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ADAPTIVE SYNCHRONIZATION IN CHAOTIC LASER DIODES SUBJECT TO OPTICAL FEEDBACK(S)

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Abstract

In this paper a proposal is made of an adaptive coupling function for achieving synchronization between two lasers subject to optical feedback. Such a control scheme requires knowledge of the systems' parameters. For the first time we demonstate that when these parameters are not available on-line parameter estimation can be applied. Generalization of the approach to the multi-feedback systems is also presented.

1. INTRODUCTION

The seminal papers by Pecora and Carroll [1] and Ott, C.Grebogi and J.A.Yorke [2] on chaos synchronization have stimulated a wide range of research activity: a recent comprehensive review of such work is found in the focussed issues on chaos control [3] and references therein. Application of chaos control theory can be found in secure communications, optimization of nonlinear system performance and modeling brain activity and pattern recognition phenomena [3]. A particular focus of the work being the development of secure optical communications systems based on control and synchronization of laser chaos [4-7]. It has been shown [8] that security cannot be guaranteed in a communications format using simple chaotic systems - i.e. those with a single positive Lyapunov exponent. It is thus appreciated that to obtain reliable communications systems attention should be directed at hyperchaotic systems - i.e. those with two or more positive Lyapunov exponents. It has been claimed previously that the number of driving variables needed for synchronization in case of hyperchaotic systems should be equal to the number of positive Lyapunov exponents [3]. However, such a requirement is highly undesirable in communication applications, as most communication schemes use just one signal for transmission [3]. More recently it was argued [9-10], that hyperchaos control is possible using fewer driving variables than the number of positive Lyapunov exponents [9], and indeed even with zero- driving variables using the method of parameter change advocated in [10]. Moreover, it has recently been shown that hyperchaos control is possible with a single variable even in the case of time delay systems, when the number of positive Lyapunov exponents, in principle, can be infinite [11]. This result is of particular importance for the use of external cavity laser diodes for chaotic optical communications [7]. In addition to applications in communications the implications of the study of synchronization phenomenon in time-delayed systems can be considered as a special case of spatio-temporal chaos control. Time-delay systems are infinite-dimensional and more interestingly by changing the time-delay one can obtain different numbers of the positive Lyapunov exponents [11].

In [11] and [12] use is made of both uni-directional and bi-directional couplings between the master and slave time-delay systems. Then an estimate is made of the coupling strength, for the given coupling function, needed for the synchronization between the drive and response time-delay systems. Usually two dynamical systems are termed synchronized if the difference between their states converges to zero for $t \to \infty$ [1-2]. Recently [13-14], a generalization of this concept was proposed, where two systems are termed as being synchronized if a functional relation exists between the states of both systems.

In this paper we propose a general adaptive coupling (linking) function needed for synchronization between two time-delay systems. The approach does not require the imposition of threshold restrictions on the coupling strength.

Laser systems with optical feedback are prominent representatives of time-delay systems [6-7,

15-16]. We thus also apply the proposed approach to the case of synchronization between two lasers subject to optical feedback(s). Our results show that one can use a time delay coupling function to accomplish synchronization between the laser systems. This synchronization method is different from that of [15, 17]. We argue that such a diversity allows for more flexibility in practical control problems.

Usually, in the context of nonlinear dynamical systems, the method of adaptive control applies a feedback loop in order to drive the system parameter (or parameters) to the values required so as to achieve a target state. This is achieved by adding the evolution of the parameter(s) to the evolution dynamics of the dynamical systems [18-20]. Such a scheme is adaptive, because the parameters which determine the nature of the dynamics self-adjust or adapt themselves to yield the desired dynamics.

In this paper we use the term 'adaptive' in a slightly different sense. The proposed method of chaos synchronization between two chaotic systems can also be interpreted as follows: we apply a control law to the process model to reach the reference model (desired or target state), which in principle can be entirely different from the process model not only due to parameter(s) mismatches, but also by structure and/or dynamics; in other words, the task is to design a control force and apply it to the process model to reach an entirely different targeted state. Such a scheme is also adaptive, as in the above procedure the linkage function depending on the nature of the systems' dynamics, and structure adapts itself to yield the desired dynamics [21]. By definition, the adaptive principle is remarkably robust and efficient in generic nonlinear systems and may therefore be of immense utility in a large variety of practical control problems.

2. ADAPTIVE COUPLING FOR CHAOS SYNCHRONIZATION

Following [11-12,21] we write the time-delay system under consideration in the form:

$$
\begin{aligned}
\frac{dx}{dt} &= f(x, x_\tau), \\
\frac{dy}{dt} &= g(y, y_\tau) + W(x, y),
\end{aligned}
\tag{1}
$$

where f and g are arbitrary time delay functions such that the corresponding dynamics exhibit chaotic behavior; $x_\tau := x(t - \tau)$, $y_\tau := y(t - \tau)$; where τ is the time-delay; the term $W(x, y)$ in equation (1) is responsible for coupling (linkage) between the master (driving) (first equation in (1)) and slave (response) (second equation in (1)) systems. The relaxation terms proportional to x and y, usually written separately in the right-hand sides of the system (1), are incorporated into the f and g. In addition, one must keep in mind that in general the dynamical variables and f, g and W can be high dimensional vectors.

Let us choose the adaptive coupling function (or control input) $W(x, y)$ in the system (1) as:

$$
W(x, y) = f(x, x_\tau) - g(y, y_\tau) - Q(x, y),
\tag{2}
$$

4

with $Q(x, y)$ such that error dynamics of $x - y = e$ will be stable. For $Q(x, y) = B(x - y) = Be$ with negative B we obtain:

$$\frac{de}{dt} = Be, \tag{3}$$

whose solution decays exponentially. Generalization to the case of different delay functions with different time delays τ_1 and τ_2 is straightforward. In this case one can choose the coupling function

$$W(x, y) = f(x, x_{\tau_1}) - g(y, y_{\tau_2}) - Q(x, y). \tag{4}$$

The idea behind the way of choosing the adaptive coupling (or control input) to achieve synchronization is to cancel the nonlinear terms of the system. We want to linearize the system to make it more tractable and to use linear control theory. Sometimes this approach is called feedback linearization, for more details and pitfalls, see e.g. [22] and references therein.

As indicated above many laser communication systems are prominent representatives of time-delay systems. In this work we apply the proposed approach to the case of synchronization between two semiconductor lasers subject to optical feedback. There can be different types of couplings between the slave and master systems. For example in [15] the light that is injected into the slave system is included in the equations in a way similar to the light coming from the external resonator. This approach is widely used to describe the effects of coherent light injection into semiconductor lasers. In this paper we propose a new type of coupling between master and slave systems to achieve synchronization between these systems. A general form for synchronization condition is obtained from a consideration of the following systems of the Lang-Kobayashi equations [15,17] for the real electric field amplitude E(t), slowly varying phase $\Phi(t)$ and the carrier number n(t) for the: master (with subscript M),

$$\begin{aligned}
\frac{dE_M}{dt} &= \frac{1}{2}Gn_M E_M + k_M E_M(t - \tau) \cos(\omega_0 \tau + \Phi_M(t) - \Phi_M(t - \tau)), \\
\frac{d\Phi_M}{dt} &= \frac{1}{2}\alpha Gn_M - k_M \frac{E_M(t - \tau)}{E_M(t)} \sin(\omega_0 \tau + \Phi_M(t) - \Phi_M(t - \tau)), \\
\frac{dn_M}{dt} &= (p - 1)J_{th} - \gamma n_M(t) - (\Gamma + Gn_M)E_M^2,
\end{aligned} \tag{5}$$

and slave lasers (with subscript S),

$$\begin{aligned}
\frac{dE_S}{dt} &= \frac{1}{2}Gn_S E_S + k_S E_S(t - \tau) \cos(\omega_0 \tau + \Phi_S(t) - \Phi_S(t - \tau)) + W, \\
\frac{d\Phi_S}{dt} &= \frac{1}{2}\alpha Gn_S - k_S \frac{E_S(t - \tau)}{E_S(t)} \sin(\omega_0 \tau + \Phi_S(t) - \Phi_S(t - \tau)), \\
\frac{dn_S}{dt} &= (p - 1)J_{th} - \gamma n_S(t) - (\Gamma + Gn_S)E_S^2,
\end{aligned} \tag{6}$$

coupled by the linkage function

$$\begin{aligned}
W &= K_W(E_M - E_S) + \frac{1}{2}G(n_M E_M - n_S E_S) + k_M E_M(t - \tau) \cos(\omega_0 \tau + \Phi_M(t) \\
&\quad - \Phi_M(t - \tau)) - k_S E_S(t - \tau) \cos(\omega_0 \tau + \Phi_S(t) - \Phi_S(t - \tau)),
\end{aligned} \tag{7}$$

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where G is the differential optical gain; τ is the master laser's external cavity round-trip time; α-the linewidth enhancement factor; γ- the carrier density rate; Γ-the cavity decay rate; p- the pump current relative to the threshold value J_{th} of the solitary laser; ω_0 is the angular frequency of the solitary laser; k is the feedback rate; K_W is the coefficient determining the speed of achieving synchronization between the master and slave lasers.

One can easily see that for the type of coupling with positive K_W that the difference signal $e_E = E_M - E_S$ approaches zero, as the error dynamics in this case obey the following equation:

$$\frac{de_E}{dt} = -K_W e_E. \tag{8}$$

(Throughout this paper we introduce the relaxation or damping term to overcome the necessity for identical initial conditions in the coupled master and slave laser systems.)

In the above scheme of synchronization the master and slave systems' parameter, namely the gain was the same for both systems. Generalization of the coupling function to the case of laser systems with different parameters is straightforward; for example, with different gain parameters the coupling function is:

$$\begin{aligned} W &= K_W(E_M - E_S) - \frac{1}{2}(G_M n_M E_M - G_S n_S E_S) \\ &\quad - k_M E_M(t-\tau)\cos(\omega_0\tau + \Phi_M(t) - \Phi_M(t-\tau)) \\ &\quad + k_S E_S(t-\tau)\cos(\omega_0\tau + \Phi_S(t) - \Phi_S(t-\tau)). \end{aligned} \tag{9}$$

As was pointed out in [23], in many representative cases, chaos synchronization can be understood from the existence of a global Lyapunov function of the difference signals. In other words, the global asymptotic stability can be investigated by the Lyapunov function approach [22]. For error dynamics e_E (8), one can use the Lyapunov function

$$L = e_E^2. \tag{10}$$

As

$$\frac{dL}{dt} = -K_W e_E^2, \tag{11}$$

can be made strictly negative for positive K_W (except for $e_E = 0$) we conclude that the asymptotic stability is global.

Thus based on our recent results we have several possibilities for achieving synchronization between chaotic laser diodes: according to [17] if the coupling between master and slave systems is of the form

$$W = \sigma E_M(t-\tau_c)\cos(\omega_0\tau_c + \Phi_S(t) - \Phi_M(t-\tau_c)), \tag{12}$$

(where σ is the coupling strength between the master and slave lasers; τ_c is the light propagation time from the right facet of the master laser to the right facet of the slave laser) then the synchronization condition is

$$k_M = k_S + \sigma. \tag{13}$$

6

In this paper we have proposed another type of linkage function (7) for synchronization purposes without strict condition on the systems' parameters.

Multi-feedback and multi-delay systems are ubiquitous in nature and technology. Prominent examples can be found in biological and biomedical systems, laser physics, integrated communications [24]. In laser physics such a situation arises in lasers subject to two or more optical or electro-optical feedback. Second optical feedback could be useful to stabilize laser intensity [25]. Chaotic behaviour of laser systems with two optical feedback mechanism is studied in recent works [26]. To the best of our knowledge chaos synchronization between the multi-feedback systems is yet to be investigated. Having in mind enormous application implications of chaos synchronization e.g. in secure communication, investigation of synchronization in multi-feedback systems is of immense importance. It is well known that laser arrays hold a great promise for space communication applications, which require compact sources with high optical intensities. The most efficient result can be achieved when the array elements are synchronized [27].

In this paper we only briefly consider the case of adaptive synchronization in semiconductor lasers with double feedback. (More detailed results on synchronization regimes in the chaotic nonlinear systems will be presented elsewhere.) In the case of double feedback in the semiconductor lasers to the right-hand sides of the first equations (5) and (6) one has to add the terms $k_{M1}E_M(t-\tau_1)\cos(\omega_0\tau_1 + \Phi_M(t) - \Phi_M(t-\tau_1))$ and $k_{S1}E_S(t-\tau_1)\cos(\omega_0\tau_1 + \Phi_S(t) - \Phi_S(t-\tau_1))$. (Of course corresponding terms should be added to the phase equations in (5) and (6).) Here $k_{M1,S1}$ are the feedback rate from the second mirrors in the master and slave lasers, respectively; τ_1 is round trip time in the lasers' second external cavity. With this the linkage function to achieve adaptive synchronization between systems (5) and (6) will be written as follows:

$W = K_W(E_M - E_S) + \frac{1}{2}G(n_M E_M - n_S E_S) + k_M E_M(t-\tau)\cos(\omega_0\tau + \Phi_M(t) - \Phi_M(t-\tau)) - k_S E_S(t-\tau)\cos(\omega_0\tau + \Phi_S(t) - \Phi_S(t-\tau)) + k_{M1}E_M(t-\tau_1)\cos(\omega_0\tau_1 + \Phi_M(t) - \Phi_M(t-\tau_1)) - k_{S1}E_S(t-\tau_1)\cos(\omega_0\tau_1 + \Phi_S(t) - \Phi_S(t-\tau_1))$.

3. ADAPTIVE SYNCHRONIZATION WITH UNKNOWN PARAMETERS

The systems parameters from eqs.(5-6) are required for the adaptive synchronization coupling function. In the case that these parameters are not available, one can apply the on-line parameter estimation method. In principle the number of unavailable parameters can be equal to the total number of systems' parameters. First, in this paper we demonstrate the case of single parameter estimation, namely to gain estimation. Next we apply the approach to the case of double parameters estimation. So let us suppose that the gain's estimated value G_1 is different from the gain value G required for synchronization. With the estimated value of gain the adaptive coupling function would be of the form

$$\begin{aligned}
W &= K_W(E_M - E_S) + \frac{1}{2}G_1(n_M E_M - n_S E_S) \\
&\quad + k_M E_M(t-\tau)\cos(\omega_0\tau + \Phi_M(t) - \Phi_M(t-\tau)) \\
&\quad - k_S E_S(t-\tau)\cos(\omega_0\tau + \Phi_S(t) - \Phi_S(t-\tau)),
\end{aligned} \tag{14}$$

Under these conditions it is easy to verify that error dynamics will now satisfy the following equation:

$$\frac{de}{dt} = -K_W e + \frac{1}{2}(n_M E_M - n_S E_S)(G - G_1), \tag{15}$$

In other words, for $G \neq G_1$ the error e will not approach zero, as required for synchronization purposes. The situation can be rectified, if we add the following equation for the parameter estimation error $e_G = G - G_1$ to the previous equation (15):

$$\frac{de_G}{dt} = -e\frac{1}{2}(n_M E_M - n_S E_S). \tag{16}$$

Now we shall demonstrate that the origin of the systems (15)-(16) is asymptotically stable, i.e. that the synchronized state is asymptotically stable. Indeed, by choosing the following Lyapunov function:

$$L = \frac{1}{2}(e^2 + e_G^2), \tag{17}$$

it is trivial to check that

$$\frac{dL}{dt} = -K_W e^2 + \frac{1}{2}(n_M E_M - n_S E_S)e_G e - \frac{1}{2}(n_M E_M - n_S E_S)e_G e = -K_W e^2 < 0. \tag{18}$$

Thus we demonstrate that the stability of the adaptive control law with unknown parameters is asymptotic. Next we suppose that apart from the gain, the master laser's estimated feedback rate value k_{Mn} is different from the value of k_M required for synchronization. Then with the estimated values of gain and feedback rate the adaptive coupling function would be:
$W = K_W(E_M - E_S) + \frac{1}{2}G_1(n_M E_M - n_S E_S) + k_{Mn} E_M(t-\tau)\cos(\omega_0\tau + \Phi_M(t) - \Phi_M(t-\tau))$
$- k_S E_S(t-\tau)\cos(\omega_0\tau + \Phi_S(t) - \Phi_S(t-\tau))$. With this W the error dynamics is of the form:
$\frac{de}{dt} = -K_W e + \frac{1}{2}(n_M E_M - n_S E_S)(G - G_1) + (k_M - k_{Mn})E_M(t-\tau)\cos(\omega_0\tau + \Phi_M(t) - \Phi_M(t-\tau))$.
So for $G \neq G_1$ and $k_M \neq k_{Mn}$ synchronization is not achieved as e with time is not approaching zero. Synchronization will take place only if the parameters' estimation errors $e_G = G - G_1$ and $e_k = k_M - k_{Mn}$ obey the dynamics: $\frac{de_G}{dt} = -e\frac{1}{2}(n_M E_M - n_S E_S)$ and $\frac{de_{Mn}}{dt} = -eE_M(t-\tau)\cos(\omega_0\tau + \Phi_M(t) - \Phi_M(t-\tau))$. This time it can be shown by using the Lyapunov function $L = \frac{1}{2}(e^2 + e_G^2 + e_k^2)$. It is evident that this approach is also applicable in the case double (multi-feedback) systems.

In this paper we have only presented the case of complete chaos synchronization. Expansion of the approach to the lag and anticipating synchronizations is straightforward.

The practical implementation of the proposed scheme can be based on the approaches developed in [28-30]. In these papers it has been shown that a scheme of chaos control for external

cavity laser diodes can be effected where a periodic state of the dynamics is selected from the chaotic dynamics. The key to that approach is the utilisation of an error signal which defines the difference between the chaotic state and the targeted state. As the target state is approached the generated error signal reduces to zero. It was shown in those papers that optoelectronic feedback provides a straightforward means for generating the requiste error signal [24-26]. It is noted that the linkage function defined in the present work can be expressed as an error signal between the dynamics of the master and slave external cavity lasers. As the linkage function brings the dynamics of the two laser systems into synchronization the corresponding error signal will diminish to zero. One approach to the practical implementation of the synchronization of the present scheme would thus again be based on the use of the optoelectronic feedback.

In other words, for the practical realization of the synchronization scheme we essentially inject the amplified difference signal between the master and slave lasers' outputs to the slave laser. Comparing our approach with other widely known methods we notice that, in general, the present synchronization procedure is different from that of [15,17] and we argue that it offers more flexibility in practical control problems.

4. CONCLUSION

In this paper we have showed how one can synchronize two chaotic time delay systems in the general case by choosing an appropriate delay adaptive coupling function. We apply the proposed approach to the case of synchronization between two semiconductor lasers subject to optical feedback. For the first time we have also demonstrated that when the parameters of the systems to be synchronized are not available, then on-line parameter estimation can be applied. Generalization of the approach to the case of multi-feedback systems is also presented.

Acknowledgments

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