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# QCD CORRECTIONS TO SQUARK PRODUCTION IN $\mathrm{e}^{+} \mathrm{e}^{-}$ANNIHILATION IN THE MSSM WITH COMPLEX PARAMETERS 

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#### Abstract

We discuss the pair production of scalar quarks in $e^{+} e^{-}$annihilation within the MSSM with complex parameters. We calculate the $S U S Y$-QCD corrections to the cross section $e^{+} e^{-} \rightarrow \tilde{q}_{i} \overline{\tilde{q}}_{j}(i, j=1,2)$ and show that the effect of the CP phases of these complex parameters on the cross section can be quite strong in a large region of the MSSM parameter space. This could have important implications for squarks searches and the MSSM parameter determination in future collider experiments.


## I Introduction

Supersymmetry is the currently best motivated extension of the Standard Model (SM) of particle physics which allows to stabilize the gauge hierarchy without getting into conflict with electroweak precision data. Among all possible supersymmetric theories, the Minimal Supersymmetric Standard Model (MSSM) occupies a special position. It is not only the simplest, i.e. most economical, potentially realistic supersymmetric field theory, but it also has just the right particle content to allow for the unification of all gauge interactions. The MSSM predicts the existence of scalar partners to all known quarks and leptons. Each fermions has two spin zero partners called sfermions $\tilde{f}_{L}$ and $\tilde{f}_{R}$, one for each chirality eigenstate: the mixing between $\tilde{f}_{L}$ and $\tilde{f}_{R}$ is proportional to the corresponding fermion mass, and so negligible except for the third generation. In particular, this model gives the possibility for one of the scalar partners of the top quark $\left(\tilde{t}_{1}\right)$ to be lighter than other scalar quarks and also than the top quark [1].

So far most phenomenological studies on supersymmetric (SUSY) particle searches have been performed in the MSSM with real SUSY parameters. Studies of the 3rd generation sfermions are particularly interesting because of the effects of the large Yukawa couplings. The lighter sfermion mass eigenstates maybe relatively light and they could be thoroughly studied at an $e^{-} e^{+}$linear collider [2]. An analysis of the QCD corrections to scalar quark pair production in $e^{+} e^{-}$annihilation in the MSSM with real parameters was performed in refs. [3, 4].

The assumption that all $S U S Y$ parameters are real, however, may be too restrictive. The higgsino mass parameter $\mu$, the gaugino masses $\tilde{M}_{i}$ and the trilinear scalar coupling parameters $A_{f}$ of the sfermions $\tilde{f}$ may be complex. In the MSSM the complex parameters provide the CP violating phases. Recently, a phenomenological study of $\tau$ sleptons $\tilde{\tau}_{1,2}$ and $\tau$-sneutrinos $\tilde{\nu}_{\tau}$ has been presented [5], and the effect of the CP phases on the $\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$ decays has been found [6] in the MSSM with complex parameters. Next, in ref. [7] the one loop vertex correction to the decay width of squark decays into $W$ and $Z$ bosons within the MSSM with complex parameters has been calculated and the numerical results are also performed.

In this article we study the effects of the phases of the complex parameters $A_{q}$ on the cross sections of the process: $e^{+} e^{-} \rightarrow \tilde{q}_{i} \overline{\tilde{q}}_{j}$. We point out that these effects can be quite strong in a large region of the MSSM parameter space. This could have an important impact on the search for squarks and the determination of the MSSM parameters at future colliders. Our present study is an extension of the corresponding one in the MSSM with real parameters in ref. [3, 4].

## II Diagonalization of Mass Matrices

We neglect generation mixing. As pointed out in refs. [8, 9] only three terms in the supersymmetric Lagrangian can give rise to CP-violating phases which cannot be rotated away: The superpotential contains a complex coefficient $\mu$ in the term bilinear in the Higgs superfields. The soft supersymmetry breaking operators introduce two further complex terms, the gaugino masses $\tilde{M}_{i}$ and the left- and right-handed squark mixing term $A_{q}$. In the MSSM one has two types of scalar quarks (squarks), $\tilde{q}_{L}$ and $\tilde{q}_{R}$, corresponding to the left and right helicity states of a quark. The mass matrix in the basis $\left(\tilde{q}_{L}, \tilde{q}_{R}\right)$ is given by [1].

$$
\mathcal{M}_{\tilde{q}}^{2}=\left(\begin{array}{cc}
m_{\tilde{q}_{L}}^{2} & a_{q} m_{q}  \tag{1}\\
a_{q} m_{q} & m_{\tilde{q}_{R}}^{2}
\end{array}\right)=\left(\mathcal{R}^{\tilde{q}}\right)^{+}\left(\begin{array}{cc}
m_{\tilde{q}_{1}}^{2} & 0 \\
0 & m_{\tilde{q}_{2}}^{2}
\end{array}\right) \mathcal{R}^{\tilde{q}},
$$

with

$$
\begin{align*}
m_{\tilde{q}_{L}}^{2} & =\mathcal{M}_{\tilde{Q}}^{2}+m_{Z}^{2} \cos 2 \beta\left(I_{3 L}^{q}-e_{q} s_{W}^{2}\right)+m_{q}^{2}  \tag{2}\\
m_{\tilde{q}_{R}}^{2} & =\mathcal{M}_{\{\tilde{u}, \tilde{D}\}}^{2}+e_{q} m_{Z}^{2} \cos 2 \beta s_{W}^{2}+m_{q}^{2},  \tag{3}\\
a_{q} & =A_{q}-\mu\{\cot \beta, \tan \beta\}, \tag{4}
\end{align*}
$$

for $\{$ up, down $\}$ type squarks, respectively. $e_{q}$ and $I_{3}^{q}$ are the electric charge and the third component of the weak isospin of the squark $\tilde{q}$, and $m_{q}$ is the mass of the partner quark. $M_{\tilde{Q}}, M_{\tilde{u}}$, and $M_{\tilde{D}}$ are soft $S U S Y$ breaking masses, and $A_{q}$ are trilinear couplings.

According to eq. (1) $\mathcal{M}_{\tilde{q}}^{2}$ is diagonalized by a unitary matrix $\mathcal{R}^{\tilde{q}}$. The weak eigenstates $\tilde{q}_{L}$ and $\tilde{q}_{R}$ are thus related to their mass eigenstates $\tilde{q}_{1}$ and $\tilde{q}_{2}$ by

$$
\begin{align*}
\binom{\tilde{q}_{1}}{\tilde{q}_{2}} & =\mathcal{R}^{\tilde{q}}\binom{\tilde{q}_{L}}{\tilde{q}_{R}},  \tag{5}\\
\mathcal{R}^{\tilde{q}} & =\left(\begin{array}{cc}
e^{\frac{i}{2} \varphi_{q}} \cos \theta_{\tilde{q}} & e^{-\frac{i}{2} \varphi_{q}} \sin \theta_{\tilde{q}} \\
-e^{\frac{i}{2} \varphi_{q}} \sin \theta_{\tilde{q}} & e^{-\frac{i}{2} \varphi_{q}} \cos \theta_{\tilde{q}}
\end{array}\right), \tag{6}
\end{align*}
$$

with $\theta_{\tilde{q}}$ is the squark mixing angle and $\varphi_{q}=\arg \left(A_{q}\right)$. The mass eigenvalues are given by

$$
\begin{equation*}
m_{\tilde{q}_{1,2}}^{2}=\frac{1}{2}\left(m_{\tilde{q}_{L}}^{2}+m_{\tilde{q}_{R}}^{2} \mp \sqrt{\left(m_{\tilde{q}_{L}}^{2}-m_{\tilde{q}_{R}}^{2}\right)+4 a_{q}^{2} m_{q}^{2}}\right) . \tag{7}
\end{equation*}
$$

By convention, we choose $\tilde{q}_{1}$ to be the lighter mass eigenstate. For the mixing angle $\theta_{\tilde{q}}$ we require $0 \leq \theta_{\tilde{q}} \leq \pi$. We thus have

$$
\begin{equation*}
\cos \theta_{\tilde{q}}=\frac{-\left|a_{q}\right| m_{q}}{\sqrt{\left(m_{\tilde{q}_{L}}^{2}-m_{\tilde{q}_{1}}^{2}\right)^{2}+a_{q}^{2} m_{q}^{2}}}, \quad \sin \theta_{\tilde{q}}=\frac{m_{\tilde{q}_{L}}^{2}-m_{\tilde{q}_{1}}^{2}}{\sqrt{\left(m_{\tilde{q}_{L}}^{2}-m_{\tilde{q}_{1}}^{2}\right)^{2}+a_{q}^{2} m_{q}^{2}}} \tag{8}
\end{equation*}
$$

## III The $\tilde{q} \tilde{q} \gamma$ and $\tilde{q} \tilde{q} z$ couplings

From the matrix we can find the interaction of a neutral gauge boson $V=\gamma, Z$ with squarks in the general forms

$$
\begin{align*}
\mathcal{L}_{\tilde{q} \tilde{q} \gamma} & =i e e_{q} A_{\mu}\left(\tilde{q}_{L}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{q}_{L}+\tilde{q}_{R}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{q}_{R}\right) \\
& =i e e_{q} A_{\mu}\left(\mathcal{R}_{i 1}^{\tilde{q}} \mathcal{R}_{j 1}^{\tilde{q}}+\mathcal{R}_{i 2}^{\tilde{q}} \mathcal{R}_{j 2}^{\tilde{q}}\right) \tilde{q}_{j}^{*} \overleftrightarrow{\partial^{\mu}} \tilde{q}_{i} \\
& \equiv i e e_{q} \delta_{i j} \tilde{q}_{j}^{*} \stackrel{\partial}{ }^{\mu} \tilde{q}_{i} \tag{9}
\end{align*}
$$

where

$$
\delta=\left(\begin{array}{cc}
\cos \varphi_{q}+i \sin \varphi_{q} \cos 2 \theta_{\tilde{q}} & -i \sin \varphi_{q} \sin 2 \theta_{\tilde{q}}  \tag{10}\\
-i \sin \varphi_{q} \sin 2 \theta_{\tilde{q}} & \cos \varphi_{q}-i \sin \varphi_{q} \cos 2 \theta_{\tilde{q}}
\end{array}\right)
$$

and

$$
\begin{align*}
\mathcal{L}_{\tilde{q} \tilde{q} Z} & =\frac{i g}{c_{w}} Z_{\mu}\left(c_{q L} \tilde{q}_{L}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{q}_{L}+c_{q R} \tilde{q}_{R}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{q}_{R}\right) \\
& \equiv \frac{i g}{c_{w}} Z_{\mu} C_{i j} \tilde{q}_{j}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{q}_{i} \tag{11}
\end{align*}
$$

where

$$
C=\left(\begin{array}{ll}
\left(I_{3 L}^{q} \cos ^{2} \theta_{\tilde{q}}-e_{q} s_{W}^{2}\right) \cos \varphi_{q} & -\frac{1}{2} I_{3 L}^{q} \sin 2 \theta_{\tilde{q}} \cos \varphi_{q}  \tag{12}\\
+i\left(I_{3 L}^{q} \cos ^{2} \theta_{\tilde{q}}-e_{q} s_{W}^{2} \cos 2 \theta_{\tilde{q}}\right) \sin \varphi_{q} & +i\left(e_{q} s_{W}^{2}-\frac{1}{2} I_{3 L}^{q}\right) \sin 2 \theta_{\tilde{q}} \sin \varphi_{q} \\
& \\
-\frac{1}{2} I_{3 L}^{q} \sin 2 \theta_{\tilde{q}} \cos \varphi_{q} & \left(I_{3 L}^{q} \sin ^{2} \theta_{\tilde{q}}-e_{q} s_{W}^{2}\right) \cos \varphi_{q} \\
+i\left(e_{q} s_{W}^{2}-\frac{1}{2} I_{3 L}^{q}\right) \sin 2 \theta_{\tilde{q}} \sin \varphi_{q} & +i\left(I_{3 L}^{q} \sin ^{2} \theta_{\tilde{q}}+e_{q} s_{W}^{2} \cos 2 \theta_{\tilde{q}}\right) \sin \varphi_{q}
\end{array}\right) .
$$

We obtain the corresponding Feynman rules from

$$
\tilde{q}_{j}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{q}_{i}=i\left(k_{i}+k_{j}\right)^{\mu}
$$

where $k_{i}$ and $k_{j}$ are the four-momenta of $\tilde{q}_{i}$ and $\tilde{q}_{j}$ in direction of the charge flow. The coupling between a gauge boson $V$ and two squarks $\tilde{q}_{i}$ and $\tilde{q}_{j}$ with $i, j=1,2$ is given by (the directions of the momenta are shown in Fig. 1):

$$
\begin{align*}
\Gamma_{\tilde{q}_{i} \tilde{q}_{j} \gamma}^{\mu} & =-i e e_{q}(\bar{k}+k)^{\mu} \delta_{i j}  \tag{13}\\
\Gamma_{\tilde{q}_{i} \tilde{q}_{j} Z}^{\mu} & =-\frac{i g}{c_{W}}(\bar{k}+k)^{\mu} c_{i j} \tag{14}
\end{align*}
$$



Figure 1

## IV Tree level formulae

In this section we discuss the pair production of squarks in $e^{+} e^{-}$collisions. The process $e^{+} e^{-} \rightarrow \tilde{q}_{i} \overline{\tilde{q}}_{j}$ proceeds via $\gamma$ and $Z$ exchange, see Fig.2.


Figure 2
Using the results just obtained we get the cross section at tree level for unpolarized beams:

$$
\begin{equation*}
\sigma^{0}=\frac{\sqrt{2} \pi \alpha^{2} \lambda_{i j}^{3 / 2}}{s}\left[e_{q}^{2}\left|\delta_{i j}\right|^{2}-\frac{e_{q} v_{e}}{4 c_{W}^{2} s_{W}^{2}}\left(c_{i j} \delta_{i j}^{+}+c_{i j}^{+} \delta_{i j}\right) \cdot D_{\gamma Z}+\frac{v_{e}^{2}+a_{e}^{2}}{16 c_{W}^{4} s_{W}^{4}}\left|c_{i j}\right|^{2} D_{Z Z}\right] \tag{15}
\end{equation*}
$$

where $s$ is the $c$. $m$. energy squared, $\lambda_{i j}=\left[\left(s-m_{i}^{2}-m_{j}^{2}\right)^{2}-4 m_{i}^{2} m_{j}^{2}\right], e_{q}$ is the charge of the squarks $\left(e_{t}=2 / 3, e_{b}=-1 / 3\right)$ in the units of $e(=\sqrt{4 \pi \alpha}), a_{e}=-1, v_{e}=-1+4 s_{w}^{2}$ (with $s_{W} \equiv \sin \theta_{W}, c_{W} \equiv \cos \theta_{W}$ ), and

$$
\begin{align*}
D_{\gamma Z} & =\frac{s\left(s-M_{Z}^{2}\right)}{\left(s-m_{Z}^{2}\right)^{2}+m_{Z}^{2} \Gamma_{Z}^{2}}  \tag{16}\\
D_{Z Z} & =\frac{s^{2}}{\left(s-m_{Z}^{2}\right)^{2}+m_{Z}^{2} \Gamma_{Z}^{2}} \tag{17}
\end{align*}
$$

## V SUSY - QCD corrections

The SUSY QCD corrected cross section, corresponding to Fig. 3 can be written as

$$
\begin{equation*}
\sigma=\sigma^{0}+\delta \sigma . \tag{18}
\end{equation*}
$$



Figure 3. Feynman diagrams for the lowest order SUSY - QCD corrections to $e^{+} e^{-} \rightarrow \tilde{q}_{i} \overline{\tilde{q}}_{j}$ Here

$$
\begin{equation*}
\delta \sigma=\frac{\pi \alpha^{2} \lambda_{i j}^{3 / 2}}{s} \frac{\alpha_{s}}{3 \pi} \operatorname{Re}\left\{\delta A_{1}+\delta A_{2}+\delta A_{3}+\delta A_{4}+\delta A_{5}\right\} \tag{19}
\end{equation*}
$$

with

$$
\begin{align*}
\delta A_{1}= & \left\{i \delta _ { i i } \delta _ { j j } 2 \left[B_{1}\left(m_{i}^{2}, m_{g}^{2}, m_{i}^{2}\right)+B_{0}\left(m_{i}^{2}, m_{g}^{2}, m_{i}^{2}\right)+B_{1}\left(m_{j}^{2}, m_{g}^{2}, m_{j}^{2}\right)\right.\right. \\
& \left.+B_{0}\left(m_{j}^{2}, m_{g}^{2}, m_{j}^{2}\right)-\left(2 s-2 m_{i}^{2}-2 m_{j}^{2}-m_{g}^{2}\right)\left(C_{11}+C_{0}\right)\right] \\
& +8 \delta_{i i}\left[B_{1}\left(m_{i}^{2}, m_{g}^{2}, m_{i}^{2}\right)+2 B_{0}\left(m_{i}^{2}, m_{g}^{2}, m_{i}^{2}\right)\right] \\
& -i \delta_{i i} \delta_{i i}\left[A\left(m_{i}^{2}\right)-m_{g}^{2} B_{0}\left(m_{i}^{2}, m_{g}^{2}, m_{i}^{2}\right)\right] \\
& +4 i \sum_{k=1}^{2}\left[\left(A\left(m_{q}^{2}\right)-m_{g}^{2} B_{0}\left(m_{i}^{2}, m_{\tilde{g}}^{2}, m_{q}^{2}\right)\right) \delta_{k i}\right. \\
& \left.\left.-\left(m_{\tilde{g}} \delta_{k i}-m_{q} m_{\tilde{g}} C_{k i}\right) B_{0}\left(m_{i}^{2}, m_{\tilde{g}}^{2}, m_{q}^{2}\right)+2 S_{i k} S_{i k} A\left(m_{k}^{2}\right)\right]\right\}^{+} . \\
& \quad\left[e_{q}^{2}\left|\delta_{i j}\right|^{2}+\frac{\left(v_{e}^{2}+a_{e}^{2}\right)\left|C_{i j}\right|^{2}}{16 C_{W}^{4} S_{W}^{4}} D_{Z Z}-\frac{e_{q} v_{e}\left[C_{i j} \delta_{i j}^{+}+C_{i j}^{+} \delta_{i j}\right]}{4 C_{W}^{2} S_{W}^{2}} D_{\gamma Z}\right], \tag{20}
\end{align*}
$$

$$
\begin{align*}
\delta A_{2} & =2\left\{\delta _ { i j } \left[\left(2 m_{\tilde{g}}^{2}+m_{i}^{2}+m_{j}^{2}+m_{q^{\alpha}}^{2}+m_{q^{\beta}}^{2}\right)\left(C_{11}^{\prime}+C_{0}^{\prime}\right)-B_{0}\left(m_{i}^{2}, m_{\tilde{g}}^{2}, m_{q^{\beta}}^{2}\right)\right.\right. \\
& \left.-B_{0}\left(m_{i}^{2}, m_{\tilde{g}}^{2}, m_{q^{\alpha}}^{2}\right)+\left(m_{q^{\alpha}}^{2}+m_{q^{\beta}}^{2}+0.5 s+0.5 m_{i}^{2}+0.5 m_{j}^{2}+2 m_{q^{\alpha}} m_{q^{\beta}}\right) \cdot C_{11}^{\prime}\right] \\
& \left.-2 m_{\tilde{g}}\left(m_{q^{\beta}}+m_{q^{\alpha}}\right) \cdot E_{i j}\left(C_{11}^{\prime}+C_{0}^{\prime}\right)\right\}^{+}\left(e_{q}^{2} \delta_{i j}\right), \tag{21}
\end{align*}
$$

$$
\begin{align*}
\delta A_{3} & =2\left\{-\delta_{i j R L}\left[\left(2 m_{\tilde{g}}^{2}+m_{i}^{2}+m_{j}^{2}+m_{q^{\alpha}}^{2}+m_{q^{\beta}}^{2}\right)\left(C_{11}^{\prime}+C_{0}^{\prime}\right)-B_{0}\left(m_{i}^{2}, m_{\tilde{g}}^{2}, m_{q^{\beta}}^{2}\right)\right.\right. \\
& \left.-B_{0}\left(m_{i}^{2}, m_{\tilde{g}}^{2}, m_{q^{\alpha}}^{2}\right)+\left(m_{q^{\alpha}}^{2}+m_{q^{\beta}}^{2}+0.5 s+0.5 m_{i}^{2}+0.5 m_{j}^{2}\right) . C_{11}^{\prime}\right] \\
& \left.+2 m_{\tilde{g}}\left(m_{q^{\beta}} C_{i j R L}+m_{q^{\alpha}} C_{i j L R}\right)\left(C_{11}^{\prime}+C_{0}^{\prime}\right)-2 m_{q^{\alpha}} m_{q^{\beta}} C_{i j} C_{11}^{\prime}\right\}+\frac{e_{q} v_{e} \delta_{i j}}{2 C_{W}^{2} S_{W}^{2}} D_{\gamma Z}, \tag{22}
\end{align*}
$$

$$
\begin{align*}
\delta A_{4} & =2 E_{i j}\left\{\delta _ { i j } \left[\left(2 m_{\tilde{g}}^{2}+m_{i}^{2}+m_{j}^{2}+m_{q^{\alpha}}^{2}+m_{q^{\beta}}^{2}\right)\left(C_{11}^{\prime}+C_{0}^{\prime}\right)-B_{0}\left(m_{i}^{2}, m_{\tilde{g}}^{2}, m_{q^{\beta}}^{2}\right)\right.\right. \\
& \left.-B_{0}\left(m_{i}^{2}, m_{\tilde{g}}^{2}, m_{q^{\alpha}}^{2}\right)+\left(m_{q^{\alpha}}^{2}+m_{q^{\beta}}^{2}+0.5 s+0.5 m_{i}^{2}+0.5 m_{j}^{2}+2 m_{q^{\alpha}} m_{q^{\beta}}\right) \cdot C_{11}^{\prime}\right] \\
& \left.-2 m_{\tilde{g}}\left(m_{q^{\beta}}+m_{q^{\alpha}}\right) \cdot E_{i j}\left(C_{11}^{\prime}+C_{0}^{\prime}\right)\right\}+\frac{e_{q} v_{e} \delta_{i j}}{2 C_{W}^{2} S_{W}^{2}} D_{\gamma Z}, \tag{23}
\end{align*}
$$

$$
\begin{align*}
\delta A_{5} & =2 E_{i j}\left\{\delta _ { i j R L } \left[\left(2 m_{\tilde{g}}^{2}+m_{i}^{2}+m_{j}^{2}+m_{q^{\alpha}}^{2}+m_{q^{\beta}}^{2}\right)\left(C_{11}^{\prime}+C_{0}^{\prime}\right)-B_{0}\left(m_{i}^{2}, m_{\tilde{g}}^{2}, m_{q^{\beta}}^{2}\right)\right.\right. \\
& \left.-B_{0}\left(m_{i}^{2}, m_{\tilde{g}}^{2}, m_{q^{\alpha}}^{2}\right)+\left(m_{q^{\alpha}}^{2}+m_{q^{\beta}}^{2}+0.5 s+0.5 m_{i}^{2}+0.5 m_{j}^{2}\right) \cdot C_{11}^{\prime}\right] \\
& \left.+2 m_{\tilde{g}}\left(m_{q^{\beta}} C_{i j R L}+m_{q^{\alpha}} C_{i j L R}\right)\left(C_{11}^{\prime}+C_{0}^{\prime}\right)-2 m_{q^{\alpha}} m_{q^{\beta}} C_{i j} C_{11}^{\prime}\right\}+\frac{v_{e}^{2}+a_{e}^{2}}{16 C_{W}^{4} S_{W}^{4}} D_{Z Z} . \tag{24}
\end{align*}
$$

and

$$
\begin{aligned}
\delta_{i j R L} & =C_{q R} \mathcal{R}_{i 1}^{\tilde{q}} \mathcal{R}_{j 1}^{\tilde{q}}+C_{q L} \mathcal{R}_{i 2}^{\tilde{q}} \mathcal{R}_{j 2}^{\tilde{q}}, \\
C_{i j L R} & =C_{q L} \mathcal{R}_{i 1}^{\tilde{q}} \mathcal{R}_{j 2}^{\tilde{q}}+C_{q R} \mathcal{R}_{i 2}^{\tilde{q}} \mathcal{R}_{j 1}^{\tilde{q}}, \\
C_{i j R L} & =C_{q R} \mathcal{R}_{i 1}^{\tilde{q}} \mathcal{R}_{j 2}^{\tilde{q}}+C_{q L} \mathcal{R}_{i 2}^{\tilde{q}} \mathcal{R}_{j 1}^{\tilde{q}}, \\
E_{i j} & =\mathcal{R}_{i 1}^{\tilde{q}} \mathcal{R}_{j 2}^{\tilde{q}}+\mathcal{R}_{i 2}^{\tilde{q}} \mathcal{R}_{j 1}^{\tilde{q}}, \\
S_{i j} & =\mathcal{R}_{i 1}^{\tilde{q}} \mathcal{R}_{j 1}^{\tilde{q}}-\mathcal{R}_{i 2}^{\tilde{q}} \mathcal{R}_{j 2}^{\tilde{q}}, \\
C_{0} & =C_{0}\left(m_{i}^{2}, s, m_{j}^{2}, m_{g}^{2}, m_{i}^{2}, m_{j}^{2}\right), \\
C_{11} & =C_{11}\left(m_{i}^{2}, s, m_{j}^{2}, m_{g}^{2}, m_{i}^{2}, m_{j}^{2}\right), \\
C_{0}^{\prime} & =C_{0}^{\prime}\left(m_{i}^{2}, s, m_{j}^{2}, m_{\tilde{g}}^{2}, m_{q^{\alpha}}^{2}, m_{q^{\beta}}^{2}\right), \\
C_{11}^{\prime} & =C_{11}^{\prime}\left(m_{i}^{2}, s, m_{j}^{2}, m_{\tilde{q}}^{2}, m_{q^{\alpha}}^{2}, m_{q^{\beta}}^{2}\right) .
\end{aligned}
$$

We now turn to the numerical analysis of the $S U S Y-\mathrm{QCD}$ corrections.


Figure $4 a$


Figure $4 b$


Figure $4 c$



Figure $4 d$
Figure 4. Cross sections and their corrections of (a) $e^{+} e^{-} \rightarrow \tilde{t}_{1} \overline{\tilde{t}}_{1}$, (b) $e^{+} e^{-} \rightarrow \tilde{t}_{2} \overline{\tilde{t}}_{2}$, (c) $e^{+} e^{-} \rightarrow \tilde{b}_{1} \overline{\tilde{b}}_{1}$, and (d) $e^{+} e^{-} \rightarrow \tilde{b}_{2} \overline{\tilde{b}}_{2}$ as a functions of $\Phi$, for $\cos \theta_{t}=\cos \theta_{b}=0.5$; $\sqrt{s}=1000 \mathrm{GeV}, m_{\tilde{t}_{1}}=m_{\tilde{b}_{1}}=400 \mathrm{GeV}, m_{\tilde{t}_{2}}=m_{\tilde{g}}=600 \mathrm{GeV}, m_{\tilde{b}_{2}}=450 \mathrm{GeV}$.

In Fig. 4 we show the $\phi \equiv \varphi_{q}$ dependence of the $\sigma_{R}^{0} / \sigma_{C}^{0}, \delta \sigma_{R}^{0} / \delta \sigma_{C}^{0}$ and $\delta \sigma_{C} / \sigma_{C}^{0}$, with $R$ and $C$ indices corresponding to the cases of real and complex parameters respectively. The input parameters are: $\sqrt{s}=1000 \mathrm{GeV}, m_{\tilde{t}_{1}}=400 \mathrm{GeV}, m_{\tilde{t}_{2}}=600$ $\mathrm{GeV}, m_{\tilde{b}_{1}}=400 \mathrm{GeV}, m_{\tilde{b}_{2}}=450 \mathrm{GeV}, m_{\tilde{g}}=600 \mathrm{GeV},\left|\cos \theta_{\tilde{t}}\right|=\left|\cos \theta_{\tilde{b}}\right|=0,5$. Here we concentrate on the range of the complex phase $\varphi_{q}$ of the $A_{q}$-parameter, it must be less than of order $10^{-2}-10^{-3}$ to avoid generating electric dipole moments for the neutron, electron, and atom in conflict with observed data [11]. In the range of $\phi$ shown, $\sigma_{R}^{0} / \sigma_{C}^{0}$ varies from $100 \%$ to $99 \%$ in cases of $\tilde{t}_{1} \bar{t}_{1}$ or $\tilde{t}_{2} \bar{t}_{2}$ productions (Fig. 4a, Fig. 4b) and is about from $100 \%$ to $99.5 \%$ and $100 \%$ for $\tilde{b}_{1} \overline{\tilde{b}}_{1}$ and $\tilde{b}_{2} \overline{\tilde{b}}_{2}$ productions (Fig. 4c, Fig. 4d).

The corrections $\delta \sigma_{C}^{0} / \sigma_{C}^{0}$ are from $-28.4 \%$ to $-25 \%$, from $-38.8 \%$ to $-36.5 \%$, and from $-45.5 \%$ to $-42.5 \%$ for the $\tilde{t}_{1} \overline{\tilde{t}}_{1}, \tilde{t}_{2} \bar{t}_{2}, \tilde{b}_{1} \overline{\tilde{b}}_{1}$ and $\tilde{b}_{2} \overline{\tilde{b}}_{2}$ productions, respectively. We have also computed $\delta \sigma_{R}^{0} / \delta \sigma_{C}^{0}$. It is about from $99.5 \%$ to $96.5 \%$, from $99.5 \%$ to $93 \%$, from $100 \%$ to $99.5 \%$, and from $100 \%$ to $93 \%$, for the above - mentioned processes, respectively.

## VI Conclusions

In this paper, we have calculated the QCD radiative corrections to the production of squarks in $e^{+} e^{-}$collisions within the MSSM with complex parameters. In particular, we focus on the CP phase dependence of the production cross sections for $e^{+} e^{-} \rightarrow \tilde{q}_{i} \overline{\tilde{q}}_{j}$. From numerical results, we have found that the effects of the phases on the cross sections can be quite significant in a large region of the MSSM parameter space. This could have important implications for $\tilde{t}_{i}$ and $\tilde{b}_{i}$ searches at future colliders and the determination of the underlying MSSM parameters.

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