LAPP-EXP-2006-05

Complementary strategy of New Physics searches in B-sector.

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Abstract

We discuss a possible strategy for studies of a particular scenario of New Physics (NP) at LHC. The NP is taken to be a). U-spin symmetric, i.e. it does not distinguish d and s quarks; b). it makes no contribution to the tree processes, but contributes differently to penguin and box diagrams and c). it does not spoil the unitarity of CKM matrix. Our analysis is based on comparison of particular CKM matrix elements, which can be obtained from the processes dominated by diagrams of different topology. We argue that the standard formalism of the overall unitarity triangle fit is not suitable for studies of NP of this kind. We also stress the utmost importance of lattice computations of some particular set of hadronic inputs relevant for NP searches.

1 Introduction

The main interest of the present day flavor physics is focused towards searching for possible signals of New Physics (NP) - the effects which are not taken into account by the Standard Model (SM). These still hypothetical effects can be roughly divided into two groups. The first, "quantitative" one consists of effects which are present in the SM but whose concrete SM prediction deviates from actual experimental results. A well known example is given by rare decays strongly suppressed in the SM but expected to be enhanced in some NP scenarios. Another, "qualitative" group, is formed by the effects which are not present in the SM at all, like possible observation of nonconservation of any charge (baryon, electric etc) strictly conserved in the SM. At the moment there are no clear indications on possible NP effects of either kind. The strong hope however is that the situation will change in the nearest future with the run of LHC. Of prime importance in flavor physics is an analysis of the CKM mixing matrix. The commonly accepted parametrization-independent language used to discuss the rich physics encoded in CKM matrix is formalism of the unitarity triangle (UT). For introduction into the subject and all details the reader is refered to the materials presented on [1, 2, 3, 4] and to excellent recent reviews [5, 6].

The issue of NP search in flavor physics context is certainly much broader than the mere check of CKM matrix unitarity, however precise it can be. Of course, any inconsistencies in the UT construction will undoubtedly indicate the presence of physics beyond SM. The opposite is far from being true - there are many reasonable NP scenarios which are well compatible with perfect unitarity of CKM matrix.

There are different possible strategies to study the CKM matrix. The most popular one, adopted in particular by UTfit and CKMfitter groups [1, 2] is to use all available experimental data to overconstrain the triangle. Besides general importance of this activity the hope is that the procedure will exhibit some inconsistencies signaling NP effects. Up to now there is an overall agreement of all constraints (see recent talks [7, 8]).

However, this approach also has some disadvantages. In our view, the most important one is the fact that the set of constraints in use is not fitted to this or that particular NP scenario. On the other hand, the relevance of this or that observable from the point of view of its possible NP content strongly depends on what kind of NP we discuss. Let us explain this point taking as a typical example $\Delta M_s / \Delta M_d$ ratio. For all scenarios where NP couples identically to s and d quarks (U-spin symmetric NP) this ratio is not sensitive to NP contributions, since in this case short-distance functions, even if modified with respect to the SM predictions for each ΔM_d, ΔM_s exactly cancel in the ratio. This pattern is typical for, e.g. constrained minimal flavor violation (CMFV) NP models (see review of MFV models in [11]). As a result this quantity informs us about ratio of couplings of t-quark to d and s-quarks and also about long-distance $SU(3)$ breaking effects in QCD (see expression (10) below), but brings no information about correctness of the short-distance SM calculation of ΔM_d or ΔM_s sepa-

rately. And it is precisely the latter short-distance piece we are interested in most of all if we are looking for deviations from the SM at small distances. On the other hand, there are NP scenarios such as, e.g. MSSM at large $\tan\beta$ (see [9] and references therein) and next-to-minimal flavor violation [10] where this is not the case and the ratio under study is sensitive to NP. Moreover, it is very natural to expect (and this is our general attitude in the present paper) that NP contributes differently to the processes of different topology (i.e. tree and penguin, penguin and box etc). Obviously, this effect can be lost in comparison of observables of the same topological type. In view of that an alternative way has been proposed (see, e.g. [12, 13] and also [1, 2, 9, 14, 15] and references therein). Generally speaking, it corresponds to construction of a few a priori not coinciding unitarity triangles, each extracted from branching ratios and asymmetries for processes of some particular kind. In this case any mismatch between these UT's, e.g the so called "reference UT" [13] and "universal UT" (see recent discussion in [9]) would be a clear signal of NP, and, moreover, one could in principle identify the place (EW penguin sector is among the most promising ones) where it has come from.

Adopting the basic idea of the latter strategy we address the following problem. Let us assume the still hypothetical NP is, in the spirit of next-to-minimal flavor violation scenario: a). U-spin symmetric; b). does not contribute to the tree processes and c). does not spoil the unitarity of CKM matrix (i.e. we work in the spirit of next-to-minimal flavor violation scenario). How can we see NP from global UT fits and what observables are the most sensitive to NP effects in this particular case?

To answer this question, we analyze theoretical and experimental (having in mind mostly the LHCb experiment) perspectives for studies of some CKM matrix parameters which can be extracted from the processes of different topology and can be sensitive to NP of the discussed type. It is worth noticing that the mismatch between $\sin 2\beta$ values from $B \to J/\psi K_S$ and from $B \to \phi K_S$ modes widely discussed in recent literature (see, e.g. [5, 16] and references therein) represents exactly a kind of effects we are interested in. We will also stress the urgent need for new refined lattice data on hadronic input parameters in order to determine the product $|V_{ts}V_{tb}^*|$.

The paper is organized as follows. The section 2 is devoted to brief overview of the existing strategies for CKM matrix analysis, while our procedure and results are presented in the section 3 and conclusion in the section 4.

2   Overview of the standard strategy

In general one can choose different sets of independent parameters which enter the basic unitarity relation[1]

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \tag{1}$$

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[1] As well as five other unitarity triangles

It is worth noticing that the term "independent" is usually used in the literature in mere algebraic sense, i.e. one assumes no relations between CKM matrix elements other than those following from the unitarity constraints. This assumption may be wrong if some more fundamental underlying structure behind CKM matrix does exist. A common choice for one of the parameters is

$$s_{12} = \lambda = |V_{us}| = \left\{ \begin{array}{ll} (0.2265 \pm 0.0020) & [1] \\ (0.2258 \pm 0.0014) & [2] \end{array} \right. \tag{2}$$

This quantity can be determined with very good accuracy from the decay mode $K \to \pi l \nu$ with the latter being dominated by tree level process. The main source of error here is the poor knowledge of the corresponding formfactor $f_+(0)$, namely, according to [17] $\delta|V_{us}|_{f_+(0)} = \pm 0.0018$, $\delta|V_{us}|_{exp} = \pm 0.0005$.

The interior angles of the triangle (1) are conventionally labeled as

$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \;,\;\; \beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \;,\;\; \gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) \tag{3}$$

The Cabbibo-suppressed angle χ important for $B_s - \bar{B}_s$ oscillations is defined by

$$\chi = \arg\left(-\frac{V_{cs}^*V_{cb}}{V_{ts}^*V_{tb}}\right) \tag{4}$$

and is also of interest. Let us briefly remind the strategy for γ. The cleanest way to extract it is from the interference of the $b \to c\bar{u}s$ and $b \to \bar{c}us$ transitions (the so called "triangle" approach). Practically, this corresponds to the study of $B^- \to K^- D^0$ and $B^- \to K^- \bar{D}^0$ modes with the subsequent analysis of the common final states for D and $\bar{D}$ mesons decays. One considers CP-eigenstates as final states for $D, \bar{D}$ mesons decays (GLW approach [18]) or combines observables from different modes ($B \to K^*D$, $B \to KD^*$, $B \to KD$, $B \to K^*D^*$) (ADS approach [19]) to overconstrain the system.[2] Notice that the interfering diagrams are the tree ones.[3]

The combined results for γ presented in [3] obtained by Dalitz plot analysis [21] are given by

$$\gamma = 67^\circ \pm 28^\circ \pm 13^\circ \pm 11^\circ \text{ [BaBar]} \;\;;\;\; \gamma = 53^\circ \pm 18^\circ \pm 3^\circ \pm 9^\circ \text{ [Belle]} \tag{5}$$

where the errors are statistical, systematic and the error resulting from the choice of D - decay model. For the discussion of the situation with γ determination in LHCb the reader is referred to [22].

Using various methods the overall uncertainty in γ at LHCb is expected to be as small as 5° in 2fb^{-1} of running and will eventually reach the level of 1° with increase of statistics.

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[2]The same strategy can be applied to the case of B_c mesons, where it has some theoretical advantages [20]; however the experimental statistics becomes the main obstacle there.

[3]This mode is not completely NP-safe since in principle the latter can enter through $D^0 - \bar{D}^0$ mixing.

With the standard assignment for the elements of CKM matrix (see, e.g., [6])

$$s_{12} = \lambda \ ; \ s_{23} = A\lambda^2 \ ; \ s_{13}\exp(-i\delta_{13}) = A\lambda^3(\rho - i\eta) \tag{6}$$

to define the apex of the unitarity triangle

$$\bar{\rho} = \rho \left[1 - \frac{1}{2}\lambda^2 \right] \ ; \ \bar{\eta} = \eta \left[1 - \frac{1}{2}\lambda^2 \right] \tag{7}$$

one needs to know at least two independent quantities out of two sides

$$R_b = \frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|} = \sqrt{\bar{\rho}^2 + \bar{\eta}^2} \ ; \ R_t = \frac{|V_{td}V_{tb}^*|}{|V_{cd}V_{cb}^*|} = \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2}$$

and three angles α, β, γ where the latter are defined by (3). In particular, the authors of [15] analyzed all ten possible strategies, distinguished by the mentioned choice of two independent parameters out of five from the point of view of their efficiency in the determination of UT. For example, our geometrical intuition tells us that it is easier to construct general non-squashed triangle taking as inputs one of its angles and adjacent side (because the variations in these parameters are approximately orthogonal) than taking the same angle and the opposite side (because the variation in these parameters are approximately parallel). Numerical simulation done in [15] fully supports this intuition, giving the highest priority[4] to the strategies based on combined use of either (γ, β) or (γ, R_b).

This result is particularly encouraging because the quantities R_b and γ define the so called reference UT [12, 13]. The latter is built from the observables that are expected to be unaffected by NP, since their dominant contributions come from tree level processes. Then assuming unitarity of CKM matrix one can compute from (1) reference values for

$$R_t = \sqrt{1 + R_b^2 - 2R_b\cos\gamma} \ ; \ \cot\beta = \frac{1 - R_b\cos\gamma}{R_b\sin\gamma} \tag{8}$$

and compare them with the ones obtained by direct measurements in the processes involving loop graphs. Any difference could be a hint for a NP signal (see recent quantitative discussion of this issue in [9]).

The elements of CKM matrix which enter the definition of R_b (up to terms $\mathcal{O}(\lambda^4)$)

$$R_b = \left[1 - \frac{1}{2}\lambda^2 \right] \frac{1}{\lambda} \frac{|V_{ub}|}{|V_{cb}|}$$

are known from semileptonic B-decays. The recent inclusive update is given by [1] $|V_{cb}| = (41.79 \pm 0.63) \cdot 10^{-3}$.

Experimental determination of $|V_{ub}|_{incl}$ suffers from uncertainties, introduced by specific cuts one has to apply in order to get rid of $b \to c$ background.

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[4]In other words, to compute $(\bar{\rho}, \bar{\eta})$ with a given precision pair (γ, β) may be known with lower accuracy than, e.g. pair (R_b, β).

As for $|V_{ub}|_{excl}$ the main source of error is lattice uncertainty in calculations of $B \to \pi, \rho$ form-factors. Up to date results are given by [3] as

$$|V_{ub}|_{incl} = (4.4 \pm 0.3) \cdot 10^{-3}$$

and

$$|V_{ub}|_{excl} = (3.8 \pm 0.6) \cdot 10^{-3}$$

At the moment the perspectives to increase the accuracy in experimental determination of R_b up to a few percent level are unclear. As can be seen from Fig.1 and Fig.2, the errors in γ and R_b play a very different role in fixing the angle β with some given precision, which is a simple consequence of the fact that the angle α is close to $90°$ and the triangle is almost rectangular. The present accuracy in β extracted from the "golden mode" $B \to J/\psi K_S$ is better than $\pm 2°$, the current world average for $\sin 2\beta$ from tree level decays provided by [3] is (see recent talk [23] and references therein):

$$\sin 2\beta = (0.674 \pm 0.026) \tag{9}$$

The corresponding penguin contribution to β is Cabibbo-suppressed (see, e.g. [16]). As shown in Fig.1 an uncertainty window of $\sim 3°$ for β corresponds to an uncertainty window of $\sim (24 \pm 5)°$ for γ and therefore the precise data (9) does not constrain γ via (8) strongly enough to make the comparison discussed above meaningful. On the other hand, since both β from (9) and γ from (5) are determined from the processes dominated by tree level decays, we do not expect to see violation of the second expression from (8) with these values of the angles. Anyway, the experimental uncertainty in γ and hadronic uncertainties in R_b make (8) not valuable.

Let us briefly discuss the side R_t. The are two ways of extracting R_t by means of relations not affected by NP contributions in some scenarios, notably CMFV. These are the computation of R_t from the first expression in (8) and the computation from the ratio $\Delta M_d / \Delta M_s$ where, again, short distance contributions to the box diagrams are canceled.[5] Concerning the former algorithm, because of the same geometrical reasons (angle α close to $90°$) R_t is sensitive to the uncertainty in the angle γ only (see Fig.3). Thus, precise knowledge of γ will constrain R_t effectively. In the latter approach one obtains the ratio

$$\frac{|V_{td}|}{|V_{ts}|} = \xi \sqrt{\frac{m_{B_s}}{m_{B_d}} \frac{\Delta M_d}{\Delta M_s}} \tag{10}$$

with the nonperturbative parameter[6]

$$\xi^2 = \frac{\hat{B}_{B_s} f_{B_s}^2}{\hat{B}_{B_d} f_{B_d}^2} \tag{11}$$

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[5]As has been already mentioned, in MSSM at large $\tan \beta$ the quantity R_t is sensitive to the different Higgs couplings to d and s quarks.

[6]$\xi = 1$ in case of exact flavor $SU(3)$.

The typical error of current lattice simulations of ξ is estimated as 6% (see [24] and recent analysis in [25]). Since up to $\mathcal{O}(\lambda^4)$

$$ R_t = \frac{\xi}{\lambda} \sqrt{\frac{\Delta M_d}{\Delta M_s}} \sqrt{\frac{m_{B_s}}{m_{B_d}}} \left[1 - \lambda \xi \cos \beta \sqrt{\frac{\Delta M_d}{\Delta M_s}} \sqrt{\frac{m_{B_s}}{m_{B_d}}} + \frac{\lambda^2}{2} \right] $$

then having at our disposal recent CDF results [26] (see (22)) we can straightforwardly extract for the mean value $R_t = 0.92$ with the uncertainty dominated by ξ.

Let us summarize this part. Suppose we would be able to measure R_b and γ with some very high precision. This defines the position of the UT apex which is universal as soon as NP does not contribute to tree processes R_b and γ have been extracted from. Let us also assume that we get R_t from $\Delta M_d/\Delta M_s$ and β from $B \to J/\psi K_S$, and these observables perfectly agree with R_b and γ via (8) (i.e. the UT apex defined from R_t and β coincides with the one found[7] from R_b and γ). Does this fact mean dramatic shrinking of NP parameter space? Not at all: for NP scenarios with U-spin invariance and without sizeable NP mixing effects[8] this coincidence is trivial and brings no any information about the parameter space. One can tell that UT is simply too rough tool to see NP of this kind. In other words, the precise knowledge of ξ is important in this case to calibrate the lattice, but not to find the NP.

3 Direct comparison of CKM matrix elements from different processes

In what follows we are going to explore a complementary strategy whose essence is the comparison of values of CKM matrix elements, obtained from processes with dominant contributions coming from diagrams of essentially different topology. Again one can consider angles and sides in this respect. We are interested in observables, corresponding to the processes whose dominant contributions come from topologically different diagrams, namely:

- radiative penguin in decay modes $B \to K^* \gamma$, $B_s \to \phi \gamma$ and $B \to (\rho, \omega)\gamma$, $B_s \to \bar{K}^* \gamma$, for s and d quarks, respectively

- oscillations of neutral B^0 and B_s mesons with the dominant contribution given by box diagram, resulting in the mass shifts ΔM_s, ΔM_d

- tree and strong penguin interference in B decays into 2-body final states made of light hadrons π, K, ρ and mixing relevant for the angle α determination

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[7]Of course, any other pair can actually be used, see discussion above.

[8]NP physics contributions in the box diagrams could in principle affect both γ and β via $D_0 - \bar{D}_0$ and $B_0 - \bar{B}_0$ mixings.

For the first and the second mode the object of our interest is the product $|V_{tb}^* V_{ts}|$. For the third mode we confine our attention to the angles α, β and χ.

The reference values for these quantities are defined from tree level processes, since we adopt the usual assumption that they are free from NP pollution. One can make use of the "tree level definition" for $|V_{tb}^* V_{ts}| \leftrightarrow |V_{tb}^* V_{ts}|_{tree}$, which up to terms $\mathcal{O}(\lambda^4)$ reads

$$|V_{ts} V_{tb}^*|_{tree} = |V_{cb}| \left[1 - \frac{\lambda^2}{2} (1 - 2R_b \cos\gamma) \right] \tag{12}$$

We have already discussed the corresponding numerical values and their uncertainties. Plugging them in, we get

$$|V_{ts} V_{tb}^*|_{tree} = (41.3 \pm 0.8) \cdot 10^{-3} \tag{13}$$

As for the angle α, its reference value is given by

$$\alpha_{tree} = \pi - \beta - \gamma \tag{14}$$

where the extraction of β and γ from tree level processes is described above. As for the angle χ, there are no experimental constraints on it at the moment. The SM prediction is $|\chi| \approx 0.02 \div 0.04$.

3.1 Analysis of $|V_{tb}^* V_{ts}|$.

Let us start with the analysis of $|V_{tb}^* V_{ts}|$. Values of these elements of the CKM matrix must exactly coincide in the SM, regardless of the way they are extracted. On the other hand, lack of such coincidence will be a definite signal of NP, contributing differently to these different types of processes. Qualitatively, one can consider ratios of the following kind

$$\zeta_{q,V}^{(1)} = \frac{|V_{tb}^* V_{tq}|_{\Delta M_q}}{|V_{tb}^* V_{tq}|_{B \to V\gamma}} \; ; \; \zeta_{q,V}^{(2)} = \frac{|V_{tb}^* V_{tq}|_{B \to V\gamma}}{|V_{tb}^* V_{tq}|_{tree}} \; ; \; \zeta_{q,V}^{(3)} = \frac{|V_{tb}^* V_{tq}|_{tree}}{|V_{tb}^* V_{tq}|_{\Delta M_q}} \tag{15}$$

where $q = d, s$ and V stands for K^*, ϕ, ρ, ω. Thus we have three ways to extract the product $|V_{tb}^* V_{tq}|$ of CKM matrix elements: via expression (18) from the process dominated by the radiative penguin diagram, via expression (19) from the process dominated by the box diagram, and via (12) from the reference tree level processes. It is obvious that by construction one has

$$\zeta_{q,V}^{(1)} \cdot \zeta_{q,V}^{(2)} \cdot \zeta_{q,V}^{(3)} \equiv 1 \tag{16}$$

In the SM however much more restricted condition has to be fulfilled:

$$\zeta_{q,V}^{(1)} = \zeta_{q,V}^{(2)} = \zeta_{q,V}^{(3)} = 1 \tag{17}$$

It is convenient to present a set of three numbers $\{\zeta_{q,V}^{(1)}, \zeta_{q,V}^{(2)}, \zeta_{q,V}^{(3)}\}$ as a single point on the ternary coordinate system with $\log \zeta_{q,V}^{(i)}$ as an (algebraic) distance

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from the i-th axis. Then the SM case corresponds to the only point on this diagram - its origin, while any deviation from it is a hint to NP.

The analysis of ratios of the mass shifts $\Delta M_d/\Delta M_s$ and branchings $Br(B \to \rho\gamma)/Br(B \to K^*\gamma)$ widely discussed in the recent literature [25, 27, 28] deals in our language not directly with the quantities $\zeta_{q,V}^{(i)}$, but with their ratios like $\zeta_{s,K^*}^{(1)}/\zeta_{d,\rho}^{(1)}$. The important advantage of these ratios is the improved accuracy of their theoretical determination, especially from the point of view of hadronic uncertainties. However the price to pay is high - the short distance factors which could contain contributions of NP are canceled in these ratios. In logarithmic coordinates it corresponds to a parallel translation, which could miss a considerable piece of NP, which is clearly seen from the analysis of [25]. In short, $\zeta_{s,K^*}^{(1)} = \zeta_{d,\rho}^{(1)} = 1$ implies $\zeta_{s,K^*}^{(1)}/\zeta_{d,\rho}^{(1)} = 1$, but not vice versa.

Generally speaking, it is meaningless to look for (short-distance) deviations from the SM predictions if one has no quantitative knowledge what the latter actually are. Therefore as soon as we are discussing absolute values of mass shifts, widths etc, these short distance parameters should be determined and not just canceled in the ratios. Corresponding loss in an accuracy for hadronic contributions is perhaps inevitable. Anyway we are stressing that one has to deal with this "less accurate" low energy hadronic inputs if one tries to capture the short-distance effects of NP. For example, it is meaningless, in our view, to consider soft quantities as free parameters to fit observable branching ratios. Any possible NP induced difference between, e.g. the SM prediction for $Br(B \to V\gamma)$ and actual experimental result would be just hidden inside such "extracted from experiment" $|\xi_\perp^{(K^*)}(0)|$, which is clearly unacceptable. Simply speaking, to discuss deviations from the SM prediction we have first to know the latter.[9]

In principle, one can discuss five expressions of the kind (16), corresponding to the following choices for (q, V): (s, K^*), (s, ϕ), (d, ω), (d, ρ), $(d, \bar{K}^*)$. However all these channels have universal short-distance structure, while the long-distance contributions are related to each other by $SU(3)$ flavor arguments. The optimal strategy therefore seems to choose just one particular case, which we take to be (s, K^*) in the rest of the paper. The results for the other ones could provide important cross-checks (like, e.g. $|V_{td}|/|V_{ts}|$ ratio), but presumably no new information about a NP content of (16).

We are using the standard SM expressions for the decay rate for $B \to K^*\gamma$ and the mass difference ΔM_s. The former can be written as [28, 29, 30]:

$$\Gamma(B \to K^*\gamma) = \frac{G_F^2 \alpha m_B^3 m_b^2}{32\pi^4}(1-r)^3 |a_7(\mu)|^2 |\xi_\perp^{(K^*)}(0)|^2 |V_{ts}V_{tb}^*|^2 \qquad (18)$$

In the above expression $r = m_{K^*}^2/m_B^2$, m_b stands for the pole mass of b-quark, $a_7(\mu) = C_7^{(0)} + A^{(1)}(\mu)$ is an absolute value of the corresponding short-distance function including Wilson coefficient $C_7^{(0)}$, hard scattering contributions and

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[9]As is correctly pointed out in [2] "a model-independent UT analysis beyond the SM cannot be carried out without some *a priori* theoretical knowledge of the relevant hadronic parameters."

annihilation corrections. The detailed computation of this function at the next-to-leading order can be found in the cited papers. Notice that we omit terms of the order of m_s^2/m_b^2. The factor $|\xi_\perp^{(K^*)}(0)|$ differs from the corresponding form-factor $T_1^{B\to K^*}(0)$ by $\mathcal{O}(\alpha_s)$ corrections; according to [31] numerically one has $|\xi_\perp^{(K^*)}(0)| \approx 0.93 \cdot T_1^{B\to K^*}(0)$.

The expression for ΔM_s reads as follows:

$$\Delta M_s = \frac{G_F^2}{6\pi^2}\eta_B[M_W^2 F_{tt}]m_{B_s}(\hat{B}_{B_s}f_{B_s}^2)|V_{ts}V_{tb}^*|^2 \tag{19}$$

where η_B is calculable short-distance QCD factor, while the m_t/M_W-dependent factor $F_{tt}M_W^2$ has come from calculation of the box diagram ([32], see also [33, 34]).

According to our strategy we invert the expressions (18) and (19) to the following form:

$$|V_{ts}V_{tb}^*|_{B\to K^*\gamma} = \frac{4\pi^2}{|a_7(\mu)|} \cdot \sqrt{\frac{2\Gamma(B\to K^*\gamma)}{G_F^2\alpha m_B^5(1-r)^3}} \cdot \left[\frac{1}{|\xi_\perp^{(K^*)}(0)|}\left(\frac{m_B}{m_b}\right)\right] \tag{20}$$

and

$$|V_{ts}V_{tb}^*|_{\Delta M_s} = \frac{\pi}{\sqrt{\eta_B F_{tt}}} \cdot \sqrt{\frac{6\Delta M_s}{G_F^2 M_W^2 m_{B_s}^3}}\left[\frac{m_{B_s}}{f_{B_s}\sqrt{\hat{B}_{B_s}}}\right] \tag{21}$$

The structure of the above expressions is clear. The first factors in the r.h.s. are the short-distance SM contribution, which have to be calculated analytically. These are just numbers of order 1 and it is assumed that we have reliable theoretical control of this part. The typical accuracy of these factors is better than 5%. The second factors (the square roots) are composed from experimentally measurable quantities. The error in these factors is dominantly experimental and is currently at the 5% level for (20) and 1-2 % level for (21) . The third factors (in the square brackets) encode information about soft QCD contributions (and related problem of b quark pole mass m_b) for which we have no systematic approach of studying. The main hope here is focussed on the lattice simulations.[10] The uncertainty of currently available data can be conservatively estimated as 10-20 %. The use of (20), (21) as probes for NP entirely depends on improvement in the determination of these hadronic factors.

The quantities of our interest are $T_1^{B\to K^*}(0)$ and $f_{B_s}\sqrt{\hat{B}_{B_s}}$. The reader is referred to the papers [35] - [40] and the papers [41, 42] for lattice and sum rule determination of $T_1^{B\to K^*}(0)$, respectively. The corresponding values are in the range $0.2 - 0.4$. The relevant references for $f_{B_s}\sqrt{\hat{B}_{B_s}}$ are given by papers [43] - [46]. Looking at the data one can see that there is no clear agreement

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[10]Notice that we have included the B-meson mass m_B to both factors in (20) to provide normalization. Indeed, it can be understood as being taken from real experiment, on the other hand since any reliable lattice simulation must be correctly normalized to this experimental value of the mass, the treatment of m_B as a lattice output should make in fact no difference.

between, e.g. lattice computations and light-cone sum rules results. Moreover, the errors given by the authors of the cited lattice papers are mostly statistical ones. The procedure of correct treatment of systematic errors in this case is not yet known. In fact, the same is true for the sum rule calculations. In general, precise determination of $T_1^{B \to K^*}(0)$ on the lattice is very difficult and reliability of the calculations done so far is debatable (see [47] and recent discussion in [48]). However the utmost importance of this measurement, which hopefully will be done in the nearest future on new improved lattices in unquenched case cannot be overestimated.

Thus, having no better strategy at the moment, we will be conservative in our error treatment and take for the input value of $f_{B_s}\sqrt{\hat{B}_{B_s}}$

$$f_{B_s}\sqrt{\hat{B}_{B_s}} = (280 \pm 40) \text{ MeV}$$

while we also consider three sets of possible values for $T_1^{B \to K^*}(0)$, where the errors correspond to those reported in the cited papers:

$$\text{Set A} \quad : \quad T_{1A}^{B \to K^*}(0) = (0.25 \pm 0.05)$$
$$\text{Set B} \quad : \quad T_{1B}^{B \to K^*}(0) = (0.30 \pm 0.05)$$
$$\text{Set C} \quad : \quad T_{1C}^{B \to K^*}(0) = (0.35 \pm 0.05)$$

The ultimate goal should be to reach the accuracy of the lattice computations comparable to the accuracy of the r.h.s. of (25).

Finally, let us recall the experimental data for B_s meson levels splitting ΔM_s [26] and branching ratios for the decay $B \to K^*\gamma$. They are given by

$$\Delta M_s = [17.33^{+0.42}_{-0.21}(stat) \pm 0.07(sys)]\text{ps}^{-1} \tag{22}$$

and

$$Br(B^- \to K^{*-}\gamma) = (4.25 \pm 0.31 \pm 0.24) \cdot 10^{-5} \quad [49] \tag{23}$$
$$Br(B^- \to K^{*-}\gamma) = (3.87 \pm 0.28 \pm 0.26) \cdot 10^{-5} \quad [50] \tag{24}$$

For the life time of B^- meson we use the value $\tau = (1.652 \pm 0.014)$ ps [51], while the masses (in MeV) are given by [51]

$$m_B = (5279.0 \pm 0.5) \; ; \;\; m_{B_s} = (5367.5 \pm 1.8) \; ; \;\; m_{K^*} = (891.66 \pm 0.26)$$

Other short-distance inputs are collected in the Table 1.

We have all input data now to estimate the ratios ζ_{s,K^*}. According to the three choices of numerical value for the form-factor $T_1^{B \to K^*}(0)$ we get three sets of $\zeta_{s,K^*}^{(i)}$. The results are presented in Table 2. For graphical presentation one can use planar ternary coordinates where the constraint $\sum_{i=1}^3 \log \zeta_{s,K^*}^{(i)} = 0$ is satisfied automatically. Each solution is represented by a single point on this plane with the distance from the i-th axis to the point given by $\log \zeta_{s,K^*}^{(i)}$. It is

taken positive for two axes forming an angle the point belongs to and negative for the remaining distant axis. With this rule each point on the plane satisfies the constraint (16). Some sample result for the case [44]-B is shown on Figure 4. Notice, that the bars correspond to 1σ deviation in $\zeta_{s,K^*}^{(i)}$, not in $\log\zeta_{s,K^*}^{(i)}$. The fact that they cross the corresponding axes means less than 1σ deviation of the actual result from the SM prediction. The origin of this ternary coordinate system corresponds to $\log\zeta_{s,K^*}^{(i)} = 0$ for all i, which is the SM solution.

The main qualitative conclusion is perhaps not surprising: with the reasonable choice of parameters we observe no evidence for NP within error bars. There are two optimistic remarks however. The first is that our errors are very conservative and significant reduction of at least some of them is foreseen in the nearest future. Secondly, the errors in the Table 2 are not independent. There are two sorts of correlations. The first is the uninteresting "kinematical" one, following from the constraint (16). The second pattern corresponds to the error correlation for lattice simulations of $T_1^{B\to K^*}(0)$ and $f_{B_s}\sqrt{\hat{B}_{B_s}}$. So far these two inputs have been measured independently, by different lattice groups and within different procedures. Correspondingly, the errors shown in the Table 2 are also treated as independent. On the other hand, it is reasonable to expect an error reduction for the simultaneous calculation of $T_1^{B\to K^*}(0)$ and $f_{B_s}\sqrt{\hat{B}_{B_s}}$ and we call the attention to importance of such simulation, using the same framework (lattice action, chiral extrapolation procedure etc) and uniform error treatment. It is reasonable to expect that this would result in a better accuracy, first of all for the quantity $\zeta_{s,K^*}^{(1)}$. Speaking differently, if one assumes no NP (i.e. $\zeta_{s,K^*}^{(1)} = 1$) one is to get

$$\frac{m_b|T_1^{B\to K^*}(0)|}{f_{B_s}\sqrt{\hat{B}_{B_s}}} = 778 \cdot [Br(B \to K^*\gamma)]^{1/2} \tag{25}$$

where the uncertainty in the numerical factor 778 is of order 5% and is mostly theoretical.[11] This SM prediction demonstrates the level of precision the lattice computations must reach in order to make reliable conclusion about NP based on the lattice results. We consider the check of (25) on the lattice as a task of primary importance.

3.2   Analysis of the angle α

The angle α can be extracted from the two-body decay modes of B into light hadrons π, ρ and K (see recent review [52]). From the theoretical point of view the best channel seems at present to be $B \to \rho\rho$ [53, 54]. The most promising channel for α at LHCb however is $B \to \rho\pi \to \pi\pi\pi$ [55, 56]. The basic idea of the analysis [57] is to study the interference of the tree amplitude proportional to the weak phase factor $e^{i\gamma}$ from $V_{ub}^*V_{ud}$ and the penguin amplitude proportional

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[11]The uncertainty in experimental value of ΔM_s is small.

to factor $e^{-i\beta}$ from $V_{tb}^* V_{td}$. Writing down also the amplitudes for CP-conjugated modes and imposing isospin relations, one can fit four amplitudes, four strong phases and one weak phase from 11 observables (see details in [56]). The expected uncertainty in α of LHCb is about $10°$ in one year of running [22]. It can be mentioned that the recent result presented by BaBar collaboration [58] for α from $B \to \rho\pi$ channel is

$$\alpha = (114 \pm 39)° \tag{26}$$

while the data uncertainty for $B \to \rho\rho$ mode is $\pm 13°$ [49]. The above analysis assumes no electroweak penguin contributions. According to the estimates [54], $\delta\alpha_{EWP} = -1.5°$. The isospin breaking effects controlled by parameter $(m_d - m_u)/\Lambda_{QCD}$ are expected to be of the same order of magnitude.

For α defined as an argument of the amplitude ratio one gets (see details in, e.g. [6])

$$2\alpha_{eff} = \arg\left[-e^{-i\theta_{12}}\frac{A(\bar{B} \to f)}{A(B \to f)}\right] = \arg\left[-e^{-i\theta_{12}}\frac{e^{-i\gamma} - re^{i\theta + i\delta\alpha}}{e^{i\gamma} - re^{i\theta - i\delta\alpha}}\right] \tag{27}$$

where θ_{12} is the $B_0 - \bar{B}_0$ mixing angle, θ is strong penguin phase, r is an absolute value of penguin-to-tree ratio and $\delta\alpha$ is possible weak NP penguin phase.[12] In the absence of penguins, i.e. if $r = 0$ and if $\theta_{12} = 2\beta$ (as in the SM), one gets $\alpha_{eff} = \alpha_{tree}$ with α_{tree} defined by (14). It is worth stressing (see early discussion of related issue in [12]) that for the discussed scenario the corresponding NP phase shift $\delta\alpha$ is to coincide up to a sign with that to the angles β and χ:

$$\delta\beta_{NP} = \beta_{(B \to \phi K_S)} - \beta_{(B \to J/\psi K_S)} = -\delta\alpha = \delta\chi \tag{28}$$

due to the assumed U-spin invariance.[13] Also it has to be noticed that the box diagram corresponding to the $B^0 - \bar{B}^0$ mixing contributes identically to the discussed decay modes and its contribution to the phase (with a possible NP part) is canceled in (28). Certainly beyond the SM one could have $\theta_{12} \neq 2\beta$, but this phase shift may have no direct relation to the discussed shift $\delta\alpha = -\delta\beta$, resulted from the penguin process. Thus we are left with the only NP contribution from the penguin-mediated decay (with respect to the tree level one). We see that the ability to extract $\delta\alpha$ from experiment (i.e. from α_{eff}) crucially depends on the value of r, since given experimental uncertainty in α_{eff} corresponds to larger uncertainty in $\delta\alpha$ smaller the ratio r is. The combined fit of the data for $B \to \rho\pi$ and other modes (notably $B_0 \to K_0^* \rho_0$) taking into account nonzero penguin NP phase $\delta\alpha$ is being performed and will be reported elsewhere. Here we would like to notice that the experimental accuracy of $\delta\beta$ is currently limited by the statistics of $B \to \phi K_S$ decay and recent update for $\sin 2\beta$ from penguin decay modes as given by [23] is $\sin 2\beta|_{peng} = 0.58^{+0.12}_{-0.09} \pm 0.13$

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[12]The above assignment is self-consistent for $r \lesssim 1$, where terms in r which are nonlinear in penguin amplitudes can be neglected.

[13]See discussion of the related issue in supersymmetric context in [60].

which correspond to about $15°$ uncertainty window in the angle β. It is worth mentioning that this penguin-dominated mode would not allow to get $\sin 2\beta$ (and hence $\delta\beta$) with competitive precision at LHCb, since the latter is expected to be about 0.2 in 2fb^{-1} of running [22]. Higher accuracy should be possible for Super-B factories.

4 Conclusion

The standard approach to study CKM matrix is to overconstrain the UT using all available experimental information. However not all constrains on the (ρ, η) plane are sensitive to NP, at least if the latter is taken in the form of next-to-minimal flavor violation. Some (such as $\Delta M_d/\Delta M_s$) do not distinguish the SM from many NP scenarios just by construction, while others (such as relation (8)) are insensitive to NP because of the specific profile of the UT (α close to $90°$). In this sense there are two possible points of view regarding the fact that up to now all constraints on (ρ, η) plane agree with each other. The first one is that there are no sizeable NP effects seen in flavor physics. The second one is that UT is simply not suitable for the purpose (since NP is not present in the angles determined from the tree processes and could also cancel from the sides) and the room for manifestations of NP in b-physics observables is in fact not so small (because the uncertainties are still rather large). Following the latter attitude, we have discussed in this paper a complementary analysis of the data on the CKM matrix elements. Its key feature is the use of CKM matrix elements ratios which are sensitive to NP provided it contributes differently to the processes of different topology. In this sense the quantities $\zeta_{s,K^*}^{(i)}$ are different from the ratios like $\Delta M_d/\Delta M_s$ since the short-distance part is kept in the former. Moreover, since we have more than one choice for observables a given CKM matrix element is extracted from, we could have relations of the form (16), leaving unconstrained more than one degree of freedom. Thus the lattice simulations must match *several* hadronic inputs simultaneously (and not just *one*). This, we believe, will allow to reduce the corresponding errors and consequently to make the proposed probes more sensitive to the NP.[14] Speaking differently, one of our main messages to the lattice community is that the importance of further reducing uncertainties in the ratio ξ is limited with respect to the calculation of hadronic inputs entering the definition of ζ's since the latter are more sensitive to NP than the former.

   Concerning the determination of UT angles which are free from lattice uncertainties, we advocate the importance of estimates of the angle $\delta\alpha$ corresponding to the penguin amplitude extracted from $B \rightarrow \rho\pi$ and other modes (and hence subject of possible NP shifts). The accuracy of such a comparison can be comparable or better at LHCb than for $\sin 2\beta$ extracted from $B \rightarrow J/\psi K_S$ and $B \rightarrow \phi K_S$ modes, while the physical meaning is the same; any discrepancy between these values would undoubtedly indicate NP.

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[14]The importance of correlating of $\Delta F = 1$ and $\Delta F = 2$ processes is stressed in another respect in [10].

In principle, nothing prevents one to include the discussed quantities $\zeta_{q,V}^{(i)}$ and $\delta(\alpha, \beta, \chi)$ into the global fit of the CKM matrix. It is clear that one gets essentially no new information in this way, since we deal with the same experimental observables the standard fitting procedure does. We feel, however, that careful analysis of the proposed observables provides an alternative and transparent way of looking at NP effects. This strategy can become useful in the nearest future when LHC data will improve the accuracy of our knowledge of the CKM matrix elements dramatically.

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Table 1: Short-distance quantities from the definition of κ.

Quantity	Mean	Error	Reference
η_B	0.551	0.008	[33]
F_{tt}	2.35	0.06	[34]
$\bar{m}_t(m_t)$, GeV	164.7	2.8	[59]
$\bar{m}_b(pole)$, GeV	4.65	0.10	[28]
$|C_7^{(0)} + A^{(1)}(\mu)|^2$	0.16	0.01	[28]
m_W, GeV	80.40	0.03	[51]

Table 2: Numerical results for $\zeta_{s,K^*}^{(i)}/\Delta\zeta_{s,K^*}^{(i)}$. The abbreviation [43]-A corresponds to the branching ratio for $B \to K^*\gamma$ from [43] and the Set A choice for $T_1^{B \to K^*}(0) = 0.25 \pm 0.05$, and analogously for other columns.

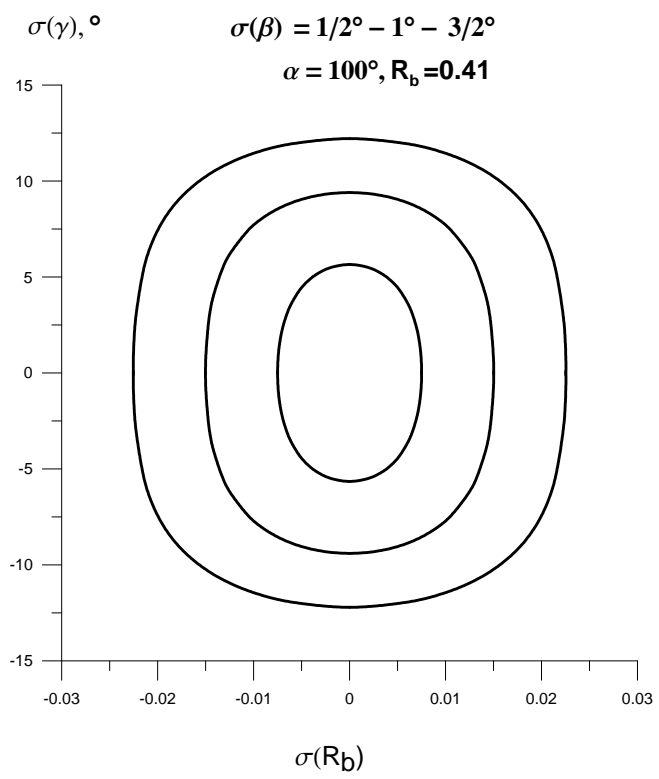|	[43]-A	[43]-B	[43]-C	[44]-A	[44]-B	[44]-C
$\zeta_{s,K^*}^{(1)}$	0.81/0.21	0.97/0.23	1.13/0.25	0.85/0.22	1.02/0.24	1.18/0.26
$\zeta_{s,K^*}^{(2)}$	1.12/0.24	0.94/0.18	0.80/0.13	1.07/0.23	0.89/0.17	0.77/0.12
$\zeta_{s,K^*}^{(3)}$	1.10/0.17	1.10/0.17	1.10/0.17	1.10/0.17	1.10/0.17	1.10/0.17

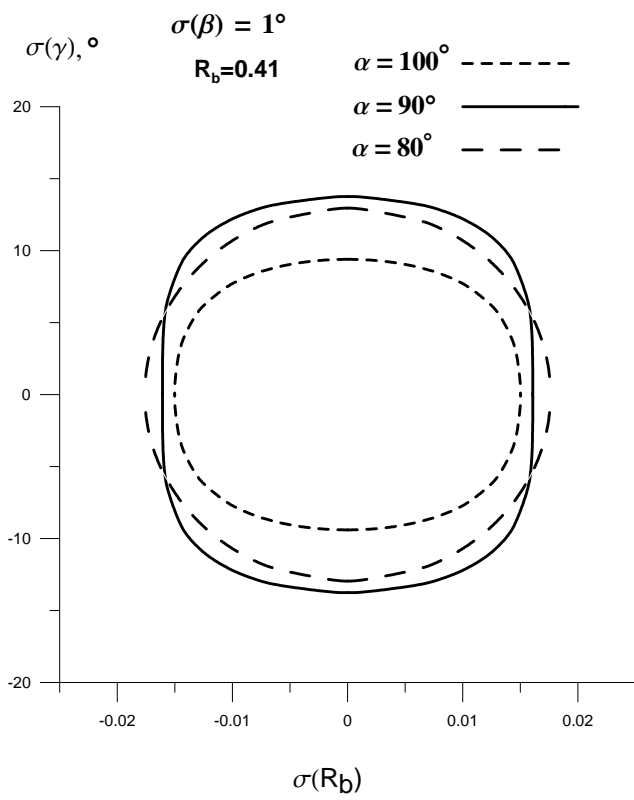Figure 1: Error propagation corresponding to the second expression from (8).

Figure 2: The same as Fig.1 for different values of the angle α.

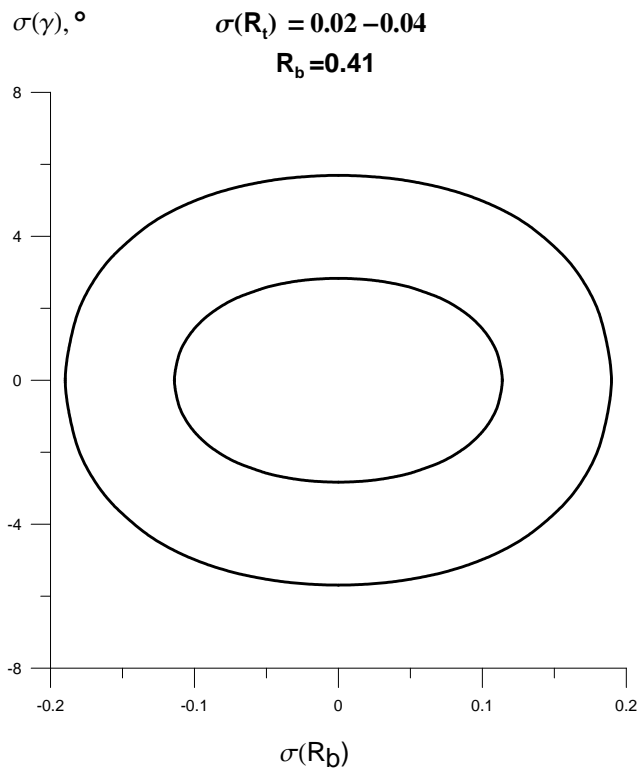Figure 3: Error propagation for R_t from (8). The curves correspond to $\sigma(R_t) = 0.02$ and 0.04. The choice for other parameters is $R_b = 0.41$, $\gamma = 1$ rad.
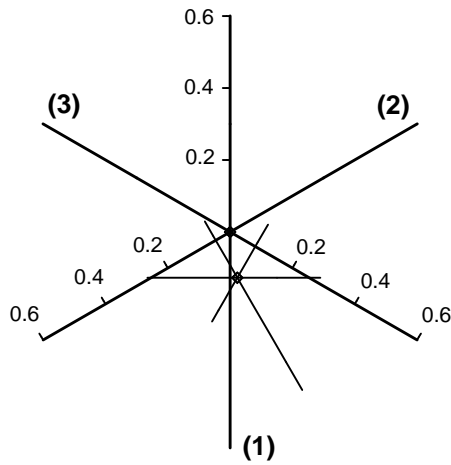
Figure 4: The results for $\log \zeta_{s,K^*}^{(i)}$, the case [44]-B plotted as a point in ternary coordinates. The SM solution is the point at the origin. The algebraic distance from the i-th axis is given by $\log \zeta_{s,K^*}^{(i)}$, positive for two axes forming an angle a point belongs to and negative for the remaining distant axis. With this rule each point on the plain satisfies the constraint (16). The bars correspond to 1σ deviation in $\zeta_{s,K^*}^{(i)}$, not in $\log \zeta_{s,K^*}^{(i)}$. The marks on axes set the scale and serve mainly for guiding eyes.