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DYNAMICAL QUARK AND GLUON CONDENSATES FROM A MODIFIED PERTURBATIVE QCD

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Abstract

As it was suggested by previous works on a modified perturbation expansion for QCD, the possibility for the generation of large quark condensates in the massless version of the theory is explored. For this purpose, it is firstly presented a way to well define the Feynman diagrams at any number of loops by just employing dimensional regularization. After that, the calculated zero and one loop corrections to the effective potential indicate a strong instability of the system under the generation of quark condensates even in the absence of the gluon one. The quark condensate dependence of particular two loop terms does not modify the instability picture arising at one loop. The results suggest a possible mechanism for a sort of Top Condensate Model to be a dynamically fixed effective action for massless QCD. The inability of lattice calculations in detecting this possibility could be related to the current limitations in treating the fermion determinants.

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I. INTRODUCTION

The explanation of chiral and flavour properties of QCD is one of the fundamental research issues in high energy physics [1–6]. A central problem in this field is the question about the origin of the quark masses, which indirectly determine the nature of most of the observable matter. More generally, the explanation of the elementary particle mass spectrum is considered as a fundamental question posed on the research in Physics. This situation gives relevance to the close examination of possible mechanisms that could be playing a role in this problem.

Specifically, a modified perturbative QCD, altered in a way that incorporates the presence of a condensate of zero-momenta gluons and quarks in the initial state used for constructing the Wick expansion, have been considered in Refs. 7–13. An interesting aspect of these works was the BCS-like modification of the gluon state studied in Refs. 7–9 which directly led to the prediction of the constituent masses for light quarks, after simply fixing to its currently estimated value, a relatively unrelated quantity, the gluon condensate parameter. Therefore, it looked reasonable to expect that a similarly constructed state for the quarks, after to be introduced in the treatment of massless QCD, could have the chance of generating the Lagrangian quark masses. The result of the preliminary consideration of this idea in Ref. 13, was positive. A modification of the free-quark propagator by introducing the zero momentum terms representing analog of the Cooper pair condensates within this problem was done. After that, the simplest approximation for the Dyson equation for quarks produced a diagonal Lagrangian mass matrix by the choice of also a diagonal structure, for the quark condensates. Henceforth, the results of the work 13, pointed out that the presence of colourless quark anti-quark condensates in the vacuum of the free theory generating the Wick expansion, is able to produce the observed quark mass matrix. Therefore, the question arose about the possibility for the generation of the necessary pattern of quark condensates by a dynamical breaking of the flavour and chiral symmetries. A preliminary step in the study of this issue was also done in Ref. [13], where the one loop contribution to the Cornwall-Jackiw-Tomboulis (CJT) effective potential for composite operators [14] was evaluated as a function of the parameters determining the quark and gluon condensates. However, it should be underlined that as the CJT potential is not directly giving the ground state energy at its minimum [14, 15], the implications of the results of the evaluation for the dynamical symmetry breaking problem under study were not clear. We estimate that in the case that a similar mechanism to the one acting in standard superconductivity, could be playing a role in the problem, the generation of large quark condensates could be expected to be produced by strong binding colour forces, linked with the interaction vertices. On other hand, the inability of the standard perturbation expansion in evidencing this effect, looks to us to be rooted in the explicit disregarding of the inclusion of zero momentum quarks and gluons in the free vacuum state, before the adiabatic connection of the interaction. The incorporation of these modified Lorentz invariant vacuums in a BCS style, allows to produce non-trivial modifications in the Wick expansion as it was argued in Ref. 8.

In the former works on the theme [7–13] a troublesome aspect was remaining about the appearance of singularities in the Feynman diagrams due to the presence of Dirac Delta terms in the free propagators. This issue will be approached here in one of the Appendices. The basic idea of the adopted procedure is simply to be consistent with the dimensional regularization and to extend the appearing $\delta(0)$ like singularities for continuous D dimensions in the previously introduced by Capper and Liebbrandt [16]. This procedure led to the outcome that these factors simply vanish in the $D \to 4$ limit. An additional recipe is also taken for the evaluation of the $1/(p^2 + i\epsilon)$ factors at zero momentum. These terms also appear due to the joining of more than $n-1$ different condensate lines in a vertex having n legs. The rule chosen in this case will be to just regularize the scalar field propagator at the zero value of the 4-momentum point to be equal to zero. These points are discussed in Section 2. The selected prescriptions, in addition, allow to identify the propagators evaluated in [13], as modified tree propagators. They also have zero order in the coupling g series expansion after considering as the independent parameters of the theory, the proper g, in addition to the gluon and quark condensate parameters multiplied by g^2. This transformation seemingly will allow to rearrange the full loop expansion to a form in which the propagators derived in Ref. 13 will play the role of new tree Green functions. However, these more formal aspects will be relegated for a further study. Here we only consider the one and two loop cases in which it is clear that the summation over the zero order in the coupling self-energy insertions can be done. The full demonstration of the coincidence of the terms evaluated here with the exact loop expansion terms of the alternative series needed only for the checking of whether the combinatorial factors of all the lower loop terms arising from the higher loop ones could obstacle the proof. The verification of this issue will be considered elsewhere.

After giving the procedure for defining the perturbative expansion, we continue in this work the study of the possibility of the dynamical generation of masses in massless QCD. Zero and one loop vacuum contributions to the effective potential are evaluated and also the full dependence on the quark condensate of a particular two loop term is presented. As noticed above, the usual definition of the effective potential was evaluated here ([15]) in order to avoid the non bounded from below property of the directly evaluated CJT potential [14]. The approximation considered consists in inserting all the condensate dependent parts of the one loop self-energy corrections into the free propagators, in the usual zero and one loop correction, as follows from the discussion in Section 2. This procedure results in employing in the usual loop diagrams, the propagators that produced the constituent masses for light fermions in the work [9], by also considering propagators associated to the gluon and quark condensates. The quark condensate dependence of only one particular two loop diagram, was also evaluated seeking to estimate the possible effects of the next corrections. The evaluation of the full two loop dependence on the condensates, in order to determine their net effect on the results will be considered in further studies. We prefer to postpone the calculation of these terms, waiting for a precise definition of the renormalization scheme in the modified expansion.

The results obtained here for the effective potential indicate a dynamical generation of quark and gluon condensates. The dependence on the potential on the quark one, gets an unbounded from below behavior, in the present approximation. This instability becomes stronger by increasing the gluon condensate. However, even in the absence of the gluon condensate, the quark condensate is dynamically generated. This result seems consistent with the fact to be expected that the finite temperature deconfinement transition should not drastically affect the masses of the heavy quarks. The unbounded from below dependence, then indicates the need of higher approximations for producing an eventual minimum of the potential. The picture arising precisely reproduces the one expected to occur in Ref. [13] as a possible consequence of the underlined analogies between the construction of the modified wave functions for QCD and the BCS states [8, 13]. The dependence of the potential on the gluon condensate starting at zero value, is a decreasing one, which becomes steeper when the quark condensate grows. Therefore, this property generates the expectation that, the yet to be determined stabilizing contribution to the potential naturally could show opposite behavior in the quark and gluon dependence, able to produce not only the stabilization, but also a value for M at the minimum laying near the known constituent mass $M = 333\ MeV$. At low values of the quark condensate parameter, the gluon one develops a minimum. However, in the present approximation, for fixed values of the quark condensate the value of the gluon condensate at the minimum tend to grow, when the value of the parameter measuring the quark condensate mq increases. This is not unexpected, since we have not evaluated the full two loop dependence on the gluon condensate, nor the possible existing stabilizing terms. As mentioned before, they could help in maintaining the gluon condensate at a minimum being near low values, allowing to be fixed to the observable value. This fixation could be done, lets say by selecting scale parameter μ. It will be clearly surprising that the magnitude of the mq required for the stabilization could be as high as of the order of hundreds, as it would be needed for predicting the top condensate mass near $m_{top} = mq = 175$ GeV (as given by the pole of the quark propagator for high mq values [13]). However, we can not yet disregard this possibility and the search for an estimation of the stabilizing terms will be undertaken.

A complementary dependence of the two loop potential as a function of the quark condensate has been evaluated in the form of a simple 2D integral depending on this condensate and the gluonic one. The outcome for this quantity turned on to be finite after including the corresponding quark condensate dependent part of the usual quark counterterm. This result gave us confidence in that the renormalization procedure can be well implemented in the modified theory. However, a careful discussion of this question should be considered in detail.

The introduction of the gauge parameter dependence is another problem which needs an additional careful consideration. Partial argues [11, 12] about the gauge invariance of the scheme have been done , and also the evaluation of the one loop gluon self-energy directly satisfied the transversality Ward identity [9]. The discussion in the present work and in Ref. [12] also make clear that the modified theory is a multi-parameter one in which the implementation of the gauge invariance can show subtleties which need to be carefully addressed. However in the Appendix we present some ideas about how to consider these questions in next studies.

The work is organized as follows: Section 2, exposes the procedure for eliminating the singularities in the diagram expansion through the employment of dimensional regularizationin. Section 3, the propagators employed in the calculations are presented and the effective action vacuum diagrams described. The zero and one-loop potential contributions are discussed in Section 4. Section 5 considers the quark condensate dependent part of the two loop effective action contribution which is evaluated. Section 6 is devoted to expose and discuss the results of the evaluations done. Finally, the Appendix discusses gauge invariance aspects and the perspectives of its implementation in the proposed scheme. In the summary the main results of the work are shortly reviewed.

II. REGULARIZATION OF THE SINGULAR TERMS

The main technical difficulty for the implementation of the modified expansion proposed in the works [7–10, 13] is related with the fact that the addition to the standard free propagator term (let us call it below the "condensate" propagator) is given by a Dirac's Delta function of the momentum. This circumstance, then can produce singular diagrams even after the theory is dimensionally regularized. These singularities are associated to the appearance after some loop integrals are performed of Dirac Delta functions, or standard Feynman propagators evaluated at zero momentum. These factors occurs due to the conservation of momentum in each vertex . Let us consider a vertex with, let us say n legs ($n = 3, 4$ for QCD). Then, when $n - 1$ different condensate lines join to this vertex, the momentum conservation at it forces the value of the momentum at the only resting line to vanish. Therefore, if a condensate line is attached to this ending, Delta functions evaluated at zero momentum will appear. On another hand, when a usual free propagator is connected to this leg the singularity appearing corresponds to $\frac{1}{p^2+i\epsilon}$ evaluated at zero momentum. This situation should be solved before a full sense could be given to the modified expansion. Below, we propose a way of considering this problem which clearly should be the subject of further investigation for its consistency. Let us consider separately the two types of singularities.

A. $\delta(0)$ singularities

A direct idea that can come to the mind after considering the appearance of these kind of terms is the following. As we will employ dimensional regularization, the Delta functions in the propagators should be also considered as dimensionally regularized forms of the Dirac Delta function. However, it has been recognized that similar Delta functions evaluated a zero spacial coordinates appearing in gravitation theories can be analytically extended to continuous dimensions D and moreover, their expression after taken in the limit of the real space $D \to 4$ tends to vanish. It can be noticed that this is not a counter-intuitive result. This is simply because it is possible to impose on the succession of functions defining the Delta distribution, the condition of to vanish at the supporting point without destroying the possibility that the limit of the integral of any continuous function multiplied by the elements of the secession, tends to the value of the function at the support. Therefore, we follow the same procedure here and interpret the Delta functions appearing as D dimensional ones. Then, it is possible to step by step reproduce the arguments of Capper and Leibbrandt [16] to conclude that these factors should vanish after removing the dimensional regularization. Let us do it below for the sake of concreteness.

For the singular $\delta(0)$ we can write

$$\delta(0) = \int_E \frac{dp^D}{(2\pi)^D}.$$

This is a singular D dimensional integral in Euclidean momentum space, being completely similar to a one in real space, considered in [16]. Then, it can be also written as follows

$$\begin{aligned}\int_E \frac{dp^D}{(2\pi)^D} &= \int_E \frac{dp^D}{(2\pi)^D}\frac{p^2}{p^2} \\ &= \int_0^\infty ds \int_E \frac{dp^D\ p^2}{(2\pi)^D} \exp(-s\ p^2)\end{aligned}$$

But, employing the redefinition of the generalized Gaussian integral for continuous values of the dimension D constructed in Ref. [16], it is possible to write

$$\begin{aligned}\int_E \frac{dp^D}{(2\pi)^D} \exp(-s\ p^2) &= \frac{1}{(4\pi)^{\frac{D}{2}}} \exp(-s\ f(\frac{D}{2})), \\ \int_E \frac{dp^D\ p^2}{(2\pi)^D} \exp(-s\ p^2) &= \frac{1}{(4\pi)^{\frac{D}{2}}} \left[\frac{D}{2} s^{-(1+\frac{D}{2})} + s^{-\frac{D}{2}}\ f(\frac{D}{2})\right] \exp(-s\ f(\frac{D}{2})),\end{aligned}$$

and

$$\delta(0) = \int_0^\infty ds \frac{1}{(4\pi)^{\frac{D}{2}}} \left[\frac{D}{2} s^{-(1+\frac{D}{2})} + s^{-\frac{D}{2}}\ f(\frac{D}{2})\right] \exp(-s\ f(\frac{D}{2}))$$

where f is the function introduced in [16] for extending the generalized Gaussian integral formula for non-integral dimension arguments. As it should be, these functions vanish for all integral values of D.

Then, after employing the integral definition of the Gamma function

$$\Gamma(z) = \int_0^\infty dt\ t^{z-1} \exp(-t),$$

for the regularized form of the Delta function at zero momentum follows

$$\delta(0) = \frac{f(\frac{D}{2})^D}{(4\pi)^{\frac{D}{2}}} \left[\frac{D}{2}\Gamma(-\frac{D}{2}) + \Gamma(1-\frac{D}{2}) \right],$$

which exactly vanish in the limit $D \to 4$.

Therefore, we will interpret that the evaluations associated to the modified expansion are done by using the above representation for the factors $\delta(0)$. Thus, as a consequence, it will be considered that all the diagrams in which only such kind of singularities appear will vanish in dimensional regularization.

B. b) $\frac{1}{p^2+i\epsilon}$ at $p = 0$ singularities

For this kind of singular behavior, let us follow the physical notion about that the modes at zero momentum are appropriately described only by the condensate propagator. Therefore, it seems reasonable to also regularize the dependence of $\frac{1}{p^2+i\epsilon}$ at exactly zero value of the momentum in a way that vanish at this single point in momentum space $p = 0$.

Following this idea, let us adopt the following particular regularization satisfying this criterion

$$\frac{1}{p^2+i\epsilon}|_{reg} = \frac{p^2}{p^2(p^2+i\epsilon)+\delta^2}.$$

This expression vanish at $p = 0$ and in the limit $\delta \to 0$ leads to the scalar Feynman propagator.

Let us consider now the general loop expansion after assuming the above two prescriptions and the original free propagators in the Feynman gauge employed in [13]

$$\begin{aligned} G^{ab}_{g\mu\nu}(p,m) &= \frac{\delta^{ab} g_{\mu\nu}}{p^2+i\epsilon} - iC\ \delta(p), \\ G^{f_1 f_2}_q(p,M,S) &= -\frac{\delta^{f_1 f_2} p_\mu \gamma^\mu}{p^2+i\epsilon} + i\ \delta^{f_1 f_2} C_f\ \delta(p), \\ \chi^{ab}(p) &= -\frac{\delta^{ab}}{p^2+i\epsilon}, \end{aligned} \tag{1}$$

and the standard vertices of QCD. Then, whenever the $\delta-$regularization is employed as described above, it directly follows that all the diagrams having a fixed number of loops showing both types of singularities will vanish by taking the limits in both regulators

$$\begin{aligned} \delta &\to 0, \\ D &\to 4. \end{aligned}$$

Therefore, after taking the limit $\delta \to 0$, before the one $D \to 4$, the remaining finite diagrams (due to the dimensional regularization) can be evaluated using the non distorted propagators (1).

At this point is useful to underline some properties of the original diagrammatic expansion based on the above propagators, assumed the above explained regularization conditions have eliminated all the singular contributions. Then it follows:

1) The appearance of a number m of the condensate propagators within a n-loop diagram will eliminate m of the n loop integrals associated to this contribution. Therefore, the considered diagram will be now an "effective" $n-m$-loop one.

2) After expressing the condensate parameters C and C_f in favor of the ones:

$$m^2 = -\frac{6g^2C}{(2\pi)^4}, \qquad (2)$$
$$S_f = \frac{g^2C_F}{4\pi^4}\ C_f,$$

which also incorporate a power of order two of the coupling constant g, it also follows that the n-loop diagrams of the effective expansion, as considered as multiple power expansion in the three parameters m^2, S_f and g, also shows the property that, given the number of external legs of the diagram, the number of loops is fixed by the power of the coupling constant appearing in it. This follows directly from the fact that each time that a condensate line appears a loop integral is annihilated and correspondingly the power of g of the diagram is reduced by two. Let us consider that p is the power of g corresponding to a n loop diagram in which the parameters are not redefined. Thus the new power of g of this diagram in which m condensate lines and the new parameters are introduced will simply be

$$p' = p - 2m. \qquad (3)$$

This property, then seems to allows for a useful reordering of the perturbation expansion. To see elements suggesting it, let us consider a particular n-loop diagram in which the change of the parameters (2) have been introduced and corresponding line symbols have been introduced for the standard and the condensate propagators separately. Therefore, for any particular standard type line in this diagram it seems possible to consider the infinite summation of all the zero order in g (tree) contributions to the connected propagator (which by construction have the same number of loops but comes from higher loops in the original expansion). This is done by considering fixed the other standard lines. Thus, if not blocked by some difficulty associated to the combinatorial and symmetry factors in the diagrams, the performed infinite additions seems that can be shown to be possible for all the normal lines. These zero order in the new expansion propagators are no other things that the expressions (5)-(8) written in the first section. This was effectively shown in Ref. [13]. Thus, it seems possible to demonstrate that the loop expansion can be reordered to produce other version of it in which the new tree propagators will be (5)-(8). The investigation of this possibility is expected to be considered in further works.

III. PROPAGATORS AND EFFECTIVE ACTION

In the next sections the evaluation of the effective potential including zero, one and two loop corrections will be considered. The contributions will be calculated by inserting the infinite ladder of condensate dependent one-loop self-energy parts in the original free propagators following the rules defined in Section 2 and Ref. [13]. These propagators for quarks and gluons, as well as for the condensate lines (defined in Section 2 and Ref. [13]) are given as

$$G^{ab}_{g\mu\nu}(p,m) = \frac{\delta^{ab}}{(p^2-m^2+i\epsilon)}(g_{\mu\nu}-\frac{p_\mu\ p_\nu}{p^2+i\epsilon})+\frac{\beta\ p_\mu\ \ p_\nu}{(p^2+i\epsilon)^2}\delta^{ab}, \tag{4}$$

$$= \frac{\delta^{ab}}{(p^2-m^2+i\epsilon)}(g_{\mu\nu}-\frac{\beta\ p_\mu\ p_\nu\ m^2}{(p^2+i\epsilon)^2}), \quad \beta=1,$$

$$G^{f_1 f_2}_q(p,M,S) = \frac{\delta^{f_1 f_2}}{\left(-p_\mu\gamma^\mu(1-\frac{M^2}{p^2})\ \ -\frac{S_{f_1}}{p^2}\right)}, \tag{5}$$

$$\chi^{ab}(p) = -\frac{\delta^{ab}}{p^2+i\epsilon}, \tag{6}$$

$$G^{ab}_m = -\frac{im^2}{g^2}\delta^{ab}\ \delta(p), \tag{7}$$

$$G_S = \frac{i4\pi^4 S_f}{g^2 C_F}\delta^{ab}\delta^{f_1 f_2}\delta(p), \tag{8}$$

where(4-6) are the gluon, quark and ghost propagators respectively and (7), (8) the gluon and quark condensate ones. In this work we will adopt the general conventions for the spinor, colour and Lorentz groups, the free propagators and interactions vertices of reference [18].

The parameters m^2, M, S_f are related to the constants C and C_f (see Ref. [13]) characterizing the gluon and quark condensates, as follows

$$-m^2 = m_g^2 = \frac{6g^2C}{(2\pi)^4}, \tag{9}$$

$$S_f = \frac{g^2 C_F}{4\pi^4}\ C_f, \tag{10}$$

$$m^2 = f\ M^2, \quad f=\pm(\frac{3}{2})^2, \tag{11}$$

$$g^2 = g_o^2\ \ \mu^{2-\frac{D}{2}}, \tag{12}$$

$$D = 4-2\epsilon.$$

In these relations the parameter $f=\pm(\frac{3}{2})^2$ will be considered for the two values of its sign. However, the negative value was the only one implied by the calculation done in [13]. In that work, it followed that if the parameter C is chosen to be positive, then the gluon mass in the simplest approximation turns to be tachyonic ($m^2=-\frac{6g^2C}{(2\pi)^4}$) and the constituent mass for light quarks M becomes real. The situation reverses for negative values of the gluon parameter C. However, it should be taken into account that these relations appeared due to the special fact that the condensate dependent part of the one gluon self-energy had no contribution from the fermions. This situation occurred precisely because they were assumed as massless. Therefore, as the very same motivation of the present discussion is related with the possibility of generating masses for the quarks, and also paying attention to the fact that the corrected propagators could be perhaps also constructed in a self-consistent manner (allowing for a self-consistently generated mass), we consider of interest to examine also the calculation for the positive choice of f . It should be mentioned that for this value of the parameter, both the gluon mass and the constituent masses are real. Moreover, the gluon mass is near the value estimated through other studies [17] $m = 0.5\ GeV$, while the constituent mass gets the reasonable value $M = 0.33\ GeV$[13]. However, a study of the physical justification of the positive choice of f needs to be done. This question could be considered elsewhere.

It should be explicitly stated that only one flavour was assumed to be condensed en the present discussion. This condition was elected because at present level of approximation , the consideration of various flavours will simply lead to the addition of identical fermions contributions to the potential. The question about the possible interference of various quark condensates, since it needs for higher order approximations for its appearance, will be relegated to further studies. It is clear that this is a relevant point, because only in the case that the presence of various kinds of such condensates will be rejected by the system, the dynamical generation of only one (main) quark condensate will be preferred. This could occurs by example, due to the presence of terms in the effective potential growing in value when more than one condensate are present. This effect seems clearly possible to occur but its considerations need for at least three loop corrections (or their descendants according to the reasoning in Appendix A) in which different quark loops can start to appear [13].

The collection of zero, and one loop diagrams which were evaluated are illustrated in Fig.1. The diagram Γ_1 is the only non having closed loops, that is a tree correction. Γ_2 and Γ_3 show the usual gluon and quark one loop corrections associated to the propagators (4) and (5) respectively. Further, diagrams $\Gamma_4, \Gamma_5, \Gamma_6$ and Γ_7 are related with the one loop corrections being "descendant" from the two loop ones due to the cancellation of one of the two loop integrals by a condensate propagator, and the insertion of all coupling g independent self-energy insertions leading to the propagators 4, 5 in the other two lines (See Section 2).

Finally in Fig. 2 the diagrams Γ_8 and Γ_9, Γ_{10} and Γ_{11} are defined as follows: Γ_8 is the standard diagram for the two loop correction including all the coupling independent self-energy insertions in its internal lines, Γ_9 is the same contribution as Γ_8 but taken in the limit $S \to 0$ which is substracted in order to consider only the quark condensate dependent part of the two loop term, the limit $S \to 0$ is indicated by the rings in the fermion lines. Finally Γ_{10} is the g^2 contribution associated to the fermion counterterm and Γ_{11} is the same contribution in the limit $S \to 0$ substracted in order to again only consider the quark condensate dependent part of the potential. The momentum integrations, in writing the diagram expressions, will be

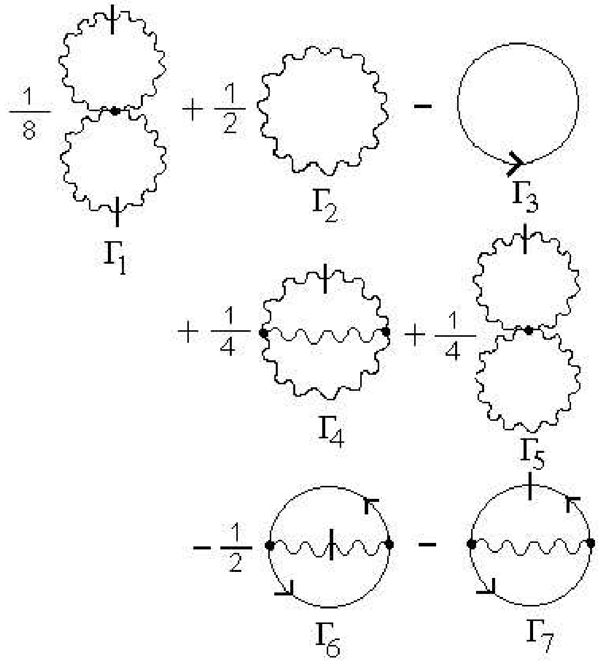

FIG. 1: The figure shows the seven Feynman diagrams defining the zero and one loop contributions evaluated in the work. The lines having cuts correspond to the condensate propagators. Therefore, although the associated diagrams may look as two loop ones the Delta functions associated to them effectively cancel one of the loop integrals.

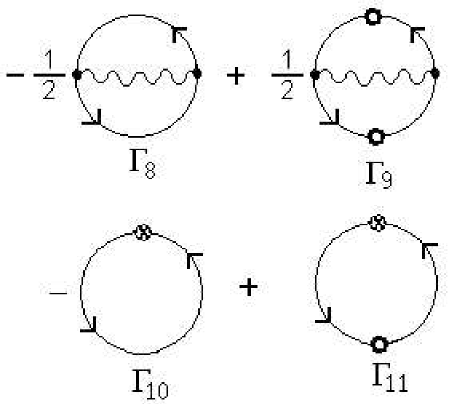

FIG. 2: The figure illustrates: a) The two fermion contribution Γ_8 considered in the work for getting a sense of the influence of the higher loops; b) The substracted diagram Γ_9 which is the same Γ_8 evaluated at $S = 0$; c) The diagram associated to the fermion counterterm Γ_{10} and d) This last diagram taken at $S = 0$ indicated by Γ_{11}.

taken in Minkowski space for after perform the Wick rotation. However, it should be made precise that in order to make the rotation without encountering poles, the sign of m^2 should be positive ($f = (\frac{3}{2})^2$). However, we will perform the Wick rotation for the two signs of f without including the terms that could be incorporated by rounding the poles in the p_o variables when deforming the integration contour if f takes its negative value. Therefore the results obtained for f negative, should be interpreted as the evaluation of the effective potential in Euclidean field theory. That is, the evaluated quantity corresponds to the thermodynamical effective potential in the limit of zero temperature.

The employed expression for the fermion renormalization constant is given by [18]

$$\begin{aligned}(Z_2 - 1) &= -\frac{g^2 C_F}{(4\pi)^2}\frac{\beta}{\epsilon}, \\ T_R &= \frac{1}{2}, \quad C_G = N, \quad C_F = \frac{N^2-1}{N}.\end{aligned}$$

As it was remarked before, only one quark flavour will be considered for the present qualitative discussion, since up to this level all the quark flavours will produce a sum of contributions all of the same functional dependence on their respective condensates. However, the fact that light quarks exist furnishes a guiding principle in the sense that the mass acquired by them, if their quark condensates do not develop, should coincide with the parameter M [13] at the minimum of the effective potential.

The value of the renormalized gauge parameter β is equal to one in accordance with the fact that the modification of the Feynman rules induced by the presence of the condensates was obtained in [8] within the Feynman gauge $\beta = 1$. As it was mentioned in the Introduction, the question of the gauge parameter invariant formulation of the modified perturbation theory under study, which clearly represents a required step in the formal completion of the approach, should be further considered. However, some general remarks on this question are given in the Appendix and a more concrete study is expected to be considered elsewhere.

IV. ZERO AND ONE-LOOP TERMS

In this section the results for the evaluation of contribution to the one loop effective potential Γ_1 to Γ_7 will exposed below in consecutive order.

A. Zero loop term

The direct substitution of the gluon condensate propagator (7) in the analytic expression associated to Γ_1, after evaluating all the Lorentz, spinor and colour traces leads to

$$\Gamma^{(0)} = -\frac{2m^4}{g^2} = -V^{(0)}.$$

That is, a positive potential proportional to m^4. As the one loop terms have zero order in the coupling g, in the expansion in powers of the parameters defined in Section 2, this term shows a power -2 of g, since the original diagram was of order two and there is two condensate lines in the diagram which reduce the power in four according to (3).

B. Standard one loop terms

The sum of the one loop terms corresponding to Γ_2 and Γ_3 in Fig.1 have the form

$$\begin{aligned}
\Gamma_{gf}^{(1)} &= \Gamma_g^{(1)} + \Gamma_f^{(1)} + \Gamma_S^{(1)} = -V_g^{(1)} - V_f^{(1)} - V_S^{(1)} \\
&= -\frac{i}{2}\, Tr\ \left[\log\left[G_g^{-1}(0)\ G_g^{-1}(m)\right]\right] + i\ Tr\ \left[\log\left[G_q^{-1}(0,0)\ G_q^{-1}(M,0)\right]\right] + \\
&\quad +i\ Tr\ \left[\log\left[G_q^{-1}(0,0)\ G_q^{-1}(M,S)\right]\right] - i\ Tr\ \left[\log\left[G_q^{-1}(0,0)\ G_q^{-1}(M,0)\right]\right],
\end{aligned}$$

which have been expressed as the sum of a S independent term corresponding to the same diagrams evaluated at $S = 0$ plus a S dependent contribution vanishing in the limit $S \to 0$. After calculating the Lorentz, spinor and colour traces for the $S = 0$ gluon and quark loops $\Gamma_g^{(1)}$ and $\Gamma_f^{(1)}$, and dimensionally regularizing the integral, it follows

$$\begin{aligned}
\Gamma_g^{(1)} &= -\frac{(N^2-1)(D-1)}{2}\int \frac{dp^D}{(2\pi)^D i}\log\left[\frac{p^2}{p^2-m^2}\right], \\
\Gamma_f^{(1)} &= 4N\int \frac{dp^D}{(2\pi)^D i}\log\left[\frac{p^2}{p^2-M^2}\right].
\end{aligned}$$

But, in both cases taking the derivative of the expressions over the parameters in the gluon and quark cases, leads to simpler expressions. Then, after performing the Wick rotation in the temporal momentum component according to

$$p_0 \to i\ p_4,$$

the derivative over the parameters expressions can be integrated in momentum space by employing the formula [18]

$$\int_E \frac{dp^D}{(2\pi)^D}\left[\frac{1}{p^2+L^2}\right] = \frac{B(\frac{D}{2},1-\frac{D}{2})}{(4\pi)^{\frac{D}{2}-2}\Gamma(\frac{D}{2})}(L^2)^{D-2},$$

in which L can be selected as m or M for the gluon or quark terms respectively. The results of the integrals can be integrated over the parameter again from their zero values to the original ones. After that, considering that the real part of $D-2$ is positive, allows to obtain

$$\Gamma_g^{(1)}(m) = \frac{(D-1)(N^2-1)B(\frac{D}{2},1-\frac{D}{2})(m^2)^{\frac{D}{2}}}{D\ (4\pi)^{\frac{D}{2}}\Gamma(\frac{D}{2})}, \tag{13}$$

$$\Gamma_f^{(1)}(M) = -\frac{8\ N\ B(\frac{D}{2},1-\frac{D}{2})(M^2)^{\frac{D}{2}}}{D\ (4\pi)^{\frac{D}{2}}\Gamma(\frac{D}{2})}, \tag{14}$$

At this point, after considering relations (9), (11) and (12) defining m and M as functions of the dimension D, and substracting the pole part in ϵ of (13) and (14), the Minimal Substraction result for the one loop effective action is

$$V_g^{(1)}(m) = -\frac{(N^2-1)}{128\ \pi^2}\ m^4 \left(-6\log\left(\frac{m^2}{4\pi\mu^2}\right) - 6\gamma + 5\right), \tag{15}$$

$$V_f^{(1)}(M) = \frac{3\ (N^2-1)}{128\ \pi^2}\ M^4 \left(-2\log\left(\frac{M^2}{4\pi\mu^2}\right) - 2\gamma + 3\right). \tag{16}$$

The S dependent correction $\Gamma_S^{(1)}$ after all the trace evaluations can be written as

$$\begin{aligned} \Gamma_S^{(1)} &= +i\ Tr\ \left[\log\left[G_q^{-1}(0,0)\ G_q^{-1}(M,S)\right]\right] - \\ &\quad -i\ Tr\ \left[\log\left[G_q^{-1}(0,0)\ G_q^{-1}(M,0)\right],\right] \\ &= 2N\int \frac{dp^D}{(2\pi)^D i}\log\left[\frac{p^2(p^2-M^2)^2}{p^2(p^2-M^2)^2-S^2}\right]. \end{aligned}$$

This integral, after the Wick rotation is convergent in the limit $D\to 4$, and takes the form

$$\begin{aligned} \Gamma_S^{(1)} &= -V_{q,S}^{(1)} = -2\ N\int_E \frac{dp^D}{(2\pi)^D}\log\left[\frac{p^2(p^2+M^2)^2}{p^2(p^2+M^2)^2+S^2}\right], \\ &= -4\ N\int_0^\infty \frac{dp\ p^3}{(4\pi)^2}\ \log\left[\frac{p^2(p^2+M^2)^2}{p^2(p^2+M^2)^2+S^2}\right]. \end{aligned}$$

C. One loop terms descending from the two loop gluon diagrams

After writing the analytical expressions for the diagram Γ_4 and evaluating the Lorentz, spinor and colour traces, the expression can be rewritten in the form

$$\begin{aligned} \Gamma_{2g}^{(1,1)} &= -V_{2g}^{(1,1)} = \frac{(N^2-1)N\ m^2}{4(2\pi)^D}\int \frac{dp^D}{(2\pi)^D i}\frac{(-6\ p^2 D + 2(D+11)\beta m^2 - 8\ m^4/p^2)}{(p^2-m^2)^2}, \\ &= \frac{(N^2-1)N\ m^2}{4(2\pi)^D}\int_E \frac{dp^D}{(2\pi)^D}\frac{(6\ p^2 D + 2(D+11)\beta m^2 + 8\ m^4/p^2)}{(p^2+m^2)^2}. \end{aligned}$$

In the second line of this equation the Wick rotation has been made. The integrals can be explicitly performed to give

$$\begin{aligned} \Gamma_{2g}^{(1,1)} &= -V_{2g}^{(1,1)} = \frac{(N-1)N\ \Gamma(1-\frac{D}{2})\left[3D^2+2\beta(D+11)(1-\frac{D}{2})\right]}{4(2\pi)^D(4\pi)^{\frac{D}{2}}}(m^2)^{\frac{D}{2}} + \\ &\quad + \frac{2(N-1)N\ \Gamma(\frac{D}{2}-1)\Gamma(3-\frac{D}{2})}{(2\pi)^D(4\pi)^{\frac{D}{2}}\Gamma(\frac{D}{2})}(m^2)^{\frac{D}{2}}. \end{aligned}$$

After applying the same procedure for the analytical expressions associated to the diagram Γ_5 the result is

$$\Gamma_{2g}^{(1,2)} = -V_{2g}^{(1,2)} = \frac{(N^2-1)N(D-1)\ m^2}{2(2\pi)^D}\int \frac{dp^D}{(2\pi)^D i}\frac{1}{(p^2-m^2)}\left[1-\frac{\beta m^2}{p^2}\right],$$
$$= -\frac{(N^2-1)N(D-1)\ m^2}{4(2\pi)^D}\int_E \frac{dp^D}{(2\pi)^D}\frac{1}{(p^2+m^2)}\left[1+\frac{\beta m^2}{p^2}\right],$$
$$= -\frac{(N-1)N\ (D-1)(D-\beta)\Gamma(1-\frac{D}{2})}{2(2\pi)^D(4\pi)^{\frac{D}{2}}}(m^2)^{\frac{D}{2}}.$$

It is an interesting outcome that after adding these two contributions and removing the dimensional regularization limit, the result remains finite, a fact that also cancel the logarithmic terms in the outcome. The total contribution of these terms for the potential at the end takes the form

$$\lim_{D\to 4}(V_{2g}^{(1,1)}+V_{2g}^{(1,2)}) = \frac{3f^2M^4}{8\pi^2}.$$

D. One loop terms descending from the two-loop quark diagram

The last one loop diagrams Γ_6 and Γ_7 correspond to the descendants of the two loop terms having a closed fermion line. The integral expression obtained for them after performing the Lorentz, spinor and colour traces are not so simple and we just numerically evaluate them in this work. The resulting integral expressions are

$$\Gamma_{2q}^{(1,1)} = -V_{2q}^{(1,1)} = \frac{(N^2-1)\ m^2}{3(4\pi)^2}\int_0^\infty dp\ p^3\frac{(2\ p^2(\ p^2+M^2)^2+D\ S^2)}{(p^2(\ p^2+M^2)^2+S^2)^2},$$
$$\Gamma_{2q}^{(1,2)} = -V_{2q}^{(1,2)} = N\ S^2\int_E \frac{dp}{(2\pi)^4}\ \frac{(D\ p^2+m^2-i\epsilon)}{(p^2(\ p^2+M^2)^2+S^2)(\ p^2+m^2-i\epsilon)}. \qquad (17)$$

In ending this section it can be noticed that all the "descendant" diagrams became finite ones.

V. TWO-LOOP QUARK TERM

Let us evaluate a sampling two loop term for checking its influence on the zero and one loop results. We will consider here the full dependence on the quark condensate associated to the diagram Γ_8 in Fig. 2 defined by a fermion loop formed with propagators (5), showing two quark-gluon interaction vertices. Therefore, the same diagram expression but evaluated at $S=0$ will be substracted from this contribution. This term is associated with Γ_9 in Fig. 2. This substraction simply corresponds to the same analytic expression of the diagram but taken for $S=0$ and is represented by a similar figure, but showing small rings on the quark propagators. As the diagram associated to the fermion counterterm of the standard massless QCD Γ_{10}(of order g^2 and therefore needed for renormalization at one loop level) is also depending on the condensate parameter S, the same kind of substraction is done for the $S=0$ counterterm term associated to Γ_{11}, in which again the ring in the quark line means the evaluation in $S=0$.

The substracted terms, exactly give the full two loop term formed by two quark propagators and one gluon line of the theory in the absence of the fermion condensate. As mentioned before we will postpone the evaluation of the full two loop gluon parameter dependence to further studies. The main reason for doing so is that for these terms, it is more relevant to precisely define the way in which the renormalization should be done within the considered scheme. As it is discussed in Section 2, at the two loop level there will appear additional quark condensate dependence coming from two loop diagrams being descendant from higher loop terms of the original expansion. From exploring evaluations done we know however, that it seems possible to cancel the two loop infinities by renormalizing the condensate parameters. However, a clearer understanding on the structure of the allowed counterterms in the modified expansion is desirable before evaluating the two loop terms.

After calculating the spinor and colour traces in the analytic expressions corresponding to the Feynman graphs shown in Fig.1, the considered contributions can be written in the form

$$\begin{aligned}
\Gamma_{fg}^{(2)}(M,S) &= -\frac{(N^2-1)g^2}{4}\int \frac{d^Dq}{(2\pi)^D i}\frac{d^Dq'}{(2\pi)^D i}\frac{1}{((q-q')^2-m^2)}\times \qquad (18)\\
&\frac{1}{(q^2(q^2-M^2)^2-S^2)(q'^2(q'^2-M^2)^2-S^2)}\times\\
&\{-4q^2q'^2(q^2-M^2)(q'^2-M^2)[(D-2)q.q'-\beta\frac{m^2}{(q'-q)^2}\times\\
&(q.q'-\frac{2q.(q'-q)q'.(q'-q)}{(q'-q)^2})]+4(D-\frac{\beta\, m^2S^2}{(q'-q)^2})\; q^2q'^2\},\\
\Gamma_{fg}^{(2)}(M,0) &= -\frac{(N^2-1)g^2}{4}\int \frac{d^Dq}{(2\pi)^D i}\frac{d^Dq'}{(2\pi)^D i}\frac{(-4)}{((q-q')^2-m^2)(q^2-M^2)(q'^2-M^2)}\times \qquad (19)\\
&[(D-2)q.q'-\beta\frac{m^2}{(q'-q)^2}(q.q'-\frac{2q.(q'-q)q'.(q'-q)}{(q'-q)^2})],\\
\Gamma_{fC}^{(2)}(M,S) &= 4N(Z_2-1)\int\frac{d^Dq}{(2\pi)^D i}\frac{(q^2)^2(q^2-M^2)}{q^2(q^2-M^2)^2-S^2}, \qquad (20)\\
\Gamma_{fC}^{(2)}(M,0) &= 4N(Z_2-1)\int\frac{d^Dq}{(2\pi)^D i}\frac{q^2}{(q^2-M^2)}. \qquad (21)
\end{aligned}$$

It can be noticed that the mass dimension of the parameter S is equal to 3, that is a relatively high value. Therefore, the terms of the expansion in powers of S for the denominator of the integrand associated to $\Gamma_{gf}^{(2)}$ will have three powers of the momentum convergence factors for each power of S appearing in the expansion. The same effect occurrs in the fermion counterterm $\Gamma_{fC}^{(2)}$.

Then, it follows that the quantity

$$\Gamma_{fg}(m,M,S,\epsilon)=\Gamma_{gf}^{(2)}(M,S,\epsilon)-\Gamma_{gf}^{(2)}(M,0,\epsilon)+\Gamma_{fC}^{(2)}(M,S,\epsilon)-\Gamma_{fC}^{(2)}(M,0,\epsilon),$$

which contains, by construction, the whole dependence of the effective action on the fermion condensate parameter S, turns to be finite in the limit $D\to 4$ ($\epsilon\to 0$). This result is simply expressing the fact that the renormalization constant Z_2 of the massless QCD (determined in the absence of any condensate) is also able to extract the infinities from the single fermion condensate dependent contribution under study. As noticed before, according to the above described substraction procedure, the substracted terms in addition with the non considered two loop ones, exactly correspond to the two loop plus counterterm contributions in the absence of the fermion condensate. These terms, including the ones descending from the higher loops (according to the reasons given in Section 2) will not be considered here.

The finite contribution Γ_{fg} before passing to Euclidean variables can be written as the sum of the following three terms

$$
\begin{aligned}
\Gamma^{(2)}_{fg} &= \Gamma^{(2,1)}_{fg} + \Gamma^{(2,2)}_{fg} + \Gamma^{(2,3)}_{fg} && (22)\\
\Gamma^{(2,1)}_{fg} &= \frac{(N^2-1)g^2}{2}\int \frac{d^Dq}{(2\pi)^D i}\int \Big(\frac{d^Dq'}{(2\pi)^D i}\frac{4\,(D-2)\,q.q'}{((q-q')^2-m^2)(q'^2-M^2)} - && (23)\\
&\quad -\frac{4}{(4\pi)^2}\frac{q^2}{\epsilon}\Big)\times\frac{S^2}{(q^2(q^2-M^2)^2-S^2)(q^2-M^2)} + \\
&\quad (N^2-1)g^2\int \frac{d^Dq}{(2\pi)^D i}\frac{d^Dq'}{(2\pi)^D i}\times\frac{2\,(D-2)q.q'S^4}{(q^2(q^2-M^2)^2-S^2)}\times \\
&\quad \frac{1}{(q'^2(q'^2-M^2)^2-S^2)(q^2-M^2)(q'^2-M^2)((q-q')^2-m^2)},\\
\Gamma^{(2,2)}_{fg} &= -\frac{\beta\, m^2(N^2-1)g^2}{4}\int \frac{d^Dq}{(2\pi)^D i}\frac{d^Dq'}{(2\pi)^D i}\times\frac{4\,q^2q'^2(q^2-M^2)}{(q^2(q^2-M^2)^2-S^2)}\times && (24)\\
&\quad \frac{(q'^2-M^2)(2\,q^2q'^2-q.q'(q^2+q'^2))}{(q'^2(q'^2-M^2)^2-S^2)((q-q')^2-m^2)((q'-q)^2)^2},\\
\Gamma^{(2,3)}_{fg} &= \frac{\beta\, m^2(N^2-1)g^2}{4}\int \frac{d^Dq}{(2\pi)^D i}\frac{d^Dq'}{(2\pi)^D i}\times\frac{4\,q^2q'^2S^2}{(q^2(q^2-M^2)^2-S^2)}\times && (25)\\
&\quad \frac{S^2}{(q'^2(q'^2-M^2)^2-S^2)((q-q')^2-m^2)(q'-q)^2},
\end{aligned}
$$

where the term showing the $\frac{1}{\epsilon}$ factor is associated to the fermion counterterm. It is responsible for the substraction of the divergent part of the remaining expressions.

After performing the Wick rotation, it is possible to eliminate the pole term in ϵ by using the identity

$$
\frac{1}{\epsilon} = \frac{(4\pi)^{\frac{D}{2}}\Gamma(\frac{D}{2})\left[-q^2\right]^{2-\frac{D}{2}}}{\epsilon B(\frac{D}{2},2-\frac{D}{2})B(\frac{D}{2}-1,\frac{D}{2}-1)}\int \frac{d^Dq'}{(2\pi)^D i}\frac{1}{(q-q')^2q'^2}.
$$

Then, the finite fermion condensate dependent contribution to the particular two loop term evaluated here, in the limit $\epsilon \to 0$, can be expressed as follows

$$
\begin{aligned}
V_{fg} &= -\Gamma_{ffg} = -v_0\left[v_f^{(1)} + v_f^{(2)} + v_f^{(3)} + v_f^{(4)} + v_f^{(5)}\right],\\
v_0 &= \frac{4}{(4\pi)^4}(N^2-1)g^2M^4,
\end{aligned}
$$

The quantities $v_f^{(i)}, i = 1,2,3,4,5$ appearing above were reduced to simple 2D integrals after performing the angular integrations in the 4-dimensional Euclidean space. They take the explicit forms

$$v_f^{(1)} = -2\ X^6 \int_0^\infty dq \int_0^\infty dq' \quad \frac{q^4 q'^4}{(q^2(q^2+1)^2 + X^6)(q'^2(q'^2+1)^2+X^6)} \times \\ \ln\left(\frac{q^2+q'^2+2qq'+f-i\epsilon}{q^2+q'^2-2qq'+f-i\epsilon}\right),$$

$$v_f^{(2)} = +\ X^6 \int_0^\infty dq \int_0^\infty dq' \quad \frac{q^3 q'^3}{(q^2(q^2+1)^2+X^6)(q^2+1)} \times \{\frac{1}{q'^2(q'^2+1)} + \\ + \frac{q^2+q'^2+f-i\epsilon}{4\ qq'(q'^2+1)} \times \ln\left(\frac{q^2+q'^2+2qq'+f-i\epsilon}{q^2+q'^2-2qq'+f-i\epsilon}\right) \\ - \frac{q^2+q'^2-i\epsilon}{4\ qq'^3} \ln\left(\frac{q^2+q'^2+2qq'-i\epsilon}{q^2+q'^2-2qq'-i\epsilon}\right)\},$$

$$v_f^{(3)} = -X^{12} \int_0^\infty dq \int_0^\infty dq' \frac{q^3 q'^3}{(q^2(q^2+1)^2+X^6)(q'^2(q'^2+1)^2+X^6)(q^2+1)(q'^2+1)} \times \\ \{-1 + \frac{q^2+q'^2-i\epsilon}{4\ q\ q'} \ln\left(\frac{q^2+q'^2+2qq'-f^2-i\epsilon}{q^2+q'^2-2qq'-f^2-i\epsilon}\right)\},$$

$$v_f^{(4)} = -\beta \int_0^\infty dq \int_0^\infty dq' \quad q^3 q'^3 (q^2+1)(q'^2+1) \times \\ \{\frac{q^2 q'^2}{(q^2(q^2+1)^2+X^6)(q'^2(q'^2+1)^2+X^6)} - \frac{1}{(q^2+1)^2(q'^2+1)^2}\} \times \\ \{1 + \frac{(q^2+q'^2 - \frac{(q^2-q'^2)^2}{f}}{4\ q\ q'} \ln\left(\frac{(q-q')^2+f-i\epsilon}{(q+q')^2+f-i\epsilon}\frac{(q+q')^2-i\epsilon}{(q-q')^2-i\epsilon}\right)\},$$

$$v_f^{(5)} = -\frac{\beta X^6}{2} \int_0^\infty dq \int_0^\infty dq' \quad \frac{q^4 q'^4}{(q^2(q^2+1)^2+X^6)(q'^2(q'^2+1)^2+X^6)} \times \\ \ln\left(\frac{(q-q')^2+f-i\epsilon}{(q+q')^2+f-i\epsilon}\frac{(q+q')^2-i\epsilon}{(q-q')^2-i\epsilon}\right),$$

in which as before the dimensionless quantities X are given as follows

$$S = M^3 X^3.$$

The ϵ parameter is retained here since it helps to regularize the integrals even in the Euclidean case when f is negative.

VI. DISCUSSION

In this section we will present the results for the evaluation of the effective potential as a function of the condensate parameters M, S and the couplings constant g. The calculations were done for the two signs of the parameter f that defines the relation between the constituent quark and gluon mass parameters m and M through

$$m^2 = f \quad M^2.$$

As it was remarked before, only the negative sign was arising in the work [9] because the constituent mass value evaluated in that work was satisfying the above relation with the negative sign. This fact was a direct consequence of the free standard quark propagator being massless. However, as it was noticed before here, we suspect that a sort of self-consistent treatment could lead to an unrestricted sign of f . Thus the evaluation for positive f values was also considered. An interesting point in this sense, is that for positive f the results for the potential are completely real, a fact that is not occurring for the more relevant case under study, that is $f = -(\frac{3}{2})^2$. Moreover, in this situation, it turned out that the value of m following, once the quark condensate $< g^2G^2 >$ is fixed, is $m = 0.5\ GeV$, which coincides with a result obtained in Ref. [17]. Unfortunately, the sign of the light quark masses which follows from the Dyson equation [9] is opposite to the sign of the gluon condensate parameter, that is basically the sign of m^2. Therefore, in case that we select the negative sign of f , then the absolute value of the constituent mass for light quarks will be also 333 MeV but the mass will be tachyonic. This perhaps is another possibility which could be needed to be also examined. In any case, neither gluons or quarks appear in Nature and perhaps both will be absent as real excitations in both descriptions in which none of them will be asymptotic states after including more corrections [13].

Let us define for the graphical illustrations the quantities $V(mq, M, g, \mu)$ and its imaginary part $V_{im}(mq, M, g, \mu)$ (where mq is defined as $mq = MX$) as the sum of all the contributions to the effective potential (the negatives of the effective action terms) evaluated in previous sections divided by the constant factor $1/(8\pi^4)$. In the various figures below the dependence of the quantity V , or its imaginary part, V_{im} are plotted as functions of two of their arguments selecting the others as given by characteristic values of interest in the present state of the discussion. The plots are associated to the relevant case $f = -(3/2)^2$ and sometimes comments about the effect of the graphs of changing the sign of f will be done. The Fig. 3 illustrates the behavior of the effective potential as a function mq and the constituent mass M. Both quantities are defined in the text in terms of the quark condensate and the gluon one through

$$\begin{aligned} X &= \frac{mq}{M} = \frac{S^{\frac{1}{3}}}{M} = \frac{1}{M}(\frac{gC_F}{4\pi^4}C_f)^{\frac{1}{3}}, \\ M &= \frac{m^2}{f}, \qquad \text{for } M \text{ real.} \end{aligned}$$

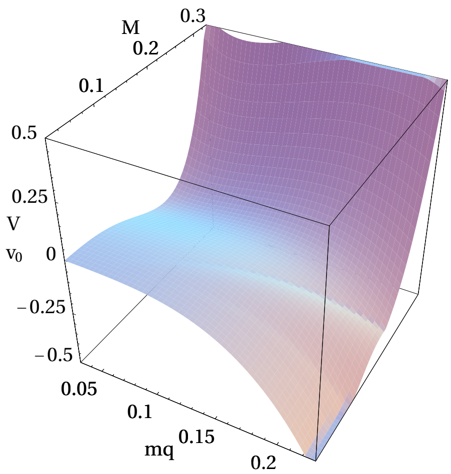

FIG. 3: The landscape picturing the dependence of the function V (the effective potential divided by $1/(8\pi^4)$) on the quark condensate (measured by mq) and the gluon one (measured by the constituent mass M). The values of the other parameters are $g = 2.74$ and $\mu = 6.8\ GeV$. At $mq = 0$ the potential develops a minimum at certain value of M which can be varied by changing the scale parameter μ. The dependence on mq indicates an instability upon the generation of values of mq which is not controlled at large mq values. It can be also seen that even at zero value of the gluon condensate ($M = 0$) the potential remains unbounded from below. That is, the sole presence of the quark condensate also makes the system unstable under the generation of mq from the state at $mq = 0$. This property suggests that the instability effect is not destroyed by the deconfinement transition at high temperatures, as it should be expected.

The value of the coupling g selected for the plot was $g = 2.74$ which corresponds to a strong coupling value $\alpha = \frac{g^2}{4\pi}$ being near 0.6. In addition the mass scale parameter value $\mu = 6.8\ GeV$ was fixed. Note that the minimum at zero quark condensate $mq = 0$ is laying near 200 MeV, which is lower but close to the constituent mass value $M = 333\ MeV$. It is interesting that to fix minimum of the potential for $mq = 0$, at this value of M requires a relatively large value of μ.

As it can be observed, the landscape of the potential makes clear that the system at $mq = 0$ dynamically develops a gluon condensate parameter with a potential similar in form to the Savvidy one in the early Chromomagnetic field models. ([19, 20]). It also can be seen that the system at zero values of both parameters shows an instability upon the generation of both gluon and quark condensates. The instability is stronger for the dynamical generation of the quark one. It can be also observed that the increasing of the gluon condensate parameter makes stronger the instability to the generation of the parameter mq. These properties are supporting the expectation expressed in [9, 13] about that the colour coupling could produce a sort superconductivity effect being able to generate intensive quark condensate values alike to the Ginzburg-Landau fields. If such effect is really occurring in Nature, the Top Condensate model could emerge as a possible effective field theory determined by the strong forces and upon

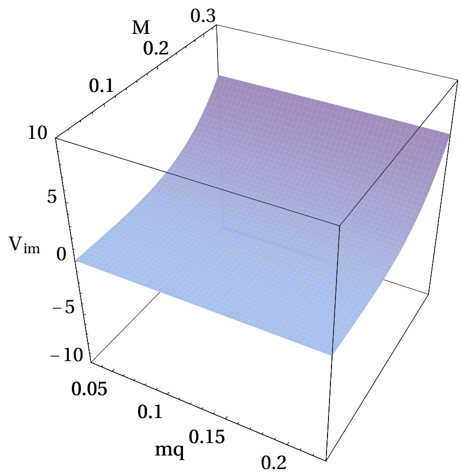

FIG. 4: The plot of the function V_{im} for the same range of mq and M used in Fig. 3 and the same fixed parameters $g = 2.74$, $\mu = 6.8\ GeV$ and $f = -(3/2)^2$. It can be seen that the ratio between the imaginary and the real part of the potential near the minimum at fixed mq decreases for the higher values of mq. Also, the imaginary part tends to be zero if the gluon condensate is disregarded in first approximation. For $f = (3/2)^2$ the potential is real for all the parameter values.

this the Higgs fields could be no other thing that the Top condensate value. [21]. This occurrence could also explain the similarities between the properties of the quark mass spectrum and the spectrum of superconductivity systems, underlined in the "Democratic Symmetry Breaking" analysis [2].

The Fig. 3 also clearly show that, in the framework of the present approximation, and for reasonable values of the coupling ($\alpha = 0.6$), there are no terms that control the instability for the generation of the quark condensate parameter, which under the shown potential will tend to grow without limit. Therefore, it becomes clear that the stabilization of the minimum of the system should come from terms higher than the ones considered here. Precisely in [13], this behavior was guessed to occur thanks to the colour interaction between quarks. Therefore, under the assumption that the technique being used in this exploration is well describing the massless QCD, it seems that this theory could dynamically develop heavy quark masses. This outcome could be another realization of the dimensional transmutation effect [22]. A requirement for the next corrections to produce helpful results for modelling, is that the stabilizing potential at large mq values behaves in such a way, that its dependence on M assures that the extreme point occurs at low values of M. Then, it could be expected that it can be fixed to the observable value near 333 MeV by selecting appropriate values for the coupling and the scale parameter. Also the value of $mq \sim 175\ GeV$ should be allowed to be fixed.

The Fig. 4 shows the value of the imaginary part V_{im} of the potential as a function of M and mq. Note that the dependence in mq is not rapidly growing, a behavior that if maintained for

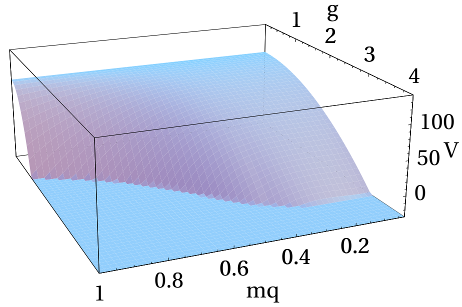

FIG. 5: The potential plotted as a function of mq and g for fixed values of $M = 333\ MeV$ and $\mu = 6.8\ GeV$. It can be observed that for each value of mq, there is critical coupling g below which the potential becomes negative, that is, lower than its value at vanishing condensates. This critical coupling decreases when X grows.

large mq values and in higher approximations will indicate an increasing stability of the vacuum being proved, for the interesting region of high values of mq. The picture is for $f = -(\frac{3}{2})^2$, as remarked before. For the positive value of f the imaginary part of the potential vanish. More generally, it can be remarked that all the other types of pictures shown in this section, after being plotted for the positive value of f, show a very similar behavior. Further Fig. 5 shows the dependence of the potential on the variable mq and the gauge coupling g. Here the mass parameter M was fixed to 333 MeV and again μ is taken as 6.8 GeV which fixes the minimum in the variable M at $mq = 0$, to be near 333 MeV. It should be recalled that we are considering that only one quark is being condensed. Therefore, the light quarks which in the present discussion do not develop their own condensates, should show at the considered level of approximation the observable value of light constituent masses. Since this quantity is fixed by the value of M, the graphics selected to be evaluated are always chosen to show a minimum near $M = 333\ MeV$ at $mq = 0$. The picture shows how the potential becomes negative (lower than its value at zero condensate state) when the coupling increases its strength over an amount fixed by the value of the quark condensate parameter mq. The greater the value of mq smaller becomes the critical coupling. The Fig. 6 show the dependence of the potential on the gluon condensate and the coupling constant for fixed values of $X = mq/M = 1$ and μ= 6.8 GeV. It can be seen that below certain critical coupling value near to 2, for all values of M, the zero gluon condensate state is stable. Increasing the value of X is not destroying this property and the value of the critical coupling is simply diminishing for larger X values.

VII. SUMMARY

The implication of a modified perturbative expansion for QCD are further investigated. Firstly, an scheme for making well defined the diagrams of the proposed expansion is intro-

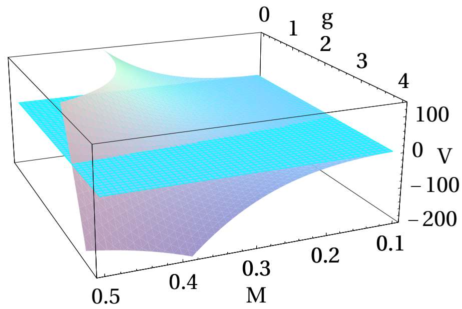

FIG. 6: The plot of V as a function of M and g fixing values of $mq = M\ GeV$ and $\mu = 6.8\ GeV$. It shows that there is critical value of strong coupling g below which the potential becomes positive for non vanishing gluon condensates ($M > 0$).

duced. After that, the zero and one loop contributions to the effective potential are evaluated. Further, the results of the zero and one loop calculation are improved by adding the quark condensate dependence of a relevant two loop term involving the quark propagators. The evaluated potential, in the considered approximation, indicates an instability of the massless QCD upon the generation of quark condensates. At this approximation also, there is no terms making the potential bounded from below. Thus, next corrections should produce such terms. Therefore, the results, could be detecting a possibility for the identification of a sort of Top Condensate model as a possible effective action for massless QCD. At this point it seems useful to remark, that the source for the indicated here effect could not had been yet detected through numerical studies, possibly since lattice QCD results are still limited in the consideration of the fermion determinants. Some questions concerning the gauge invariance implementation and the renormalization procedure in the scheme are commented as well as some further more detailed investigations expected to be considered.

Finally, let us comment below, about possible connections of the discussion given here and other considerations already advanced in the literature about the fermion mass problem.

It should be admitted massless QCD is currently accepted as being determined by the single parameter Λ_{QCD}, which is expected to fix in turn all the physical quantities. Thus, assuming this point of view, it would be surprising that the radically different scales for the six quarks masses could be predicted by this unique parameter. However, it can be argued in this respect that Λ_{QCD} perhaps is a quantity mostly determined by gluon condensate effects, and that, on other hand the large quark masses could be associated to more relevant quark condensate effects, being probably able to introduce another independent physical parameter. In fact, during long time, it has been argued in the literature by H. Fritzsch, [2], in the context of the so called Democratic Symmetry breaking scheme, that the presence of a large mass for one of the quarks, can in the next steps of approximations, imply the observed lower mass values for the other five quark. Moreover, indications about the presence of superconductive system types of properties in the quark mass spectrum, has also been also stressed in this approach. The main role played by the BCS like states in the steps motivating the modified expansion (see [8]) is therefore supporting the exploration continued in the present work. The strong nature of the colour forces makes imaginable the appearance of strong Ginzburg-Landau like fields for massless QCD, in close analogy with the ones produced by the weak phonon interactions in the BCS superconductivity. A next aspect to be worth commenting is related with possibilities that the validity of the investigated mechanism for the generation of the quark masses, could open ways for the additional production of the lepton masses. About this issue, it can be underlined that precisely the quark condensate in itself can play the role of the Higgs field. This possibility was already pointed out in the classic paper of W.A. Bardeen [21], were it was argued that the effective Lagranagian of the Top condensate model adopts a similar expression to that of the Higgs model, in which the quark condensate plays the role of the Higgs field. Therefore, it should be naturally expected that the mechanism examined here can also generate Yukawa terms for quarks as well as for leptons thanks to the underlaying gauge invariance of the systems. The opportunity for this interesting outcome to occur was already conjectured in one of the previous works: [9]. The idea advanced is the following: Let us assume the generation of a large quark condensate, for one of the flavour values. This quantity could (as indicated in Ref. [21]) act as composite Higgs field, which through effective gauge invariant interactions can induce Yukawa terms for quarks and leptons. Further, the fact that the leptons do not directly strongly interact could explain the lower mass values of the electron, muon and tau leptons. Finally, the additional lack of electromagnetic interactions of the corresponding three types of neutrinos could determine their even smaller mass values.

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APPENDIX A

Let us comment here the on the question about the implementation of the gauge invariance in the proposed scheme. This problem can be divided in two main lines: a) The satisfaction of the Slavnov-Ward-Takahashi identities given a fixed quantum gauge condition, and b) The invariance of the physical quantities under changes in the quantum gauge condition, in particular for different values of the gauge parameter α [23]. Both issues need for additional attention within the modified expansion under study, since the new appearing elements, the condensate propagators, being neat distributions, make the discussion more subtle than in the normal situation. This study is planned for a next more basic work. However, some points of interest can be remarked here below.

In connection with the satisfaction of the Slavnov-Ward-Takahashi identities, already in the work [7], it followed that the one loop correction to the polarization operator satisfies the simplest identity; that is, is exactly transverse. This is a non trivial result that suggests the possibility of its occurrence at higher loops approximations. Also in Refs. [10–12] some general argues and particular checks of this property in particular processes were given. The study of this problem will be continued.

On more general grounds, an observation that also indicates the possibility of implementing the invariance under the changes of the quantum gauge condition is the following one.

It can be noticed that due to the following identities in the sense of the generalized functions:

$$\begin{aligned} p^2\left(\frac{1}{p^2+i\epsilon}-iC\ \delta(p)\right) &= 1, \\ -\gamma_\mu p^\mu\left(\frac{-\gamma_\mu p^\mu}{p^2+i\epsilon}+iC_f\ I\ \delta(p)\right) &= I, \end{aligned}$$

the propagators of the modified expansion are also inverse kernels of the second variational derivative of the tree level action of massless QCD. Therefore, formal steps can be done to transform the full generating functional of the Green functions of massless QCD as written in the Wick expansion representation, to a functional integral representation over the gluon, quark and ghost fields. Henceforth, there is the possibility that some of the changes of variables which are employed to show the quantum gauge independence of physical quantities could be also implemented in the case of our interest, since the functional integral will only differ in the boundary conditions. A particular interesting way of considering the problem seems to employ the Yokoyama modification [24] of the Nakanishi-Lautrup B field [25] quantization of the interaction free version of massless QCD. In this scheme the variation of the gauge parameter α can be implemented as a quantum gauge transformation between the field operators. Thus, the transformation properties of the free generating functional associated to the condensate states employed in Ref. [8], under the gauge parameter modifications could be more efficiently investigated. These issues requires careful study that we expect to be able of perform in next works.

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