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IC/2004/101

United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency

THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

LIMIT DISTRIBUTION FUNCTION OF INHOMOGENEITIES IN REGIONS WITH RANDOM BOUNDARY IN THE HILBERT SPACE

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Abstract

The interaction of charged particle systems with a membrane consisting of nonhomogeneities which are randomly distributed by the same law in the vicinity of appropriate sites of a planar crystal lattice is studied. A system of equations for the self-consistent potential $U_1(x, \xi^0, ..., \xi^N, ...)$ and the density of induced charges $\sigma(x, \xi^0, ..., \xi^N, ...)$ is solved on Hilbert space.

MIRAMARE – TRIESTE
October 2004

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The mathematical model of reverse osmosis-the process of passing ions through a membrane-can be applied to discribe many physical, biological processes. For this reason, special attention is given to the building up of such a model. One of the well developed and consistent models is the model created by D. Ya. Petrina [6] . To discribe the reverse osmosis, the model deals with a system of diffusion equation in the self-consistent approximation and the electrostatic equation with boundary conditions.

The solution of a set of equations reduces to the solution of the Poisson-Boltzmann's equation. The latter one reduced to the solution of a set of integral equations for the self-consistent potential and the density of induced charges. The next step towards the solution of the system of equations on Banach space is to apply Schauder's theorem of contracting operators. In so doing, it is essential that periodically located ball-like inhomogeneities created by a membrane and the subtracting procedure applied in quantum mechanics should be used . In practice , the in homogeneities of a membrane can be randomly distributed. The authors of a series of articles [1-7] devoted to study of the interaction of membrane with charges in its vicinity used a membrane as a set of inhomogeneities which are distributed by the same functions .

In used models [1],[4],[6], the description of other physical processes, in particular, the illustration of the interactions of ions or particles with a micromedium consisting of microinhomogeneities or particles, say, electrons, must take both microccopic nature of the particles and the fact that in a real system inhomogeneities may be randomly distributed around the lattice sites . In this case the approach to the problem of understanding interaction is with in the scope of the quantum theory. Hence, it is important that the desired space to solve a system of equations for the self-consistent potential and the density of induced charges should be a space of square-integrable functions. Here, the model of reverse osmosis developed in [1], [4], [6] is generalized to a case of the interaction between a system of charged particles and a membrane that contains inhomogeneities distributed in the neighbourhood of sites of a planar cryslal lattice. The problem of the determination of the charge distribution is reduced to that of a system of integral equations for the mean density of induced charges of microinhomogeneities. To solve the problem, a space of square-integrable functions is introduced and the subtracting procedure along with Schauder's theorem are applied. Then the system of integral equations is solved on this space. It has been proved that under certain conditions on the density of charges and the density of microinhomogeneities the unique solution of the system of equations exists. The results obtained also allow qualitative analyses of reverse osmosis in a studied system to be made.

In [6] the integro-differential equation that provides a picture of the interaction of systems of charges particles with a membrane consisting of inhomogeneities was investigated. The authors of [6] proved the existence and the uniqueness of the solution for the self-consistent potential under certain conditions if the membrane consists of inhomogeneities located at sites of a planar crystal lattice on Banach space. Such a problem was studied on Hilbert space in work [8] in which the same results were obtained. M.Yu.Rasulova [9-11], who studied a complicated process of interaction, examined a membrane located by the same law in the visinity of sites in a planar crystal lattice . Under such condition a system of equations for the self-consistent potential $U_1(x;\xi^0,\xi^1,...\xi^N,...)$ and the density of induced charges $\sigma(x;\xi^0,\xi^1,...,\xi^N,...)$ were introduced and the chief results proved on Banach space.

The present work is an extention of that discussed in [9-11]. It was of interest to verify the above model on another space. The work under study was made on Hilbert space. The main results are the following: 1) a system of equations for the self-consistent potential and the density of induced charges is solved; 2) the existence and the uniqueness of the solution is proved on Hilbert space.

1.THE STATEMENT OF THE PROBLEM.

Let's consider a medium filling the whole three-dimensional space R^3 with the dielectric constant ε_1 and the diffusion coefficients D_1^+ and D_1^- for particles with the charges e^+ and e^-. We introduce into this medium the inhomogeneities-balls of a radius R with the dielectric constant ε_2 and the diffusion coefficients D_2^+ D_2^-.

Suppose these balls are distributed in the ranges region $r_k - \frac{a}{2} + R_{ef.} < \xi^k < r_k + \frac{a}{2} - R_{ef.}$ of the sites of a planar crystal lattice whose unit cells are squares with side a in accordance with a distribution law with density $f(\xi^k)$. Here $R_{ef.}$-the effective radius of interaction.Let the densities of distribution functions $f(\xi^k)$ satisfy the following conditions: $f(P_i\xi^k) = f(\xi^k)$, and $\int_{\Lambda^k} f(\xi^k)d\xi^k = 1$. Here ξ^k is the random deviation of the center of ball k from site k and P_i are transformations of reflection of the axes x_1, x_2, x_3 separately: $(P_ix)_j = x_j; i \neq j; (P_ix)_i = -x_i; \Lambda^k = \Lambda$, Λ are the squares of side $a - 2R_{ef.}$. The set of the balls makes up a dynamic membrane .

Let $x = (x_1; x_2; x_3) \in R^3$, we dispose the origin in the center of one of the balls;x_1, x_2 axes are in the membrane plane, i. e. centres of all balls are in the plane $x_3 = 0$.

Denote the density distribution functions of particles with charges e^+ and e^- trough $W_+(x,\xi^0,...,\xi^N,...)$ and $W_-(x,\xi^0,...,\xi^N,...)$, the self-consistent potential via $U(x,\xi^0,...,\xi^N,...)$, and by $e^+U_2(x,\xi^0,...,\xi^N,...)$ and $e^-U_2(x,\xi^0,...,\xi^N,...)$ the potentials of the interaction of the point charges e^+ and e^- with the charges induced by them on the surface of the membrane.

Suppose that charged particles may exist only in a layer $|x_3| \leq L < \infty$ and they are retained in it by an external field $e^\pm\overline{U(x,\xi^0,...,\xi^N,...)}$;

$$e^\pm\overline{U(x,\xi^0,...,\xi^N,...)} = \begin{cases} \infty & \text{if} \quad |x_3| > L, \\ 0 & \text{if} \quad |x_3| \leq L, \end{cases}$$

We shall assume that the potential

$$|U_2(x,\xi^0,...,\xi^N,...)| < \infty$$

is known. To solve the problem we write down the system of nonlinear diffusion equations for the distribution functions $W_+(x,\xi^0,...,\xi^N,...)$ and $W_-(x,\xi^0,...,\xi^N,...)$ and the electrostatic equation for self-consistent potential $U_1(x,\xi^0,...,\xi^N,...)$ [1].

$$D^{\pm}\Delta W_{\pm}(x,\xi^0,...,\xi^N,...)+e^{\pm}\beta D^{\pm}\nabla(\nabla U(x,\xi^0,...,\xi^N,...)W_{\pm}(x,\xi^0,...,\xi^N,...))=0, \tag{1}$$

$$e^{\pm}U(x,\xi^0,...,\xi^N,...)=e^{\pm}(U_1(x,\xi^0,...,\xi^N,...)+e^{\pm}U_2(x,\xi^0,...,\xi^N,...)+\overline{U(x,\xi^0,...,\xi^N,...)}),$$

$$W_+(x,\xi^0,...,\xi^N,...)=W_-(x,\xi^0,...,\xi^N,...)=0,|x_3|>L,D^{\pm}=D_1^{\pm},x\in R^3/M,$$

$$D^{\pm}=D_2^{\pm},x\in M;$$

$$\Delta U_1(x,\xi^0,...,\xi^N,...)=-\frac{1}{\varepsilon_1}(e^+W_+(x,\xi^0,...,\xi^N,...)+e^-W_-(x,\xi^0,...,\xi^N,...)),$$

$$\Delta U_2(x,\xi^0,...,\xi^N,...)=-\frac{1}{\varepsilon_1}\delta(x-y),y\in R^3, \tag{2}$$

$$U_2(x,\xi^0,...,\xi^N,...)=(U_2(x,\xi^0,...,\xi^N,...)-(\frac{1}{4\pi\varepsilon_1|x-y|})_{x=y} \tag{3}$$

with the following conjugation conditions on the membrane surface:

$$D_1^+\frac{\partial W_+(x,\xi^0,...,\xi^N,...)}{\partial n_x^+}=D_2^+\frac{\partial W_+(x,\xi^0,...,\xi^N,...)}{\partial n_x^-} \tag{4}$$

$$D_1^-\frac{\partial W_-(x,\xi^0,...,\xi^N,...}{\partial n_x^+}=D_2^-\frac{\partial W_-(x,\xi^0,...,\xi^N,...)}{\partial n_x^-},x\in\partial M,$$

$$\varepsilon_1\frac{\partial U_1(x,\xi^0,...,\xi^N,...)}{\partial n_x^+}=\varepsilon_2\frac{\partial U_1(x,\xi^0,...,\xi^N,...)}{\partial n_x^-},$$

$$\varepsilon_1\frac{\partial U_2(x,\xi^0,...,\xi^N,...)}{\partial n_x^+}=\varepsilon_2\frac{\partial U_2(x,\xi^0,...,\xi^N,...)}{\partial n_x^-},x\in\partial M.$$

Here $\frac{\partial}{\partial n_x^{\pm}}$ denotes the limiting values of the normal derivations on the outside and on the inside of the balls, n_x is the vector of the unit normal at the point $x\in\partial M$, and β is the inverse temperature.

Besides the conditions (4), we require that

$$W_+(x,\xi^0,...,\xi^N,...),W_-(x,\xi^0,...,\xi^N,...),U_1(x,\xi^0,...,\xi^N,...),U_2(x,y,\xi^0,...,\xi^N,...)$$

be continuous functions on the membrane surface (for y outside the membrane) and that

$$W_+(x,\xi^0,...,\xi^N,...),W_-(x,\xi^0,...,\xi^N,...)$$

satisfy the condition of electrical neutrality:

$$\int_{R^3/M}\prod_{i=0}^{\infty}\int_{\Lambda^i}(e^+W_+(x,\xi^0,...,\xi^N,...)+e^-W_-(x,\xi^0,...,\xi^N,...))f(\xi^i)dx=0. \tag{5}$$

As it is well known [1], the particular solution

$$W_{\pm}(x,\xi^0,...,\xi^N,...) = \frac{1}{v^+} exp(-\beta e^{\pm} U(x,\xi^0,...,\xi^N,...)), x \in R^3/M \quad (6)$$

$$W_+(x,\xi^0,...,\xi^N,...) = W_-(x,\xi^0,...,\xi^N,...) = 0, x \in M,$$

of the diffusion equation satisfies the conditions (4) and (5) if $U_1(x,\xi^0,...,\xi^N,...)$ is bounded together with its derivatives. Here, $\frac{1}{v^+}$ and $\frac{1}{v^-}$ are constants, equal in the spatially homogeneous case to the densities of the particles. Make up the equation for the mean values with followed by the solution [3].

Substituting (6) in (2), we obtain a generalized Poisson-Boltzmann's equation for the self-consistent potential:

$$\Delta < U_1(x,\xi^0,...,\xi^N,...) >= -\frac{1}{\varepsilon_1} < [e^+ \frac{1}{v^+}(U_1(x,\xi^0,...,\xi^N,...) + e^+ U_2(x,\xi^0,...,\xi^N,...))+ \quad (7)$$

$$+e^- \frac{1}{v^-} exp(-\beta e^-(U_1(x,\xi^0,...,\xi^N,...) + e^- U_2(x,\xi^0,...,\xi^N,...)))] >,$$
$$x \in R^3/M \bigcap |x_3| < L, \Delta < U_2(x,\xi^0,...,\xi^N,...) >= 0, x \in M.$$

The solution of the problem (7), (4) is represented in [1] as a sum of volume potentials and simple-layer potentials created by the induced charges $< \sigma_k(y,\xi^0,...,\xi^N,...) >$ on the surface S_k of the balls k :

$$< U_1(x,\xi^0,...,\xi^N,...) >= \sum_k \int_{S_k} \frac{< \sigma_k(y(\xi^k),\xi^0,...,\xi^N,...) >}{4\pi\varepsilon_1 |x- < y(\xi^k) > |} ds_{<y(\xi^k)>} + \quad (8)$$

$$+ \int_{R^3/M} \frac{< e^+ W_+(y,\xi^0,...,\xi^N,...) + e^- W_-(y,\xi^0,...,\xi^N,...) >}{4\pi\varepsilon_1 |x-y|} dy, x \in R^3/M \bigcap |x_3| < L,$$

the densities of the surface charges $< \sigma_\alpha(x,\xi^0,...,\xi^N,...) >$ being solutions of the integral equations

$$< \sigma_\alpha(x(\xi^\alpha),\xi^0,...,\xi^N,...) >= -\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2} \sum_k \int_{S_k} \frac{< \sigma_k(y(\xi^k),\xi^0,...^{\xi^k}...,\xi^N,...) >}{2\pi| < x(\xi^k) > - < y(\xi^k) > |^2} \times \quad (9)$$

$$\times \cos(< x(\xi^\alpha) > - < y(\xi^k) >, n_{\alpha,<x(\xi^\alpha)>}) ds_{<y(\xi^k)>} - \frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2} \times$$
$$\times \int_{R^3/M} \frac{\cos(< x(\xi^\alpha) > -y, n_{\alpha,<x(\xi^\alpha)>})}{2\pi| < x(\xi^\alpha) > -y|^2} \times$$
$$\times \{< e^+ W_+(y,\xi^0,...,\xi^N,...) + e^- W_-(y,\xi^0,...,\xi^N,...)\} > dy,$$

where

$$< \sigma_k(y,\xi^0,...,\xi^N,...) >=< \sigma_k(y(\xi^k),\xi^0,...^{\xi^k},...,\xi^N,...) >=$$

$$= \prod_{i=0}^{\infty} \int_{\Lambda^i} \sigma_k(y(\xi^k), \xi^0, ..., \xi^{k-1}, \xi^{k+1}, ..., \xi^N, ...) f(\xi^i) d\xi^i,$$

$$< x(\xi^\alpha) > - < y(\xi^k) >$$

is the vector directed from the point $< y(\xi^k) >$ to the point $x < \xi^\alpha) >, n_{\alpha, <x(\xi^\alpha)>}$ is the outher normal to S_α at the point

$$< x(\xi^\alpha) >= \int_{\Lambda^i} x(\xi^\alpha) f(\xi^\alpha) d\xi^\alpha.$$

Since the total induced charge on each ball on the average is zero,we must have

$$\int_{S_k} \prod_{i=0}^{\infty} \int_{\Lambda^i} \sigma_k(y(\xi^k), \xi^0, ...^{\xi^k} ..., \xi^N, ...) f(xi^i) d\xi^i dS_y = 0.$$

We divide the space R^3 into parallelepipeds:

$$V_k(x||x_1 - ak_1| \leq \frac{a}{2}, |x_2 - ak_2| \leq \frac{a}{2}, -\infty < x_3 < \infty).$$

Since

$$e^{\pm} U(x, \xi^0, ..., \xi^N, ...) = \infty, W_{\pm}(x, \xi^0, ..., \xi^N, ...) = 0$$

for $|x_3| \geq L$, we determine $U_1(x, \xi^0, ..., \xi^N, ...)$ in the region

$$V_k(x||x_1 - ak_1| \leq \frac{a}{2}, |x_2 - ak_2| \leq \frac{a}{2}, |x_3| < L, |x| > \frac{d}{2}).$$

Then the matematical expectations

$$< U_1(x, \xi^0, ..., \xi^N, ...) >, < \sigma_\alpha(x, \xi^0, ..., \xi^N, ...) >$$

have the form

$$\prod_{i=0}^{\infty} \int_{\Lambda^i} U_1(x, \xi^0, ..., \xi^N, ...) f(\xi^i) d\xi^i = \tag{10}$$

$$= \sum_k \int_{S_k(<\xi^k>)} \frac{< \sigma_k(y^k + \xi^k - ak, \xi^0, ...,^{\xi^k} ..., \xi^N, ...) >}{4\pi\varepsilon_1 |x - y^k - < \xi^k > -ak|} dS_{y^k} +$$

$$+ \sum_k \int_{V_k} \frac{< e^+ W_+(y^k + ak, \xi^0, ..., \xi^N, ...) + e^- W_-(y^k + ak, \xi^0, ..., \xi^N, ...) >}{4\pi\varepsilon_1 |x - y^k - ak|} dy^k,$$

$$x = x^\alpha + a\alpha, \alpha = 0, 1, 2, ..., \ y(\xi^k) = y^k + \xi^k + ak, k = 0, 1, 2, ...$$

$$\prod_{i=0}^{\infty} \int_{\Lambda^i} \sigma_\alpha(x(\xi^\alpha), \xi^0, ..., \xi^N, ...) \times \tag{11}$$

$$\times f(\xi^i) d\xi^i = -\frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$

$$\sum_{k=0}^{\infty} \int_{S_k(<\xi^k>)} \frac{\cos(< x(\xi^\alpha) > - - y^k - < \xi^k > -ak, n_{\alpha, <x(\xi^\alpha)>})}{2\pi| < x(\xi^\alpha) > -y^k - < \xi^k > -ak|^2} \times$$

$$\times < \sigma_k(y^k+\xi^k+ak,\xi^0,...^{\xi^k}...,\xi^N,...) > dS_{y^k}-$$

$$-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\sum_k\int_{V_k} < e^+W_+(y^k+ak,\xi^0,...,\xi^N,...)+e^-W_-(y^k+ak,\xi^0,...,\xi^N,...) > \times$$

$$\times\frac{\cos(< x(\xi^\alpha) > -y^k-ak, n_{\alpha,< x(\xi^\alpha) >)})}{2\pi| < x(\xi^\alpha) > -y^k-ak|^2}dy^k,$$

where

$$x(\xi^\alpha)=x^\alpha+\xi^\alpha+a\alpha, \alpha=0,1,2,..., y(\xi^k)=y^k+\xi^k+ak, k=0,1,2,...$$

(see in [3]).

It follows from the symmetry of the problem that the mean density of the surface changes

$$< \sigma(x,\xi^0,...,\xi^N,...) >$$

is invariant with respect to the period a, i.e.,

$$< \sigma(x+ak,\xi^0,...,\xi^N,...) > = < \sigma(x,\xi^0,...,\xi^N,...) > .$$

It also follows from this symmetry that $< U_1(x+ak,\xi^0,...,\xi^N,...) > = < U_1(x,\xi^0,...,\xi^N,...) >$. In addition,it follows from the property $f(\xi)=f(P\xi)$ that

$$< \xi > = \int_{-\frac{a}{2}+R_{ef.}}^{\frac{a}{2}-R_{ef.}}\xi f(\xi)d\xi=0.$$

Therefore, Eqs. (10) and (11) can be written in the form:

$$< U_1(x^\alpha+\alpha a,\xi^0,...,\xi^N,...) > = < U_1(x^\alpha,\xi^0,...,\xi^N,...) > = \tag{12}$$

$$=\sum_{k=0}^{\infty}\int_{S_k}\frac{< \sigma_k(y^k+\xi^k,\xi^0,...,\xi^N,...) >}{4\pi\varepsilon_1|x^\alpha-y^\alpha-ak|}dS_{y^k}+$$

$$+\sum_k\int_{V_k}\frac{< e^+W_+(y^k,\xi^0,...,\xi^N,...)+e^-W_-(y^k,\xi^0,...,\xi^N,...) >}{4\pi\varepsilon_1|x^\alpha-y^\alpha-ak|}dy^k,$$

$$< \sigma_k(x^\alpha+\xi^k+\alpha,\xi^0,...^{\xi^\alpha}...,\xi^N,...) > = \tag{13}$$

$$= < \sigma_\alpha(x^\alpha+\xi^\alpha,\xi^0,...^{\xi^\alpha}...,\xi^N,...) > =$$

$$=-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\sum_{k=0}^{\infty}\int_{S_k}\frac{\cos(x^\alpha-y^k-ak,n_{\alpha,x^\alpha}) < \sigma_k(y^k+\xi^k,\xi^0,...^{\xi^k}...,\xi^N,...) >}{2\pi|x^\alpha-y^k-ak|^2}dS_{y^k}-$$

$$-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\sum_{k=0}^{\infty}\int_{S_k}\frac{\cos(x^\alpha-y^k-ak,n_{\alpha,x^\alpha})}{2\pi|x^\alpha-y^k-ak|^2}\times$$

$$\times < e^+W_+(y^k,\xi^0,...,\xi^N,...)+e^-W_-(y^k,\xi^0,...,\xi^N,...) > dy^k.$$

Since the value of a definite integral does not depend on the symbol used for the variable of integration, in the expressions in the integrand we can make successive changes of variables:

$$\xi^k|_{k=0,1,2,...}\equiv\xi, \xi^\alpha|_{\alpha=0,1,2,...}\equiv\zeta,$$

and then

$$y^k|k=0,1,2,...\equiv y, x^\alpha|_{\alpha=0,1,2,...}\equiv x.$$

After these substitutions $\Lambda^0=\Lambda^1=...=\Lambda^N=...\equiv\Lambda, S_0=S_1=S_2=...=S_N=...\equiv S$ and Eqs. (12) and (13) take the form

$$< U_1(x,\xi^0,...,\xi^N,...) >= \sum_k \int_S \frac{<\sigma(y(\xi),\xi^0,...\xi^k...,\xi^N,...)>}{4\pi\varepsilon_1|x-y-ak|}dS_y+ \tag{14}$$

$$+\sum_k \int_V \frac{dy}{4\pi\varepsilon_1|x-y-ak|}\{< e^+W_+(y,\xi^0,...,\xi^N,...)+$$

$$e^-W_-(y,\xi^0,...,\xi^N,...)>\}.$$

$$<\sigma(y+\xi,\xi^0,...^\xi...,\xi^N,...)>= -\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\times \tag{15}$$

$$\times\sum_k \int_S \frac{\cos(x-y-ak,n_x)}{2\pi|x-y-ak|^2}<\sigma(y+\xi,\xi^0,...^\xi...,\xi^N,...)>dS_y-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\times$$

$$\times\sum_k \int_V \frac{\cos(x-y-ak,n_x)}{2\pi|x-y-ak|^2}\{< e^+W_+(y,\xi^0,...,\xi^N,...)+e^-W_-(y,\xi^0,...,\xi^N,...)>\}dy,$$

where $V=V_k|_{k=0,1,2,...}$.

We shall consider the system of equations (14) and (15) in the space whose elements are the pairs of functions

$$\rho(x,\xi^0,...,\xi^N,...)=\begin{pmatrix} U(x,\xi^0,...,\xi^N,...)\\ \chi(x,\xi^0,...,\xi^N,...)\end{pmatrix},$$

where $U(x,\xi^0,...,\xi^N,...)$ is concentrated in $\bar V=\{x\in V,\xi^0\in\Lambda,...,\xi^N\in\Lambda,...\}$, and $\chi(x,\xi^0,...,\xi^N,...)$ is concentrated in $x\in S,\xi^0\in\Lambda,...,\xi^N\in\Lambda,...$ and satisfies the condition

$$\int\prod_{i=0}^{\alpha}\int_{\Lambda^i}\chi(x,\xi^0,...,\xi^N,...)f(\xi^i)d\xi^i dS_x=0. \tag{16}$$

The set of such pairs of functions is a linear space if we perform linear operations over the columns per component. Introduce a scalar product in the space that:

$$(\rho_1\rho_2)=\int[U_1(x,\xi^0,...,\xi^N,...)\overline{U_2(x,\xi^0,...,\xi^N,...)}+ \tag{17}$$

$$+\chi_1(x,\xi^0,...,\chi^N,...)\overline{\chi_2(x,\xi^0,...,\chi^N,...}]dxd\xi,$$

where

$$\rho_1(x,\xi^0,...,\xi^N,...)=\begin{pmatrix} U_1(x,\xi^0,...,\xi^N,...)\\ \chi_1(x,\chi^0,...,\chi^N,...)\end{pmatrix},\ \rho_2(x,\chi^0,...,\xi^N,...)=\begin{pmatrix} U_2(x,\xi^0,...,\xi^N,...)\\ \chi(x,\xi^0,...,\xi^N,...)\end{pmatrix},$$

and $d\xi=d\xi^0d\xi^1...d\xi^N...$. The integration goes over all spaces. Next, we introduce the norm in accordance with the formula

$$||\rho||=[\int[U(x,\xi^0,...,\xi^N,...)\overline{U(x,\xi^0,...,\xi^N,...)}+\chi(x,\xi^0,...,\xi^N,...)\overline{\chi(x,\xi^0,...,\xi^N,...)}]dx]^{\frac{1}{2}}. \tag{18}$$

The set of such functions with norm (18) is a Hilbert space and we denoted this space by H.

Suppose also that $< U_1(x, \xi^0, ..., \xi^N, ...) >$ and $\chi(x, \xi^0, ..., \xi^N, ...) >$ are invariant with respect to the transformations of reflections of the axes x_1, x_2, x_3 in separate:

$$< U(Px, P\xi^0, ..., P\xi^N, ...) >=< U(x, \xi^0, ..., \xi^N, ...) >,$$

$$< \chi(Px, P\xi^0, ..., P\xi^N, ...) >=< \chi(x, \xi^0, ..., \xi^N, ...) > .$$

We examine Eqs. (14) and (15) in this Hilbert space H. We find divergences associated with the series

$$\sum_k \frac{1}{4\pi\varepsilon_1 |x-y-ak|}, \sum_k \frac{\cos(x-y-ak, n_x)}{2\pi|x-y-ak|^2}$$

when $< \sigma(x, \xi^0, ..., \xi^N, ...) >, < e^+ W_+(y, \xi^0, ..., \xi^N, ...) + e^- W_-(y, \xi^0, ..., \xi^N, ...) >$. To eliminate these divergences,we use the substractional procedure of [1]. We expande the kernels of eqs. (14) and (15) in Taylor series in the variables $x_i - y_i$ at the point 0. The remainder terms of these expansions are expressed as:

$$K_1(x,y) = \frac{1}{4\pi\varepsilon_1}[\frac{1}{|x-y-ak|} - \frac{1}{|ak|} - \sum_{i=0}^{3}(\frac{1}{|x-y-ak|})_i|_{x-y=0}(x_i - y_i)], \qquad (19)$$

$$x \in V, y \in S, \text{ or } x \in V, y \in V.$$

$$K_2(x,y) = \sum_{i=0}^{3} cos(n_x, x_i)(\frac{x_i - y_i - ak_i}{|x-y-ak|^3} + \frac{ak_i}{|ak|^3}), k_3 = 0, x, y \in S \text{ or } x \in S, y \in V, \qquad (20)$$

where the symbol $(.)_i$ denotes the derivative with respect to $(x-y)_i$. We estimate

$$\sum_{|k|\geq 2} \int_S \prod_{i=0}^{\infty} \int_{\Lambda^i} K_1(x,y) f(\xi^i) d\xi^i dS_y, \ \sum_{|k|\geq 2} \int_S \prod_{i=0}^{\infty} \int_{\Lambda^i} K_2(x,y) f(\xi^i) d\xi^i dS_y.$$

First we establish some lemmas here.To derivate our theorem;

LEMMA 1. For inhomogeneities in the form of balls of diameter $d = 2R$ takes place:

$$\sup_{\substack{x\in V \\ \xi^0\in\Lambda \\ \cdots \\ \xi^N\in\Lambda \\ \cdots}} |\sum_{|k|\geq 2} \int_S \prod_{i=0}^{\infty} \int_{\Lambda^i} f(\xi^i) d\xi^i K_1(x,y) dS_y| \leq \sup_{\substack{x\in V \\ \xi^0\in\Lambda \\ \cdots \\ \xi^N\in\Lambda \\ \cdots}} \prod_{i=0}^{\infty} \int_{\Lambda^i} |f(\xi^i)| d\xi^i \times$$

$$\times \sum_{|k|\geq 2} \int_S |K_1(x,y)| dS_y \leq c_1 d^2, c_1 < \infty.$$

LEMMA 2. The following estimation takes place:

$$sup_{\substack{x\in V \\ \xi^0\in\Lambda \\ \cdots \\ \xi^N\in\Lambda \\ \cdots}} |\sum_{|k|\geq 2} \int_S \prod_{i=0}^{\infty} \int_{\Lambda^i} \frac{\cos(x - -y - ak, n_x)}{2\pi|x-y-ak|^2} f(\xi^i) d\xi^i dS_y| < c_2 \frac{d^3}{a^3}, c_2 < \infty.$$

For the proof of lemmas 1 and 2 see [1], [3].

THEOREM. For sufficiently low values of the parameters $\frac{1}{v^+}$, $\frac{1}{v^-}$, $\frac{4\pi d^3}{3a^3}$ and $\frac{|\varepsilon_1-\varepsilon_2|}{\varepsilon_1+\varepsilon_2}$, the system of Eqs. (14) and (15) has a unique solution in the ball $||\rho-\rho^0||\leq R$ of the Hilbert space H.

Proof. As a starting point for our proof we apply the substractional procedure to Eqs. (14) and (15) and consider the equations

$$\sum_{|k|\geq 2}\int_S <\sigma(y,\xi^0,...,\xi^N,...)>[\frac{1}{|ak|}+\sum_{i=1}^{3}(\frac{1}{|x-y-ak|})_j|_{x-y=0}(x_i-y_i)]dS_y=0,$$

$$\sum_{|k|\geq 2}\int_V <e^+W_+(y,\xi^0,...,\xi^N,...)+e^-W_-(y,\xi^0,...,\xi^N,...)>\times$$

$$\times[\frac{1}{|ak|}+\sum_{i=1}^{3}(\frac{1}{|x-y-ak|}_j|_{x-y=o}(x_i-y_i)]dy=0,$$

$$\sum_{|k|\geq 2}\int_S <\sigma(y,\xi^0,...,\xi^N,...)>\sum_{i=1}^{3}\cos(n_x,x_i)\frac{ak_i}{|ak|^3}dS_y=0,$$

$$\sum_{|k|\geq 2}\int_V <e^+W_+(y,\xi^0,...,\xi^N,...)+e^-W_-(y,\xi^0,...,\xi^N,...)>\times$$

$$\times\sum_{i=1}^{3}cos(n_x,x_i)\frac{ak_i}{|ak|^3}dy=0,$$

which follow from the conditions (5) and (16) and also from the invariance of $<\sigma(x,\xi^0,...,\xi^N,...)>$, $<W_\pm(x,\xi^0,...,\xi^N,...)>$ with respect to reflections.

We then obtain in the Hilbert space H the equation

$$<F>=<KF>, \tag{21}$$

where

$$<F(x,\xi^0,...,\xi^N,...)>=\begin{pmatrix}<U_1(x,\xi^0,...,\xi^N,...)>\\ <\sigma(x,\xi^0,...,\xi^N,...)>\end{pmatrix},$$

$$<(KF)(x,\xi^0,...,\xi^N,...)>=\begin{pmatrix}<(KF_1)(x,\xi^0,...,\xi^N,...)>\\ <(KF_2)(x,\xi^0,...,\xi^N,...)>\end{pmatrix}.$$

Here

$$<KF_1)(x,\xi^0,...,\xi^N,...)>=\sum_{|k|<2}\int_S\frac{<\sigma(y+\xi,\xi^0,...^{\xi}...,\xi^N,...)>}{2\pi\varepsilon_1|x-y-ak|}dS_y+$$

$$+\sum_{|k|\geq 2}\int_S K_1(x,y)<\sigma(y+\xi,\xi^0,...^{\xi}...,\xi^N,...)>dS_y+$$

$$+\sum_{|k|<2}\int_V\frac{<e^+W_+(y,\xi^0,...,\xi^N,...)+e^-W_-(y,\xi^0,...,\xi^N,...)>}{4\pi\varepsilon_1|x-y-ak|}dy+$$

$$+\sum_{|k|\geq 2}\int_V K_1(x,y)<e^+W'_+(y,\xi^0,...,\xi^N,...)+e^-W'_-(y,\xi^0,...,\xi^N,...)>dy+$$

$$+<U_1^0(x,\xi^0,...,\xi^N,...)>,x\in V,$$

$$<(KF_2)(x,\xi^0,...,\xi^N,...)>=-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\times$$

$$\times \sum_{|k|<2} \int_S \frac{<\sigma(y+\xi,\xi^0,...,\xi^N,...)>}{2\pi|x-y-ak|^2} \cos(x-y-ak,n_x)dS_y - \frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\times$$

$$\times \sum_{|k|>2} \int_S K_2(x,y) <\sigma(y+\xi,\xi^0,...,\xi^N,...)> dS_y - \frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\times$$

$$\times \sum_{|k|<2} \int_V \frac{\cos(x-y-ak,n_x)}{2\pi|x-y-ak|^2} < e^+W_+(y,\xi^0,...,\xi^N,...) + e^-W_-(y,\xi^0,...,\xi^N,...) > dy-$$

$$-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2} \sum_{|k|<2} \int_V \frac{\cos(x-y-ak,n_x)}{2\pi|x-y-ak|^2} < e^+W_+(y,\xi^0,...,\xi^N,...) + e^-W_-(y,\xi^0,...,\xi^N,...) > dy-$$

$$-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2} \sum_{|k|\geq 2} \int_V K_2(x,y)\{< e^+W'_+(y,\xi^0,...,\xi^N,...) + e^-W'_-(y,\xi^0,...,\xi^N,...) >\}dy+$$

$$+ <\sigma^0(x,\xi^0,...,\xi^N,...)>, x\in S;$$

$$W'_\pm(y,\xi^0,...,\xi^N,...) = \frac{1}{v^\pm} exp(-\beta(e^\pm)^2 U_2(y,\xi^0,...,\xi^N,...))[exp(-\beta e^\pm U(y,\xi^0,...,\xi^N,...)) - 1];$$

$$U^0(x,\xi^0,...,\xi^N,...) = \sum_{|k|<2} \int_V \frac{1}{4\pi\varepsilon_1|x-y-ak|}\{e^+\frac{1}{v^+}exp(-\beta(e^+)^2U_2(y,\xi^0,...,\xi^N,...))+$$

$$+e^-\frac{1}{v^-}exp(-\beta(e^-)^2U_2(y,\xi^0,...,\xi^N,...))\}dy + \sum_{|k|\geq 2}\int_V K_1(x,y)\{e^+\frac{1}{v^+}exp(-\beta(e^+)^2\times$$

$$\times U_2(y,\xi^0,...,\xi^N,...)) + e^-\frac{1}{v^-}exp(-\beta(e^-)^2U_2(y,\xi^0,...,\xi^N,...))\}dy;$$

$$\sigma^0(x,\xi^0,...,\xi^N,...) = -\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\sum_{|k|<2}\int_V \frac{\cos(x-y-ak,n_x)}{2\pi|x-y-ak|^2}\{e^+\frac{1}{v^+}exp(-\beta(e^+)^2\times$$

$$\times U_2(y,\xi^0,...,\xi^N,...)) + e^-\frac{1}{v^-}exp(-\beta(e^-)^2U_2(y,\xi^0,...,\xi^N,...))\}dy - \frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\sum_{|k|\geq 2}\int_V K_2(x,y)\times$$

$$\times\{e^+\frac{1}{v^+}exp(-\beta(e^+)^2U_2(y,\xi^0,...,\xi^N,...) + e^-\frac{1}{v^-}exp(-\beta(e^-)^2U_2(y,\xi^0,...,\xi^N,...))\}dy.$$

We show that for sufficiently small $\frac{1}{v^+}$, $\frac{1}{v^-}$, $\frac{4\pi d^3}{3a^3}$, $\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}$ the operator K is contractive in the ball

$$||\rho - \rho^0|| \leq R, \tag{22}$$

where

$$\rho^0 = \begin{pmatrix} <U^0> \\ <\chi^0> \end{pmatrix},$$

$$|| <K\rho> - <\rho^0> || = (\int [\lim_{N\to\infty}\int_{\Lambda^N}[| <(KF)_1(x,\xi^0,...,\xi^N,...)> -$$

$$- <u^0(x,\xi^0,...,\xi^N,...)> |^2 + | <(KF)_2(x,\xi^0,...,\xi^N,...)> -$$

$$- <\chi^0(x,\xi^0,...,\xi^N,...)> |^2]d\xi^0...d\xi^N...]dx)^{\frac{1}{2}} =$$

$$= (\int_{x\in V}[|\sum_{|k|<2}\int_S[\lim_{N\to\infty}\int_{\Lambda^N}\frac{<\sigma(y+\xi,\xi^0,...^\xi...,\xi^N,...)>}{2\pi\varepsilon_1|x-y-ak|}d\xi^0...d\xi^N...]dS_y+$$

$$+\sum_{|k|\geq 2}\int_S[\lim_{N\to\infty}\int_{\Lambda^N} K_1(x;y) <\sigma(y+\xi,\xi^0,...^\xi...,\xi^N,...)> d\xi^0...d\xi^N...]dS_y+$$

$$+\sum_{|k|<2}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}\frac{< e^+W'_+(y,\xi^0,...,\xi^N,...)+e^-W'_-(y,\xi^0,...,\xi^N,...) >}{4\pi\varepsilon_1|x-y-ak|}d\xi^0d\xi^1...d\xi^N...]dy+$$

$$+\sum_{|k|\geq 2}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}K_1(x;y)\{< e^+W'_+(y,\xi^0,...,\xi^N,...)+$$

$$+e^-W'_-(y,\xi^0,...,\xi^N,...) >\}d\xi^0...d\xi^N...]dy+[\lim_{N\to\infty}(< U_1^0(x,\xi^0,...,\xi^N,...) > -$$

$$-< u_1^0(x,\xi^0,...,\xi^N,...) >)d\xi^0...d\xi^N...]|^2+|-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\times$$

$$\times\sum_{|k|<2}\int_S[\lim_{N\to\infty}\int_{\Lambda^N}\frac{<\sigma(y+\xi,\xi^0,...^{\xi}...,\xi^N,...)}{2\pi|x-y-ak|^2}\cos(x-y-ak,n_x)d\xi^0d\xi^1...d\xi^N...]dS_y-$$

$$-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\sum_{|k|>2}\int_S[\lim_{N\to\infty}\int_{\Lambda^N}K_2(x;y)<\sigma(y+\xi,\xi^0,...^{\xi}...,\xi^N,...)>d\xi^0d\xi^1...d\xi^N...]dS_y-$$

$$-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\sum_{|k|<2}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}\frac{\cos(x-y-ak,n_x)}{2\pi|x-y-ak|^2}\{< e^+W_+(y,\xi^0,...,\xi^N,...)+$$

$$+e^-W_-(y,\xi^0,...,\xi^N,...) >\}d\xi^0d\xi^1...d\xi^N...]dy-\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\sum_{|k|\geq 2}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}K_2(x;y)\times$$

$$\times\{< e^+W'_+(y,\xi^0,...,\xi^N,...)+e^-W'_-(y,\xi^0,...,\xi^N,...) >\}d\xi^0d\xi^1...d\xi^N...]dy+$$

$$+[\lim_{N\to\infty}\int_{\Lambda^N}(<\chi^0(x,\xi^0,...,\xi^N,...)>-<\sigma^0(x,\xi^0,...,\xi^N,...)>)d\xi^0...d\xi^N...]|^2]dx)^{\frac{1}{2}}\leq$$

$$\leq(\int_V[\sum_{|k|<2}\int_S[\lim_{N\to\infty}\int_{\Lambda^N}\frac{<\sigma(y+\xi,\xi^0,...^{\xi}...,\xi^N,...)>}{2\pi\varepsilon_1|x-y-ak|}d\xi^0...d\xi^N...]dS_y]|^2dx)^{\frac{1}{2}}+$$

$$+(\int_V[\sum_{|k|\geq 2}\int_S[\lim_{N\to\infty}\int_{\Lambda^N}|K_1(x;y)<\sigma(y_+\xi,\xi^0,...^{\xi}...,\xi^N,...)>d\xi^0...d\xi^N...]dS_y]|^2dx)^{\frac{1}{2}}+$$

$$+(\int_V[\sum_{|k|<2}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}\frac{|< e^+W'_+(y,\xi^0,...,\xi^N,...)+e^-W'_-(y,\xi^0,...,\xi^N,...) >|}{4\pi\varepsilon_1|x-y-ak|}d\xi^0...$$

$$...d\xi^N...]dy]^2dx)^{\frac{1}{2}}+(\int_V[\sum_{|k|\geq 2}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}|K_1(x;y)\{e^+W'_+(y,\xi^0,...,\xi^N,...)+$$

$$+e^-W'_-(y,\xi^0,...,\xi^N,...) >\}|d\xi^0...d\xi^N...]dy]^2dx)^{\frac{1}{2}}+(\int_V[\lim_{N\to\infty}\int_{\Lambda^N}|(< U_1^0(x,\xi^0,...,\xi^N,...) > -$$

$$-< u_1^0(x,\xi^0,...,\xi^N,...) >)|^2d\xi^0...d\xi^N...]dx)^{\frac{1}{2}}+(\int_V|[\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}\times$$

$$\times|\sum_{|k|<2}\int_S[\lim_{N\to\infty}\int_{\Lambda^N}|\frac{<\sigma(y+\xi,\xi^0,...^{\xi}...,\xi^N,...)>}{2\pi|x-y-ak|^2}\cos(x-y-ak,n_x)|d\xi^0...d\xi^N...]dS_y]^2dx)^{\frac{1}{2}}+$$

$$+(\int_V|[\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}|\sum_{|k|>2}\int_S[\lim_{N\to\infty}\int_{\Lambda^N}|K_2(x;y)<\sigma(y+\xi,\xi^0,...^{\xi}...,\xi^N,...)>|d\xi^0...d\xi^N...]dS_y]^2dx)^{\frac{1}{2}}+$$

$$+(\int_V[|\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}|\sum_{|k|<2}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}|\frac{cos(x-y-ak,n_x)}{2\pi|x-y-ak|^2}$$

$$\{< e^+W_+(y,\xi^0,...,\xi^N,...)+e^-W_-(y,\xi^0,...,\xi^N,...) >\}|d\xi^0...d\xi^N...]dy]^2dx)^{\frac{1}{2}}+$$

$$+(\int_V[|\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1+\varepsilon_2}|\sum_{|k|\geq 2}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}|K_2(x;y)\{< e^+W'_+(y,\xi^0,...,\xi^N,...)+$$

$$+e^-W'_-(y,\xi^0,...,\xi^N,...)>\}|d\xi^0...d\xi^N...]dy]^2dx)^{\frac{1}{2}}+$$

$$+(\int_V[[\lim_{N\to\infty}\int_{\Lambda^N}|<\chi^0>-<\sigma^0(x,\xi^0,...,\xi^N,...)>|d\xi^0...d\xi^N...]]^2dx)^{\frac{1}{2}}\leq$$

$$\leq(\int_V[\int_S[\lim_{N\to\infty}\int_{\Lambda^N}<\sigma(y+\xi,\xi^0,...^{\xi}...,\xi^N,...)>d\xi^0...d\xi^N...]dS_y]^2dx)^{\frac{1}{2}}\times$$

$$\times[(\int_V[\sum_{|k|<2}\int_S\frac{1}{2\pi\varepsilon_1|x-y-ak|}dS_y]^2dx)^{\frac{1}{2}}+(\int_V[\sum_{|k|\geq 2}\int_S|K_1(x;y)|dS_y]^2dx)^{\frac{1}{2}}]+$$

$$+[(\int_V[\sum_{|k|<2}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}\frac{exp(\beta(e^+)^2U_2(y,\xi^0,...,\xi^N,...)}{4\pi\varepsilon_1|x-y-ak|}d\xi^0...d\xi^N...]dy)^2dx)^{\frac{1}{2}}+$$

$$+(\int_V[\sum_{|k|\geq 2}[\lim_{N\to\infty}\int_{\Lambda^N}|K_1(x;y)|exp(\beta(e^+)^2U_2(y,\xi^0,...,\xi^N,...)d\xi^0...d\xi^N...]dy)^2dx)^{\frac{1}{2}}]\times$$

$$\times\{[\int_V[\beta(e^+)^2\frac{1}{v^+}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}exp(\beta e^+|U_2(y,\xi^0,...,\xi^N,...)|d\xi^0...d\xi^N...]dy]^2dx]^{\frac{1}{2}}\times$$

$$\times(\int_V[\int_V[\lim_{N\to\infty}\int_{\Lambda^N}|U_2(y,\xi^0,...,\xi^N,...)|d\xi^o...d\xi^N...]dy]^2dx)^{\frac{1}{2}}\}+$$

$$+[(\int_V[\sum_{|k|<2}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}\frac{exp(\beta(e^-)^2U_2(y,\xi^0,...,\xi^N,...)}{4\pi\varepsilon_1|x-y-ak|}d\xi^0...d\xi^N...]dy)^2dx)^{\frac{1}{2}}+$$

$$+(\int_V[\sum_{|k|\geq 2}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}|K_1(x;y)|exp(\beta(e^-)^2|U_2(y,\xi^0,...,\xi^N,...)|d\xi^0...d\xi^N...]dy)^2dx)^{\frac{1}{2}}]\times$$

$$\times\{[\int_V[\beta(e^-)^2\frac{1}{v^-}[\int_V[\lim_{N\to\infty}\int_{\Lambda^N}exp(\beta e^-|U_2(y,\xi^0,...,\xi^N,...)|d\xi^0...d\xi^N...]dy]^2dx)^{\frac{1}{2}}\times$$

$$\times(\int_V(\int_V[\lim_{N\to\infty}\int_{\Lambda^N}|U_2(y,\xi^0,...,\xi^N,...)|d\xi^0...d\xi^N...]dy)^2dx)^{\frac{1}{2}}\}+(\frac{|\varepsilon_1-\varepsilon_2|}{\varepsilon_1+\varepsilon_2}\times$$

$$\times\{[\int_V(\int_S\sum_{|k|<2}\frac{|\cos(x-y-ak,n_x)|}{2\pi|x-y-ak|^2}dS_y)^2dx]^{\frac{1}{2}}+(\frac{|\varepsilon_1-\varepsilon_2|}{\varepsilon_1+\varepsilon_2}[\int_V(\int_S\sum_{|k|>2}|K_2(x;y)|dS_y)^2dx]^{\frac{1}{2}}\}\times$$

$$\times[\int_V[\lim_{N\to\infty}\int_{\Lambda^N}|<\sigma(y+\xi,\xi^0,...^{\xi}...,\xi^N,...>|^2d\xi^0...d\xi^N...]dy]^{\frac{1}{2}}+\{\frac{|\varepsilon_1-\varepsilon_2|}{\varepsilon_1+\varepsilon_2}\times$$

$$\times[\int_V[\sum_{|k|<2}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}\frac{|\cos(x-y-ak,n_x)|}{2\pi|x-y-ak|^2}exp(-\beta(e^+)^2U_2(y,\xi^0,...,\xi^N,...))d\xi^0...d\xi^N...$$

$$...]dy]^2dx)^{\frac{1}{2}}+(\int_V\sum_{|k|\geq 2}(\int_V[\lim_{N\to\infty}\int_{\Lambda^N}|K_2(x;y)|exp(-\beta(e^+)^2U_2(y,\xi^0,...,\xi^N,...)dy)^2dx)^{\frac{1}{2}}\}\times$$

$$\times\{\beta(e^+)^2\frac{1}{v^+}(\int_V[\lim_{N\to\infty}\int_{\Lambda^N}exp(-\beta e^+|U_2(y,\xi^0,...,\xi^N,...)|)d\xi^0...d\xi^N...]dy)^2dx)^{\frac{1}{2}}\times$$

$$\times(\int_V[\int_V|K_2(x;y)|dy)^2dx)^{\frac{1}{2}}+(\frac{|\varepsilon_1-\varepsilon_2|}{\varepsilon_1+\varepsilon_2}(\int_V(\sum_{|k|<2}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}\frac{|\cos(x-y-ak,n_x)|}{2\pi|x-y-ak|^2}\times$$

$$\times exp(-\beta(e^-)^2U_2(y,\xi^0,...,\xi^N,...)d\xi^0...d\xi^N...]dy)^2dx)^{\frac{1}{2}}+(\int_V\sum_{|k|\geq 2}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}|K_2(x;y)|\times$$

$$\times exp(-\beta(e^-)^2U_2(y,\xi^0,...,\xi^N,...)d\xi^0...d\xi^N...]dy)^2dx)^{\frac{1}{2}}\{\beta(e^-)^2\frac{1}{v^-}[\int_V[\lim_{N\to\infty}\int_{\Lambda^N}exp(-\beta|e^-|\times$$

$$\times|U_2(y,\xi^0,...,\xi^N,...)|d\xi^0...d\xi^N...]dy]^2dx)^{\frac{1}{2}}(\int_V[\int_V|K_2(x;y)dy]^2dx)^{\frac{1}{2}} =$$

$$= \int_V[\int_S[\lim_{N\to\infty}\int_{\Lambda^N} < \sigma(y+\xi,\xi^0,...^{\xi}...,\xi^N,...) > d\xi^0...d\xi^N...]dS_y]^2dx)^{\frac{1}{2}}c_5+$$

$$+[\int_V[\beta(e^+)^2\frac{1}{v^+}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}exp(\beta e^+|U_2(y,\xi^0,...,\xi^N,...)|d\xi^0...d\xi^N...]dy]^2dx]^{\frac{1}{2}}\times$$

$$\times(\int_V[\int_V[\lim_{N\to\infty}\int_{\Lambda^N}|U_2(y,\xi^0,...,\xi^N,...)|d\xi^0...d\xi^N...]dy)^2dx)^{\frac{1}{2}}c_6+$$

$$+\{\int_V[\beta(e^-)^2\frac{1}{v_-}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}exp(\beta e^-|U_2(y,\xi^0,...,\xi^N,...)|d\xi^0...d\xi^N...]dy]^2dx)^{\frac{1}{2}}\times$$

$$\times(\int_V[\int_V[\lim_{N\to\infty}\int_{\Lambda^N}|U_2(y,\xi^0,...,\xi^N,...)|d\xi^0...d\xi^N...]dy)^2dx)^{\frac{1}{2}}c_7+$$

$$+(\int_V[\lim_{N\to\infty}\int_{\Lambda^N}| < \sigma(y+\xi,\xi^0,...^{\xi}...,\xi^N,...) > |^2d\xi^0...d\xi^N...]dy)^{\frac{1}{2}}c_8+$$

$$+(\beta(e^+)^2\frac{1}{v^+}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}exp(-\beta e^+|U_2(y,\xi^0,...,\xi^N,...)|)d\xi^0...d\xi^N...]dy)^2dx)^{\frac{1}{2}}c_9+$$

$$+\{\beta(e^-)^2\frac{1}{v^-}[\int_V[\lim_{N\to\infty}\int_{\Lambda^N}exp(-\beta e^-|U_2(y,\xi^0,...,\xi^N,...)|d\xi^0...d\xi^N...]dy]^2dx)^{\frac{1}{2}}\times$$

$$\times\int_V[\int_V|K_2(x;y)|dy)^2dx)^{\frac{1}{2}}c_{10} \leq ||\rho||\{\max(c_5,c_8)+\beta(e^+)^2\frac{1}{v^+}exp(\beta e^+||\rho||)\times$$

$$\times\max(c_6,c_9)+\beta(e^-)^2\frac{1}{v^-}exp(\beta|e^-|||\rho||)max(c_7,c_{10})\},$$

where

$$c_5 = (\int_V\sum_{|k|<2}(\int_S\frac{dS_y}{2\pi\varepsilon_1|x-y-ak|})^2dx)^{\frac{1}{2}}+\int_V\sum_{|k|\geq 2}(\int_S|K_2(x;y)|dS_y)^2dx)^{\frac{1}{2}},$$

$$c_6 = (\int_V[\sum_{|k|<2}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}\frac{exp(\beta(e^+)^2U_2(y,\xi^0,...,\xi^N,...)}{4\pi\varepsilon_1|x-y-ak|}d\xi^0...d\xi^N...]dy)^2dx)^{\frac{1}{2}}+$$

$$+(\int_V\int_V[\sum_{|k|\geq 2}[\lim_{N\to\infty}\int_{\Lambda^N}|K_2(x;y)|exp(\beta(e^+)^2U_2(y,\xi^0,...,\xi^N,...)d\xi^0...d\xi^N...]dy)^2dx)^{\frac{1}{2}},$$

$$c_7 = (\int_V[\sum_{|k|<2}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}\frac{exp(\beta(e^-)^2U_2(y,\xi^0,...,\xi^N,...)}{4\pi\varepsilon_1|x-y-ak|}d\xi^0...d\xi^N...]dy)^2dx)^{\frac{1}{2}}+$$

$$+(\int_V[\sum_{|k|\geq 2}[\lim_{N\to\infty}\int_{\Lambda^N}|K_2(x;y)|exp(\beta(e^-)^2U_2(y,\xi^0,...,\xi^N,...))d\xi^0...d\xi^N...]dy)^2dx)^{\frac{1}{2}},$$

$$c_8 = \frac{|\varepsilon_1-\varepsilon_2|}{\varepsilon_1+\varepsilon_2}(\int_V(\int_S\sum_{|k|<2}[\lim_{N\to\infty}\int_{\Lambda^N}\frac{|cos(x-y-ak,n_x)|}{2\pi|x-y-ak|^2}d\xi^0...d\xi^N...]dS_y)^2dx)^{\frac{1}{2}}+$$

$$+\frac{|\varepsilon_1-\varepsilon_2|}{\varepsilon_1+\varepsilon_2}[\int_V(\int_S\sum_{|k|\geq 2}|K_2(x;y)|dS_y)^2dx)^{\frac{1}{2}}, c_9 = \frac{|\varepsilon_1-\varepsilon_2|}{\varepsilon_1+\varepsilon_2}\times$$

$$\times[\int_V[\sum_{|k|<2}\int_V[\lim_{N\to\infty}\int_{\Lambda^N}\frac{|cos(x-y-ak,n_x)|}{2\pi|x-y-ak|^2}exp(-\beta(e^+)^2U_2(y,\xi^0,...,\xi^N,...))d\xi^0...d\xi^N...]dy)^2dx)^{\frac{1}{2}}+$$

$$+(\int_V \sum_{|k|\geq 2} \int_V [\lim_{N\to\infty} \int_{\Lambda^N} |K_2(x;y)| exp(-\beta(e^+)^2 U_2(y,\xi^0,...,\xi^N,...)) d\xi^0...d\xi^N...]dy)^2)^{\frac{1}{2}}\},$$

$$c_{10} = \frac{|\varepsilon_1 - \varepsilon_2|}{\varepsilon_1 + \varepsilon_2} (\int_V \sum_{|k|<2} \int_V [\lim_{N\to\infty} \int_{\Lambda^N} \frac{|cos(x-y-ak, n_x)|}{2\pi |x-y-ak|^2} \times$$

$$\times exp(-\beta(e^-)^2 U_2(y,\xi^0,...,\xi^N,...) d\xi^0...d\xi^N...]dy)^2 dx)^{\frac{1}{2}} +$$

$$+(\int_V \sum_{|k|\geq 2} \int_V [\lim_{N\to\infty} \int_{\Lambda^N} |K_2(x;y)| exp(-\beta(e^-)^2 U_2(y,\xi^0,...,\xi^N,...)) d\xi^0...d\xi^N...]dy)^2 dx)^{\frac{1}{2}}.$$

In the notation $b = \max(c_5, c_8)$, $g = \max(c_6, c_7, c_9, c_{10})$, the inequality (21) takes the form $|| < K\rho > - < \rho^0 > || = (||\rho^0|| + R)\{b + g[\beta(e^+)^2 \frac{1}{v^+} exp(\beta e^+(||\rho^0|| + R)) + \beta(e^-)^2 \frac{1}{v^-} exp(\beta |e^-|(||\rho^0|| + R))\}$. It is possible to choose $b, g, \frac{1}{v^+}, \frac{1}{v^-}$ so that $|| < K\rho > - < \rho^0 > || \leq R$, i. e. $b + g[\beta(e^+)^2 \frac{1}{v^+} exp(\beta e^+(||\rho^0|| + R)) + \beta|e^-|^2 \frac{1}{v^-} exp(\beta|e^-|(||\rho^0|| + R))] < \alpha < 1$. Then K is a contractive operator and, Eq. (21) has a unique solution in the ball (22).

Acknowledgments

The authors would like to thank the INTAS (2000-015, field 2). M. Rasulova would also like to thank the Abdus Salam International Centre for Theoretical Physics (Trieste, Italy), Prof. V. Kravtsov and Prof. K.R. Sreenivasan for hospitality.

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