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EXQUISITIVELY SENSITIVE MATTER-WAVE INTERFEROMETRY USED TO MEASURE THE CHANGE IN VELOCITY OF A MOVING MAGNET TRAVERSING AN ELECTRIC FIELD

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Abstract

Motivated by the quite recent interferometric experiment of Shinohara, Aoki and Morinaga (Phys. Rev. **A66**, 042106, 2002) in which the scalar Aharonov-Bohm effect was studied, we re-open the extension to neutral particles carrying a magnetic moment and passing through a region of intense electric field, treated theoretically by Aharonov and Casher (AC) and independently by Anandan. An alternative interpretation of results on (a) neutrons and (b) TℓF molecules to that afforded by AC is shown to involve only (i) the de Broglie wavelength of matter waves and (ii) the prediction from Maxwell's equations for the change in velocity of a neutral moving magnet as it enters or leaves an electric field. The exquisite sensitivity of experiment (b) allows a fractional change in velocity of order 10^{-15} to be quantitatively determined.

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The motivation for this Brief Report comes from the recent study of Shinohara, Aoki and Morinaga [1] in this journal, using matter-wave interferometry. Their immediate concern was with the scalar Aharanov-Bohm effect for ultracold atoms. As they pointed out, both scalar and vector Aharonov-Bohm [AB] effects [2] were extended to neutral particles carrying a magnetic moment by Aharonov and Casher [3: termed AC below] and independently by Anandan [4].

As stressed in ref.[1], for neutral particles the first experiment bearing on the theoretical studies of [3] and [4] above employed slow neutrons [5] and used real-space interferometry. This was complemented later by spin-space interferometry with neutral molecules [6-8], out of which we choose to discuss the specific experiment of Sangster et al. [6]. The authors selectively flipped the magnetic moments of some of the F nuclei in a beam of TℓF molecules which was traversing a strong electric field and this will be discussed further below. Later, in an experiment also relevant to the theory presented in [3] and [4], Yanagimachi et al. [9] successfully observed the AC phase for a relatively short measurement time by means of an atomic polarizing white color interferometer.

While, of course, the AC formula for the phase shift experienced by a neutral particle with magnetic moment $\boldsymbol{\mu}$ travelling through an electric field $\boldsymbol{\mathcal{E}}$ is both illuminating and elegant (see also [4]), there is an alternative physical, argument which, as we will expand on below, demonstrates very strikingly the exquisite sensitivity of the interferometric experiments detailed above.

In the light of the substantial current interest in matter waves, even involving huge organic molecules such as the buckyball C_{60} [10], we emphasize first, and expand on below, the fact that the measured phase shift, as predicted by AC [3] and Anandan [4], can be interpreted in the experiments [5] and [6] referred to specifically below, by invoking two very basic physical results. These are (i) the de Broglie relation h/p for the wavelength of the matter wave corresponding to momentum p and (ii) the change in velocity, denoted below by Δv, of a moving magnet as it enters or leaves an electric field.

To focus on the sensitivity required to detect $\Delta v/v$, we stress first of all that under the conditions obtaining in the experiment of Sangster et al. [6] for the TℓF molecular beam, the ratio $\Delta v/v$ was $\sim 10^{-15}$, a value to be discussed a little further below. It is truly remarkable to the present writer that spin-space interferometry has reached the stage where it cannot just detect, but also quantitatively measure, such an increment Δv in the velocity v.

The basic electromagnetism needed to calculate (ii) above can be found in various places (see, for example, [11-13]) but for completeness we summarize below, and quite briefly, the essential points needed for the present purposes. Consider such a neutral moving magnet having moment $\boldsymbol{\mu}$ and entering a region of electric field $\boldsymbol{\mathcal{E}}$. The initial velocity $\mathbf{v}$, as the magnet enters the field $\boldsymbol{\mathcal{E}}$, is changed by a very small increment, Δv as introduced already, due to the interaction of the moving dipole with the magnetic flux density given by

$$\mathbf{B} = \mathbf{v} \times \boldsymbol{\mathcal{E}}/c^2 \tag{1}$$

where c is the velocity of light. When such a magnet, having mass m, moves from a region where $\boldsymbol{\mathcal{E}} = 0$ to an intense field region of strength $\mathcal{E}$, the magnet gains or loses momentum $\Delta\mathbf{p}$ given by

$$\Delta\mathbf{p} = \boldsymbol{\mu} \times \boldsymbol{\mathcal{E}}/c^2, \tag{2}$$

the change in velocity being $\Delta\mathbf{p}/m$. For common magnets, the fractional change of velocity $\Delta v/v$ is incredibly small. But for, say, a beam of neutrons, which has, of course, accurately measurable de Broglie wavelengths, minute changes in velocity are observable by interferometry.

Taking therefore specifically the coherent neutron beam experiment of Cimmino et al. [5], their experimental conditions were such that

$$\Delta v = \mu F/mc^2 = 1.87 \times 10^{-9} ms^{-1} \tag{3}$$

But $v = h/m\lambda$ where the de Broglie wavelength $\lambda = 148$ pm and $v = 2680$ ms^{-1}. Hence from eqn.(3) it follows that

$$\Delta v/v = 7 \times 10^{-13} \tag{4}$$

The extent of the electric field region in the experiment of Cimmino et al. [5] was 0.0253 m and thus the neutrons experience the field for a time $t = 9.4 \times 10^{-6}s$. The change of phase, $\Delta\phi$ say, of the neutrons while traversing the electric field is given by

$$\Delta\phi = 2t\Delta v/(\lambda/2\pi), \tag{5}$$

since the beam is split into two branches and the phase changes there are equal and opposite. Thus one finds, inserting $t = 9.4 \times 10^{-6}s$ into eqn.(5), a phase shift $\Delta\phi = 1.5 \times 10^{-3}$ radians. This phase shift, and consequently from eqn.(5) the velocity change Δv, is what is detected by the delicate interferometric technique employed by Cimmino et al. [5]. These workers reported an observed phase shift of 2.2 ± 0.5 m rad., which is quite close to embracing the value of 1.5×10^{-3} rad. obtained above. Thus, the de Broglie relation for matter waves, plus standard electromagnetism embodied in Maxwell's equations, yield by simple physical arguments close agreement with this neutron experiment [5] in which the unpolarized coherent beam was split by diffraction in a Si crystal and the two branches then passed through opposite electric fields having magnitudes $\mathcal{E}$ of $+2.92 \times 10^7 Vm^{-1}$ and $-2.92 \times 10^7 Vm^{-1}$.

As a second numerical example, let us return to the interferometric experiment carried out on thallium fluoride (TℓF) molecules. Here Sangster et al. [6] employed as their moving magnets the fluorine nuclei in a TℓF molecular beam, again in a strong externally applied electric field of magnitude $\mathcal{E} = 0.5 - 3MVm^{-1}$. For numerical estimates, made again as for the neutron experiment above, the TℓF molecules have mass $m = 3.71 \times 10^{-25}kg$. For a field strength $\mathcal{E} = 2.01 \times 10^6 v/m$, Sangster et al. obtained a phase shift $\Delta\phi$ of 2.47×10^{-3} rad. From eqn.(5) above, using eqn.(3) for Δv, the values we obtained were $\Delta v = 3.39 \times 10^{-13} m/s$ with corresponding fractional change $\Delta v/v = 1.33 \times 10^{-15}$, the order of magnitude of which was already anticipated above. The corresponding phase shift from eqn.(5) agrees with the measured phase shift to within a few percent.

In summary, by means of the de Broglie relation for the wavelength h/p associated with matter waves, plus the prediction from Maxwell's equations for the change in velocity of a moving magnet as it enters or leaves an electric field, one can quantitatively interpret the interferometric experiments using both a beam of neutrons and molecular beam of TℓF molecules. The exquisite sensitivity of the interferometry used is plain from the fractional changes in velocity, $\Delta v/v$, which are for the neutron experiment $\sim 10^{-12}$ and for the TℓF molecules 10^{-15}. This in no way denies the interest and elegance of the AC work in [2] and the study of Anandan in [4]; and in particular their finding that the phase shift $\Delta\phi$ calculated above can be represented in a precise path-independent formula.

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