

LHC Magnet Polarities

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Abstract

The LHC magnet polarity conventions as laid-out in the CERN-EDMS document 90042 are reviewed. The same multipole field error definition in two different reference frames (magnet measurement frame and moving frame of Beam 1 in the LHC machine) may result in confusion about the polarity of the vertical orbit correctors and the treatment of measured skew multipole errors in the beam physics program MAD. This document aims at establishing the coherence between the field error definitions in the different reference frames on one side and the LHC magnet polarity conventions on the other side.

INTRODUCTION

The magnet powering system for the LHC is very complex. About 10000 magnets will be connected in 1612 electrical circuit. Several 10000 superconducting connections have to be produced during the installation of the magnets in the LHC tunnel. The power converters have to be connected to the current leads in the DFBs, the local current leads for orbit corrector magnets or directly to the magnet terminals of the normal conducting magnets. Any wrong connection of a magnet can seriously compromise LHC operation. Examples for wrong connections are the inversion of polarity, the connection of a magnet in a wrong circuit, the inversion of the polarity at the level of the power converter or the current lead, etc. Detecting a wrong connection will be very difficult, once the machine is in operation.

The coherence between magnet construction and measurement on one side, and the magnet interconnection according to the layout database and the hard and software for the electrical quality assurance (ELQA) on the other side must be established. The same understanding and application of the engineering specification for LHC magnet polarities by all teams involved has to be ensured.

The same multipole field error definition in two different reference frames (magnet measurement frame and moving frame of Beam 1 for beam physics calculations) may result in confusion about the polarity of the skew magnet elements. This report is therefore intended to provide a firm basis on which further investigations can be launched.

MAGNET POLARITY CONVENTIONS

The rules which follow the conventions in the CERN-EDMS document 90042, [1], are conceived to yield a simple identification of the polarities of the magnets installed in the LHC tunnel, without reference to the different coordinate systems used for beam tracking and field measure-

ments. The set of rules allows magnets of a given type to be manufactured and assembled without prior knowledge of their position or function in the accelerator.

The polarity of the excitation current, and thus the optical function of the magnet, will be determined by the connection of the magnet terminals.

Contrary to a statement in an older version of the EDMS document 90042, the set of rules does **not** follow the conventions of the beam optics program MAD or the conventions for magnetic field computations and measurements. Appropriate transformations into the moving frame of the circulating beam or the magnet reference frame have to be applied.

The conventions for the LHC arc magnets are summarized below.

- The reference beam is called Beam 1 rotating clockwise in the LHC main ring seen from above. Beam 2 is rotating counter-clockwise seen from above.
- The observer is looking in direction of Beam 1 so that the center of the machine is to his right.
- In the two-in-one magnets (or magnet assemblies) the left aperture seen from the connection terminals is called Aperture 1 the right aperture is named Aperture 2.
- The LHC main dipole and quadrupole magnets are installed in the LHC tunnel with their connection terminals upstream of Beam 1. Then Aperture 2 becomes the internal aperture and Aperture 1 the external aperture.
- The magnet connection terminals are marked with *A* and *B* (not + and -).
- The fields¹ and gradients are said to be positive if the current enters the *A* terminal. A positive field is defined to point upwards (deflecting Beam 1 to the inside of the ring) while a positive gradient is defined such that the vertical field increases along the outward pointing machine radius (thus it is focusing Beam 1 in the horizontal plane), see Figs. 1, 2.
- The skew multipole magnets of order *N* are tilted clockwise by an angle of $\pi/2N$ degrees where $N=1$ for the dipole, $N=2$ for the quadrupole etc. Thus a positive skew dipole is deflecting Beam 1 downwards, see Fig. 3.
- Twin aperture magnets in the arcs have only one pair of terminals serving the two apertures. The external aperture (Aperture 1) takes priority for the application

¹The usual slang for the vector field of magnetic flux density.

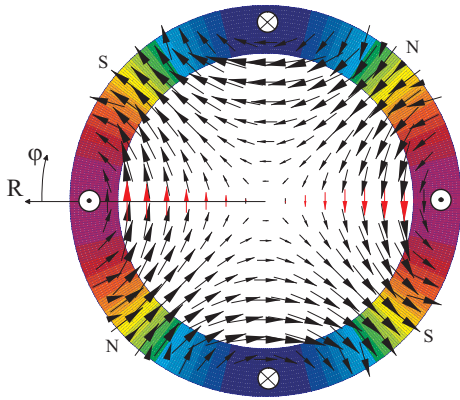
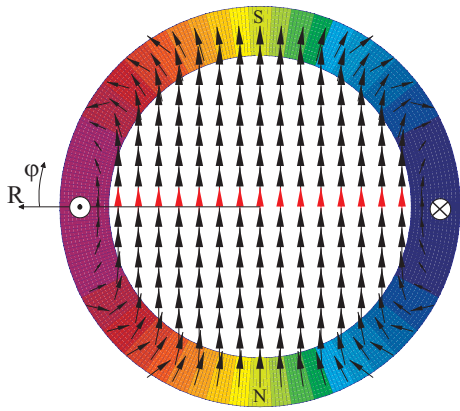


Figure 1: Current and field distribution in a single aperture dipole (top) and single aperture quadrupole with current entering the *A* terminal (positive dipole field and gradient in the quadrupole). The observer is looking in direction of Beam 1. The clockwise rotating Beam 1 is deflected in the dipole toward the center of the machine and it is horizontally focused in the quadrupole. Notice the field distribution on the mid plane (icons in red color).

of the rules. Consequently, if the current enters the *A* terminal of the LHC main dipoles, the main field direction is downward in the internal aperture (Aperture 2) and upward in the external aperture, see Fig. 4. In the main quadrupoles the field gradient is identical in both apertures. Thus, if the current enters the *A* terminal in the two-in-one quadrupoles, the fields in both apertures increase along the outward pointing machine radius. In case Beam 1 is in the external aperture (in sectors 1-2, 5-6, 6-7, 7-8) it will be focused in the horizontal plane. Beam 2 (circulating counter clockwise in these sectors) will be defocused, see Fig. 5.

- The position of the external connection terminals of magnet assemblies defines the *normal* installation direction in the tunnel, i.e., with the external connection terminals upstream of Beam 1. Notice that multipole correctors within the magnet assembly might

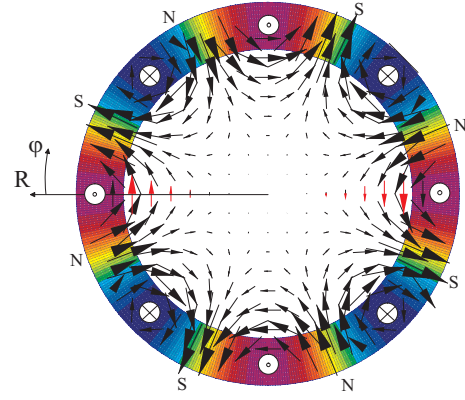
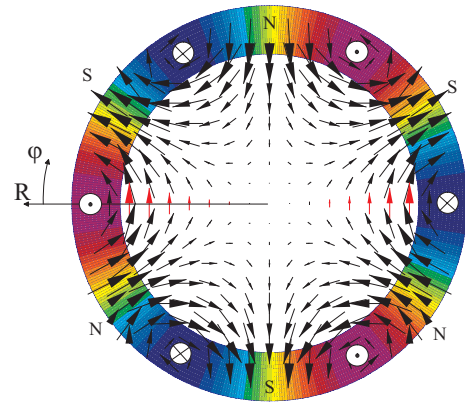


Figure 2: Current and field distribution in a single aperture sextupole (top) and single aperture octupole with current entering the *A* terminal (positive sextupole and octupole fields). Notice the field distribution on the mid plane (icons in red color).

have their connection terminals facing downstream of Beam 1, e.g., MCB in the SSS, MCS in the MB. For various reasons a magnet assembly may be turned with respect to the normal direction. The construction and internal connections of these magnets are not changed. Also the naming of the connection terminals *A* and *B* are not changed. However, the magnet polarity may change depending on the multipole order. In this case the polarity is inverted at the warm side of the powering circuit which is reflected in the electrical layout database and layout drawing.

- In case of magnets where both beams pass through a single aperture, e.g., in the MQXA and MQXB magnets, Beam 1 is used to define the polarity.

For the polarity of corrector magnets powered from a bipolar power supply the conventions follow those of the main magnets. Thus a positive sextupole will compensate for the persistent current effect in the main dipole.

- Current entering the *A* terminal of the sextupole and decapole corrector magnets integrated in the main

dipole cold masses have an upward pointing field direction in the horizontal plane, see Fig. 2 (top) for the sextupole.

- Current entering the *A* terminal of the octupole correctors results in a vertical field increasing along the outward pointing machine radius, Fig. 2 (bottom).
- In sectors 1-2, 5-6, 6-7 and 7-8, where Beam 1 is in the external aperture and where the current of the main dipole enters the *A* terminal, all the corrector magnets in the external aperture have their current entering the *A* terminal. The corrector magnets in the internal aperture have their current entering the *B* terminal.

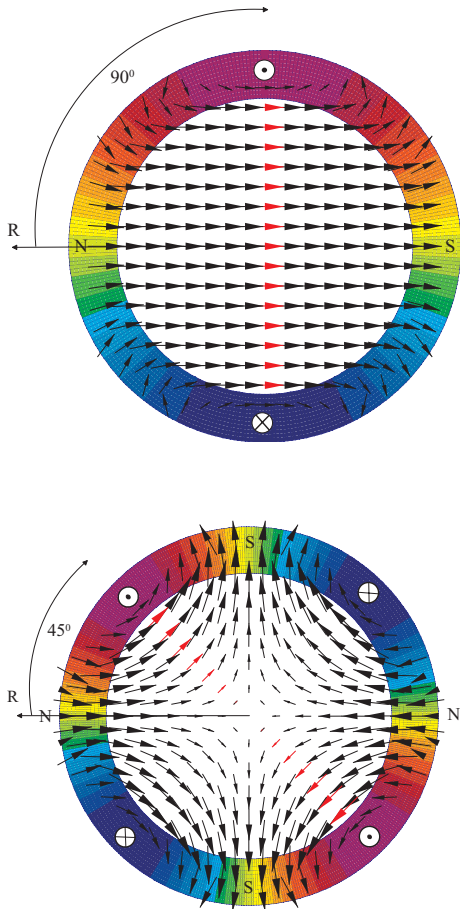


Figure 3: Current and field distribution in a single aperture skew dipole (top) and single aperture skew quadrupole with current entering the *A* terminal (positive skew dipole and skew quadrupole fields). The magnets are rotated clockwise (looking in direction of Beam 1) by $\pi/2N$ where *N* is the multipole order of the magnet (*N*=1 dipole, *N*=2 quadrupole, etc.). A positive skew dipole field is deflecting Beam 1 downwards.

For twin aperture separator dipoles and twin aperture magnets with the same optical functions (e.g. MQWB), the following conventions apply:

- For twin aperture quadrupoles with the same function

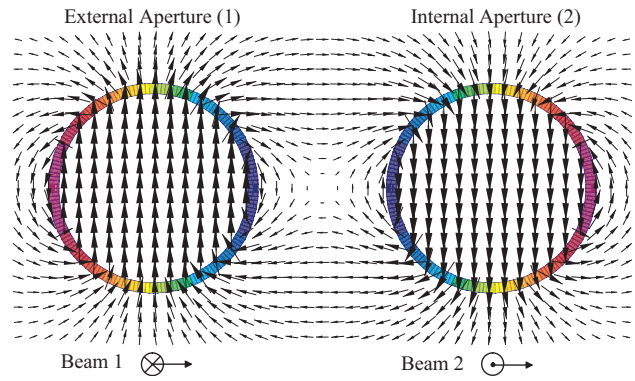


Figure 4: Current and field distribution in the twin aperture main dipole (MB) with current entering the *A* terminal.

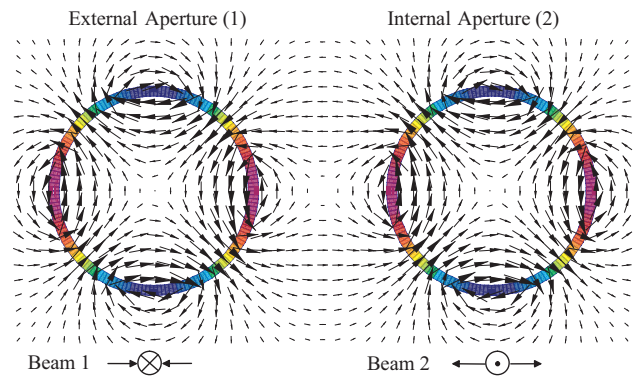


Figure 5: Current and field distribution in the twin aperture main quadrupole (MQ) with current entering the *A* terminal, focusing Beam 1 and defocusing Beam 2 on the horizontal plane.

in both apertures, the polarity convention is as follows: In case Beam 1 is in the external aperture (in sectors 1-2, 5-6, 6-7, 7-8) both beams will be focused in the horizontal plane if the current enters the *A* terminal.

- When the current enters the *A* terminal in the twin aperture separator dipoles, the field is pointing upwards both in the external and the internal aperture.

FRAMES

In accordance with the document [1] we have avoided frames up to now. They cannot be ignored, however, if only to make contact with the all-time honored concept in superconducting magnet design, the so-called $\cos n\varphi$ current distribution. It is well known that a $\cos \varphi$ current distribution generates an ideal dipole field within the aperture. Obviously, the direction of the current at $\varphi = 0$ has to be the same for both the dipole as well as the quadrupole (and higher order multipole) magnets with $\cos n\varphi$ current distribution. The conventions, i.e., a positive dipole field is

bending Beam 1 inwards, a positive quadrupole is focusing Beam 1 on the horizontal plane can only be met if the angle φ is counted positively as indicated in Figs. 1 and 2 which corresponds to the reference frame used for particle tracking, see Fig. 6.

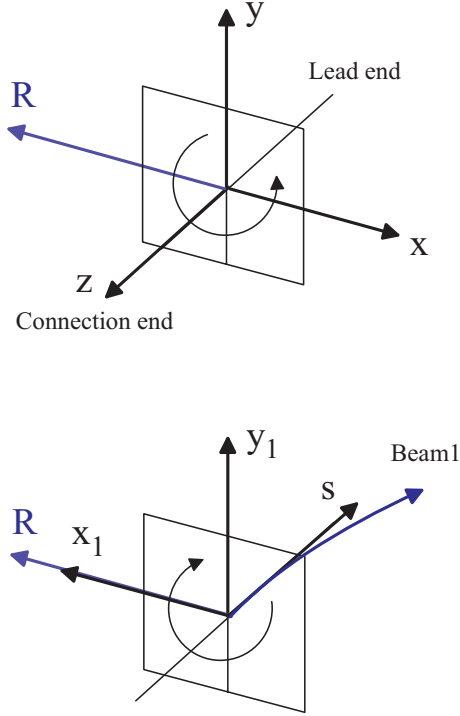


Figure 6: Top: Reference frame used in magnet design and measurement. The x -axis is pointing towards the center of the machine. Bottom: Reference frame used for particle tracking with the x_1 -axis pointing in the direction of the machine radius. Notice the orientation of the plane \mathbb{R}^2 which we identify with the complex plane.

MULTIPOLE EXPANSIONS

Magnet frame

The cartesian components of the magnetic field \mathbf{B} in the aperture of the LHC magnets are combined in the complex function $\tilde{B} = B_y + jB_x$, holomorphic in $U = \{z \mid |x + jy| < \rho\}$, where ρ is the radius of the magnet aperture. \tilde{B} can then be expanded as

$$\begin{aligned} B_y + jB_x &= \sum_{n=1}^{\infty} (B_n + jA_n) \left(\frac{z}{r_0}\right)^{n-1} \\ &= B_N \sum_{n=1}^{\infty} (b_n + ja_n) \left(\frac{z}{r_0}\right)^{n-1}. \end{aligned} \quad (1)$$

The normal and skew multipole coefficients $B_n(r_0), A_n(r_0)$ are given in units of tesla at a reference radius r_0 of 17 mm. The small $b_n(r_0), a_n(r_0)$ denote

the normal and skew relative multipole coefficients at the reference radius, related to the main field component $B_N(r_0)$ which is $B_1(r_0)$ for the dipole, $B_2(r_0)$ for the quadrupole, etc. The $b_n(r_0), a_n(r_0)$ are dimensionless and are given in units of 10^{-4} . The scaling of the multipoles with respect to the reference radius and the transformation under frame changes is discussed in [5].

Remark 1: Obviously, we think of the complex plane with its real axis to the right. For the observer looking downstream of Beam 1 this implies that the x -axis is pointing towards the machine center. However, we can identify the complex plane with \mathbb{R}^2 by means of

$$\mathbb{R}^2 \xrightarrow{\cong} \mathbb{C}, (x, y) \rightarrow x + jy \quad (2)$$

and consequently regard a complex function f as a mapping $f : U \rightarrow \mathbb{R}^2$ with $U \subset \mathbb{R}^2$. The plane \mathbb{R}^2 can then be outer oriented by selecting a crossing direction through it (the one of Beam 1, for example). With ambient space being oriented in the sense of the right handed screw, this would imply that the x -axis is pointing into outward direction, see Fig. 6. In order to emphasize our **choice** of orientation we shall write the magnetic field in the magnet frame (here with the x -axis pointing inwards) as

$$\begin{aligned} B_y + jB_x &= \sum_{n=1}^{\infty} (B_n^{\text{mag}} + jA_n^{\text{mag}}) \left(\frac{z}{r_0}\right)^{n-1} \\ &= B_N^{\text{mag}} \sum_{n=1}^{\infty} (b_n^{\text{mag}} + ja_n^{\text{mag}}) \left(\frac{z}{r_0}\right)^{n-1} \end{aligned} \quad (3)$$

with a bit of additional typesetting worth the effort. Recall that we have done our ink-saving by writing in shorthand B_n, A_n for the radius dependent $B_n(r_0), A_n(r_0)$.

With $B_\varphi + jB_r = (B_y + jB_x)e^{j\varphi}$ we obtain in the magnet frame

$$\begin{aligned} B_\varphi + jB_r &= \frac{r_0}{r} \sum_{n=1}^{\infty} (B_n^{\text{mag}} + jA_n^{\text{mag}}) \left(\frac{z}{r_0}\right)^n \\ &= B_N^{\text{mag}} \frac{r_0}{r} \sum_{n=1}^{\infty} (b_n^{\text{mag}} + ja_n^{\text{mag}}) \left(\frac{z}{r_0}\right)^n \end{aligned} \quad (4)$$

and for the field components at any radius $r < \rho$:

$$B_r(r, \varphi) = \sum_{n=1}^{\infty} \left(\frac{r}{r_0}\right)^{n-1} (B_n^{\text{mag}} \sin n\varphi + A_n^{\text{mag}} \cos n\varphi), \quad (5)$$

$$+ A_n^{\text{mag}} \cos n\varphi, \quad (6)$$

$$B_\varphi(r, \varphi) = \sum_{n=1}^{\infty} \left(\frac{r}{r_0}\right)^{n-1} (B_n^{\text{mag}} \cos n\varphi - A_n^{\text{mag}} \sin n\varphi). \quad (7)$$

$$- A_n^{\text{mag}} \sin n\varphi). \quad (8)$$

and

$$B_x(r, \varphi) = \sum_{n=1}^{\infty} \left(\frac{r}{r_0}\right)^{n-1} (B_n^{\text{mag}} \sin(n-1)\varphi + A_n^{\text{mag}} \cos(n-1)\varphi), \quad (9)$$

$$+ A_n^{\text{mag}} \cos(n-1)\varphi, \quad (10)$$

$$B_y(r, \varphi) = \sum_{n=1}^{\infty} \left(\frac{r}{r_0} \right)^{n-1} (B_n^{\text{mag}} \cos(n-1)\varphi) \quad (11)$$

$$-A_n^{\text{mag}} \sin(n-1)\varphi), \quad (12)$$

where again we have written in shorthand $B_n^{\text{mag}}, A_n^{\text{mag}}$ for $B_n^{\text{mag}}(r_0), A_n^{\text{mag}}(r_0)$. The definition (3) (in the magnet frame shown in Fig. 6, top) results in the following sign conventions for the multipole coefficients:

- B_1^{mag} is the dipole field pointing into positive y direction, i.e., a positive field bends a positively charged particle inwards. It corresponds to the conventions of EDMS 90042.
- A_1^{mag} is the skew dipole field pointing into positive x direction, i.e., inwards. A positive skew dipole field bends a positively charged particle downwards. It corresponds to the conventions of EDMS 90042.
- A positive B_2^{mag} is a quadrupole field that implies horizontal defocussing of Beam 1. This is a sign reversion with respect to the conventions of EDMS 90042.
- A positive A_2^{mag} is a skew quadrupole that implies defocussing of Beam 1 in the (x, z) plane rotated clockwise by $\pi/4$ (points in this rotated plane have coordinates $y = -x$). This is a sign reversion with respect to the conventions of EDMS 90042 (resulting from the sign reversion of the quadrupole field).

The local reference frame of Beam 1

In the local reference frame of Beam 1 (shown in Fig. 6, bottom) the field is expanded as

$$\begin{aligned} B_{y_1} + jB_{x_1} &= \sum_{n=1}^{\infty} (B_n^{\text{Beam1}} + jA_n^{\text{Beam1}}) \left(\frac{z_1}{r_0} \right)^{n-1} \\ &= B_N^{\text{Beam1}} \sum_{n=1}^{\infty} (b_n^{\text{Beam1}} + ja_n^{\text{Beam1}}) \left(\frac{z_1}{r_0} \right)^{n-1} \end{aligned} \quad (13)$$

with $z_1 = x_1 + jy_1$. This definition results in the following sign conventions for the multipole coefficients:

- B_1^{Beam1} is the dipole field pointing into positive y_1 direction, i.e., a positive field bends a positively charged particle inwards. It corresponds to the conventions of EDMS 90042.
- A_1^{Beam1} is the skew dipole field pointing into positive x_1 direction, i.e., outwards! A positive skew dipole field bends a positively charged particle upwards. This is a sign reversion with respect to the conventions of EDMS 90042.
- A positive B_2^{Beam1} is a quadrupole field that implies horizontal focusing of Beam 1. It corresponds to the conventions of EDMS 90042.
- A positive A_2^{Beam1} is a skew quadrupole that implies defocussing of Beam 1 in the (x_1, s) plane rotated

clockwise by $\pi/4$ (points in this rotated plane have coordinates $x_1 = y_1$). This is a sign reversion with respect to the conventions of EDMS 90042.

Remark 2: Both in the magnet frame and in the local frame of Beam 1, the mapping $B_n \rightarrow A_n$ implies a rotation of the magnet element in mathematically negative (!) sense. For the observer looking downstream of Beam 1 this is a clockwise rotation of the magnet element in case of $B_n^{\text{mag}} \rightarrow A_n^{\text{mag}}$ and a counter clockwise rotation of the magnet element in case of $B_n^{\text{Beam1}} \rightarrow A_n^{\text{Beam1}}$. Rossbach and Schmüser [2] expand the field as

$$B_y + jB_x = \sum_{n=1}^{\infty} (B_n - jA_n) \left(\frac{z}{r_0} \right)^{n-1}, \quad (14)$$

where the mapping $B_n \rightarrow A_n$ implies a rotation of the magnet in mathematically positive sense which seems more natural. Nothing canonical, however, as the multipole coefficients have to be matched to the calculated or measured ones at a given reference radius. If we measure B_r , expressed according to Eq. (5), by means of the flux linkage through a rotating tangential coil and perform a Fourier analysis of the voltage signal thus obtained, then because of the usual convention $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$, it is rather the plus sign that has to be evoked.

Field error definition in the accelerator design program MAD

The MAD program uses a Maclaurin series expansion of the integrated (along the magnet axis) field at the mid-plane $y_1 = 0$ with

$$B_{y_1}(x_1) = B_0 + \frac{dB_{y_1}}{dx_1} \Big|_{x_1=y_1=0} x_1 + \dots \quad (15)$$

$$\dots + \frac{1}{n!} \frac{d^n B_{y_1}}{dx_1^n} \Big|_{x_1=y_1=0} x_1^n + \dots \quad (16)$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} B_{n,n}^{\text{MAD}} x_1^n \quad (17)$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} B\rho K_{n,n} x_1^n, \quad (18)$$

where $B\rho$ is the magnetic rigidity of the beam and the roman type subscript n denotes the normal multipole coefficient. The same definition holds for the skew field components (denoted as $B_{n,s}^{\text{MAD}}$) with the reference frame (!) rotated clockwise around the beam axis by $\pi/2N$.

Thus the sign conventions for the multipole coefficients follow those of the Beam 1 reference frame:

- $B_{0,n}^{\text{MAD}}$ is the dipole field pointing into positive y_1 direction, i.e., a positive field bends a positively charged particle inwards.

- $B_{0,s}^{\text{MAD}}$ is the skew dipole field pointing into positive x_1 direction (in the Beam 1 frame, i.e., outwards). A positive skew dipole field thus deflects Beam 1 upwards.
- A positive $B_{1,n}^{\text{MAD}}$ is a quadrupole field that implies horizontal focusing of Beam 1.
- A positive $B_{1,s}^{\text{MAD}}$ is a skew quadrupole that implies defocussing of Beam 1 in the (x_1, s) plane rotated clockwise by $\pi/4$ (points this rotated plane have coordinates $x_1 = y_1$).

- [3] Iselin, F.C.: The MAD program, User's Reference Manual, CERN/SL/98-XX (AP), 1998
- [4] MAD-X Home Page: <http://mad.home.cern.ch/mad/>
- [5] Wolf, R.: Field error naming conventions for LHC magnets, Engineering Specification, EDMS document No. 90250, CERN, 2001.

Remark 3: Only the User's Reference Manual of the (obsolete) MAD 9.01 program [3] gives the sign conventions for the multipole coefficients (page 11,12) unfortunately with type-setting errors (cut-and-paste of text segments). MAD-X obeys the same rules, as can be seen in Tables 1 and 2². The MAD-X manual [4] gives the sign conventions for the $K_{n,n}$ and $K_{n,s}$ on page 10 for the quadrupole and on page 11 for the sextupole. The description of the TILT angle is erroneous, however. It should read: A TILT= $\pi/4$ turns a normal into a *negative* skew quadrupole.

PUTTING IT ALL TOGETHER

Tables 1 and 2 in the Appendix show the transformations for MAD-X input, multipole coefficients in the moving frame of Beam 1, the polarity conventions (EDMS Doc. Nr. 90042), and the multipole coefficients in the magnet frame.

It can be seen that unlike stated in an older version of the EDMS document No. 90042, the polarity conventions do **not** follow the beam physics conventions. The polarity conventions are compatible with the conventions in the Beam 1 frame as far as the normal multipoles are concerned. They are not coherent with the MAD conventions for the skew multipoles. Remember that a clockwise rotation of the magnet element (observer looking downstream of Beam 1) results in mappings $B_n^{\text{mag}} \rightarrow A_n^{\text{mag}}$ but $B_n^{\text{Beam1}} \rightarrow -A_n^{\text{Beam1}}$.

The transformation laws between the multipole coefficients read for $n = 1, 2, 3, \dots$:

$$\frac{r_0^{n-1}}{(n-1)!} B_{(n-1),n}^{\text{MAD}} = B_n^{\text{Beam1}} = (-1)^{n-1} B_n^{\text{mag}} \quad (19)$$

$$\frac{r_0^{n-1}}{(n-1)!} B_{(n-1),s}^{\text{MAD}} = A_n^{\text{Beam1}} = (-1)^n A_n^{\text{mag}}. \quad (20)$$

REFERENCES

- [1] Proudlock P., Russenschuck, S., Zerlauth, M.: LHC Magnet Polarities, Engineering Specification, EDMS Document Nr. 90041, CERN, 2004
- [2] Rossbach, J., Schmüser, P.: Basic course on accelerator physics, CERN Accelerator School, Proceedings Vol. 1, CERN 94-01, 1994

²Thanks to Frank Schmidt AB-ABP for running the test cases.

APPENDIX

MAD-X Input	HKICKER	VKICKER	$B_{0,n}^{\text{MAD}}$	$B_{0,n}^{\text{MAD TILT}}$	$B_{0,s}^{\text{MAD}}$	$B_{0,s}^{\text{MAD TILT}}$
$B_n^{\text{Beam1}}, A_n^{\text{Beam1}}$	$-B_1^{\text{Beam1}}$	A_1^{Beam1}	B_1^{Beam1}	$-A_1^{\text{Beam1}}$	A_1^{Beam1}	B_1^{Beam1}
Deflecting Beam 1	Outwards	Upwards	Inwards	Downwards	Upwards	Inwards
Polarity Conv. EDMS 90042	Negative dipole	Negative skew dipole	Positive dipole	Positive skew dipole	Negative skew dipole	Positive dipole
$B_n^{\text{mag}}, A_n^{\text{mag}}$	$-B_1^{\text{mag}}$	$-A_1^{\text{mag}}$	B_1^{mag}	A_1^{mag}	$-A_1^{\text{mag}}$	B_1^{mag}

Table 1: Transformation table for MAD-X input, polarity convention (EDMS Doc. Nr. 90042) and dipole coefficients in the magnet frame. TILT is the so-called roll angle about the longitudinal axis (a positive angle represents a clockwise rotation of the magnet. Notice that in MAD-X a tilted dipole corresponds to a negative skew dipole.

MAD-X Input	$B_{2,n}^{\text{MAD}}$	$B_{2,n}^{\text{MAD TILT}}$	$B_{2,s}^{\text{MAD}}$	$B_{2,s}^{\text{MAD TILT}}$
$B_n^{\text{Beam1}}, A_n^{\text{Beam1}}$	B_2^{Beam1}	$-A_2^{\text{Beam1}}$	A_2^{Beam1}	B_2^{Beam1}
Effect on on Beam 1	Focusing in horizontal plane	Q1 + Q2 -	Q1 - Q2 +	Focusing in horizontal plane
Polarity Conv. EDMS 90042	Positive quadrupole	Positive skew quad.	Negative skew quad.	Positive quadrupole
$B_n^{\text{mag}}, A_n^{\text{mag}}$	$-B_2^{\text{mag}}$	$-A_2^{\text{mag}}$	A_2^{mag}	$-B_2^{\text{mag}}$

Table 2: Transformation table for MAD-X input, polarity convention (EDMS Doc. Nr. 90042) and quadrupole coefficients in the magnet frame. TILT is the so-called roll angle about the longitudinal axis (a positive angle represents a clockwise rotation of the magnet. Notice that in MAD-X a tilted quadrupole corresponds to a negative skew quadrupole.