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COMPARATIVE STUDY OF INTER-STRAND COUPLING CURRENT MODELS FOR ACCELERATOR MAGNETS

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Inter-strand coupling currents (ISCCs) contribute to field errors and losses in Rutherford-type superconducting cables in the time-transient regime. A field change induces eddy currents in loops formed by the superconducting twisted strands and the resistive matrix. The implementation of ISCC models in ROXIE allows to combine ISCC calculations with models for persistent current sand inter- filament coupling currents. Saturation effects in iron can be taken into account as well. The predictions of different ISCC models with regard to losses and field errors are compared for two design versions of the LHC main dipole.

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Abstract

Inter-strand coupling currents (ISCCs) contribute to field errors and losses in Rutherford-type superconducting cables in the time-transient regime. A field change induces eddy currents in loops formed by the superconducting twisted strands and the resistive matrix. The implementation of ISCC models in ROXIE allows to combine ISCC calculations with models for persistent currents and inter-filament coupling currents. Saturation effects in iron can be taken into account as well. The predictions of different ISCC models with regard to losses and field errors are compared for two design versions of the LHC main dipole.

INTRODUCTION

The two layers of transposed strands in Rutherford type cables are forming loops in which eddy currents can be induced by time-transient fields, e.g., during the ramping of magnets. These so-called inter-strand coupling currents (ISCCs) are responsible for additional heat losses and magnetic field errors. The ability to predict ISCCs is especially important in the context of fast-ramping superconducting synchrotron facilities, e.g., the FAIR project at GSI, Germany.

A network model for the calculation of ISCCs based on work by Devred and Ogitsu [1] has been implemented in the ROXIE program [2]. In this model, contact resistances between strands can be assumed as being constant or with random fluctuations along the cable. The network model with lumped elements is solved using mesh analysis based on matrix algebra. Spatial periodic boundary conditions can be applied in order to model an infinitely long cable. The calculations presented in this paper are steady-state calculations, although the model is equally suited to perform transient calculations and predict time-constants.

In this paper we introduce the ability of the CERN ROXIE program [5] to perform ISCC analysis during an integrated design process of superconducting magnets. The ISCC effects can be studied with non-linear iron-yoke magnetization, persistent-current magnetization and inter-filament coupling current effects, simultaneously. We compare the results of the network model in ROXIE with results of a stand-alone network code by A. Verweij [3], and with an equivalent-cable-magnetization model by M. N. Wilson [4].

THE NETWORK MODEL

We represent the Rutherford-type superconducting cable as a lumped-element network model. The strands are represented by perfectly conductive thin wires and the adjacent- and cross-over-resistances by lumped resistive elements, compare Fig. 1.

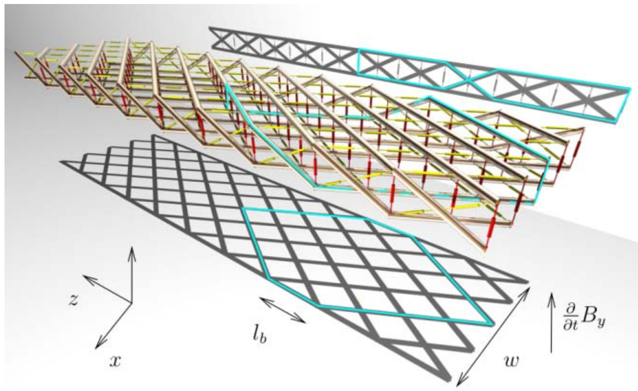

Figure 1: A typical current loop in a cable with 10 strands. The current is driven by a field sweep $\partial_t B_y$. Adjacent resistances are displayed in yellow and cross-over resistances in red.

The field-sweep as a driving force is represented in the network model by voltage sources. Each branch b_j has a source defined by

$$\{U_\mathrm{s}\}^j = \int_{b_j} \partial_t \mathbf{A}_\mathrm{s} \cdot \mathbf{t}_{b_j} \,\mathrm{d}l,$$

with the source magnetic vector potential $\mathbf{A}_\mathrm{s}$ and the branch tangent-vector $\mathbf{t}_{b_j}$. As a consequence we have to solve the following steady-state linear equation system:

$$\{U\} = [\mathrm{R}]\{I\} + \{U_\mathrm{s}\}, \tag{1}$$

where $\{U\}$, $\{I\}$, $\{U_\mathrm{s}\} \in \mathbb{R}^{N_\mathrm{b}}$ represent branch voltages, -currents, and -sources, and $[\mathrm{R}] = [\mathrm{G}]^{-1} = \mathrm{diag}\left(\frac{1}{g_j}\right) \in \mathbb{R}^{N_\mathrm{b}\times N_\mathrm{b}}$ is the resistance matrix. N_b denotes the number of branches and N_n will denote the number of nodes.

With the branch-node incidence matrix $[\mathrm{A}] \in \mathbb{R}^{N_\mathrm{n}\times N_\mathrm{b}}$, which is defined by

$$a_{ij} := \begin{cases} 1 & \text{if branch } j \text{ exits from node } i, \\ -1 & \text{if branch } j \text{ enters in node } i, \\ 0 & \text{otherwise,} \end{cases}$$

we write the first law of Kirchhoff (current law) in matrix form $[\mathrm{A}]\{I\} = 0$, and the potential formulation of the electric voltage $\{U\} = [\mathrm{A}]^\mathrm{T}\{\varphi\}$. Consequently we find

$$\{\varphi\} = \left([\mathrm{A}][\mathrm{G}][\mathrm{A}]^\mathrm{T}\right)^{-1}[\mathrm{G}]\{U_\mathrm{s}\}.$$

The problem with this formulation lies in the near-zero conductivity of superconducting strands which renders an inversion of the conductivity matrix [G] impossible. We therefore adopt a method which uses a mesh-branch incidence matrix $[\mathrm{M}] \in \mathbb{R}^{N_\mathrm{m} \times N_\mathrm{b}}$: For topological considerations we divide the network into a spanning tree of $N_\mathrm{n} - 1$ branches and its complement co-tree. We find that there are as many linearly independent meshes or loops in the network as there are co-tree branches, $N_\mathrm{m} = N_\mathrm{b} - N_\mathrm{n} + 1$. Each elementary mesh contains one, and only one, co-tree branch, which defines the mesh orientation. The mesh-branch incidence matrix is defined by

$$m_{ij} := \begin{cases} 1 & \text{if branch } j \text{ belongs to mesh } i \\ & \text{with same orientation,} \\ -1 & \text{if branch } j \text{ belongs to mesh } i \\ & \text{with opposite orientation,} \\ 0 & \text{otherwise.} \end{cases}$$

Kirchhoff's voltage law can be written in matrix form $[\mathrm{M}]\{U\} = 0$. We express the branch currents as a linear superposition of so-called mesh currents $\{I_\mathrm{M}\} \in \mathbb{R}^{N_\mathrm{m}}$ by $\{I\} = [\mathrm{M}]^\mathrm{T}\{I_\mathrm{M}\}$ and obtain from Eq. 1

$$\{I_\mathrm{M}\} = -\left([\mathrm{M}][\mathrm{R}][\mathrm{M}]^\mathrm{T}\right)^{-1}[\mathrm{M}]\{U_\mathrm{S}\}.$$

In order to simulate an infinitely long cable, we set up the network equations for only one band of length l_b, compare Fig. 1. Periodic boundary conditions are applied to the resulting linear equation system. Calculations for the LHC main dipole are shown in Fig. 2.

EQUIVALENT-MAGNETIZATION MODEL

The equivalent-magnetization model for ISCCs represents the main loops in the network model that contribute to ISCCs. It consists of three vectorial contributions, [4]:

$$\begin{aligned} \mathbf{M}_\mathrm{c}^\perp &= \frac{1}{120}\frac{\partial_t B^\perp}{R_\mathrm{c}}\, l_\mathrm{p}\, N_\mathrm{s}(N_\mathrm{s}-1)\,\frac{w}{b}\mathbf{e}^\perp, \\ \mathbf{M}_\mathrm{a}^\perp &= \frac{1}{3}\frac{\partial_t B^\perp}{R_\mathrm{a}}\, l_\mathrm{p}\,\frac{w}{b}\mathbf{e}^\perp, \\ \mathbf{M}_\mathrm{a}^\parallel &= \frac{1}{8}\frac{\partial_t B^\parallel}{R_\mathrm{a}}\, l_\mathrm{p}\,\frac{b}{w}\mathbf{e}^\parallel, \end{aligned}$$

where $\mathbf{M}_\mathrm{c}^\perp$ is the magnetization due to currents in the cross-over resistances induced by a field perpendicular to the broad side of the conductor, $\mathbf{M}_\mathrm{a}^\perp$ is the magnetization due to currents in the adjacent resistances from a perpendicular field, and $\mathbf{M}_\mathrm{a}^\parallel$ is the magnetization due to currents in the adjacent resistances from a parallel field. The cable twist-pitch length l_p features in the equations as well as the cable width w, the cable height b, the number of strands N_s, and the cross-over- and the adjacent-resistance R_c and R_a. Figure 3 shows the field distribution due to the equivalent-magnetization model. We observe that both models, network and equivalent magnetization, predict ISCC-fields in the aperture that contribute to the exciting field, an effect that is known as "field advance" due to ISCCs.

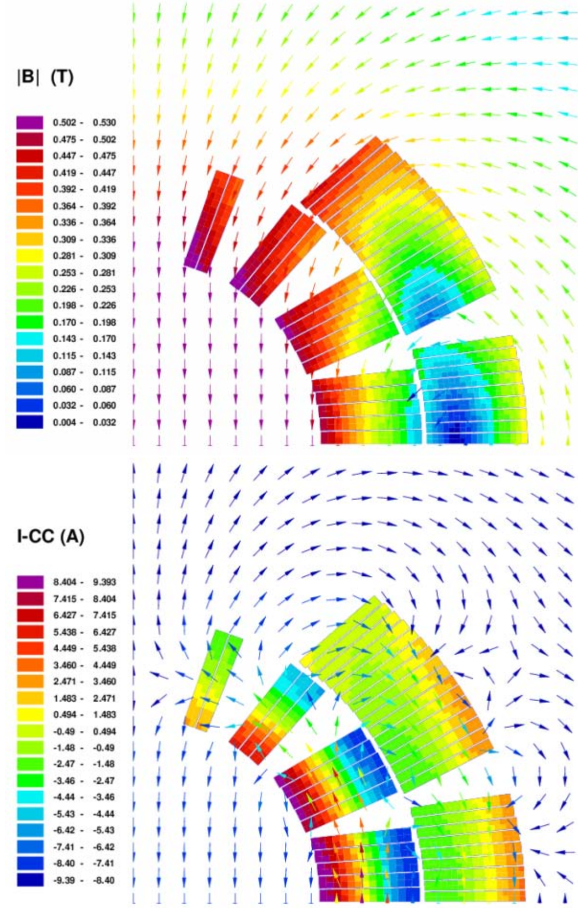

Figure 2: Top: Magnetic flux density in the cross-section of the LHC main dipole. Bottom: ISCCs in steady-state condition at $\partial_t B_\mathrm{central}$ = 0.094Ts^{-1}. Icons represent the magnetic flux density generated by ISCCs. Note that the icons in both plots are not to scale.

COMPARISON OF MODELS

Both models, the network model and the equivalent-magnetization model for ISCCs, have been implemented in ROXIE. This allows for a combined simulation of ISCCs and other effects. For a verification we compare the ROXIE results to a network-model by A. Verweij [3]. The Verweij code has been used to determine cross-over and adjacent resistances in LHC pre-series dipoles from measured field quality and losses.

In [6] we find data on the ISCCs' impact on field-quality. In [3], page 153, we find loss calculations. Both documents present calculations of the so-called "White-Book"- and "Pink-Book"-dipoles. The naming refers to two LHC design reports and, thus, two different design stages of the LHC main dipoles.

Tables 1, 2, and 3 summarize the results. We find that the network models agree within a few percent. The difference is due to a slightly different implementation of the adjacent resistances on the cables' narrow edges. The equivalent-

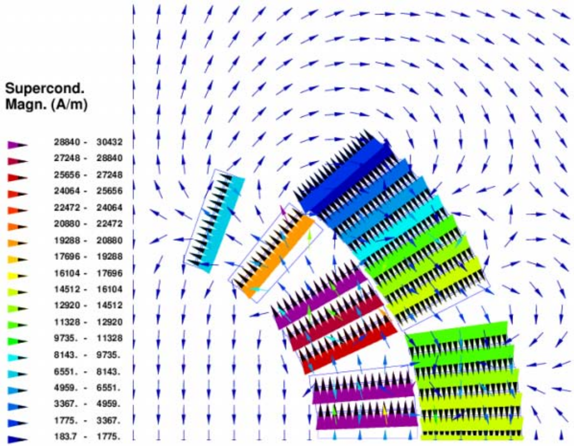

Figure 3: Equivalent cable magnetization from ISCCs. Icons represent the magnetic flux density generated by ISCCs, compare the field pattern outside the coil cross-section in Fig. 2.

magnetization model generally yields lower estimates for ISCCs. The reason for this lies in the simplicity of the model, and in the averaging of the field-sweep over a cable-width in the computation of the equivalent cable magnetization. This averaging implies that for cables which are exposed to a field with opposite direction over the cable cross-section (compare the outer layer in upper picture of Fig. 2) the model predicts too small magnetization values. The problem is more severe for 1-layer coils. The effect might be remedied by averaging the field over parts of the cable cross-section only.

Table 1: Pink-book dipole: ISCC influence on the relative field harmonics (in units 10^{-4} at a reference radius of 10 mm) at a center field of 0.58 T. The field increases at a rate of 0.00667 T/s (8 T in 1200 s). We use $\mathrm{R_c} = 1\ \mu\Omega$ and $R_\mathrm{a} = 10\ \mathrm{m}\Omega$.

n	**ROXIE Equiv.-Magn.** Δb_n	**ROXIE Network Model** Δb_n	**Verweij Network Model** Δb_n
1	-86.15	-95.28	-97.6
3	3.41	3.88	3.11
5	-0.31	-0.21	-0.32
7	-0.01	0.01	0.00
9	0.01	0.01	0.01

CONCLUSIONS

We have shown that the models for inter-strand coupling currents which have been implemented in the ROXIE program yield predictions that are consistent with prior work on the topic carried out at CERN. We can therefore proceed to use the models in combination with other ROXIE features such as non-linear iron-yoke magnetization, superconductor magnetization, inter-filament coupling currents, etc.

Table 2: White-book dipole: ISCC influence on the relative field harmonics (in units 10^{-4} at a reference radius of 10 mm) at a center field of 0.58 T. The field increases at a rate of 0.00667 T/s (8 T in 1200 s). We use $\mathrm{R_c} = 1\ \mu\Omega$, $R_\mathrm{a} = 10\ \mathrm{m}\Omega$.

n	**ROXIE Equiv.-Magn.** Δb_n	**ROXIE Network Model** Δb_n	**Verweij Network Model** Δb_n
1	-73.23	-75.82	-77.2
3	1.92	2.22	2.11
5	-0.09	-0.05	-0.07
7	-0.01	-0.01	-0.01
9	0.0	0.00	0.00

Table 3: Losses in joule/meter for a field sweep between 0.6 and 8.4 T with a ramp rate of 0.00667 T/s for the pink-book dipole (PBD) and a white-book dipole (WBD). We use $\mathrm{R_c} = 2\ \mu\Omega$ and $R_\mathrm{a} = 10\ \mathrm{m}\Omega$.

	ROXIE Equiv.-Magn.	**ROXIE Network M.**	**Verweij Network M.**
PBD	74.39	88.82	85.0
WBD	91.31	105.1	105

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