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KERR ELLIPTICITY EFFECT IN A BIREFRINGENT OPTICAL FIBER

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Abstract

An intensity-dependent change in the ellipticity of an input light beam leads to a characteristic shift in polarization instability. Dichroism gives rise to a self-induced ellipticity effect in the polarization state of an intense input light oriented along the fast axis of a birefringent optical fiber. The critical power at which the fiber effective beat length becomes infinite is reduced considerably in the presence of dichroism.

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1 Introduction

When two or more optical waves copropagate inside a birefringent single-mode fiber they can differ not only in their wavelengths but also in their states of polarization. The optical pulses can further couple with each other through fiber nonlinearity and the polarization of each field can change during propagation as a result of optically induced nonlinear birefringence. The coupling of two waves with the same frequency but different polarizations gives rise to a number of interesting effects in optical fibers. One such effect is the polarization instability which develops when the nonlinear change in refractive index grows to be comparable with linear birefringence. This instability manifests as large changes in the output state of polarization when the input power or the input polarization state is changed slightly. Then, at a critical value of input power, nonlinear birefringence can cancel intrinsic birefringence completely and the effective beat length becomes infinite. It has been discovered and demonstrated that induced nonlinear birefringence is responsible for polarization instability [1-5].

As early as 1964, it was observed that an intense elliptically polarized beam propagating within an isotropic medium induces a nonlinear birefringence effect known as ellipse rotation which is a continuous precession of the orientation angle of the polarization ellipse, while leaving its shape and handedness unchanged assuming no dichroism [6]. Since then, self-induced polarization effects have been studied extensively, particularly in the context of optical kerr effect or nonlinear refractive index and remains a topic of much interest given its practical and important consequences for laser propagation characteristics as well as its use as the basis for a wide variety of applications [7]. These nonlinear polarization effects in a birefringent single-mode optical fiber have led to practical applications through intensity discriminators, fiber-optic gates and kerr shutters [8]. Also, a new class of devices, among which a linear coherent amplifier mixer and an optically activated polarization switch, have been envisaged due to the intensity-dependent changes in the light polarization as it evolves along a lossless birefringent single mode optical fiber [9]. The problem of the interaction between a nonlinear and a dc-induced birefringence has been analytically solved by Sala for an isotropic medium [10]. The form of the refractive index change induced by the elliptically polarized pump beam corresponds to elliptical birefringence with the important consequence that an arbitrarily polarized probe beam in addition to orientation experiences a change in the shape and handedness of its polarization ellipse. Winful presented exact solutions for the intensity-dependent polarization state of a light wave in a birefringent optical fiber taking into account both the intrinsic linear polarization evolution and the nonlinear ellipse rotation and showed that self-induced polarization changes can occur with equal excitation of the fiber principal axes [11]. While these effects on optical properties and propagation characteristics in birefringent fibers have long been known and examined in details, the problem of the resulting effects due to an intensity-dependent change in the ellipticity of the polarization ellipse has received very little attention.

In their work, the authors assumed a lossless fiber thereby neglecting dichroism which is related to the imaginary part of the nonlinear susceptibility tensor and is also responsible for ellipticity which is acquired by the different absorption rates of the right and left circularly polarized modes due to the imginary part of the complex refractive index. Contrary to the above assumption, dichroism is always present in a real fiber. The purpose of this paper is to evaluate the influence of dichroism on the polarization nonlinear evolution after including the effects of both intrinsic linear birefringence and optically induced nonlinear birefringence. We find as was reported in [1] that if dichroism is neglected, the fiber's effective beat length becomes infinite when total input power P_0 reaches a critical power level P_{cr}. However, we further observe that in the presence of dichroism, the effective beat length becomes infinite when $P_0 = 2P_{cr}$. We note that this result is in agreement with the experiments conducted by [4]. Thus, there is a characteristic shift in the critical power value at which polarization instability occurs when dichroism is considered present in the fiber. We also want to state that the shift in polarization instability results from an intensity-dependent change in the ellipticity of the input beam. We report the presence of Kerr ellipticity whose origin is due mainly to the presence of dichroism in the fiber and manifested as a change in the shape of the polarization ellipse. Kerr ellipticity modulates the critical input power value for polarization instability. Infact, the critical power at which polarization instability occurs is decreased considerably in the presence of increased dichroic effects. This in our opinion can be useful for efficient optical switching.

The theoretical method used to calculate and show the polarization state dependence on the nonlinear susceptibility tensor is presented in section 2, while in section 3 we present and discuss our main results. Our conclusions are summarized in section 4.

2 Theoretical Background

The monochromatic electric fields propagating along a birefringent single-mode optical fiber can be expressed as a superposition of the two orthogonal polarization of the fundamental mode

$$\mathbf{E} = \sum_{j=x,y} A_j\left(z,t\right) \mathbf{E}_j\left(x,y\right) \exp\left[i\left(k_j z - \omega t\right)\right] \quad + \quad c.c. \tag{1}$$

where $A_j\left(z,t\right)$ are the field amplitudes along the principal axes of the fiber, $\mathbf{E}_j\left(x,y\right)$ are the transeverse mode distributions, and ω, t, and k_j have the usual interpretation of respectively frequency, time, and propagation constant. The longitudinal coordinate z is chosen to coincide with the fiber axis of symmetry along which the light wave propagates.

The nonlinear wave equation which describes the transverse electric fields evolution along a birefringent optical fiber is expressed as

$$\frac{d^2 E_i}{dz^2} + \left(\frac{\omega}{c}\right)^2 \epsilon_{ij} \mathbf{E}_j = -4\pi \left(\frac{\omega}{c}\right)^2 P_i^{NL}\left(\omega, \mathbf{r}\right) \tag{2}$$

where $P_i^{NL}(\omega, \mathbf{r})$ is the spectral amplitude of the nonlinear induced polarization and ϵ_{ij} is the dielectric tensor which is diagonal in the principal axis representation.

Since the anisotropy of birefringent fibers is relatively weak $(\delta n \simeq 10^{-4})$, the nonlinear polarization will not differ much from that of an isotropic medium, at least for effects third order in the electric field [11]. Thus, we can write the nonlinear induced polarization as

$$P_i^{NL}(\omega, \mathbf{r}) = 3\chi_{ijkl}^{(3)}(\omega; \omega, \omega, -\omega) E_j(\omega, \mathbf{r}) E_k(\omega, \mathbf{r}) E_l^*(\omega, \mathbf{r}) \tag{3}$$

where the indices run over (x, y).

The evolution equations describing the state of the light polarization can thus be expressed in compact form as [12, 13]

$$\frac{d\mathbf{S}}{dz} = [\Omega_L(z) + \Omega_L(z, \mathbf{S})] \times \mathbf{S} \tag{4}$$

Here, $\mathbf{S} = (S_1, S_2, S_3)$ is the Stokes vector which is associated with the polarization state of the fields and Ω with projections Ω_1, Ω_2, and Ω_3 is a vector in stokes space related to the material characteristics and depend on the Stokes parameters if the medium is nonlinear. In one of the standard conventions [14], the Stokes parameters are defined as

$$S_i = A_j^* (\sigma_i)_{jk} A_k \qquad i = 1, 2, 3 \tag{5}$$

where σ_i are the Pauli spin matrices.

If we assume small anisotropy along the direction of propagation and that the cubic nonlinearity is isotropic, the nonlinear evolution equations will take the following form

$$\frac{dS_1}{dz} = -\left(\frac{12\pi\omega}{nc}\chi_{xxyy}^{(3)}\right) S_2 S_3 \tag{6}$$

$$\frac{dS_2}{dz} = \frac{\omega}{2nc}\left(\epsilon_{xx} - \epsilon_{zz} + 24\pi\chi_{xxyy}^{(3)} S_1\right) S_3 \tag{7}$$

$$\frac{dS_3}{dz} = -\left\{\frac{\omega}{2nc}(\epsilon_{xx} - \epsilon_{zz})\right\} S_2 \tag{8}$$

The exact analytical solutions to this system of equations have been obtained in terms of elliptic functions [10]

$$S_3 = \frac{2pkf}{q}\mathbf{cn}(R_0 f z + \mathcal{C}\ ; \mathrm{k}) \tag{9}$$

$$S_2 = \frac{2pkf^2}{q}[\mathbf{sn}(R_0 f z + \mathcal{C}\ ; \mathrm{k})\,\mathbf{dn}(R_0 f z + \mathcal{C}\ ; \mathrm{k})] \tag{10}$$

$$S_1 = \frac{f^2}{q}\left\{1 - 2m\left[\mathbf{sn}^2(R_0 f z + \mathcal{C}\ ; \mathrm{k})\right]\right\} - 1 \tag{11}$$

where **cn**, **sn**, and **dn** are Jacobi elliptic functions and $R_0 = \omega(\epsilon_{xx} - \epsilon_{zz})/2nc$ with $f = \left[(1 + qS_{10})^2 - q^2 S_{20}^2\right]^{\frac{1}{4}}$. Also, $q = 24\pi\chi_{xxyy}^{(3)}/(\epsilon_{xx} - \epsilon_{zz})$ and $p = sgn(S_{30})$. The Jacobi modulus k is given by

$$\mathrm{k} = \left[\frac{1}{2} + \frac{1}{4f^2}\left(q^2 - 1 - f^4\right)\right]^{\frac{1}{2}} \tag{12}$$

and

$$- \operatorname{Re}[K(m)] \leq \mathcal{C} \leq + \operatorname{Re}[K(m)] \tag{13}$$

with $[K(m)]$ denoting the Jacobian quarter-period. Here, the Jacobian parameter $m = \mathrm{k}^2$.

Clearly, the output polarization state of the optical beam is dependent on the Jacobian modulus k which in turn depends on the input polarization of the light as well as the nonlinear susceptibility tensor and the anisotropic tensor.

3 Results and Discussion

Since the period of the elliptic function determines the effective beat length of the fiber [1], the beat length is dependent on both input intensity through the nonlinear susceptibility tensor and input polarization. The variation of the effective beat length as a function of input power for beams polarized along the fast and slow axes of the fiber has been obtained [1]. We obtain in Fig.(1), as was deduced in [1] that, barring fiber losses, when the input beam is polarized along the fast axis of the fiber, the effective beat length becomes infinite at a critical power value because of complete cancellation between the intrinsic and nonlinear birefringences. However, we further observe as is shown also in Fig.(1) that, in the presence of dichroism, the effective beat length becomes infinite at a different critical power value. Infact, in terms of the normalized input power defined as $p = P_0/P_{cr}$, when dichroism is absent $p = 1$ and in the presence of dichroism $p = 2$. The latter result is in agreement with experiments conducted by [4]. Thus, there is a characteristic shift in the critical power value at which polarization instability occurs when dichroism is considered present in the fiber. This shift in polarization instability results from an intensity-dependent change in the ellipticity of the polarization ellipse. The polarization ellipse of the propagating beam experiences an intensity-dependent change in its shape through the nonlinear susceptibility tensor due to dichroism. The change in ellipticity (Kerr ellipticity) of polarization is due mainly to the presence of dichroism in the fiber and manifested as a change in the shape of the polarization ellipse. Kerr ellipticity will modulate the critical input power value for polarization instability. Infact, the critical power at which polarization instability occurs is reduced considerably in the presence of increased dichroic effects. In our opinion, this can be useful for efficient optical switching.

4 Conclusions

We have studied the resulting effects of dichroism on polarization instability after including both intrinsic linear birefringence and optically induced nonlinear birefringence. It was found that the critical power at which polarization instability occured was reduced considerably in the presence of dichroism. Our results are in agreement with the theory which neglects dichroism [1] and with experiments [4] when dichroic effects are intrinsically present and cannot be discarded. More importantly, we found that the shift in polarization instability is due to Kerr ellipticity which results from an intensity-dependent change in the ellipticity of the propagating beam when dichroism is present. In our opinion, this can be useful for efficient optical switching. Therefore, eventhough polarization instability is a birefringence related nonlinear effect, we have been able to stress the role played by the presence of dichroism.

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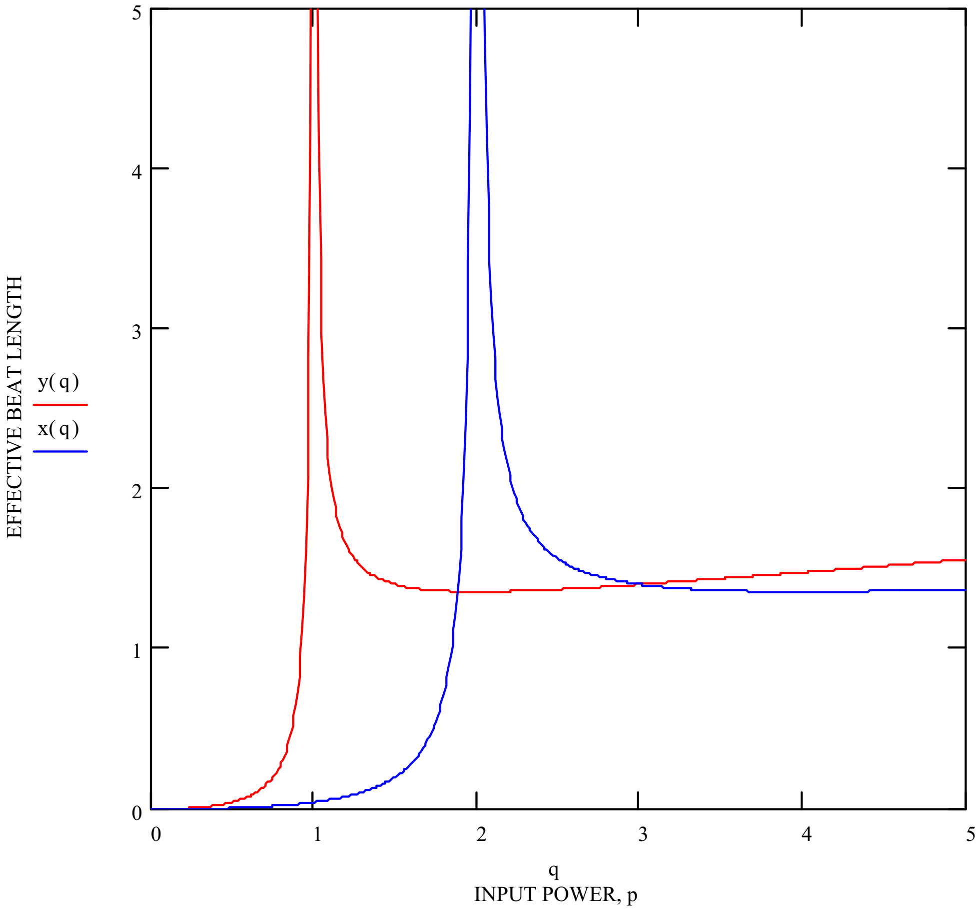

Figure 1: Effective beat length as a function of input power when dichroism is neglected (solid line) and in the presence of dichroism (dashed line).