United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency

THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

LOW-FREQUENCY ELECTROSTATIC DUST-MODES IN A NONUNIFORM MAGNETIZED DUSTY PLASMA

S.K. Paul, S.S. Duha and A.A. Mamun[1]
Department of Physics, Jahangirnagar University, Savar, Dhaka, Banglasdesh
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.

Abstract

A self-consistent and general description of obliquely propagating low frequency electrostatic dust-modes in a inhomogeneous, magnetized dusty plasma system has been presented. A number of different situations, which correspond to different low-frequency electrostatic dust-modes, namely, dust-acoustic mode, dust-drift mode, dust-cyclotron mode, dust-lower-hybrid mode, and other associated modes (such as, accelerated and retarded dust-acoustic modes, accelerated and retarded dust-lower-hybrid modes, etc.), have also been investigated. It has been shown that the effects of obliqueness and inhomogeneities in plasma particle number densities introduce new electrostatic dust modes as well as significantly modify the dispersion properties of the other low-frequency electrostatic dust-modes. The implications of these results to some space and astrophysical dusty plasma systems, especially to planetary ring-systems and cometary tails, are briefly mentioned.

MIRAMARE – TRIESTE
July 2004

[1]Regular Associate of the Abdus Salam ICTP.

Nowadays there has been a great deal of interest in understanding different types of collective processes in dusty plasmas (plasmas with extremely massive and negatively charged dust grains), because of their vital role in the study of space and astrophysical environments, such as, asteroid zones, planetary atmospheres, interstellar media, circumstellar disks, dark molecular clouds, cometary tails, nebulae, earth's environment, etc. [1-12]. These dust grains are invariably immersed in the ambient plasma and radiative background. The interaction of these dust grains with the other plasma particles (viz. electrons and ions) is due to the charge carried by them. The dust grains are charged by a number of competing processes, depending upon the local conditions, such as, photoelectric emission stimulated by the ultraviolet radiation, collisional charging by electrons and ions, disruption and secondary emission due to the Maxwellian stress, etc. [3-12].

It has been found that the presence of static charged dust grains modifies the existing plasma wave spectra [13-20]. Bliokh and Yaroshenko [13] studied electrostatic waves in dusty plasmas and applied their results in interpreting spoke-like structures in Saturn's rings (revealed by Voyager space mission [21]). Angelis *et al.* [14] investigated the propagation of ion-acoustic waves in a dusty plasma, in which a spatial inhomogeneity is created by a distribution of immobile dust particles [22]. They [14] applied their results in interpreting the low frequency noise enhancement observed by the *Vega and Giotto* space probes in the dusty regions of Haley's comet [23].

On the other hand, it has been shown both theoretically and experimentally that the dust charge dynamics introduces different new eigen modes, such as, dust-acoustic mode [24,25], dust-ion-acoustic mode [26], lower-hybrid mode [27,28], dust-cyclotron mode [29], etc. The linear and nonlinear properties of these modes are studied by considering a uniform dusty plasma model with or without external magnetic field [30-41]. But, in practice, all dusty plasma systems contain some region of inhomogeneity capable of causing drift motions and associated drift waves [42-44]. Shukla *et al.* first considered a nonuniform dusty plasma system and have shown the existence of dust-drift mode [42]. Recently, Mamun *et al.* [45], Salimullah and Shukla [46], and Rao [47] also considered this dust-drift mode in self-gravitating [45,46] and non-ideal [47] dusty plasma systems. Mamun *et al.* [45] mainly studied gravitational instability (of ultra-low-frequency electrostatic dust-modes) due to the self-gravitational field in a nonuniform dusty plasma. Salimullah and Shukla [47] extended this work of Mamun *et al.* [46] to some other frequency limits, where the inhomogeneity in the dust particle density has been neglected. On the other hand, Rao [47] mainly studied density and temperature gradient Kelvin-Helmholtz instability in a nonideal dusty plasma where electron and ion density inhomogeneities have been neglected. Rao [47] also assumed Boltzmann distribution of electrons and ions, i.e., neglected the dynamics of electrons and ions. This assumption is a good approximation for extremely low-frequency modes (considered by Rao [47]) and for a situation where electron and ion density inhomogeneities are not important, but not for a mode like dust-lower-hybrid mode or for a situation where electron and ion density inhomogeneities are important.

The present work has considered an obliquely propagating low-frequency electrostatic dust-mode in a nonuniform, magnetized dusty plasma system, and systematically, investigated the effects of obliqueness and inhomogeneities in plasma particle number densities on different low-frequency electrostatic dust-modes, namely, dust-acoustic mode, dust-drift mode, dust-cyclotron mode, and dust-lower-hybrid mode, by accounting for the dynamics of electrons, ions, and dust grains, and the density inhomogeneities in all these plasma species. It has been shown that the effects of obliqueness and nonuniformity in plasma particle densities introduce new eigen modes as well as significantly modify the dispersion properties of the other low-frequency electrostatic dust-modes.

We consider a three-component, nonuniform magnetized dusty plasma system consisting of negatively charged (extremely massive) dust grains and electrons, and positively charged ions. This plasma system is assumed to be immersed in an external static magnetic field. The macroscopic state of this nonuniform, magnetized dusty plasma system may be described by

$$\frac{\partial N_s}{\partial t} + \nabla \cdot (N_s \mathbf{U_s}) = 0, \tag{1}$$

$$N_s\left(\frac{\partial}{\partial t} + \mathbf{U}_s \cdot \nabla\right)\mathbf{U_s} = \frac{N_s q_s}{m_s}\left[-\nabla\boldsymbol{\Phi} + \frac{1}{c}(\mathbf{U}_s \times \mathbf{B_0})\right] - \frac{\gamma_s k_B T_s}{m_s}\nabla N_s, \tag{2}$$

$$\triangledown^2\Phi = -4\pi \sum_s q_s N_s, \tag{3}$$

where $\mathrm{m_s}$, $\mathrm{q_s}$, and $\mathrm{N_s}$ are, respectively, mass, charge, and number density of the species s (dust grains, ions, and electrons); $\mathbf{U}_s$ is the hydrodynamic velocity; $\mathrm{k_B T_s}$ is the thermal energy; γ_s is the adiabatic constant; Φ is the electrostatic wave potential; c is the speed of light in vacuum. We are interested in looking at different low-frequency electrostatic modes (ω, $\mathbf{k}$) propagating obliquely with an external magnetic field $\mathbf{B}_0$ immeresed in such a dusty plasma system. We assume that $\mathbf{B}_0$ is along the z-axis, (i.e., $\mathbf{B}_0 \parallel \hat{\mathbf{z}}$ and propagation vector $\mathbf{k}$ lies in y-z plane, i.e., $\mathbf{k} = \hat{\mathbf{y}}k_y + \hat{\mathbf{z}}k_z$). We also assume that equilibrium plasma density gradient and equilibrium electrostatic potential gradient are along the x-axis. To study obliquely propagating electrostatic modes in such a nonuniform, magnetized, dusty plasma system, we shall carry out a normal mode analysis. We first express our dependent variables N_s, U_s, and Φ in terms of their equilibrium and perturbed parts as

$$\left.\begin{aligned} N_s &= n_{s0}(x) + n_s, \\ \mathbf{U}_s &= \mathbf{u}_{s0} + \mathbf{u}_s, \\ \Phi &= \phi_0(x) + \phi. \end{aligned}\right\} \tag{4}$$

$\mathrm{n_{s0}(x)}$ and $\phi_0(x)$ are the equilibrium density and electrostatic potential, which are not constant, but vary with x. $\mathbf{u}_{s0}$ is the drift velocity of the species s which can be obtained from the zeroth-order solution of Eq. (2) as

$$\left.\begin{aligned} \mathbf{u}_{s0} &= \hat{\mathbf{y}}(u_{Ds}+u_E), \\ u_{Ds} &= \tfrac{k_{Ls}v_{ts}^2}{\omega_{cs}}, \\ k_{Ls} &= \tfrac{1}{n_{s0}}\tfrac{dn_{s0}}{dx}, \\ u_E &= \tfrac{c}{B_0}\tfrac{d\phi_0}{dx}, \end{aligned}\right\} \tag{5}$$

where $\omega_{\mathrm{cs}} = \mathrm{q_s B_0/m_s c}$ and $v_{ts} = (\gamma_{\mathrm{s}}\mathrm{k_B T_s/m_s})^{1/2}$. It should be noted here that the transverse shear in the fluid flow velocity component parallel to the ambient magnetic field has been neglected. Thus, $\mathbf{u}_{\mathrm{s0}}$ contains two contributions: the electric field drift due to the equilibrium electric field and the diamagnetic drift due to the equilibrium pressure of the specis s. k_{Ls} represents the inverse of the density inhomogeneity scale-length of the specis s. It may be noted that if the electric field at equilibrium $(-\nabla\phi_0)$ is assumed to be constant, from Eq. (3) one can obtain $Z_i\mathrm{n_{i0}(x) = Z_d n_{d0}(x) + n_{e0}(x)}$, where $\mathrm{n_{i0}}$ $(\mathrm{n_{d0}})$ is the equilibrium ion (dust) number density, $\mathrm{Z_d}$ $(\mathrm{Z_i})$ is the number of electrons (protons) residing on the dust grain (ion) and $\mathrm{n_{e0}(x)}$ is the equilibrium electron number density.

Now, using Eqs. (4) ans (5) we first linearize our basic equations, Eqs. (1) - (3), to a first order approximation and then solve them in order to obtain n_s as

$$n_s = -\left(\frac{k^2\chi_s}{4\pi q_s}\right)\phi, \tag{6}$$

where χ_{s} is the susceptibility of the species s and is given by

$$\chi_s = -\frac{1}{k^2}\sum_s \frac{\omega_{ps}^2}{\Omega_s^2}\left(k_z^2 + (\frac{\omega_{\star s}^2}{\omega_{\star s}^2-\omega_{cs}^2})k_y^2 + (\frac{\omega_{\star s}\omega_{cs}}{\omega_{\star s}^2-\omega_{cs}^2})k_y k_{Ls}\right), \tag{7}$$

$$\Omega_s^2 = \omega_{\star s}^2 - k_z^2 v_{ts}^2 - (\frac{\omega_{\star s}^2}{\omega_{\star s}^2-\omega_{cs}^2})k_y^2 v_{ts}^2 - 2(\frac{\omega_{\star s}\omega_{cs}}{\omega_{\star s}^2-\omega_{cs}^2})k_y k_{Ls} v_{ts}^2 - (\frac{\omega_{\star s}^2}{\omega_{\star s}^2-\omega_{cs}^2})k_{Ls}^2 v_{ts}^2, \tag{8}$$

and $\omega_{\star\mathrm{s}} = \omega - \mathrm{k_y u_{s0}}$. The substitution of Eq. (6) into the linearized poisson's equation yields $\varepsilon\phi = 0$, where $\varepsilon = 1 + \chi_{\mathrm{e}} + \chi_{\mathrm{i}} + \chi_{\mathrm{d}}$ is the dielectric constant for the nonuniform, magnetized dusty plasma system. This equation $\varepsilon\phi = 0$ is compatible with non-zero ϕ only if $\varepsilon = 0$, i.e., ω and k are related by

$$1 + \chi_e + \chi_i + \chi_d = 0. \tag{9}$$

It is obvious from Eqs. (7) and (8) that if we assume $k_z v_{te,i} >> \omega$; $\omega << \omega_{ce,i}$; $k_y v_{te,i} << \omega_{ce,i}$; $k_{Le,i} \to 0$; $u_E \to 0$, then $\chi_{e,i} = 1/k^2\lambda_{De,i}^2$, where $\lambda_{De,i} = [\gamma_{e,i}k_B T_{e,i}/(4\pi n_{e0,i0}q_{e,i}^2)]^{1/2}$. These susceptibilities with $\gamma_{e,i} = 1$, $\chi_{e,i} = 1/k^2\lambda_{De,i}^2$, are for Maxwellian distribution of electrons and ions [47]. This means that for extremely low frequency mode, uniform electron and ion distributions, and thermal velocity of electrons and ions being almost along the external magnetic field, the assumption of Maxwellian distribution of electrons and ions is a good aproximation, but not for any situation where any of these three conditions are not valid. Thus, Eq. (9) [with Eqs. (7) and (8)] represents a general dispersion relation for any obliquely propagating electrostatic mode where the effects of inhomogeneities in electron, ion, and dust particle number densities, obliqueness, external magnetic field, and dust fluid temperature are included. However, our present interest is to study low-frequency electrostatic dust-modes in different low frequency ranges of interest, namely, $\omega << \omega_{\rm cd}$, $\omega \sim \omega_{\rm cd}$, and $\omega_{\rm cd} << \omega << \omega_{\rm ci}$.

A. $\omega << \omega_{cd}$:

To study electrostatic dust-mode in this frequency range ($\omega << \omega_{cd}$), for simplicity, we assume that the dust fluid is cold ($v_{td} \to 0$), but electron (ion) thermal velocity along the external magnetic field is large enough to satisfy the approximations $k_z v_{te,i} >> \omega, k_y u_{e0,i0}$ and $\omega_{ce,i} >> k_y v_{te,i}$. These approximations reduce the general dispersion relation, Eq. (9), to a simple form:

$$\omega^2 - k_y V_D \omega - k_z^2 C_{sd}^2 = 0, \tag{10}$$

where

$$C_{sd} = \left[\frac{1}{\alpha}\left(\frac{Z_d n_{d0}}{Z_i n_{i0}}\right)\frac{Z_d \gamma_i k_B T_i}{Z_i m_d}\right]^{1/2},$$

$$V_D = \frac{1}{\alpha}\frac{\gamma_i k_B T_i}{m_i \omega_{ci}}\frac{Z_d}{Z_i n_{i0}}\left(\frac{dn_{d0}}{dx}\right), \tag{11}$$

$$\alpha = 1 + k^2\lambda_{Di}^2 + \frac{n_{e0}}{Z_i n_{i0}}\frac{\gamma_i T_i}{Z_i \gamma_e T_e}.$$

Eq. (10) represents a dispersion relation for the dust-acoustic mode modified by the obliqueness and inhomogeneity or dust-drift mode modified by the obliqueness. If we consider parallel propagation or homogeneous dusty plasma system and put $Z_i = 1$ and $\gamma_{i,e} = 1$, the dispersion relation reduces to that for the dust-acoustic mode studied by Rao *et al.* [24] and if we further put $n_{e0} = 0$ (all electrons are depleted to the surface of the dust grains), this dispersion relation stands for the dust-acoustic waves studied by Mamun *et al.* [32,33] and others [34-36]. On the other hand, if we consider perpendicular propagation ($k_z = 0$), this dispersion relation represents the dust-drift mode studied by Shukla *et al.* [42]. One can express the solution of this equation as

$$\omega = \frac{1}{2}\left[k_y V_D \pm \sqrt{k_y^2 V_D^2 + 4k_z^2 C_{sd}^2}\right]. \tag{12}$$

It is obvious from this dispersion relation that for small k_z (i.e., for $\mathrm{k_y V_D >> k_z C_{sd}}$) there are two modes, namely, $\omega = k_y V_D$ and $\omega = -k_z^2 C_{sd}^2 / k_y V_D$. The phase velocity of the upper mode is V_D, which is the diamagnetic ion-drift speed for $\mathrm{n_{e0}} = 0$. It is also shown that for large k_z, the drift waves appear as an accelerated (retarded) dust-acoustic mode for $+(-)$ sign. These accelerated and retarded dust-acoustic modes are analogous to accelerated and retarded ion-acoustic modes that exist in nonuniform electron-ion plasma [48].

B. $\omega \sim \omega_{cd}$:

We now study low-frequency electrostatic dust mode whose frequency is comparable to the dust-cyclotron frequency. We, again, for simplicity, assume that the dust fluid is cold ($v_{td} \to 0$), but electron (ion) thermal velocity along the external magnetic feild is large enough to satisfy the approximations $k_z v_{te,i} >> \omega, k_y u_{e0,i0}$ and $\omega_{ce,i} >> k_y v_{te,i}$. These approximations allow us to write the general dispersion relation, Eq. (9), as

$$\omega^3 - (\omega_{cd}^2 + k_y^2 C_{sd}^2)\omega + \omega_{cd}^2 k_y V_D = 0, \tag{13}$$

where C_{sd} and V_D are given by Eqs. (11). There are three solutions of this cubic equation, i.e., there exists three modes, namely,

$$\omega = \omega_1 + \omega_2, \tag{14}$$

$$\omega = -\frac{1}{2}(\omega_1 + \omega_2) + i\frac{\sqrt{3}}{2}(\omega_1 - \omega_2), \tag{15}$$

$$\omega = -\frac{1}{2}(\omega_1 + \omega_2) - i\frac{\sqrt{3}}{2}(\omega_1 - \omega_2), \tag{16}$$

with

$$\omega_{1,2} = \left[-\frac{1}{2}\omega_{cd}^2 k_y V_D \pm \sqrt{\frac{1}{4}\omega_{cd}^4 k_y^2 V_D^2 - \frac{1}{27}(\omega_{cd}^2 + k_y^2 C_{sd}^2)^3} \right]^{1/3}, \tag{17}$$

where + ($-$) corresponds the first (second) quantity involved. These are three different low-frequency electrostatic modes with frequency comparable to the dust-cyclotron frequency or dust-drift wave frequency. The last two also represent the dispersion relation for the dust-cyclotron mode modified by the inhomogeneity. If we consider homogeneous dusty plasma system, i.e., put $\mathrm{V_D} = 0$, the dispersion relation reduces to the dust cyclotron mode studied by Shukla and Rahman [29]. It should be noted that if we consider $\omega, \mathrm{k_y C_{sd}} << \omega_{\mathrm{cd}}$, Eq. (13) stands for dust drift mode studied in our earlier case.

C. $\omega_{cd} < \omega < \omega_{ci}$:

We now study low-frequency electrostatic dust-mode, whose frequency is in between dust and ion-cyclotron frequencies (i.e., $\omega_{cd} < \omega < \omega_{ci}$), in a cold dusty plasma (i.e., $v_{ts} \to 0$). These assumptions ($\omega_{cd} << \omega << \omega_{ci}$ and $v_{ts} \to 0$) allow us to write Eq. (9) as

$$1+\frac{\omega_{pi}^2}{\omega_{ci}^2}\left(1+\frac{m_e}{m_i}\frac{n_{e0}}{n_{i0}}\right)\frac{k_y^2}{k^2}+\frac{k_y}{k^2\omega'}\left(\frac{\omega_{pe}^2}{\omega_{ce}}k_{Le}+\frac{\omega_{pi}^2}{\omega_{ci}}k_{Li}-\frac{\omega_{pd}^2}{\omega_{cd}}\frac{\omega_{cd}^2}{\omega'^2}k_{Ld}\right)-\delta\frac{\omega_{pd}^2}{\omega'^2}=0, \tag{18}$$

where $\delta = 1+(\omega_{pe}^2/\omega_{pd}^2+\omega_{pi}^2/\omega_{pd}^2)k_z^2/k^2$ and $\omega' = \omega - k_y u_E$. It may be pointed out here that if we put $k_{Ld} = 0$ and $u_E = 0$, this equation (the right hand side of it is ε) becomes compatible with Eq. (31) of Ref. 46 where the inhomogeneity in the dust fluid density is neglected (i.e. where n_{d0}=constant). If we consider n_{d0}=constant, i.e., use $dn_{d0}/dx = 0$ or $k_{Ld} = 0$ and the quasineutrality condition (i.e., use $\mathrm{Z_i(dn_{i0}/dx) = Z_d(dn_{d0}/dx) + dn_{e0}/dx}$), we can express $(\omega_{\mathrm{pe}}^2/\omega_{\mathrm{ce}})\mathrm{k_{Le}} + (\omega_{\mathrm{pi}}^2/\omega_{\mathrm{ci}})\mathrm{k_{Li}} = 0$. This means that for homogeneous dust fluid (n_{d0}= constant) and for $\mathrm{Z_i(dn_{i0}/dx) = Z_d(dn_{d0}/dx) + dn_{e0}/dx}$ being valid [46], the combined effects of electron and ion density inhomogeneities on this dispersion relation is zero. However, if the inhomogeneity in the dust fluid is included, we can express by using the qausineutrality condition that $(\omega_{\mathrm{pe}}^2/\omega_{\mathrm{ce}})\mathrm{k_{Le}} + (\omega_{\mathrm{pi}}^2/\omega_{\mathrm{ci}})\mathrm{k_{Li}} = -(\omega_{\mathrm{pd}}^2/\omega_{\mathrm{cd}})\mathrm{k_{Ld}}$. It is obvious from Eq. (18) that to study the dust-lower-hybrid mode one can keep k_z up to a value such that k_z^2/k^2 is the order of $\omega_{pd}^2/(\omega_{pe}^2+\omega_{pi}^2)$. However, for most of the dusty plasma situations $\omega_{pd}^2/(\omega_{pe}^2+\omega_{pi}^2) << 1$. Thus, using the appropriate approximations for dust-lower-hybrid mode, $\omega_{\mathrm{cd}}^2/\omega^2 << 1$, $\omega >> k_y u_E$, and $k_z/k << 1$, Eq. (18) can be expreesed as

$$1+\frac{\omega_{pi}^2}{\omega_{ci}^2}\left(1+\frac{m_e}{m_i}\frac{n_{e0}}{n_{i0}}\right)-\frac{\omega_{pd}^2}{\omega\omega_{cd}}\frac{k_{Ld}}{k_y}-\frac{\omega_{pd}^2}{\omega^2}=0. \tag{19}$$

This is the dispersion relation for the dust-lower-hybrid mode modified by the inhomogeneities in electron, ion and dust fluid densities. To derive a simple form of the dispersion relation for the dust-lower-hybrid mode, we assume that all electrons are absorbed by the dust-grains [26,32] (i.e., $\mathrm{n_{e0}} = 0$). Thus, the simplified form of the dispersion relation for the dust-lower-hybrid mode becomes

$$\omega^2+\omega_{ci}\left(\frac{k_L}{\beta k_y}\right)\omega+\frac{1}{\beta}\omega_{cd}\omega_{ci}=0, \tag{20}$$

where $\beta = 1+\omega_{ci}^2/\omega_{pi}^2$. To derive Eq. (20) we have used $k_{Ld} = k_{Li} = k_L$ and $\omega_{\mathrm{pi}}^2/\omega_{\mathrm{ci}} = -\omega_{\mathrm{pd}}^2/\omega_{\mathrm{cd}}$, since $\mathrm{Z_d n_{d0} = Z_i n_{i0}}$. The solution of this quadratic equation for ω is given by

$$\omega=\frac{1}{2}\left[-\omega_{ci}\left(\frac{k_L}{\beta k_y}\right)\pm\sqrt{\omega_{ci}^2\left(\frac{k_L}{\beta k_y}\right)^2+\frac{4}{\beta}|\omega_{cd}|\omega_{ci}}\right], \tag{21}$$

where $|\omega_{cd}|$ is the absolute value of ω_{cd}. It is obvious that for $4\beta\omega_{\mathrm{cd}}/|\omega_{\mathrm{ci}}| << (\mathrm{k_L^2/k_y^2})$ the two roots of Eq. (21) become $\omega_+ \simeq |\omega_{\mathrm{cd}}|(\mathrm{k_y/k_L})$ and $\omega_- \simeq -\omega_{\mathrm{ci}}(\mathrm{k_L/k_y})[1/\beta + (|\omega_{\mathrm{cd}}|/\omega_{\mathrm{i}})(\mathrm{k_L^2/k_y^2})]$. These indicate that the density inhomogeneity introduces a new mode: $\omega_+ \simeq |\omega_{\mathrm{cd}}|(\mathrm{k_y/k_L})$ (when the density gradient is positive) or $\omega_- \simeq \omega_{\mathrm{ci}}(|\mathrm{k_L}|/\mathrm{k_y})[1/\beta + (|\omega_{\mathrm{cd}}|/\omega_{\mathrm{i}})(\mathrm{k_L^2/k_y^2})]$ (when the density gradient is negative). On the other hand when $4\beta(|\omega_{\mathrm{cd}}|/\omega_{\mathrm{ci}}) >> (\mathrm{k_L^2/k_y^2})$, Eq. (21) provides a retarded dust-lower-hybrid mode for positive density gradient and accelerated dust-lower-hybrid mode for negative density gradient.

It should be mentioned that this analysis has neglected the variation of the dust grain charge which arises the damping of the dust-acoustic mode [31,35]. It has been shown by Rosenberg and Krall [44] that such damping effects may need to be considered when the wave frequency (ω) is of the order of the charging frequency (ν_{ch}) of the dust grain. Thus, our analysis is correct for the low-frequency waves where the charging frequency is much smaller than the frequency of the waves involved, i.e., $\omega >> \nu_{ch}$ which has been estimated to be valid for space plasma situations [44].

It may be stressed here that the present investigation may be useful for understanding the low-frequency electrostatic noise enhancement observed by the *Vega* and *Giotto* space probes [23] in the dust regions of Haley's comet and spoke-like structures revealed by the Voyager space mission [21] in Saturn's rings.

It may be added that the nonlinear propagation and stability analysis of these modes are also problems of great importance, but beyond the scope of the present work.

Acknowledgments. This work was done within the framework of the Associateship Scheme of the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy. The financial support from the Swedish International Development Cooperation Agency (SIDA) is acknowledged. The useful discussion with Prof. M. H. A. Hassan is also gratefully acknowledged.

References

[1] M. Horanyi and D. A. Mendis, Astrophys. J. **294**, 357 (1985).

[2] M. Horanyi and D. A. Mendis, Astrophys. J. **307**, 800 (1986).

[3] C. K. Goertz, Rev. Geophys. **27**, 271 (1989).

[4] D. A. Mendis and M. Rosenberg, Annu. Rev. Astron. Astrophys. **32**, 419 (1994).

[5] F. Verheest, Space Sci. Rev. **77**, 267 (1996).

[6] F. Verheest, *Waves in Dusty Space Plasmas* (Kluwer Academic Publishers, Dordrecht, 2000).

[7] P. K. Shukla and A. A. Mamun, *Introduction to Dusty Plasma Physics* (Institute of Physics Publishing Ltd., Bristol, 2002).

[8] B. Feuerbacher, R. T. Willis, and B. Fitton, Astrophys. J. **181**, 101 (1973).

[9] H. Fechting, E. Grün, and G. E. Morfill, Planet. Space Sci. **27**, 511 (1979).

[10] O. Havnes, C. K. Goertz, G. E. Morfill, E. Grün, and W. Ip, J. geophys. Res. **92**, 2281 (1987).

[11] M. S. Barnes, J. H. Keller, J. C. Forster, J. A. O'Neil, and D. K. Coultas, Phys. Rev. Lett. **68**, 313 (1992).

[12] B. Walch, M. Horanyi, and S. Robertson, Phys. Rev. Lett. **75**, 838 (1995).

[13] P. V. Bliokh and V. V. Yaroshenko, Sov. Astron. (Engl. Transl.) **29**, 330 (1985).

[14] U. de Angelis, V. Formisano, and M. Giordano, J. Plasma Phys. **40**, 399 (1988).

[15] U. de Angelis, R. Bingham, and V. N. Tsytovich, J. Plasma Phys. **42**, 445 (1989).

[16] N. D'Angelo, Planet. Space Sci. **38**, 9 (1990).

[17] R. Bingham, U. de Angelis, V. N. Tsytovich, and O. Havnes, Phys. Fluids B **3**, 811 (1991).

[18] P. K. Shukla and L. Stenflo, Astrophys. Space Sci. **190**, 23 (1992).

[19] U. de Angelis, A. Forlani, R. Bingham, P. K. Shukla, A. Ponomarev, and V. N. Tsytovich, Phys. Plasmas **1**, 236 (1994).

[20] P. K. Shukla and S. V. Vladimirov, Phys. Plasmas **2**, 3179 (1995).

[21] B. A. Smith *et al.*, Science **215**, 504 (1982).

[22] E. C. Whipple, T. G. Northrop, and D. A. Mendis, J. geophys. Res. **90**, 7405 (1985).

[23] R. Grard *et al.*, Nature **321**, 290 (1986).

[24] N. N. Rao, P. K. Shukla, and M. Y. Yu, Planet. Space Sci. **38**, 543 (1990).

[25] A. Barkan, R. L. Merlino, and N. D'Angelo, Phys. Plasmas **2**, 3563 (1995).

[26] P. K. Shukla and V. P. Silin, Phys. Scripta **45**, 508 (1992).

[27] P. K. Shukla, Phys. Scripta **45**, 504 (1992).

[28] M. Salimullah, Phys. Lett. A **215**, 296 (1996).

[29] P. K. Shukla and H. U. Rahman, Planet. Space Sci. **46**, 541 (1998).

[30] M. Rosenberg, Planet. Space Sci. **41**, 229 (1993).

[31] F. Melandsø, T. K Aslaksen, and O. Havnes, Planet. Space Sci. **41**, 321 (1993).

[32] A. A. Mamun, R. A. Cairns, and P. K. Shukla, Phys. Plasmas **3**, 702 (1996).

[33] A. A. Mamun, R. A. Cairns, and P. K. Shukla, Phys. Plasmas **3**, 2610 (1996).

[34] S. G. Tagare, Phys. Plasmas **4**, 3167 (1997).

[35] J. X. Ma and J. Liu, Phys. Plasmas **4**, 253 (1997).

[36] B. Xie, K. He, and Z. Hung, Phys. lett. **A247**, 403 (1998).

[37] N. Y. Kotsarenko, S. V. Koshevaya, G. A. Stewart, and D. Maravilla, Planet. Space Sci. **46**, 429 (1998).

[38] M. Salimullah and G. E. Morfill, Phys. Rev. E **59**, 2558 (1999).

[39] R. Roychoudhury and P. Chatterjee, Phys. Plasmas. **6**, 406 (1999).

[40] D. Winske, M. S. Murillo, and M. Rosenberg, Phys. Rev. E **59**, 2263 (1999).

[41] B. Xie, K. He, and Z. Hung, Phys. Lett. **A253**, 83 (1999).

[42] P. K. Shukla, M. Y. Yu, and R. Bharuthram, J. geophys. Res. **96**, 21343 (1991).

[43] P. K. Shukla and K. Varma, Phys. Fluids B **5**, 236 (1993).

[44] M. Rosenberg and N. A. Krall, Planet. Space Sci. **43**, 619 (1995).

[45] A. A. Mamun, M. Salahuddin, and M. Salimullah, Planet. Space Sci. **47**, 79 (1999).

[46] M. Salimullah and P. K. Shukla, Phys. Plasmas **6**, 686 (1999).

[47] N. N. Rao, Phys. Plasmas. **6**, 2349 (1999).

[48] N. A. Krall and A. W. Trivelpiece, Principles of Plasma Physics (McGraw-Hill, New York, 1973) p. 209.