# Effect of Nuclear Size on the Stopping Power of Ultrarelativistic Heavy Ions

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A new formulation for the theory of electronic stopping power of ions at relativistic energies has been proposed by Lindhard and Sørensen (LS). In it, they find that, at sufficiently high energy, nuclear size effects should act to reduce the momentum transfer to electrons and hence the stopping power. To test this result, we passed beams of 33.2-TeV $^{208}$Pb ions ($\gamma = 168$) from the CERN-SPS through targets of C, Si, Cu, Sn, and Pb, and measured energy loss and beam broadening. The LS theory for stopping power is confirmed, but with a slight drift upward from theory for high-$Z$ targets. A drastic decrease in energy straggling (factor of $\sim$4) predicted by LS cannot be deconvoluted from the multiple Coulomb scattering distribution. [S0031-9007(96)01272-0]

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The slowing down of energetic ions in matter is dominated by momentum exchanging collisions with electrons. The theory of this venerable subject was formulated early on by Bohr [1] and Bethe [2]. It was modified by Bloch [3], and it was shown that at relativistic velocities a "Mott" correction for spin changing collisions [4] was required. The proper combination of these effects was shown to match quantitatively with experiments on the stopping power of heavy ions with energies from 700 to 1000 MeV $A$ [5].

The energy loss $\Delta E$ of a totally stripped ion $Z_1$ passing through a thickness $\Delta x$ with an electron density $n_e$ can be expressed as

$$\Delta E = \Delta x\left(\frac{4\pi Z_1^2 e^4}{m_0 V^2}\right) n_e L\,. \tag{1}$$

At relativistic energies the term $L$ is given by

$$L = \ln\left(\frac{2\gamma^2 m_0 v^2}{I}\right) - \beta^2 - \delta/2 + \Delta L\,, \tag{2}$$

where $\gamma$ is the Lorentz factor $(1-\beta^2)^{-1/2}$, $\beta = v/c$, $m_0$ is the electron rest mass, and $I$ is the mean ionization potential of the target electrons. The term $\delta$ arises from the so-called density effect [6,7] due to the relativistic increase in the transverse field and the attendant target screening of the projectile charge in distant collisions. For $\gamma \geq 100$ as in our experiments, the density effect correction can be closely approximated by

$$\delta/2 \cong \ln\left(\frac{\gamma\hbar\omega_p}{I}\right) - 1/2\,, \tag{3}$$

where $\omega_p$ is the plasmon frequency of the total density of the electrons of the medium. Then with $\beta \cong 1$ and

$$L = \ln\left(\frac{2\gamma m_0 c^2}{\hbar\omega_p}\right) - 1/2 + \Delta L \tag{4}$$

and

$$\Delta L = \Delta L_{\mathrm{Bloch}} + \Delta L_{\mathrm{Mott}} + \Delta L_{\mathrm{NS}}\,. \tag{5}$$

The new term here is $\Delta L_{\mathrm{NS}}$, the correction for nuclear size effect that has just recently been proposed by Lindhard and Sørensen [8,9].

Lindhard and Sørensen (LS) performed exact quantum mechanical calculations on the basis of the Dirac equation to produce values for the average energy loss and straggling which are stated to be accurate for any value of projectile charge. Note that the various $\Delta L$ terms are just a consequence of the calculation. Using a point Coulomb potential, they are able to reproduce the results of Bohr, Bethe, Bloch, and Mott. However, they show that at sufficiently high energies the finite nuclear size effects the stopping power.

It is convenient to view the projectile nucleus as a stationary scattering point for a flux of electrons moving at the velocity of the ion in the laboratory system. According to LS, an electron will encounter the nucleus when its angular momentum $pR \cong \gamma m_0 c R$ where $R$ is the nuclear radius ($R \sim 1.2 \times 10^{-13} A^{1/3}$ cm, where $A$ is the atomic weight). When $pR \sim \hbar/2$ modification of the first few quantum phase shifts will be needed, i.e., nuclear size effects should be important when $2\gamma m_0 c R/\hbar = \gamma A^{1/3}/160 \cong 1$. Alternatively, one may consider that the effect will become important when the deBroglie wavelength of the electron $\lambda\!\!\!^{-} = \hbar/\gamma m_0 c$ becomes comparable to the nuclear size.

  

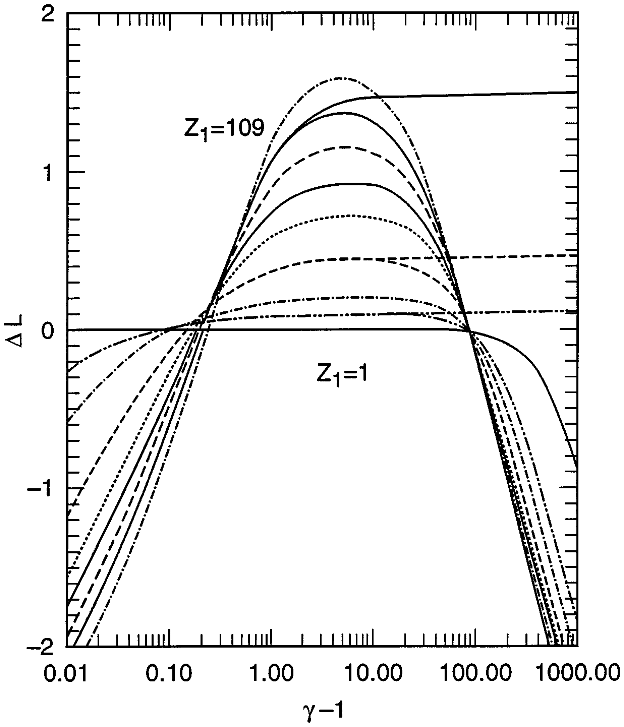


FIG. 1. Stopping for finite nuclear size. The curves show computed values of $\Delta L$ for atomic numbers $Z_1 = 1$, 10, 18, 54, 66, 79, 92, and 109. The thin lines to the right show $\Delta L$ predicted for point nuclei for $Z_1 = 10$, 36, and 92 (from Lindhard and Sørensen, Ref. [7]).

The result of the LS theory is shown in Fig. 1, where $\Delta L$ is plotted versus $\gamma - 1$ for projectiles of various $Z_1$. A negative value equivalent to the Bloch correction is seen to dominate at low $\gamma$. It then diminishes and gives way to a growing positive correction that is related to the Mott term. In the absence of a nuclear size effect, $\Delta L$ would asymptotically approach constant values as indicated by the horizontal lines at the right of the figure. Instead, because of an effective cutoff in momentum transfer for small impact parameter collisions, $\Delta L$ decreases sharply with increasing $\gamma$.

In this paper, we report on measurements of $dE/dx$ for 160-GeV $A\,^{208}Pb^{82+}(\gamma = 168)$ ions, obtained from the SPS facility at CERN, in targets of C, Si, Cu, Sn, and Pb, and compare the results with the predictions of the LS theory. For lead, at $\gamma = 168$ the value of $\Delta L$ not including nuclear size effect is $+1.40$; the inclusion of nuclear size gives a $\Delta L = -0.72$, hence with $L \approx 14$ a possible 15% effect on stopping power.

The experimental setup is pictured in Fig. 2. The 33.2-TeV $^{208}Pb^{82+}$ beam is delivered from the CERN SPS accelerator and is monitored by secondary emission detectors made from thin foils placed in the way of the beam. The beam is $\sim$3 mm wide when it passes through the target. It is then bent 42 mr by an array of dipoles and is momentum analyzed using a collimator slit $\sim$150 m downstream. After a passage of $\sim$300 m, it is bent again and focused onto a detector $\sim$350 m further downstream. The detector used was a fast Cherenkov counter. The slits are $\sim$1 m thick; they can be set to a width as low as 2 mm and can be moved in 2 mm steps. The momentum calibration can either be calculated from the beam optics or it can be experimentally determined from the positions registered for $^{208}$Pb and $^{207}$Pb in a single scan of the slits. The latter is copiously formed by neutron stripping in all targets. The measured resolution of the system is $\sim 7 \times 10^{-4}$, which permits the location of a peak to be determined with a precision of $\sim 1 \times 10^{-4}$. The targets are mounted on a ladder in two parallel arrays that can be moved vertically and horizontally for positioning. Because the ladder is located almost 1 km from the control room in an inaccessible and high radiation area, a special PC control and data acquisition system was created and is described elsewhere [10]. Four targets of each element mounted on the ladder were selected to give energy losses of approximately 0.1%, 0.2%, 0.4%, and 0.8% of the primary beam energy.

Figure 3 shows a set of beam profiles for an open beam and four C targets demonstrating the shift of position due to energy loss versus target thickness and beam broadening as a function of target thickness. The stopping

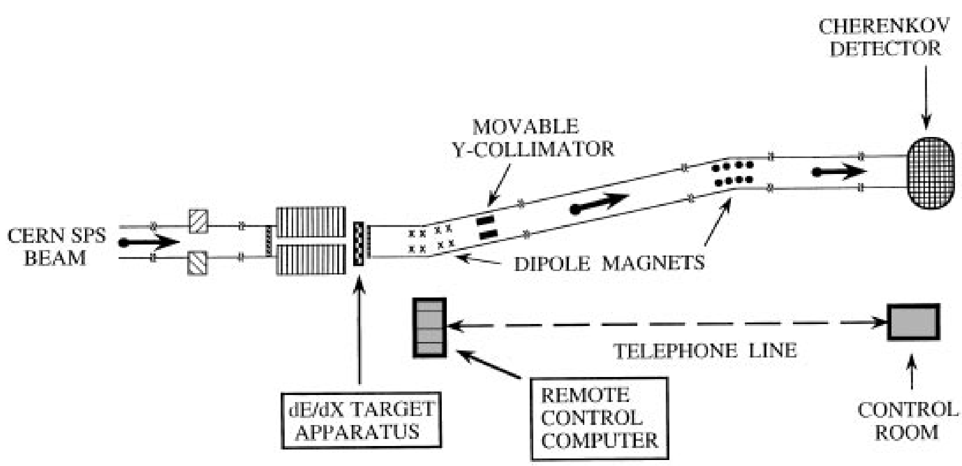


FIG. 2. Schematic of the "magnetic spectrograph" system used at CERN.

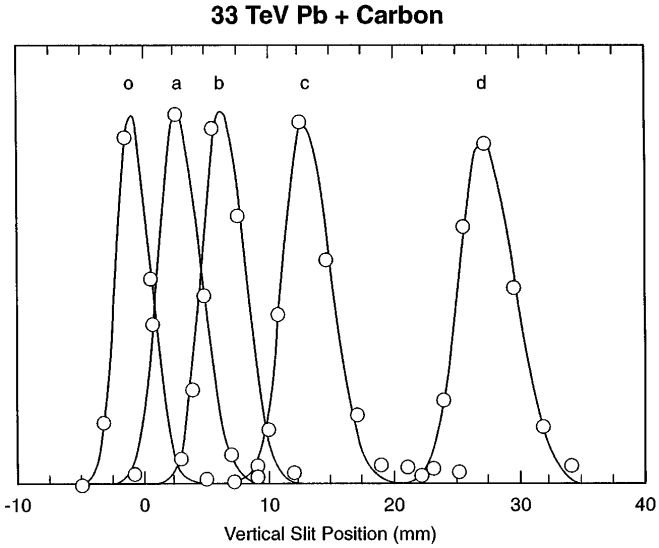


FIG. 3. Lead beam profiles and positions as a function carbon target thickness. $\phi =$ no target, a $= 1.5$ g/cm$^2$, $b =$ 3.0 g/cm$^2$, c $= 6.1$ g/cm$^2$, and d $= 12.2$ g/cm$^2$.

power is determined from the slope of the line for energy loss from the four samples for each of the elements. The measured stopping powers were [in MeV/(mg/cm$^2$)] 15.20 for C, 15.09 for Si, 13.05 for Cu, 12.38 for Sn, and 11.69 for Pb. The error derived from the error in the slope ranged from 0.5% to 1%.

The experimentally determined value of $L_{\rm exp}$ and values calculated with nuclear size effect included $L_{\rm calc}$ (NS), and for a point charge, $L_{\rm calc}$ (PC) are shown in Fig. 4. The values of $\hbar\omega_p$ used in the calculation were 27.63 eV for C ($\rho = 1.84$ g/cm$^2$), 31.05 eV for Si, 58.27 eV for Cu, 50.52 eV for Sn, and 61.13 eV for Pb.

It is evident that the LS prediction of nuclear size effect is confirmed. For low target $Z_2$ such as C and Si, the agreement is within experimental error. However, there is a drift toward higher stopping power in Cu and Sn and a definite deviation in the case of the Pb target. The deviation of $L$ above LS theory for the Pb target corresponds to an increase of $\sim$650 keV/(mg/cm$^2$) in stopping power. Several possible mechanisms for this increase can be considered. One such is electron-positron pair production. The total cross section for pair production for Pb-Pb at $\gamma = 168$ is 3500 b and the mean energy per pair is $\sim$10 MeV [11]. Therefore, pair production can account only for $\sim$110 keV/(mg/cm$^2$) of Pb and proportionately less $(1/Z_2^2)$ for the remaining targets. Another even less likely candidate is Coulomb excitation of the projectile Pb nucleus followed by emission of a photon. Such excitations could be as high as $\sim$3–5 MeV that when multiplied by $\gamma$ could lead to projectile energy losses of $\sim$500–800 MeV per event. However, the loss of 700 keV/(mg/cm$^2$) would require the impossible cross sections of $2-3 \times 10^3$ b for nuclear Coulomb excitation. Thus the cause for the observed increase remains unresolved.

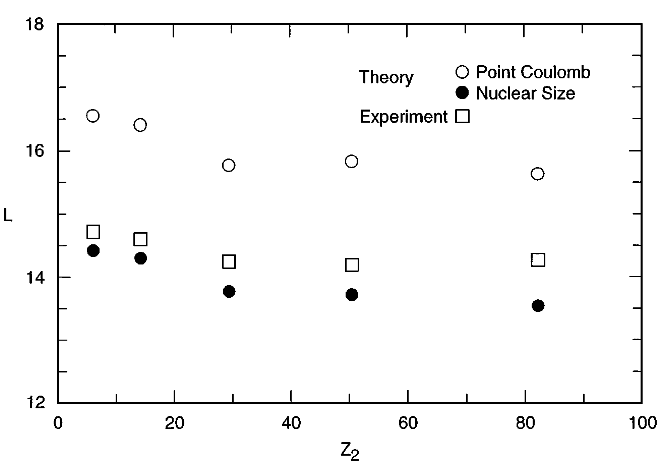


FIG. 4. Values of $L$ calculated from Ref. [7] for point nuclei, ○; for finite nuclear size, •; and the experimental points □ versus target $Z_2$.

*Peak widths.*—Since small impact parameters contribute most to energy straggling, the predicted effect of nuclear size is enormous. The average square fluctuation in energy loss due to electron collisions is given by

$$\frac{d\Omega^2}{dx} = 4\pi Z_1^2 e^4 n_e \gamma^2 X\,. \tag{6}$$

The parameter $X$ is calculated in LS theory for our situation of $\gamma = 168$ Pb ions to be $X = 1.7$ for a point nucleus, but reduces to $X = 0.12$ when one takes into account the nuclear size effect [8].

The peak widths are also affected by multiple Coulomb scattering (MCS) from target nuclei. Both of these effects contribute to the observed width and the experimental system is unable to distinguish between the two.

The two factors can be differentiated by their functional dependence. The multiple Coulomb scattering (MCS) angle $\theta_0$ is determined by nuclear collisions and is given in terms of $T_0$, the radiation length in the absorber,

$$\theta_0 = \frac{13.6\ \mathrm{MeV}}{\beta cp} Z_1 (T/T_0)^{1/2}[1 + 0.038\ln(T/T_0)]\,, \tag{7}$$

where $Z_1$ is the projectile nuclear charge, $cp$ is the projectile momentum in MeV, $T$ is the target thickness, and $T_0$ is the "radiation length"

$$T_0 \cong \frac{716.4\ \mathrm{g}/(\mathrm{cm}^2 A)}{Z_2(Z_2 + 1)\ln(287/Z_2^{1/2})}\,, \tag{8}$$

where $Z_2$ is the absorber atomic number.

There is no question that MCS must contribute to peak width. The question is whether the width, due to energy straggling, can be observed against this background. This is an important question since it is a direct test of Lindhard and Sørensen theory.

The measured angular peak widths are obtained from the measured width with the open beam width deconvoluted. The conversion to corresponding angular spread is obtained from a knowledge of the beam optics starting from the target to the position of the slits 150 m downstream.

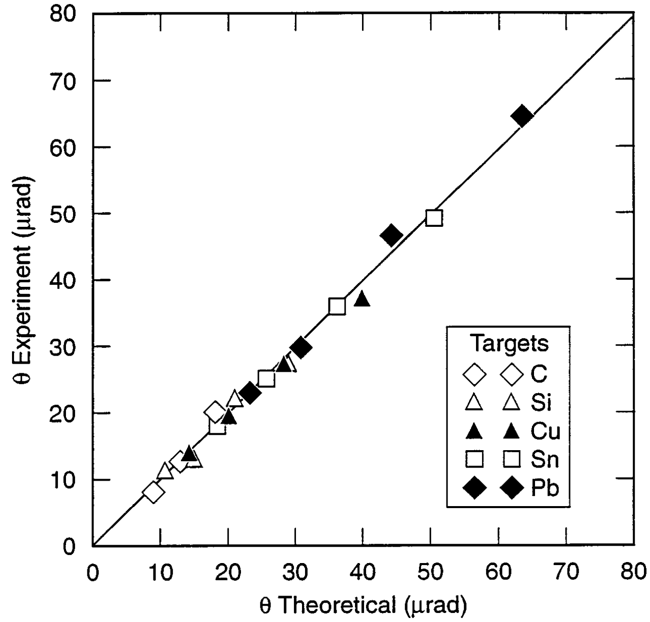


FIG. 5. Calculated multiple Coulomb Scattering (MCS) angle versus measured peak broadening.

The experimentally determined widths for all targets are plotted in Fig. 5 versus the MCS widths calculated from Eqs. (7) and (8). The fit is remarkably good, but this is the minimum broadening that can occur. The question remains: Would additional broadening, due to energy straggling, be observable?

Unlike MCS, which predicts widths that increase with $Z_2$ at fixed energy loss (as observed), energy straggling depends only on the total energy loss. Combining Eq. (1) for the energy loss with Eq. (6) for the energy fluctuation we obtain

$$\Omega^2 = \gamma^2 m_0 c^2 (dE/L) X \,. \tag{9}$$

As an example, a silicon target gives

$$\frac{\Omega}{dE} = \sqrt{\frac{984X}{dE}} \,. \tag{10}$$

If we take a favorable specific case, e.g., Si 15.96 $\mathrm{g/cm^2}$ thick with an energy loss of $242 \times 10^3$ MeV, we would obtain a straggling width of 5350 MeV for $X = 0.12$, and 20 110 MeV for $X = 1.7$. From the known energy calibration, these correspond to widths of 0.77 and 2.88 mm, respectively. When convoluted with the measured width of 6.5 mm, these would correspond to a change in width of 0.06 and 0.68 mm, respectively. We observe no significant increase in width above that given by MCS. However, since even the high value of $X$ gives only a barely discernible width increase, we are unable to draw any firm conclusions concerning energy straggling, other than that the high value is extremely improbable.

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