# Clarification of the Three-Body Decay of ${ }^{12} \mathrm{C}(12.71 \mathrm{MeV})$ 

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(Received 15 April 2003; published 21 August 2003)


#### Abstract

Using $\beta$ decays of a clean source of ${ }^{12} \mathrm{~N}$ produced at the IGISOL facility, we have measured the breakup of the ${ }^{12} \mathrm{C}(12.71 \mathrm{MeV})$ state into three $\alpha$ particles with a segmented particle detector setup. The high quality of the data permits solving the question of the breakup mechanism of the 12.71 MeV state, a longstanding problem in few-body nuclear physics. Among existing models, a modified sequential model fits the data best, but systematic deviations indicate that a three-body description is needed.


DOI: 10.1103/PhysRevLett. 91.082502
PACS numbers: 21.45.+v, 23.40.-s, 27.20.+n

The breakup of a quantum system into three particles presents challenges that have still not been fully met, in particular, when long-range Coulomb forces are combined with short-range stronger forces as in nuclear and particle physics; see, e.g., the recent proceedings [1,2]. Experimentally, such studies are challenging since complete specification of the final state (complete kinematics) requires detection of the momenta of at least two of the three particles. The kinematics is not completely restricted by conservation laws as in the two-particle case. Instead the energy and angular distributions of the fragments reflect different possible breakup mechanisms.

In nuclear and particle physics, the distinction is often made between direct and sequential decay. In sequential decay, two of the particles form an intermediate state and the only correlation between the first emitted particle and the later ones are those due to conservation laws. In contrast, in direct decay there can be dynamic correlations between all particles. (This delineation focuses on a physical interpretation of the two decay modes since it is known that general reaction formalisms, e.g., the $R$ matrix [3,4], in principle can describe all processes.)

The excited states of ${ }^{12} \mathrm{C}$ decaying into $3 \alpha$ final states provide an ideal model case for tests of the breakup mechanism [5]. The normal parity states can decay sequentially via the narrow $0^{+}$ground state of ${ }^{8} \mathrm{Be}$, whereas for the $1^{+}$and $2^{-}$states this decay mode is forbidden by parity conservation. This leaves only sequential decay via the broad $2^{+}$excited state of ${ }^{8} \mathrm{Be}$ or direct decay possible. In fact, due to the short lifetime of the $2^{+}$state, this seems a very unlikely case for sequential decay. Theoretically, the system of three $\alpha$ particles is attractive since the $\alpha$ particle is a spin- 0 boson with well-known
interaction, the charges of all particles have the same sign, and no binary bound states exist.

The $\alpha$ spectrum from the $1^{+}$state at 12.71 MeV in ${ }^{12} \mathrm{C}$ (formed in the reaction ${ }^{13} \mathrm{C}\left({ }^{3} \mathrm{He}, \alpha\right){ }^{12} \mathrm{C}^{*}$ [6]) has been analyzed both in terms of direct decay [5] and sequential decay taking into account Bose symmetry effects [6]. The same data has also been analyzed by Takahashi [7] in a three-body calculation taking into account the final state interactions with the Faddeev equations. Although none of these models succeeded in completely reproducing the data, this case is often mentioned in the literature as an example of a direct decay [8,9]. References [5-7] suggest that complete kinematics data covering the full phase space is needed for a better test of the models, and a clarification of this problem.

We have met this challenge by producing the 12.71 MeV state in the $\beta$ decay of ${ }^{12} \mathrm{~N}$ at the IGISOL facility of the Jyväskylä Accelerator Laboratory, Finland. An important advantage in using $\beta$ decay is that the initial state is produced unpolarized, whereas in [6] the polarization had to be determined in a separate measurement. The activity was produced with the ${ }^{12} \mathrm{C}(p, n){ }^{12} \mathrm{~N}$ reaction with the $10 \mu \mathrm{~A}$ proton beam from the cyclotron. The produced nuclei were subsequently extracted with the IGISOL method [10], accelerated to 40 keV , mass separated, and finally transferred to the detection system where they were stopped in a $50 \mu \mathrm{~g} / \mathrm{cm}^{2}$ carbon collection foil. In this approach, the ${ }^{12} \mathrm{~N}$ beam is implanted at a well-defined depth in the collection foil and the breakup $\alpha$ particles following the $\beta$ decay suffer reduced energy loss compared to the reaction approach of [6].

The detection system consisted of two double sided Si strip detectors (DSSSDs) placed on either side of the
collection foil. This detection system is described in detail in [11]. Important here is that the system provides an efficient detection (total solid angle in the order of $25 \%$ ) of both energy and position of the emitted particles, allowing us to record the breakup in complete kinematics and covering the full phase space. A further crucial advantage is the fact that the energy loss in the collection foil can be calculated and corrected for event by event using the position information from the DSSSDs, as was demonstrated in [11].

A preliminary report on the data from the present experiment has been presented in [12]. With both energy and position information for each detected particle, the final state is completely determined when two particles are detected as the energy and position of the third particle can be reconstructed from energy and momentum conservation. In the upper part of Fig. 1, we represent the complete kinematics data by the Dalitz coordinates $\eta_{1}=E_{1}$ and $\eta_{2}=\left(E_{1}+2 E_{2}\right) / \sqrt{3}$ [13]. The circle marks the boundary of the kinematically allowed region. The lower part of Fig. 1 shows the $\alpha$ spectrum generated from the same events (solid line). The latter cannot be directly compared to the spectra presented in [6] due to the motion of the center-of-mass system induced by the reaction in that experiment. Also shown on this plot are the $\alpha$ spectra obtained from Monte Carlo simulations based on three models to be discussed in the following.

First, we test the assumption that the breakup proceeds directly to the final state (democratic decay) ${ }^{12} \mathrm{C} \rightarrow \alpha_{1}+$ $\alpha_{2}+\alpha_{3}$ with no influence of the $\alpha-\alpha$ interaction. This is shown with the dot-dashed (blue) curve in Fig. 1 as obtained from Eq. (18) of [5]. In this model, the decay amplitude is calculated as the lowest order term in an expansion in hyperspherical harmonics functions [5], which is symmetrized due to the presence of identical bosons in the final state. Note that absence of interactions does not infer a phase-space distribution of the events as the conservation of angular momentum induces correlations among the fragments. Also, quantum correlations from the effects of the symmetrization are present. This model qualitatively describes the presence of three peaks in the $\alpha$-particle spectrum, but it clearly fails to reproduce the position of these peaks. Hence, the $\alpha-\alpha$ interaction does play a role in this breakup. The only existing threebody model [7] taking this interaction into account could not reproduce the previous data [6]. We therefore turn to sequential models.

In the sequential model, one $\alpha$ particle is emitted leaving the two remaining particles in a resonant state of ${ }^{8} \mathrm{Be}$, which subsequently breaks up: ${ }^{12} \mathrm{C} \rightarrow \alpha_{1}+{ }^{8} \mathrm{Be} \rightarrow \alpha_{1}+$ $\alpha_{2}+\alpha_{3}$. The amplitude is fully determined by specifying the relative energy of the two secondary $\alpha$ particles, $E_{23}$, and the angle between the directions of the primary and secondary $\alpha$-particle emissions, $\Theta$. Neglecting Bose symmetry, the $E_{23}$ dependence of the decay amplitude is in the $R$-matrix formalism [3] given by


FIG. 1 (color). The upper part is a Dalitz plot where complete kinematics data for the breakup of the $12.71 \mathrm{MeV} 1^{+}$state in ${ }^{12} \mathrm{C}$ are plotted in the coordinates $\eta_{1}=E_{1}$ and $\eta_{2}=\left(E_{1}+\right.$ $\left.2 E_{2}\right) / \sqrt{3}$. The circle marks the boundary of the kinematically allowed region. The lower part is the $\alpha$ spectrum generated from the same events (solid black line ). The data is compared to Monte Carlo simulations based on three models as discussed in the text. The dot-dashed curve (blue) is a direct breakup model; the dashed (red) and dotted (green) curves are sequential models with and without Bose symmetry effects included.

$$
\begin{equation*}
|f|^{2} \propto \frac{\Gamma_{1} \Gamma_{2}}{\left(E_{23}-E_{0}-\gamma_{2}^{2}\left[S_{l^{\prime}}\left(E_{23}\right)-S_{l^{\prime}}\left(E_{0}\right)\right]\right)^{2}+\Gamma_{2}^{2} / 4}, \tag{1}
\end{equation*}
$$

where $\Gamma=2 P_{l}(E) \gamma^{2}$ with $P_{l}(E)$ the penetrability for either the $\alpha{ }_{-}{ }^{8} \mathrm{Be}\left(\Gamma_{1}\right)$ or $\alpha-\alpha\left(\Gamma_{2}\right)$ breakup, $\gamma^{2}$ is the reduced width, $S_{l}$ is the shift function, and $E_{0}$ is the formal energy of the ${ }^{8} \mathrm{Be}$ resonance. The angular distribution can
be calculated from Ref. [14] to be $W(\Theta)=\sin ^{2}(2 \Theta)$. For the breakup of a $1^{+}$state via ${ }^{8} \mathrm{Be}\left(2^{+}\right)$, the first $\alpha$ particle is emitted with angular momentum $l=2$, while for the secondary emission $l^{\prime}=2$. The dotted (green) curve in Fig. 1 is obtained from Monte Carlo simulations based on Eq. (1). We use the recent parameters from Ref. [15] for the description of the ${ }^{8} \mathrm{Be}\left(2^{+}\right)$resonance. In this model, the central peak is dominated by the first emitted $\alpha$ particle while the other two peaks originate from the
secondary emitted $\alpha$ particles. The angular correlation $W(\Theta)$ determines the shape of these peaks. Although the position of the peaks agrees with data, this model fails to reproduce the depths of the minima. Note that all previous analyses of the $\alpha$ decay of the 12.71 MeV state fed in the $\beta$ decay of ${ }^{12} \mathrm{~N}[16]$ are based on Eq. (1).

The importance of Bose symmetry interference effects in the decay spectrum of the 12.71 MeV state was already discussed in [6] where a modified $R$-matrix expression was also given [Eq. (3) of [6]],

$$
\begin{equation*}
f=\sum_{m_{b}}\left(l m_{a}-m_{b} j_{b} m_{b} \mid j_{a} m_{a}\right) Y_{l}^{m_{a}-m_{b}}\left(\Theta_{1}, \Phi_{1}\right) Y_{l^{\prime}}^{m_{b}}\left(\theta_{2}, \phi_{2}\right) \frac{\sqrt{\Gamma_{1} \Gamma_{2} / \sqrt{E_{1} E_{23}}} e^{i\left(\omega_{l}-\phi_{l}\right)} e^{i\left(\omega_{l^{\prime}}-\phi_{l^{\prime}}\right)}}{E_{0}-\gamma_{2}^{2}\left[S_{l^{\prime}}\left(E_{23}\right)-S_{l^{\prime}}\left(E_{0}\right)\right]-E_{23}-i \frac{1}{2} \Gamma_{2}}, \tag{2}
\end{equation*}
$$

where $\left(\Theta_{1}, \Phi_{1}\right)$ is the direction of emission of the first $\alpha$ particle in the center of mass, $\left(\theta_{2}, \phi_{2}\right)$ is the direction of one of the secondary emissions in the recoil center of mass, $j_{a}$ and $j_{b}$ are the spins of the states in ${ }^{12} \mathrm{C}$ and ${ }^{8} \mathrm{Be}$, and $\omega_{l}-\phi_{l}$ is the Coulomb minus hard sphere phase shift. The $\sqrt{E_{1}}$ and $\sqrt{E_{23}}$ factors are introduced to remove the two-body phase-space factors inherent in the penetrabilities. The final amplitude is obtained by symmetrizing in the coordinates of the three $\alpha$ particles, then squaring, and finally averaging over the initial spin direction $m_{a}$. This result is then multiplied by the appropriate three-body phase-space factor. Because of this procedure, interference effects are introduced into the total breakup amplitude. If the symmetrization step is neglected, Eq. (1) is recovered. The dashed (red) curve in Fig. 1 is obtained from the Monte Carlo simulation based on Eq. (2). The effect of the Bose symmetry interference is to deepen the minima between the three peaks such that the agreement with the data is significantly improved although small deviations remain.

The comparison between data and models is until now based solely on comparing the $\alpha$ spectra. Before turning to the discussion of the deviations between Eq. (2) and the data, we wish to introduce a procedure for extracting more of the information available in the complete kinematics data as presented in the Dalitz plot. The $\alpha$ spectrum corresponds to projecting the Dalitz plot onto the ordinate. An alternative projection scheme is suggested by the spherical symmetry inherent in the Dalitz plot, namely, to separate the radial and angular distribution of the points in the Dalitz plot. Figure 2 shows how an angle and radius are defined from the Dalitz plot, and gives the resulting projections on these two new coordinates. Again the results from Monte Carlo simulations based on the models discussed above are compared to the data. The two models including Bose symmetrization both reproduce perfectly the angular dependence, while Eq. (1) deviates in the depths of the minima. In the radial dependence, Eqs. (1) and (2) are much closer to each other and to the data while the democratic model deviates strongly. Obviously, differences between data and Eq. (2) are still visible in Fig. 2. This indicates that by analyzing the

Dalitz plot in these coordinates effects of Bose symmetry and interactions can be separated to a large extent.

We now turn to a discussion of the deviations between the data and the sequential model. The difference between the data and Eq. (2) is mainly at the low energy and high energy sides of the $\alpha$ spectrum and is kinematically equivalent with there being too few events in the data with a small opening angle between pairs of $\alpha$ particles. This is the expected signature for Coulomb repulsion in the final state. In the parametrization of Eq. (2), the interaction between the first emitted particle and the two particles formed after the breakup of the ${ }^{8} \mathrm{Be}$ intermediate state is ignored. This interaction is expected to be small when the first emitted particle travels a long distance during the lifetime of the intermediate state. In a first estimate, based on a kinetic energy of the order of 1 MeV , the typical distance traveled by the first emitted particle is only $\sim 5 \mathrm{fm}$. At that distance, the electrostatic energy between the first and second emitted $\alpha$ particles is $\sim 1 \mathrm{MeV}$, and therefore the relevant Coulomb barrier for the secondary breakup ought to be significantly modified from that assumed in the $R$-matrix formalism. Alternatively, in the tunneling picture of the first emission process, the $\alpha$ particle emerges outside the Coulomb barrier of the $\alpha-{ }^{8} \mathrm{Be}$ system at a distance closer to $\sim 10 \mathrm{fm}$ with a corresponding reduction of the electrostatic energy. By simply introducing extra barrier penetrabilities in the numerator of Eq. (2) for each of the two secondary $\alpha$ particles in the final state, and adjusting the radius used in the calculation of the penetrability to $\sim 10 \mathrm{fm}$ between the $\alpha$ particles, near perfect agreement can be achieved. These estimates are rather crude, but serve to underline that dynamic correlations are present, i.e., a full three-body model is needed to reproduce the data fully. Still, we believe the main physical ingredients are identified.

Related final state Coulomb effects have previously been identified for the $3 \alpha$ system $[17,18]$ where the energy shifts were calculated by solving the classical Hamilton equations for the three particles in the final state. Interestingly, these calculations could only reproduce


FIG. 2 (color). The angle (top) and radius (lower) projections from the Dalitz plot of Fig. 1. On the projections are also given the corresponding results from Monte Carlo simulations from three models for the breakup as discussed in the text. The lines are marked as in Fig. 1.
the data with a seemingly unphysically large distance traveled by the first emitted particle.

In conclusion, we have measured the $3 \alpha$ breakup of the 12.71 MeV state in ${ }^{12} \mathrm{C}$ in complete kinematics with unprecedented precision. The sequential model Eq. (2) can describe the data rather well when correlations due to conservation laws and Bose symmetry are included. The remaining deviations indicate that dynamic correlations beyond the sequential model are important for this
breakup. To get a complete description of the data, quantum mechanical three-body models will be needed.

Although it might seem surprising that the sequential model fits the data relatively well, we note that many other nuclear breakup processes can be described in a similar way [19-22]. It will be interesting to see if in the future ground state two-proton radioactivity $[23,24]$ can also be described in a sequential model.

We thank D.V. Fedorov, A. S. Jensen, I. Mukha, and D. Schwalm for fruitful discussions on this work. This research was supported by the Academy of Finland under the Finnish Centre of Excellence Programme 2000-2005 (Project No. 44875, Nuclear and Condensed Matter Physics Programme at JYFL), by the Spanish Agency CICYT under Contract No. AEN99-1046-C02-02, and by the European Union Fifth Framework Programme "Improving Human Potential-Access to Research Infrastructure."
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