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RANDOM ERRORS IN SUPERCONDUCTING DIPOLES

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The magnetic field in a superconducting magnet is mainly determined by the position of the conductors. Hence, the main contribution to the random field errors comes from random displacement of the coil with respect to its nominal position. Using a Monte-Carlo method, we analyze the measured random field errors of the main dipoles of the LHC, Tevatron, RHIC and HERA projects in order to estimate the precision of the conductor positioning reached during the production. The method can be used to obtain more refined estimates of the random components for future projects.

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Abstract

The magnetic field in a superconducting magnet is mainly determined by the position of the conductors. Hence, the main contribution to the random field errors comes from random displacement of the coil with respect to its nominal position. Using a Monte-Carlo method, we analyze the measured random field errors of the main dipoles of the LHC, Tevatron, RHIC and HERA projects in order to estimate the precision of the conductor positioning reached during the production. The method can be used to obtain more refined estimates of the random components for future projects.

INTRODUCTION

Magnets for circular colliders and storage rings have to satisfy stringent requirements on the field harmonics: imperfections must be of the order of 10^{-4} with respect to the main component, and must be controlled within 10^{-5}. In superconducting magnets the uncertainty in the coil position in the transverse cross-section generated by mechanical tolerances gives rise to geometric random errors. This is one of the main sources of random components of the field harmonics, limiting the possibility of obtaining a perfect field quality.

Estimates of the geometric random errors are usually based on computing field perturbations induced by a random displacement of the coil blocks (Monte-Carlo) with a spread of ~50 μm r.m.s. In this paper we review the data of the production of dipoles relative to four accelerators [1-5] to analyse the agreement of the Monte-Carlo estimates with the measured values.

The above quoted Monte-Carlo method, widely used in the past, gives similar estimates for normal and skew harmonics of the same order. However, already in the Tevatron production it has been observed [6] that random components of normal and skew harmonics of the same order can differ of a factor 4 to 6. Here, following the approach of [6, 7], we propose to associate different amplitudes to generate normal and skew harmonics, in order to better fit the experimental data. The final result of the analysis is an improved phenomenological model based on the acquired experience of the four large scale dipole productions to describe and forecast the random errors in a superconducting dipole.

FIELD HARMONICS DEFINITION

The magnetic field in the plane (x,y) perpendicular to the reference orbit is expanded in a power series

$$B_y + iB_x = 10^{-4} B_{ref} \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x+iy}{R_{ref}} \right)^{n-1} \qquad (1)$$

where R_{ref} is the reference radius, usually taken as 2/3 of the magnet aperture, B_{ref} is the main field component, and (b_n, a_n) are the multipolar coefficients expressed in terms of units (one unit is 10^{-4} times the reference field).

The field is measured along consecutive positions with a rotating coil to cover all the magnet length, and an integral value of the multipoles is built by computing an average over the positions, weighted with the main field component. Having a set of magnets, one defines the random component as the standard deviation of the integral multipoles. The random component can be taken over all the production of magnets with the same design, or it can be split according to the different manufacturers to evaluate the influence of the production tooling and procedures on the field quality.

There are four families of multipoles, (odd normal - b_{2n+1}; even normal- b_{2n}; odd skew - a_{2n+1}; even skew - a_{2n}) which are generated by different symmetries of the coil imperfections, in the four quadrants, as shown in Fig. 1. We recall that according to the Biot-Savart law, a current line at a radius r generates multipoles that decay following the power law:

$$b_n, a_n \propto \left(\frac{R_{ref}}{r} \right)^n \qquad (2)$$

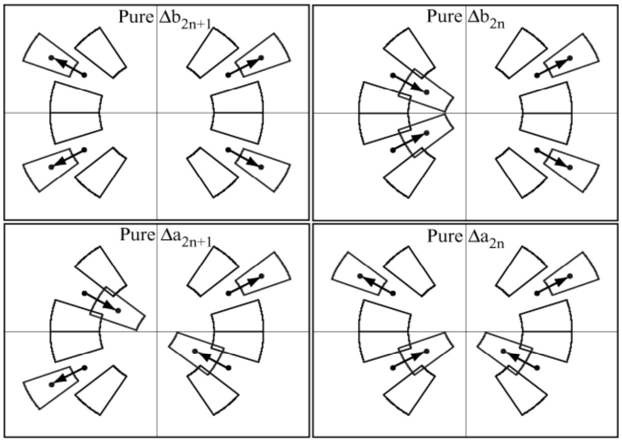

Figure 1: Coil imperfections generating multipole families.

PHENOMENOLOGY

We analysed data relative to superconducting dipoles in Tevatron, HERA, RHIC and LHC (see Table I). We considered homogeneous sets of magnets with the same cross-section. The standard deviation of the integral field harmonics is shown in Figs. 2-5 versus the multipole order in a semi-logarithmic scale. One can make the following remarks.

- The random components follow the expected decay law (2) for all machines but saturate at 0.3 units for Tevatron and at 0.1 units for HERA. This is probably given by the precision of the measurement system. Therefore, we used Tevatron data up to order 6, HERA up to order 7, RHIC up to order 8 and LHC up to order 9.
- In all cases one observes a saw-tooth pattern (but for Tevatron skew harmonics), where the standard deviation of the odd normal is larger than the odd skew of the same order, and vice-versa for the even. LHC data show an anomalous behaviour since the skew harmonic have no saw-tooth. The ratio between the standard deviation of the normal and skew of the same order is at most 4 for the LHC and 6 for the other machines.

Table 1: Parameters of the analysed dipoles

	Number	**aperture radius [mm]**	**coil width [mm]**	**Layers**	**Blocks**
Tevatron	774	38.1	16.3	2	2
HERA	416	37.5	21.2	2	4
RHIC	296	40.0	10.06	1	4
LHC	1232	28.0	30.8	2	6

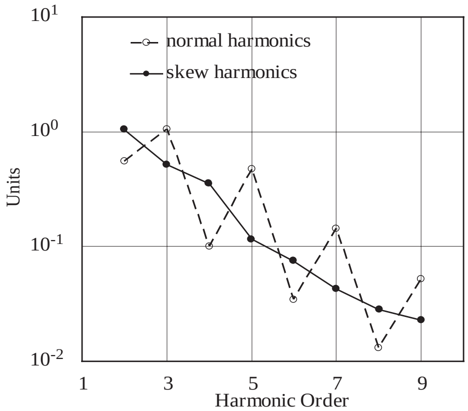

Figure 2: Random components measured in the main LHC dipoles. Reference radius: 17mm.

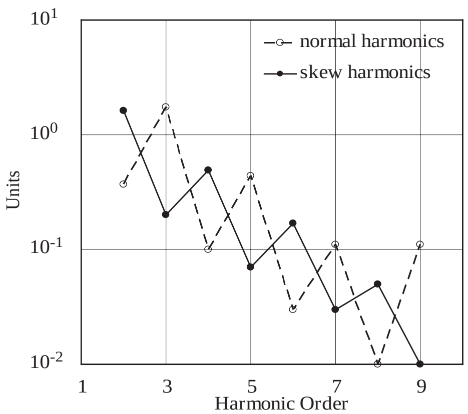

Figure 3: Random components measured in the main RHIC dipoles. Reference radius: 25 mm.

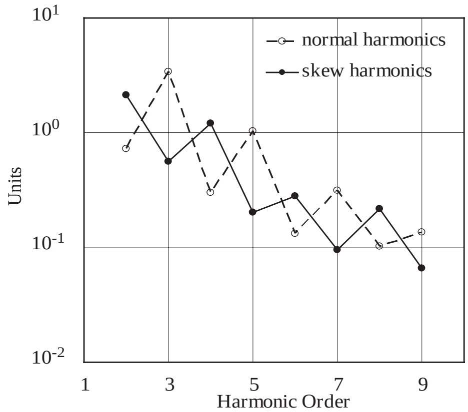

Figure 4: Random components measured in the main HERA dipoles. Reference radius: 25 mm.

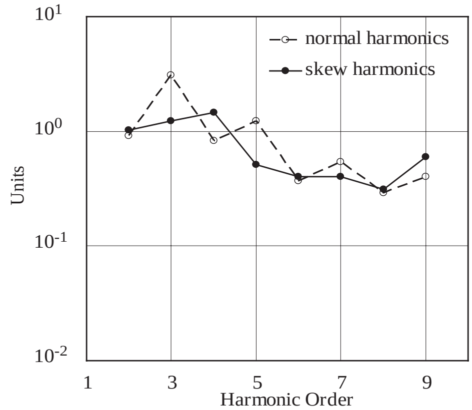

Figure 5: Random components measured in the main Tevatron dipoles. Reference radius: 25.4 mm.

INTERPRETATION

We carry out a Monte-Carlo simulation to interpret the data shown in Figs. 2-5 in terms of coil displacements. Each coil block is displaced in the transverse plane randomly along 3 degrees of freedom as shown in Fig. 6: radial and azimuth translation, and tilt.

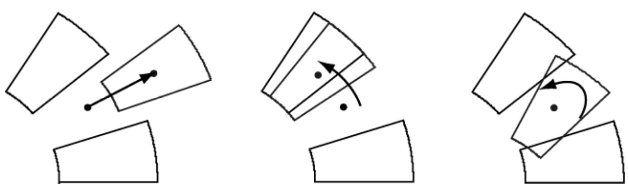

Figure 6: Block displacements used in the simulation.

The movements are considered to be rigid, i.e. no coil deformation is considered. The amplitude of each displacement is drawn from a Gaussian distribution with spread $d/\sqrt{3}$ and zero mean value. 1000 realizations have been used for each computation, reaching a precision in the estimate of the multipole standard deviations of 3%.

The simulations for a given amplitude d provide a set of random components $\sigma_{bn}^{s}(d)$ and $\sigma_{an}^{s}(d)$. The dependence on d is linear for our range of interest (10 to 100 μm).

We evaluate the discrepancy of the standard deviation of the measurements $\sigma_{bn}{}^{m}$, $\sigma_{an}{}^{m}$ with respect to simulations as:

$$\eta = \sum_{n=2}^{N} \left[\ln \sigma_{bn}^{m} - \ln \sigma_{bn}^{s}(d)\right]^2 + \left[\ln \sigma_{an}^{m} - \ln \sigma_{an}^{s}(d)\right]^2 \quad (3)$$

In order to minimize the relative error on the spreads we used the logarithms of the r.m.s.; without the logarithm the minimization would ignore the higher orders. We define d_0 ("coil waviness") as the amplitude that minimizes η. In Table 2 we give d_0 for the dipoles of four accelerators, together with the average relative error between the Monte-Carlo and measurements. According to this analysis, the smaller r.m.s. of the measured multipoles corresponds to an improvement in the precision in positioning the blocks. The RHIC dipole, whose lay-out is a single layer with a thin cable width (10 mm), assembled by the same firm, has reached the smallest random displacement (16 μm r.m.s.). The LHC case, notwithstanding the more complicated 2-layer structure, with a 15 mm cable width, and three different assemblers, corresponds to amplitudes which are only 50% larger (25 μm r.m.s.).

Table 2: Displacement amplitude estimated from measurements and average relative error on each harmonic

	d_0 [μm]	**error [%]**
Tevatron	65	35
HERA	41	40
RHIC	16	63
LHC	25	43

The average discrepancy between the measured values and the estimate based on the Monte-Carlo is 35% to 60% (see Table 2). Therefore, this method can be used only to give an estimate of the order of magnitude of the random geometric errors. This is mainly due to the saw-tooth, i.e. to the asymmetry between normal and skew components, as first observed in [6].

If in the Monte-Carlo the block tilt (Fig. 6, right) is not implemented, one obtains exactly the same normal and random components for the same order. The block tilt has an effect which is considerably smaller than the translations (~a factor 4, depending on the block size), and it also induces an asymmetry between normal and skew components of the same order. For instance, in the LHC case one finds that the random a_2 is 10% larger that b_2, whereas the random b_3 is 5% larger than a_3. This is the same qualitative pattern observed in the experimental data, but there is no quantitative agreement since the difference between normal and skew of the same order is much larger.

To get the observed difference between normal and skew components, we split the random movements in the four orthogonal families shown in Fig. 1, associating a different amplitude to each family, as suggested in [6, 7]. We therefore define four η functions, two for the normal harmonics:

$$\eta_1(d) = \sum_{n=2}^{N} \left[\ln \sigma_{b2n-1}^{m} - \ln \sigma_{b2n-1}^{s}(d)\right]^2 \quad (4)$$

$$\eta_2(d) = \sum_{n=2}^{N} \left[\ln \sigma_{b2n}^{m} - \ln \sigma_{b2n}^{s}(d)\right]^2 \quad (5)$$

and two similar expressions for the skew, computing four parameters d_1, d_2, d_3, d_4 (see Table 3). Optimizing the data with these four parameters the error between simulation and model drops to 10-30% for four cases analysed.

If the LHC the data are split according to the dipole assembler, the random component of odd normal and skew is nearly 1/3 less, corresponding to smaller amplitudes (see Table 3, last 3 rows).

Table 3: Displacement amplitude estimated from measurements, separated for each multipole family

	d_1 [μm]	**d_2 [μm]**	**d_3 [μm]**	**d_4 [μm]**	
	b2n+1	**b2n**	**a2n+1**	**a2n**	**error [%]**
Tevatron	128	52	70	52	30
HERA	122	20	24	58	25
RHIC	52	6	8	32	30
LHC	54	12	18	26	25
LHC Firm1	38	10	12	22	19
LHC Firm2	42	8	12	24	13
LHC Firm3	32	10	14	22	17

CONCLUSIONS

We have analysed the data of the random geometric harmonics in the main dipoles of Tevatron, HERA, LHC and RHIC. We showed that for the more recent productions (LHC and RHIC), the order of magnitude of geometric random components is compatible with a random movement of the blocks of ~50 μm r.m.s. for the odd normal multipoles, ~30 μm for the even skew, and 5 to 20 μm for the even normal and for the odd skew. Such parameters allow estimating the random geometric errors with an average error of ~20%.

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