$$

This leads to a total number N of supercapacitors equal to 16.

4.2 Discharge at constant power

The requirements in Table 1 lead to a 10 kW constant discharge power. Two equations have to be considered, where 2600 F/0.7 mΩ/2.5 V supercapacitors are used. Equation (29) is used to describe the possible choices for the number N of supercapacitors versus the voltage discharge ratio d that match 90% energy efficiency during the discharge. Equation (28) describes the possible choices for the number N of supercapacitors versus the voltage discharge ratio d that match a discharge time of five seconds. Once the identification of the number N of supercapacitors and of the voltage discharge ratio d is made, one must check whether the ratio d is compatible with the condition (17) that defines the limitation of the minimum voltage ratio d_{min}.

The final choice should then be the intersection of the curves defining the energy efficiency and the time for discharge: $N = 16.25$ supercapacitors (17) and $d = 76\%$. It should be noted that this choice is compatible with the limiting condition (17), where $d > d_{min}$.

4.3 Power availability

In order to compare the different sizes that are proposed in accordance with the specifications of Table 1, a summary of the results is proposed in Table 3 for a storage tank using supercapacitors with the following characteristics: 2600 F, 0.7 mΩ, 2.5 V.

Table 3: Sizing a supercapacitive tank taking into account the energy efficiency

Sizing criteria	*N*	*d* (%)	Volume (l)	Weight (kg)	W_{bM} (kJ)
Energy requirement	9	50	3.78	4.72	73.12
Constant current discharge	16	75.84	6.72	8.4	130
Constant power discharge	17	76	7.14	8.9	138.12

$W_{bu} = 50$ kJ (13.9 Wh), discharge time: 5 s,
$C = 2600$ F, $R_s = 0.7$ mΩ, $U_m = 2.5$ V.

The number of supercapacitors has to be strongly increased for the discharge at constant current or at constant power, as compared to the sizing that takes only energy requirements into account. This is due to the requirement of a 90% of energy efficiency during the discharge. This increased number of supercapacitors is necessary in order to obtain a supercapacitive tank compatible with the power availability needed in this application, even if this power availability is not clearly expressed in the requirements of Table 1. But the energy efficiency has to be taken into account to make a supercapacitive tank 'power compatible'.

In the example we have been considering, this leads to an oversized supercapacitive tank regarding the usable energy that is stored in it: the voltage discharge ratios have to be higher than 75% to maintain the energy efficiency higher than or equal to 90%. This represents only 43.75% of the total stored energy.

The result of this oversizing is illustrated in Fig. 4, where the energy efficiency of the discharges is plotted versus the time, at constant current discharge and at constant power discharge, for the sizing of Table 3.

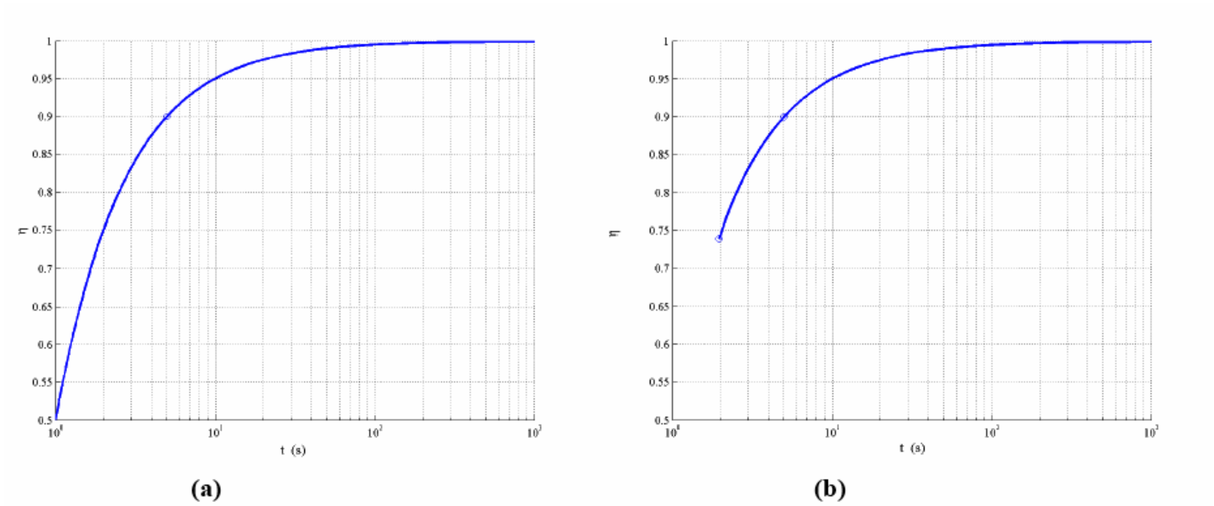

Fig. 4: Energy efficiency of the supercapacitive tank. a) Constant current. b) Constant power.

It can be seen that the energy efficiency is effectively 90% in the case of a discharge in five seconds. If this time is longer, then the energy efficiency will be higher. In the opposite case, the energy efficiency will also be affected.

Finally, we present in Table 4 a comparison of the main performances of the 2600 F/0.7 mΩ/2.5 V supercapacitor, between the characteristics proposed by the manufacturer and the characteristics issued from the previous sizing; the energy efficiency is taken into account according to the specifications in Table 1.

Table 4: Comparison of the performances of supercapacitors

	Datasheet	**90% efficiency**	
		Constant current	**Constant power**
Maximum current	600 A	314 A	309 A
Usable energy	../..	0.96 Wh	0.95 Wh
Energy density	4.8 Wh/kg	1.83 Wh/kg	1.82 Wh/kg
Maximum power	../..	785 W	588 W
Power density	4300 W/kg	1500 W/kg	1120 W/kg

While the manufacturers specify a 600 A maximum current, the previous sizing shows that this current has to be kept lower than 314 A or 309 A to obtain a 90% energy efficiency during the discharge. As the voltage discharge ratio has to be kept higher than 75% (Table 3), the usable energy is only a part of the stored energy (theoretically 2.25 Wh). Then it seems logical to define the energy density of the supercapacitor thanks to this usable energy instead of the total stored energy. The consequence is an energy density much lower than the specifications given by manufacturers.

For the sizing of the supercapacitive tank at constant power discharge, if the total power is 10 kW, then each of the 17 supercapacitors will have to provide 588 W. Then the power density is only 1120 W/kg instead of the theoretical 4300 W/kg. This demonstrates that, depending on requirements, the sizing of a supercapacitive tank taking into account the energy efficiency can lead to a strong derating of the component properties as compared to the performances announced by manufacturers, especially when supercapacitors have to be used as power sources. In this case, the needed power availability of the tank is more significant than the energy availability.

5 Conclusion

We have presented a method for sizing a supercapacitive tank. Two main parameters are important for sizing a supercapacitive tank dedicated to storing and providing energy: the usable energy, and the power availability. This last parameter mainly defines the energy efficiency of the storage tank.

The usable energy stored into a tank and the power availability are two parameters that are not directly linked. In the sizing process, these two requirements have to be satisfied. In general applications, this is usually the case. But in special applications, where supercapacitors are used as power sources to deliver high power peaks for a short time, the power availability requires more supercapacitors in order to obtain a high energy efficiency. Thus the energy requirements will not be the determining factor for the final weight and volume of the storage tank.

References

[1] P. Barrade and A. Rufer, Supercapacitors as energy buffers: a solution for elevators and for electric busses supply, PCC 2002: Power Conversion Conference, Osaka, Japan.

[2] R. Kötz *et al.*, Supercapacitors for peak-power demand in fuel-cell-driven cars, ECS Electro-Chemical Society, 52nd Meeting, San Francisco (2001).

[3] A. Rufer and P. Barrade, Supercapacitor-based energy-storage system for elevators with soft commutated interface, *IEEE Trans. Ind. Appl.*, **38** (2002) 1151.

[4] P. Barrade and A. Rufer, High-power fast energy exchange between storage systems: supercapacitors as energy buffer in transportation systems, EVS-18: The 18th International Vehicle Symposium, International Congress Centre (ICC), Berlin, Germany.

[5] T. Christen and M.W. Carlen, Theory of Ragone plots, *J. Power Sources*, **91** (2000) 210.

[6] S. Buller, D. Kok and R.W. De Doncker, Modeling the dynamic behaviour of supercapacitors using impedance spectroscopy, *IEEE Trans. Ind. Appl.*, **38** (2002) 1622.

[7] P. Barrade and A. Rufer, Considerations on the energy efficiency of a supercapacitive tank, MAGLEV 2002: International Conference on Magnetically Levitated Systems and Linear Drives, Lausanne, Switzerland.