Accelerated expansion from structure formation

Syksy Räsänen

CERN, Physics Department Theory Unit, CH-1211 Geneva 23, Switzerland

E-mail: syksy *dot* rasanen *at* iki *dot* fi

Abstract.
We discuss the physics of backreaction-driven accelerated expansion. Using the exact equations for the behaviour of averages in dust universes, we explain how large-scale smoothness does not imply that the effect of inhomogeneity and anisotropy on the expansion rate is small. We demonstrate with an analytical toy model how gravitational collapse can lead to acceleration. We find that the conjecture of the accelerated expansion being due to structure formation is in agreement with the general observational picture of structures in the universe, and more quantitative work is needed to make a detailed comparison.

PACS numbers: 04.40.Nr, 95.36.+x, 98.80.-k, 98.80.Jk

arXiv:astro-ph/0607626 v3   9 Nov 2006

Contents

1. Introduction

Evidence for acceleration. There is a large body of observational evidence supporting the claim that the expansion of the universe has accelerated in the recent past, and may be accelerating today. This conclusion has been bolstered by the verification of the prediction of the location of the baryon acoustic peak in the matter power spectrum [1], in a convincing demonstration of concordance. In addition, the worrisome feature that nearby and distant populations of type Ia supernovae used to have different absolute magnitudes and were both individually consistent with deceleration has disappeared with new and better data [2] (though see [3, 4]).

The ΛCDM model, where the acceleration is driven by vacuum energy (or the cosmological constant, which is the equivalent modification of gravity) agrees well with most observations, with the notable exception of the low CMB multipoles [5–9] (it has also been argued that cluster observations support a non-accelerating universe [10]). However, given the lack of theoretical understanding about the parameters of the ΛCDM model (notably the vacuum energy density), it is a phenomenological fit rather than a well-founded theory, and its success does not rule out the possibility that quite a different model can also be a good fit to the data. (The values obtained for the parameters of a cosmological model by fitting to observations should not be mistaken for measurements, as model selection studies show; see e.g. [3, 4, 11].) In particular, while the observation that there is accelerating expansion seems robust, the nature of the acceleration is not well constrained. In the ΛCDM model, the transition to acceleration is gradual, but a rapid transition is not ruled out [12, 13]. In fact, from the SNIa data it is difficult to say anything beyond that the universe has accelerated in the recent past, even whether the expansion is still accelerating [3, 4, 14].

Keeping to the assumption that the universe is completely homogeneous and isotropic, any explanation of the acceleration has to involve either a medium with negative pressure or modified gravity. Such models in general, and the ΛCDM model

in particular, suffer from *the coincidence problem*: why does the acceleration happen around a redshift of unity, at around 10 billion years? In other words, why are we seeing a very particular phase in the evolution of the universe, when the inferred energy density of the source driving the acceleration has recently become equal to the energy density of matter? The clearest qualitative change in the late-time universe is the formation of non-linear structures. It therefore seems a natural possibility that the observed deviation from the prediction of homogeneous and isotropic cosmological models with normal matter and gravity could be related to the known breakdown of the assumption that the universe is homogeneous and isotropic (rather than to a speculated failure in the description of the matter content or the theory of gravity).

The inhomogeneous universe.   One possible avenue is trying to explain the observations without having any accelerated expansion. Cosmological information is borne to us by light along null geodesics (apart from information carried by neutrinos and cosmic rays). The standard analysis of light propagation assumes that the universe is perturbatively near a homogeneous and isotropic Friedmann–Robertson–Walker (FRW) model, which is manifestly not true on scales smaller than 70-100 h^{-1}Mpc [15–17].

It is therefore possible that the propagation of light would be affected by the inhomogeneities and/or anisotropies in a way that looks like acceleration when interpreted in the context of an FRW model. Studies of the Lemaître–Tolman–Bondi (LTB) model [18] (see [19] for a review), the spherically symmetric dust solution of the Einstein equation, have demonstrated that the effect of inhomogeneity on the luminosity distance can mimic acceleration [20, 21] (see [21] for more references). Even though spherical symmetry is a questionable assumption for the entire universe, it could be a good first approximation for the local region. In any case, one would expect a qualitatively similar effect to be present also in more realistic and less symmetric spacetimes [22] – arguably, the effect of clumpiness could even be stronger when there is less symmetry. The effect of inhomogeneity and anisotropy on the luminosity distance has also been studied in perturbed FRW models [23, 24].

An explanation of the apparent acceleration in terms of inhomogeneity and/or anisotropy could solve the coincidence problem, since inhomogeneity and anisotropy become important only in the late-time universe. However, inhomogeneity and anisotropy affect different observations in different ways, and it would require an odd coincidence for all the various indicators of expansion rate (SNIa luminosity distances, the cosmological microwave background (CMB) anisotropies, large scale structure (LSS), and so on) to be affected in a way that would be consistently interpreted as acceleration when fitting to a FRW model.

One proposed possibility is that we live in an underdense region, a 'Hubble bubble' [25] (for more references, see [21]). In this proposal, the local matter density today is $\Omega_{m0} \approx 0.15\text{-}0.35$, as indicated by local observations [26], while the global value is $\Omega_m = 1$. The SNIa luminosity distances as well as the difference between the local and global values of the expansion rate could be explained in terms of inhomogeneity, while

a global model with no acceleration can fit most other observations, including the CMB and LSS [27] (though it is not supported by studies of the local and global expansion rate [28]). However, it would be difficult to explain the baryon acoustic peak [29]: one would have to appeal to inhomogeneities (or features in the primordial power spectrum) to supply a pattern that by coincidence happens to fit the expectations of an accelerating FRW model.

One way to phrase the issue is that cosmological observations involve a larger number of a priori independent parameters than the ΛCDM model. Therefore the ΛCDM model implies relations between observationally independent parameters. (For an early discussion of cosmological observations in an inhomogeneous and anisotropic spacetime which makes the issue transparent, see [30].) It is not surprising that models with more degrees of freedom, such as the LTB model which involves two arbitrary functions, could fit the data as well as FRW models. However, it would be an unlikely coincidence for them to also produce the same relations between observables as FRW models. This is generally true for any models where the explanation of the luminosity distances is decoupled from the explanation of the low matter density, baryon acoustic peak and so on (such as mixing of photons with axions [31] or with the gauge bosons of a new $U(1)$ gauge group [32]).

The fitting problem. While the FRW scale factor has been very successful in fitting observations, it is difficult to understand the matter content implied by the FRW equations which relate the scale factor to the energy–momentum tensor. Note that the fact that the mean properties of the universe are well described by an overall scale factor does not imply the stronger statement that the scale factor evolves according to the FRW equations, since the universe is not completely homogeneous and isotropic.

The idea that the average behaviour of inhomogeneous and/or anisotropic spacetimes is in general different from the behaviour of homogeneous and isotropic spacetimes goes back to at least 1963 [33]. The first comprehensive discussion was given in 1983 by George Ellis [34], who called the task of finding the smooth metric which best fits the real clumpy universe *the fitting problem.* The influence of inhomogeneity and/or anisotropy on the average behaviour is also known as backreaction [23, 35–82]; see [19, 54] for further references, in particular early ones, and [83] for an overview.

The idea that perturbations with wavelengths smaller than the Hubble radius could lead to acceleration for the scale factor (as opposed to merely mimicking the appearance of acceleration via changing null geodesics) was studied in the context of linear perturbation theory in [54] (the possibility had been earlier touched upon in [44, 48]; see also [50]). The calculation was then extended to second order [23, 58], and it was suggested that linear perturbations with wavelengths much larger than the Hubble radius could lead to acceleration [23, 58–60]. It is now agreed that this is not possible [61–63, 65, 69]‡. As for perturbations smaller than the Hubble radius, there

‡ Even though super-Hubble perturbations do not contribute to acceleration in a dust universe, they could lead to deceleration during inflation driven by a scalar field or a cosmological constant,

is no acceleration to at least second order in perturbation theory [54, 58, 69, 77, 78]. If inhomogeneity and anisotropy are to explain the observed acceleration, the only possibility is via non-linear sub-Hubble perturbations, that is, the process of structure formation, as proposed in [50, 54].

It has been analytically shown in the LTB toy model how backreaction of non-linear perturbations can modify the Hubble law [57], and acceleration has also been numerically demonstrated in the LTB model [76, 79, 80], but the physical meaning of inhomogeneity- and anisotropy-driven acceleration and the connection to structure formation has been unclear. We will discuss the relation between homogeneity and isotropy, the overall scale factor, the FRW metric and the equations which describe the average expansion of the universe. We will then look at an exact toy model of structure formation, explain the physics of acceleration driven by inhomogeneity and anisotropy, and note that structure formation involves a preferred time near the observed acceleration era. In particular, we will clarify two apparent paradoxes of backreaction-driven acceleration: how the average expansion of a manifold can accelerate even though the local expansion rate decelerates everywhere, and how collapse implies acceleration. These issues were earlier discussed in the brief essay [81].

In section 2 we discuss the assumptions underlying FRW models, and go through the derivation of the Buchert equations which describe the average behaviour of an inhomogeneous and/or anisotropic dust spacetime. In section 3 we consider an exact toy model where gravitational collapse produces acceleration, discuss how this mechanism may operate in the real universe, and compare this picture with some observations and simulations of structures in the universe. In section 4 we summarise the situation with regard to the conjecture that the observed acceleration is due to backreaction.

2. Smoothness and variance

2.1. The Friedmann–Robertson–Walker assumptions

The three assumptions. The assumption of homogeneity and isotropy that underlies the Friedmann–Robertson–Walker models of cosmology can be broken down into three distinct parts.

Assumption 1: *FRW scale factor.* The observables characterising the mean properties of the universe can be computed from an overall scale factor.

Assumption 2: *FRW dynamics.* The overall scale factor evolves according to the FRW equations.

Assumption 3: *FRW + perturbations.* Deviations from homogeneity and isotropy evolve according to linear perturbation theory around the average.

and super-Hubble scalar field perturbations left over from inflation could still be important today [37, 39, 49, 52, 53, 67, 68, 72, 73, 75].

Since the universe is not exactly homogeneous and isotropic, it is clear that the above assumptions do not hold exactly. However, there is usually no attempt to quantify the deviation, and the first two assumptions are often conflated. A notable exception is the program of observational cosmology (and related work) by George Ellis and collaborators, which aimed at formulating cosmological theory in a manner that does not involve a priori assumptions about homogeneity and isotropy and that is as close to the observations as possible [84–87]; see [88, 89] for an overview.

We will look at the above assumptions from the slightly different point of view of backreaction studies, where the emphasis is not so much on establishing the level of inhomogeneity and isotropy of the universe as on quantifying their effect on the expansion rate and other observables. Let us discuss the three assumptions in turn.

Assumption 1: FRW scale factor. Observations indicate that the universe is statistically homogeneous and isotropic on large scales. It has recently become possible to directly establish from the Sloan Digital Sky Survey, for the first time, the average homogeneity of the universe by looking at the fractal dimension of the point set of luminous red galaxies [15, 16]. The related homogeneity scale has been quantified as 70-100 h^{-1}Mpc, though analysis based on the morphology of structures indicates a homogeneity scale that is larger than 100-200 h^{-1}Mpc [17]. Dividing the observational volume into 10 regions with individual volume 2×10^7 $(h^{-1}\text{Mpc})^3$ (corresponding to a ball with radius ≈ 170 h^{-1}Mpc), the density variance is 7% in the redshift range $0.2 < z < 0.35$, quantifying the degree of statistical homogeneity in present observations. The largest structure known is the Sloan Great Wall at 420 h^{-1}Mpc, which has superseded the old 240 h^{-1}Mpc Great Wall. These sizes are 14% and 8%, respectively, of the Hubble radius; in the Einstein-de Sitter universe (the spatially flat matter-dominated FRW model) this would be 7% and 4%, respectively, of the visual horizon. Structures this large are rare, and the typical size of observed collapsing structures and voids is ≈ 20-40 h^{-1}Mpc, of the order 10^{-3} to 10^{-2} of the visual horizon [90–92] (though some simulations suggest that a significant fraction would be larger [93] or smaller [94]).

Statistical isotropy is in turn supported by the high degree of isotropy in the CMB, along with the 'almost Ehlers–Geren–Sachs theorem', which states that a universe where the CMB looks almost isotropic everywhere is almost FRW on large scales [95, 96] (see [97, 98] for discussion and caveats). The applicability of the 'almost EGS theorem' to the real universe is, in fact, somewhat unclear. The proof of the theorem requires the assumption that the expansion rate is positive everywhere, which is not valid in the real universe unless scales where structure formation by gravitational collapse is relevant are smoothed over. Even then, it is unclear how the strict limits (essentially given by the CMB anisotropy of 10^{-5}) on spatial variation of the exansion rate and other observables can be reconciled with the observed (and theoretically expected) differences of order one in the expansion rate and energy density in non-linear regions (we will discuss the observations in section 3.3). Nevertheless, it seems intuitively clear that the isotropy of the CMB (coupled with the assumption that we do not occupy a preferred position in

the universe) indicates a high degree of average isotropy in the geometry on large scales.

Statistical homogeneity and isotropy show that a description in terms of an overall scale factor could be consistent. In physical terms, because the size of typical non-linear regions is small, light rays coming to us cosmological distances pass through several such regions on the way to us, and the differences could be expected to average out. However, the small size of non-linear regions does not prove that a description in terms of an overall scale factor is necessarily correct. We receive almost all cosmological information along null geodesics, and from the fact that inhomogeneities and anisotropies are small when averaged over large scales it does not follow that a description in terms of an overall scale factor will correctly capture the physics of light propagation, as discussed in section 1.

The FRW metric is conformally flat, so the Weyl tensor which embodies the non-local effects of gravity vanishes, while the Ricci tensor, determined by the local matter distribution, is non-zero. However, photons in the real universe only occasionally encounter matter and mostly travel in vacuum, where the Weyl tensor is non-zero but the Ricci tensor vanishes (neglecting the small contribution of the microwave and neutrino backgrounds). The FRW description is exactly the opposite of the real situation, and it is not obvious why it would correctly capture the physics of the passage of light in the real inhomogeneous and anisotropic universe. From a theoretical point of view, this issue has not been satisfactorily settled [99–102]; for an overview, see [83] and for further references, see [21, 83].

A related issue is that cosmological models are usually discussed in terms of hypersurfaces of proper time, whereas observations are mostly made along the past lightcone. This issue was considered in detail in the program of observational cosmology [85]. In FRW models this distinction is not important, and they enjoy (for monotonous expansion) a simple one-to-one correspondence between time and redshift. In an inhomogeneous and/or anisotropic universe, the issue is more complicated, and is related to the problem of choosing the hypersurface of proper time [49, 53, 54, 57]. Looking only at an average scale factor has been criticised in [71] (see also [74]) on the grounds that one obtains different behaviour for different choices of time slicing.

In FRW models with only a single fluid, there is a preferred time coordinate given by the proper time measured by observers comoving with the fluid. This notion can be straightforwardly extended to inhomogeneous and/or anisotropic dust spacetimes which do not have any symmetries. If the matter consists of irrotational dust, then such hypersurfaces of constant proper time are everywhere orthogonal to the fluid flow lines, and the situation is analogous to the FRW case. If vorticity is present, the hypersurfaces of proper time will not mesh together to fill the spacetime [103, 104], and the situation is more complicated [63, 65, 96]. (During inflation driven by a cosmological constant or a scalar field, the issue of time slicing and observables is more involved [39, 49, 53, 68].) In any case, averaging does involve a loss of covariance; see [105] for discussion.

The approximation that the matter is an irrotational, pressureless ideal fluid will necessarily break down at small scales once structure formation has started [106, 107], as otherwise collapsing structures could not stabilise, given that the local expansion rate

of irrotational dust can never increase. In other words, structure formation and the associated vorticity need to be considered and may be important for backreaction. As we will see, zero vorticity is an important mathematical assumption for the definition of averages and an overall scale factor. However, from a physical point view one would not expect the vorticity associated with structure formation to make a quantitative difference for the expansion of the universe. The issue should be carefully considered, but we shall simply assume that the small-scale breakdown of the picture of matter as an irrotational ideal fluid with zero pressure is not important.

Note that the problem of time and the averaging hypersurface is not an artifact of inhomogeneous and/or anisotropic models, but a feature of the general relatistic description of the real universe. It is a virtue of the observational cosmology and backreaction approaches that they make these issues explicit and provide tools for studying and quantifying them, unlike FRW models.

A seemingly worrisome aspect of the hypersurfaces of constant proper time is that observables averaged over them depend on regions outside the visual horizon, since the hypersurfaces extend beyond the past lightcone. It may seem unphysical that an observational quantity would change if we changed the definition of the hypersurface in regions beyond our past lightcone. However, if the universe is statistically homogeneous and isotropic on large scales, we do not have the freedom to independently adjust the hypersurface of proper time in the regions inside and outside our past lightcone, since the state of matter and geometry inside and outside the lightcone is required to be statistically identical for the same proper time. (See [84,86] for discussion of homogeneity in cosmology.) This puzzle is therefore just the homogeneity and isotropy problem, which is solved (or at least alleviated [108]) by inflation. The statistical equality of widely separated regions was set up in the early universe when they were in causal contact, so the particle horizon is much larger than the visual horizon. (For clarification of the different horizons, see [109, 110].) In spacetimes which are not statistically homogeneous and isotropic, the averaging procedure would not be expected to be useful, and a description in terms of an overall scale factor would probably not make sense. For example, this would be the case if the size of typical non-linear structures were a sizeable fraction of the visual horizon.

We will simply extend the FRW notions of proper time and scale factor in the most straightforward manner (as we will discuss in section 2.2) and assume that redshift is related to the scale factor in the same way as in FRW models. Dependence on proper time could then be expressed in terms of redshift, as in the FRW case, so that the problem discussed above is not apparent. Of course, this is simply a matter of rewriting the equations in a manner that makes the assumption of statistical homogeneity and isotropy less transparent.

Properly, one should derive the quantitive conditions under which the scale factor approximation is valid and see to which extent they are realised in the universe. This would involve defining redshifts, luminosity distances and other observables using null geodesics in the inhomogeneous and anisotropic universe, and determining under

which conditions these reduce to the quantities defined with the overall scale factor. As discussed above, the issue of average light propagation in inhomogeneous and/or anisotropic spacetimes has not been satisfactorily settled from the theoretical point of view.

Nevertheless, from an observational point of view, the scale factor ansatz has been very successful in fitting a range of observations. This success is all the more remarkable as different observations depend on null geodesics in a different manner. For example, there are several different definitions of the expansion rate (such as the volume expansion rate, the rate of deviation between neighbouring geodesics, the expansion rate inferred from the luminosity distance and so on), which agree for a homogeneous and isotropic space but differ when inhomogeneities and/or anisotropies are present. Yet, expansion rates inferred from different observations agree to within $\approx 20\%$ [28]. The approximation of considering only the scale factor has worked very well in practice, whatever its theoretical status, and the observed statistical homogeneity and isotropy on large scales at least guarantees that this approximation is consistent.

Note that even small corrections to the approximation of looking only at the scale factor can be interesting for the light they shed on the average behaviour. For example, the dipole of the angular power spectrum of the inhomogeneous luminosity distance in a linearly perturbed FRW model yields a direct measure of the Hubble parameter as a function of redshift [24]. Also, even if the passage of light through cosmological distances, and thus many non-linear regions, is well described in terms of a scale factor, this does not rule out the possibility that local structures could affect null geodesics in a way that is not captured by that approximation. In particular, the anomalies of the low CMB multipoles [5–9] could be related to the local breakdown of the description in terms of a linearly perturbed FRW metric [111, 112].

(Another distinct issue, which we will not discuss, is the "dressing" of cosmological parameters, i.e. the feature that the usual interpretation of observations does not account for the geometrical inhomogeneities and/or anisotropies in the averaging domain when considering observables [46, 51, 55].)

Even if the effects of inhomogeneity and anisotropy cancel on large scales so that the FRW *scale factor* is a good average description, this does not mean that the FRW *metric* would be a good approximation of the average geometry. The reason is that the FRW metric also contains the spatial curvature, which is assumed to be homogeneous and isotropic. In a general spacetime, the spatial curvature is inhomogeneous and anisotropic, and does not evolve like the FRW spatial curvature, even on average. The reason is that the evolution of non-linear regions with positive spatial curvature and those with negative spatial curvature is different, and they are not correlated so as to produce the FRW behaviour. In particular, even if a universe is perturbatively close to spatial flatness early on, this condition is not necessarily preserved once density perturbations become non-linear.

In mathematical terms, this is related to the fact that the FRW evolution $\propto a^{-2}$ of the spatial curvature in terms of the scale factor a arises from the integrability

condition between the Raychaudhuri equation and the Hamiltonian constraint. The two receive different corrections from the inhomogeneities and anisotropies, so the general integrability condition does not agree with the FRW case and the average spatial curvature does not in general evolve like a^{-2}. From a physical point of view, regions with negative spatial curvature expand faster than regions with positive spatial curvature, so one would expect that they will come to dominate the volume and the average spatial curvature will become negative. The non-FRW evolution of the spatial curvature and the competition between overdense and underdense regions is central to the proposed backreaction explanation for accelerated expansion, and we will discuss these issues in detail in sections 2.2 and 3.1. The behaviour of spatial curvature is at the heart of the distinction between the scale factor being a good description and the scale factor following the FRW equations, an issue to which we now turn.

Assumption 2: FRW dynamics. As discussed above, we will simply assume that the average properties of the universe can be described in terms of an overall scale factor, without deriving the conditions under which this assumption is valid. In contrast, when considering the dynamics, we will write down the exact equations governing the evolution of the scale factor, quantify the domain of validity of the FRW equations and discuss the impact of the corrections. Indeed, while there is strong observational support for the FRW scale factor, the time evolution given by the FRW equations is quite different from what is observed, unless the equations are amended by adding a source term with negative pressure or by modifying gravity.

It has been suggested that in order to emphasise the difference between the geometry and the dynamics, the names Robertson–Walker would be associated with the assumption that the geometry is approximately homogeneous and isotropic, while the stronger assumption that the dynamics of the scale factor is given by the Einstein equation applied to a completely homogeneous and isotropic metric would bear the names Friedmann–Lemaître [88, 89]. (As discussed above, the present situation is slightly different in that we are not even assuming that the geometry is described by a homogeneous and isotropic metric, simply that we can use an overall scale factor.)

Simply inserting the overall scale factor into the Einstein equation is not the correct way to find the dynamical equations which it satisfies. Instead, one should insert the full inhomogeneous and/or anisotropic metric into the Einstein equation and only then take an average, since the average behaviour of an inhomogeneous and/or anisotropic spacetime is not the same as the behaviour of the corresponding smooth spacetime (where "corresponding" means that the smooth and average quantities have the same initial conditions). In other words, the average properties of an inhomogeneous and/or anisotropic spacetime do not satisfy the Einstein equation. The fact that taking an average metric and plugging it into the Einstein equation and plugging the real metric into the equation and then averaging are not equivalent is sometimes expressed by saying that because the Einstein tensor $G_{\mu\nu}$ is non-linear in the metric, we have $\langle G_{\mu\nu}(g_{\alpha\beta}) \rangle \neq G_{\mu\nu}(\langle g_{\alpha\beta} \rangle)$, where $\langle \rangle$ stands for averaging. It would be more accurate

to say that the problem arises because time evolution and averaging do not commute. (The statement about averaging the Einstein equation is anyway rather heuristic because tensors cannot be straightforwardly averaged on curved manifolds; though see [113].) This is the origin of the fitting problem discussed in section 1.

The standard assumption is that even after non-linear structures have started forming, the average evolves according to the FRW equations if smoothing on the scale of the non-linearity is performed. However, the equations for the mean expansion which are actually derived, rather than assumed, do not bear out this expectation [36, 41, 47]. As we will discuss in section 2.2, inhomogeneities and anisotropies affect the behaviour of the scale factor, and neither statistical homogeneity and isotropy nor the small size of the inhomogeneous and anisotropic regions (relative to the visual horizon) is a sufficient condition for recovering the FRW behaviour.

A different reason why the FRW equations could be invalid even though the universe is very homogeneous and isotropic on large scales has also been advanced. It has been suggested that the breakdown of the approximation of treating the matter as an ideal fluid involves negative pressure which might explain the acceleration [50]. An argument against the treatment of matter as an ideal fluid goes as follows. Small-scale processes in the universe are in general not thermodynamically irreversible but instead produce entropy. Since there is no production of negative entropy, this entropy generation does not vanish upon averaging. In contrast, in the approximation of an adiabatically expanding ideal fluid, the entropy of the universe is constant, and such small-scale effects are completely absent. In numerical terms, a single 3×10^6 solar mass black hole, such as the one at the center of our galaxy, has an entropy of 10^{90}, of the same order of magnitude or more as the total entropy ascribed to the observable universe in the ideal fluid picture. Supermassive black holes are abundant in the universe, so the entropy associated with them alone completely overwhelms the entropy associated with the adiabatic fluid. On the other hand, in the average description of an ideal fluid spacetime it may look as if the entropy was increasing, even though there is no local entropy production [56]. The issue of gravitational entropy is tied up with coarse-graining and thus backreaction, and is not well understood [83]. Whether these problems of the ideal fluid description are important for the expansion rate is not clear.

A related issue is whether one can neglect the influence of the structure within stabilised regions on the overall expansion, i.e. whether one can continuously and consistently "renormalise" the scale of the stable regions which are treated as particles of the dust fluid [114].

We will not study these two issues, and keep to the assumption that deviations from the dust behaviour are not important for the overall expansion.

Assumption 3: FRW + perturbations. The region of validity of the assumption that inhomogeneities and anisotropies on a FRW background evolve according to linear perturbation theory is well-known, and its breakdown at the end of the linear regime is well understood. As long as the density contrast $\delta \equiv (\rho - \langle \rho \rangle)/\langle \rho \rangle$ of a perturbation

is small, it evolves according to linear perturbation theory. In a spatially flat matter-dominated FRW background, the growing mode is proportional to the scale factor, $\delta \propto a$. As δ becomes of order ± 1, the linear approximation breaks down. An overdensity will collapse and the density contrast will grow faster than in the linear regime, whereas an underdensity will grow more slowly than in the linear regime, asymptotically approaching emptiness.

For intermediate regime perturbations which have not yet gone non-linear but are nevertheless small enough to be located inside a non-linear structure, one would expect the evolution to depend on the environment and not just on the overall average expansion of the universe. For example, the evolution of a 20 h^{-1}Mpc radius perturbation would be expected to be different inside a 50 h^{-1}Mpc void or a 200 h^{-1}Mpc wall. Such perturbations, which are larger than the size of typical structures and yet fit inside non-linear regions, are by definition untypical, and one can argue that their effect on the mean evolution of perturbations is small.

However, it is not obvious that linear perturbation theory around the average in a space which is highly inhomogeneous and anisotropic correctly captures the evolution even for perturbations with wavelength longer than the size of the largest structures. Regions with large overdensities or underdensities presumably contribute differently to the evolution of the long-wavelength modes, and it is not clear that these effects would cancel to give the same answer as linear perturbation theory around the average.

The separation into background and perturbations is more involved in a universe with large inhomogeneities and anisotropies than in the FRW case, and in section 2.2 below we will show that linear perturbations do not in general satisfy the same perturbation equation as in FRW models. We will not discuss the issue further, but in a realistic model of backreaction, the sensitivity of perturbations to smaller scale inhomogeneities and anisotropies should be looked at in more detail, particularly when these have a large effect on the background. For work on perturbations in a backreaction context, see [38, 40, 42, 43, 48].

2.2. The Buchert equations

The metric and the local equations. We assume that the matter content of the universe can be described as dust, i.e. a pressureless ideal fluid. We further assume that the dust is irrotational (i.e. the vorticity is zero). Then the metric can be written in the synchronous gauge [103, 104]

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + {}^{(3)}g_{ij}\mathrm{d}x^i\mathrm{d}x^j \ , \tag{1}$$

where t is the proper time measured by observers comoving with the dust and ${}^{(3)}g_{ij}(t, \boldsymbol{x})$ is the metric on the hypersurface of constant t. The Einstein equation reads

$$\begin{aligned} G_{\mu\nu} &= 8\pi G_N T_{\mu\nu} \\ &= 8\pi G_N \rho \, u_\mu u_\nu \ , \end{aligned} \tag{2}$$

where $G_{\mu\nu}(t, \boldsymbol{x})$ is the Einstein tensor, G_N is Newton's constant, $T_{\mu\nu}(t, \boldsymbol{x})$ is the energy–momentum tensor, $\rho(t, \boldsymbol{x})$ is the dust energy density and $u^\mu = (1, \mathbf{0})$ is the velocity of comoving observers.

We wish to find the equations for average quantities. Since only scalars can be straightforwardly integrated on a curved manifold (though see [113]), we should project (2) to obtain a set of scalar equations. In addition to u^μ and $g^{\mu\nu}$, we have the covariant derivative ∇^μ available. From these we can build three independent rank two tensors to project with§, so the Einstein equation (2) yields the following three exact, local, covariant scalar equations [103, 104, 115]

$$\dot\theta + \frac{1}{3}\theta^2 = -4\pi G_N \rho - 2\sigma^2 \tag{3}$$

$$\frac{1}{3}\theta^2 = 8\pi G_N \rho - \frac{1}{2}{}^{(3)}R + \sigma^2 \tag{4}$$

$$\dot\rho + \theta\rho = 0 , \tag{5}$$

where a dot stands for derivative with respect to t, $\theta(t, \boldsymbol{x}) = (\sqrt{{}^{(3)}g})^{-1}\partial_t(\sqrt{{}^{(3)}g})$ is the expansion rate of the local volume element, $\sigma^2(t, \boldsymbol{x}) = 2\,\sigma^{ij}\sigma_{ij} \geq 0$ is the scalar built from the shear tensor σ_{ij}, and ${}^{(3)}R(t, \boldsymbol{x})$ is the Ricci scalar of the hypersurface of constant t (i.e. the spatial curvature). The acceleration equation (3) is known as the Raychaudhuri equation, and (4) is the Hamiltonian constraint.

The price for reducing the Einstein equation down to a set of scalar equations is that the system is not closed: there are three equations for four independent variables. Essentially, the propagation of the shear tensor (or equivalently, of the Ricci tensor on the hypersurface of constant t) does not reduce to a scalar equation. The integrability condition between (3) and (4) reads

$$\partial_t({}^{(3)}R) + \frac{2}{3}\theta \; {}^{(3)}R = 2\,\partial_t\sigma^2 + 4\theta\sigma^2 , \tag{6}$$

so specifying either the shear or the spatial curvature fixes the other.

Note that no approximations have been made: the equations (3)–(5) are exact for irrotational dust, with arbitrarily large density variations.

Deriving the Buchert equations. We are interested in the evolution of average quantities: specifically, we want to know how the average expansion rate behaves. Our discussion follows the original derivation by Buchert [41]. When (and only when) the vorticity is zero, the rest spaces of constant proper time of comoving observers mesh together to form a family of hypersurfaces which fills spacetime. The spatial average of a scalar quantity f is then straightforwardly defined on these hypersurfaces as

$$\langle f\rangle(t) \equiv \frac{\int d^3x \sqrt{{}^{(3)}g(t, \boldsymbol{x})}\, f(t, \boldsymbol{x})}{\int d^3x \sqrt{{}^{(3)}g(t, \boldsymbol{x})}} , \tag{7}$$

where the integral is over the hypersurface of constant t. An important property of the averaging (7) is that it does not commute with time evolution,

$$\partial_t\langle f\rangle = \langle\partial_t f\rangle + \langle f\theta\rangle - \langle f\rangle\langle\theta\rangle . \tag{8}$$

§ Since $\nabla^\nu G_{\mu\nu}$ vanishes identically, $\nabla^\mu\nabla^\nu$ does not give anything new compared to $u^\mu\nabla^\nu$.

In order to describe the average evolution and compare it to FRW models, we have to define a scale factor. The simplest extension of the notion of an overall scale factor to an inhomogeneous and/or anisotropic spacetime is to define it with the volume of the hypersurface of constant t:

$$a(t) \equiv \left(\frac{\int d^3x \sqrt{^{(3)}g(t, \boldsymbol{x})}}{\int d^3x \sqrt{^{(3)}g(t_0, \boldsymbol{x})}} \right)^{\frac{1}{3}} , \tag{9}$$

where the normalisation has been chosen as $a(t_0) = 1$ at some time t_0 (which we shall take to be today). In words, $a(t)^3$ is the volume of the hypersurface of constant t (up to the usual multiplicative constant). We could equivalently define the scale factor with the average of the volume expansion rate, by $3\dot{a}/a \equiv \langle \theta \rangle$. We will also use the notation $H \equiv \dot{a}/a$.

By taking the average (7) of the scalar equations (3)–(5) and commuting the time derivatives as shown in (8) we obtain the equations satisfied by the scale factor (9), first derived by Thomas Buchert in 1999 [41]:

$$3\frac{\ddot{a}}{a} = -4\pi G_N \langle \rho \rangle + \mathcal{Q} \tag{10}$$

$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \langle \rho \rangle - \frac{1}{2} \langle ^{(3)}R \rangle - \frac{1}{2}\mathcal{Q} \tag{11}$$

$$\partial_t \langle \rho \rangle + 3\frac{\dot{a}}{a} \langle \rho \rangle = 0 , \tag{12}$$

and the integrability condition between the average Raychaudhuri equation (10) and the average Hamiltonian constraint (11), analogous to (6), reads

$$\partial_t \langle ^{(3)}R \rangle + 2\frac{\dot{a}}{a} \langle ^{(3)}R \rangle = -\dot{\mathcal{Q}} - 6\frac{\dot{a}}{a}\mathcal{Q} , \tag{13}$$

where the backreaction variable $\mathcal{Q}$ is a new term compared to the FRW equations, containing the effect of inhomogeneity and anisotropy:

$$\mathcal{Q} \equiv \frac{2}{3} \left(\langle \theta^2 \rangle - \langle \theta \rangle^2 \right) - 2\langle \sigma^2 \rangle . \tag{14}$$

The Buchert equations (10)–(13) are exact for the averages when matter consists of irrotational dust. (The Newtonian limit was derived by Buchert and Ehlers in 1995 [36] and the case with non-zero pressure by Buchert in 2001 [47].) The backreaction variable $\mathcal{Q}$ consists of two terms: the variance of the expansion rate and the shear. Shear is also present in the local equation (3) and decelerates expansion. In contrast, the variance is only present in the averaged equations, arising from the the non-commutation of averaging and taking a time derivative as shown in (8), and acts to accelerate the expansion rate. The presence of this term makes it possible for the average equations to display behaviour which is qualitatively different from the local behaviour. In the Newtonian limit, $\mathcal{Q}$ can be written in terms of the Minkowski functionals, which are a statistical measure of the morphological properties of cosmic structure, relating $\mathcal{Q}$ directly to structure formation [44].

Recovering the FRW equations. As with the system (3)–(6), there are only three independent equations in (10)–(13) for the four independent functions $a, \langle \rho \rangle, \mathcal{Q}$ and $\langle {}^{(3)}R \rangle$. Physically this means that different inhomogeneous and/or anisotropic spacetimes can evolve differently even if they have the same average initial conditions. While the equations (10)–(13) cannot be solved, they can be used to check whether a given scale factor can result from backreaction [65].

Specifying one more function (or relation between the four functions) leads to a soluble system. In particular, in the limit when the shear and the variance of the expansion rate are small compared to the contribution of the energy density, the Buchert equations reduce to the FRW equations. The integrability condition (13) between (10) and (11) then leads to the standard behaviour $\langle {}^{(3)}R \rangle \propto a^{-2}$. (Since the integrability condition ties the evolution of spatial curvature to the backreaction variable $\mathcal{Q}$, the average spatial curvature evolves like $\langle {}^{(3)}R \rangle \propto a^{-2}$ *only* in the FRW limit or in the special case when $\mathcal{Q} \propto a^{-6}$, in agreement with the discussion of assumption 1 in section 2.1.) Conversely, if the shear or the variance of the expansion rate are large compared to the contribution of the energy density in a large fraction of space, the FRW equations are not a good approximation for the average behaviour. This is the way to derive the FRW equations and quantify their domain of validity.

The average behaviour also reduces to the FRW equations when the shear and the variance of the expansion rate cancel, even if they are not small. (For examples of such solutions, see [35].) In the Newtonian limit, this happens generically for periodic boundary conditions [36] and for spherically symmetric spaces [42]. (With Minkowski functionals, the latter result can be understood to follow from the property that the backreaction variable $\mathcal{Q}$ in the Newtonian limit measures the deviation of morphology from that of a ball [44].) This cancellation is also present in relativistic perturbation theory when expanding to second order, as shown in [54] and recently rederived in [77]. This is not evidence towards a theorem that backreaction would not in general affect the acceleration, as claimed in [77]. Instead, it is related to the vanishing of backreaction for periodic boundary conditions in the Newtonian limit. It was correctly identified in [64] that in the perturbative expansion the Newtonian terms are those with the largest number of spatial gradients, as they are accompanied by the largest number of powers of the speed of light. Therefore, the terms with the highest number of spatial gradients at each order in perturbation theory vanish for periodic boundary conditions (which are implicit in the use of Fourier decomposition). Since the highest number of gradients at order N in perturbation theory is $2N$, the second order term with four gradients vanishes upon averaging. However, at fourth order there is no reason for the term with six derivatives to vanish, and this term becomes large after non-linear structures start forming, so the perturbation expansion is expected to break down.

An important point is that the expansion described by the Buchert equations (10)–(13) does *not* reduce to the FRW behaviour simply when the spatial size of the inhomogeneities and anisotropies is small. If the shear or the variance of the expansion rate is comparable to the contribution of the energy density in a sizeable

fraction of space, the behaviour will deviate from the FRW case (barring the sort of cancellation discussed above), regardless of the size of the individual inhomogeneous and/or anisotropic regions. The variance of the expansion rate is non-negative, and only vanishes if the expansion rate is completely homogeneous: the contributions from different inhomogeneous regions cannot cancel to zero. The same is true for the shear. (Of course, the shear and variance from different regions could cancel each other.) The Buchert equations (10)–(13) make explicit and quantify the statement made in section 2.1 that large-scale statistical homogeneity and isotropy does not guarantee that the dynamics of the scale factor follows the FRW equations.

Acceleration without acceleration. The average Raychaudhuri equation (10) shows that if the variance of the expansion rate is large enough compared to the shear and the energy density, the average expansion accelerates, even though the local expansion rate decelerates at every point according to the local Raychaudhuri equation (3). The possibility of acceleration is a property of the averaged system which is not present in the local behaviour, and is due to the non-commutation of averaging and time evolution.

Since $\mathcal{Q}$ contributes positively to the acceleration (10), but negatively to the Hubble rate (11), it might seem that negative curvature is needed to balance the negative contribution of $\mathcal{Q}$ to the Hubble rate, as claimed in [65]. In fact, it is possible to have acceleration even when the spatial curvature is positive, as all that is needed is that the sum $-\langle^{(3)}R\rangle - \mathcal{Q}$ increases (i.e. becomes less negative). This just means that $\langle^{(3)}R\rangle$ has to decrease (i.e. $-\langle^{(3)}R\rangle$ has to increase) faster than $\mathcal{Q}$ is growing. Acceleration necessarily involves (non-FRW) spatial curvature, as (13) shows: if $\langle^{(3)}R\rangle \propto a^{-2}$, we have $\mathcal{Q} \propto a^{-6}$, and there is no acceleration. This feature demonstrates the difference between having a description in terms of a scale factor and the FRW metric being valid, discussed in the context of assumption 1 in section 2.1.

One consequence of the fact that local expansion can only decelerate is that once a shell of matter has started collapsing, it cannot turn around and stabilise. It follows that the formation of stable structures involves vorticity and/or breakdown of the dust approximation [106, 107]. The contribution of vorticity to the local acceleration (3) (which we did not include) is always positive, so it cannot disappear upon averaging, as the contribution from each region has the same sign. This is also the case for shear, and the two can cancel each other. Indeed, in stabilised regions, the positive contribution of vorticity has to exactly balance the negative contributions of the shear and the energy density to produce net zero acceleration. One would thus naively expect our approximation of neglecting vorticity to lead to a lower bound on the acceleration. However, vorticity would also complicate the averaging procedure, as discussed in section 1, so the issue should be carefully studied.

We will discuss the physical meaning of the average acceleration and the relation to structure formation in section 3, but let us first complete the overview of the FRW assumptions by looking at assumption 3 concerning the behaviour of small perturbations in an inhomogeneous and/or anisotropic universe.

The evolution of perturbations. We will briefly consider perturbation theory to see what are the corrections to the FRW picture. For linearly perturbed FRW models, one writes the equations of motion as the sum of the background part plus a small perturbation. Discarding terms beyond first order in the perturbation and taking the spatial average leads to the equations for the FRW background (since the average of the perturbation is taken to vanish). Deducting these from the perturbed equations then gives the evolution equation for the perturbations.

When backreaction is important, the same procedure does not work, as the difference between the local equations (3)–(5) and the averages (10)–(12) is large by definition. We will do the closest thing, which is to separate the local terms into a 'large' part which determines the average behaviour and a 'small' part which does not contribute to the averages. We write $\theta(t, \boldsymbol{x}) = \theta_0(t, \boldsymbol{x}) + \Delta\theta(t, \boldsymbol{x})$, and assume that $|\theta_0| \gg |\Delta\theta|$. We similarly split $\rho(t, \boldsymbol{x}) = \rho_0(t, \boldsymbol{x}) + \Delta\rho(t, \boldsymbol{x})$, $\sigma^2(t, \boldsymbol{x}) = \sigma_0^2(t, \boldsymbol{x}) + \Delta(\sigma^2)(t, \boldsymbol{x})$ and $^{(3)}R(t, \boldsymbol{x}) = {}^{(3)}R_0(t, \boldsymbol{x}) + \Delta(^{(3)}R)(t, \boldsymbol{x})$. The assumption that the 'large' part is responsible for the average behaviour, $\langle\theta\rangle = \langle\theta_0\rangle$, implies that $\langle\Delta\theta\rangle = 0$, and similarly for the other quantities. We then insert this ansatz into the local equations (3)–(5) and keep only terms up to linear order in the 'small' quantities. Unlike in the FRW case, there is no rigorous way to separate the perturbation from the background, as both have spatial dependence. A similar issue arises in deriving the perturbation equations for 'tilted' cosmological models, i.e. models where the fluid velocity is not everywhere normal to the hypersurface of constant proper time [116]. (If we had non-zero vorticity, the model would necessarily be tilted.) If we simply assume that the equations for the 'large' and 'small' parts decouple, then the 'large' parts satisfy the local equations (3)–(5) and their average gives the Buchert equations (10)–(12). For the 'small' terms, we then obtain the following evolution equation

$$\ddot{\delta} + \frac{2}{3}\theta_0\dot{\delta} - 4\pi G_N \rho_0 \delta = 2\Delta(\sigma^2) \ , \tag{15}$$

where $\delta \equiv \Delta\rho/\rho_0$, along with the consistency condition $\langle\theta_0\Delta\theta\rangle = 0$. In addition to the shear source term on the right-hand side, (15) differs from its FRW counterpart in that θ_0, ρ_0 are position-dependent.

It is possible to obtain evolution equations where the average expansion rate and energy density appear instead of the local quantities. Let us assume that δ is separable, $\delta(t, \boldsymbol{x}) = D(t)h(\boldsymbol{x})$; in the linearly perturbed FRW case, this would correspond to considering pure growing or decaying modes. Dividing (15) by $h(\boldsymbol{x})$ and averaging, we have

$$\ddot{D} + 2\frac{\dot{a}}{a}\dot{D} - 4\pi G_N \langle\rho\rangle D = 2\left\langle\frac{\Delta(\sigma^2)}{h}\right\rangle \ . \tag{16}$$

The evolution equation (16) agrees with the linearly perturbed FRW case‖, apart from the shear source term (note that $\langle\rho\rangle \propto a^{-3}$ by (12)). In the spatially flat FRW limit, we recover the standard behaviour $D \propto a$ for the growing mode. Let us write

‖ For a given $a(t)$; the evolution of a (and thus the evolution of D) as a function of time will in general be different, since it is governed by the Buchert equations and not by the FRW equations.

$D(t) = a(t)d(t)$ in order to analyse the deviation from this limiting case. Using equations (10) and (11) we can write (16) as

$$\frac{\ddot{d}}{d} + 4\frac{\dot{a}}{a}\frac{\dot{d}}{d} = \frac{1}{3}\langle^{(3)}R\rangle + 2\frac{1}{ad}\left\langle\frac{\Delta(\sigma^2)}{h}\right\rangle . \tag{17}$$

If the expansion accelerates, the spatial curvature will become more negative, tending to decrease d, just like acceleration and negative spatial curvature act against the growth of perturbations in FRW models. The shear term can have either sign: writing $\sigma^{ij} = \sigma_0^{ij} + \Delta\sigma^{ij}$ for the shear tensor, we have $\Delta(\sigma^2) = 4\,\sigma_0^{ij}\Delta\sigma_{ij}$.

Just as the equations for the averages are not closed, neither are the perturbation equations. The perturbation equations require more information about the inhomogeneities and anisotropies (specifically about the shear perturbation) in addition to that needed for closure of the average equations. Once this information is supplied, the backreaction equations for the averages and the perturbations are analogous to modified gravity: there is no new energy component, but the relation between the average energy density and the average expansion rate, as well as between the averages and the perturbations, is different from the FRW case. For the averages, any model of modified gravity can of course be formally written as a general relativistic model with extra sources, and vice versa. Likewise, in the case of backreaction one can write the average behaviour in terms of a scalar field, the "morphon" [82]. It would be interesting to see how far the analogy can be extended to the perturbations.

We will not discuss perturbation theory further. In a realistic model of backreaction the issue should be studied in detail, in order to be able to compare with observations of CMB and LSS. For work on perturbations in a backreaction context, see [38,40,42,43,48].

3. Acceleration from collapse

3.1. A toy model for backreaction

Breakdown of the FRW approximation. The proposal [54] that structure formation leads to accelerating expansion implies in the context of the Buchert equations (10), (14) that the relative variance of the expansion rate is of order one. This behaviour has been numerically demonstrated in specific examples of LTB models [76, 79, 80]. However, the physical meaning of having average acceleration even though the local expansion decelerates everywhere has not been clear [71] (see also [74]).

We will clarify the physical meaning of the acceleration with a simple toy model, and show that it is not unreasonable for structure formation to involve a variance in the expansion rate that is large enough to produce acceleration. A shorter treatment was presented in the essay [81], and the role of collapsing regions has also been discussed in [79].

As mentioned in section 2.1 when discussing assumption 3, the breakdown of perturbation theory at $\delta \sim \pm 1$ is well understood. The simplest model used to describe the non-linear evolution is the spherical collapse model for overdense regions

(see e.g. [117, 118]), and the equivalent for underdense regions (the appendix of [119] has a useful summary of both cases). The model consists of a spherically symmetric density perturbation embedded in a FRW universe (surrounded by an empty region to make the mean density agree with the background), studied in the Newtonian limit. The perturbation behaves on average like an independent FRW universe with positive or negative spatial curvature in the case of an overdense or underdense region, respectively. In terms of the Buchert equations (10)–(13), this comes about because the backreaction variable $\mathcal{Q}$ vanishes for spherical symmetry in the Newtonian limit [42]. (For some work on the general relativistic case, see [57, 80].)

According to the spherical collapse model, the expansion of an overdense domain slows down with respect to the background until the structure stops expanding, turns around and starts collapsing. The spherical collapse model indicates that matter collapses to a singularity in a finite time, whereas in practice structures stabilise at a finite radius, usually taken to be half the radius at turnaround. The spherical collapse model is not a good description of the final stages of collapse, as departures from spherical symmetry are important for real structure formation, and collapse amplifies asymmetries. There are more accurate and realistic treatments of collapse, see e.g. [43, 45, 106, 120], but as we are mostly interested in qualitative features, the spherical collapse model will be an adequate description of non-linear density perturbations.

What is not generally appreciated is that just as linear perturbation theory around the FRW universe breaks down as perturbations become non-linear, the FRW universe itself breaks down as a description of the average behaviour when the non-linear perturbations occupy a sizeable fraction of space, as discussed in section 2.2. We want to look at this breakdown of FRW equations with a simple model of structure formation, in analogy with the description of the breakdown of perturbation theory in the spherical collapse model.

Two-region toy model of structure formation. In the real universe, structure formation consists of small overdensities and underdensities developing into stable structures with fixed density and voids which are constantly becoming emptier, respectively. We will consider the simplest possible toy model of structure formation, with two separate spherically symmetric dust regions, one overdense and one underdense. We will consider the Newtonian limit, so that the regions evolve according to the spherical collapse model. The regions are taken to be disjoint, and we ignore their embedding into the whole space. We are then essentially just comparing two FRW universes. The same kind of toy model was qualitatively discussed in [71]; we will take a more detailed look and add an understanding of the physics behind the equations.

We denote the scale factors of the regions by a_1, a_2 and the corresponding Hubble parameters by H_1, H_2, where region 1 is underdense and region 2 is overdense, so $H_1 > H_2$. Since the regions are disjoint, the total volume is simply the sum of the volumes a_1^3 and a_2^3, and the overall scale factor is $a = (a_1^3 + a_2^3)^{1/3}$. The overall Hubble and deceleration parameters are (they can be computed from (10), (11), (14) or directly

from the definition of a)

$$H = \frac{a_1^3}{a_1^3 + a_2^3} H_1 + \frac{a_2^3}{a_1^3 + a_2^3} H_2 \equiv H_1 \left(1 - v + vh\right) \tag{18}$$

$$q \equiv -\frac{1}{H^2}\frac{\ddot{a}}{a} = q_1 \frac{1 - v}{(1 - v + hv)^2} + q_2 \frac{vh^2}{(1 - v + hv)^2} - 2\frac{v(1 - v)(1 - h)^2}{(1 - v + hv)^2} \ , \tag{19}$$

where q_1, q_2 are the deceleration parameters of regions 1 and 2, and we have denoted the fraction of space in the overdense region by $v \equiv a_2^3/(a_1^3 + a_2^3)$ and the relative expansion rate of the two regions by $h \equiv H_2/H_1$.

The Hubble rate (18) is simply the volume-weighted average of the Hubble rates in regions 1 and 2. Not so for the deceleration parameter: in addition to the first two terms in (19), there is a third term related to the variance of the expansion rate, corresponding to the backreaction variable $\mathcal{Q}$ in the average Raychaudhuri equation (10). While q_1, q_2 are positive or zero, the last term is always negative, corresponding to the fact that $\mathcal{Q}$ is positive.

For simplicity, we take the underdense region to be completely empty, so $a_1 \propto t$. Region 2 behaves like a closed FRW universe, with $a_2 \propto 1 - \cos\phi$, $t \propto \phi - \sin\phi$, where the parameter ϕ is called the development angle. The overdense region starts expanding from the big bang singularity at $\phi = 0$ and slows down until it turns around at $\phi = \pi$ and starts collapsing, finally shrinking to zero size and infinite density at $\phi = 2\pi$. In practice, overdense regions stabilise at fixed size and density. In the spherical collapse model, this is often implemented by hand at $\phi = 3\pi/2$, when the radius of the structure is half the radius at turnaround. We will therefore follow the evolution only until $\phi = 3\pi/2$.

There is one free parameter in this toy model, the relative size of the two regions at some given time. Taking this time to be at turnaround at $\phi = \pi$ and denoting the fractions of space in regions 1 and 2 at that moment by $f_1 = 1 - f_2, f_2$, we have

$$v = \frac{f_2\pi^3(1 - \cos\phi)^3}{8(1 - f_2)(\phi - \sin\phi)^3 + f_2\pi^3(1 - \cos\phi)^3}$$

$$h = \frac{\sin\phi(\phi - \sin\phi)}{(1 - \cos\phi)^2} \ . \tag{20}$$

Inserting (20) into (19), it is easy to establish that the acceleration can be positive.

In figure 1 (a) and (b), we have plotted the deceleration parameter q and the Hubble rate multiplied by t, as dimensionless measures of the acceleration and the expansion rate, respectively. In addition to the toy model, we show the behaviour in the ΛCDM model, just to make the qualitative features of backreaction-driven acceleration easier to grasp by comparison; the toy model is not meant to be taken seriously in a quantitative sense. We have chosen the value of the free parameter to be $f_2 = 0.3$, so that the value of q at $\phi = 3\pi/2$ in the toy model approximately equals the value in the ΛCDM model when $\Omega_m = 0.3, \Omega_\Lambda = 0.7$. In figure 1 (c) we show the density parameters for matter, spatial curvature and the backreaction variable $\mathcal{Q}$ (defined in section 3.3 after (21), (22)) in the toy model. Note that negative Ω_R corresponds to positive spatial curvature, and vice versa.
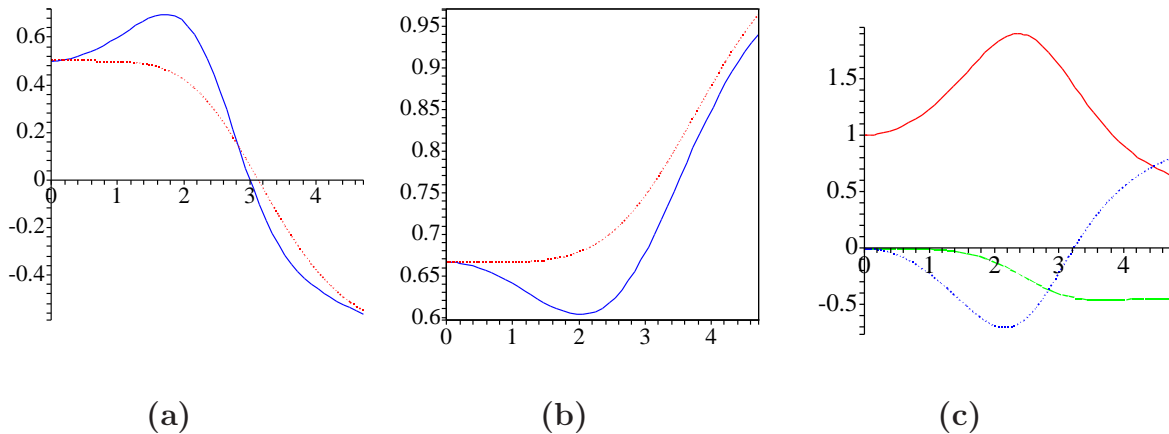
Figure 1. The evolution of the toy model as a function of the development angle ϕ. (a): The deceleration parameter q in the toy model (blue, solid) and in the ΛCDM model (red, dash-dot). (b): The Hubble parameter multiplied by time, Ht, in the toy model (blue, solid) and in the ΛCDM model (red, dash-dot). (c): The density parameters of matter, spatial curvature and the backreaction variable $\mathcal{Q}$ in the toy model. Red (solid) is Ω_m, blue (dash-dot) is Ω_R and green (dash) is $\Omega_{\mathcal{Q}}$.

Figure 1 (a) shows that the expansion accelerates, particularly after region 2 starts collapsing at $\phi = \pi$. It may seem paradoxical that gravitational collapse induces acceleration. However, the explanation is simple. Initially, the expansion is similar to the Einstein–de Sitter case, with $q = 1/2, Ht = 2/3$. The contribution of the overdense region vH_2 gradually slows down the average expansion rate H. The relative volume v occupied by the overdense region decreases monotonously, and eventually it becomes so small that the contribution of the underdense region begins to dominate and the expansion rate, as measured by Ht, grows. The related change from positive to negative spatial curvature is clearly seen in figure 1 (c). This effect is particularly pronounced and easy to understand once the overdense region has started collapsing: then the Hubble rate H_2 is actually negative, and its contribution shrinks rapidly as a_2 contracts, so the average expansion rate rises. (The fraction of volume in the overdense region at $\phi = 3\pi/2$ is only $v \approx 0.01$.) The increase in the absolute value of the collapse rate cannot compensate for the decrease in volume, as the FRW Hubble law shows: $a_1^3 H_1 = a_1^3 \sqrt{8\pi G_N \rho_{10} a_1^{-3}/3 - K_1 a_1^{-2}}$. This also means that the results are not sensitive to the diverging behaviour in the final stages. The contribution of the collapsing region decreases as it approaches the singularity, and the mean quantities would remain finite even if we followed the system to the end of the collapse.

Note that we have first averaged over the internal behaviour of each region, and only then taken the average over the two regions. This neglects the variance within each region, thus underestimating the total variance by the volume-weighted sum of the individual variances. Since the regions are spherically symmetric, this contribution to the backreaction variable $\mathcal{Q}$ (defined in (14)) is exactly cancelled by the shear contribution, so the two-step method does give the right $\mathcal{Q}$.

General lessons. The reason that the average expansion can accelerate even though the local expansion decelerates everywhere is that the growth of the relative volume occupied by the faster expanding regions contributes to the average acceleration. This makes the physical content of the Buchert equation (10) clear: the larger is the variance of the expansion rate, the wider is the difference between the fastest and the slowest expanding regions, so the more rapidly the relative volume of the fastest region can grow, and the stronger is the acceleration (assuming that the shear is not too large).

It is also transparent why acceleration from backreaction involves decreasing spatial curvature, as indicated by (10), (11), (14) and shown by figure 1 (c): the more negatively curved a region is, the faster it expands, and the larger its relative volume will become. Therefore the most negatively curved regions will come to dominate the average curvature. For underdense FRW regions, Ht is above $2/3$ and approaches unity from below. So, the decrease of spatial curvature is expected to be accompanied by Ht approaching 1, as seen in figure 1 (b).

While the rise of Ht beyond the Einstein–de Sitter value of $2/3$ is due to the underdense region, it is the overdense region which is essential for acceleration, i.e. the fall of q. (It is possible, but harder, to have acceleration without a collapsing region.) Unlike in the ΛCDM model, these two effects are distinct. For example, if we replaced the empty void obeying $a_1 \propto t$ with the Einstein–de Sitter universe for which $a_1 \propto t^{2/3}$, there can still be acceleration because of the collapse, even though Ht is at most $2/3$. The important thing is that Ht first slows down, so that it can later rise rapidly due to the collapsing region.

In fact, a more slowly expanding underdense region can even lead to stronger acceleration. For example, if instead of $a_1 \propto t$ we were to take $a_1 \propto t^{4/5}$, which is arguably closer to the behaviour of real voids [121], q would be more negative than in the case when region 2 is empty or an Einstein–de Sitter universe. The reason is that there are two competing effects: if the expansion of the faster expanding region is too slow, the contrast with the overdense region won't be strong enough to give a large variance, while if it's too rapid, the faster expanding region will dominate the Hubble rate before the overdense region can have any impact.

The fact that gravity is attractive (for matter satisfying the strong energy condition) implies that the local expansion rate is bounded from above, as can be shown by integrating the Raychaudhuri equation (10) as an inequality. The same holds for the mean expansion rate, resulting in the bound $Ht < 1$ for acceleration driven by backreaction [65]. In contrast, the collapse rate is not bounded from below, so collapsing regions can lead to arbitrarily rapid change in the expansion rate, and q can become arbitrarily negative. In fact, q can diverge to negative infinity in a finite time. In the two-region toy model, for sufficiently large f_2 the negative contribution vH_2 will at some point equal the positive contribution $(1-v)H_1$, so H passes zero from above and q diverges to positive infinity. When H later passes zero from below on its way back to positive values, q will diverge to negative infinity.

Such behaviour is in contrast to FRW models, where Ht can grow without limit, but

$q < -1$ requires violating the null energy condition (or the modified gravity equivalent). Acceleration with $q < -1$ is consistent with observations, and at present there is no statistically significant evidence either for it or against it [3, 4, 12].

3.2. Backreaction in the real universe

Start of the acceleration. The toy model studied above shows that gravitational collapse is intimately associated with accelerating expansion, and makes transparent the physical content of the Buchert equations (10)–(13). Obviously, the real universe does not consist of two disjoint spherically symmetric regions. Let us clarify which features of the toy model are expected to be relevant for the real universe. The density distribution of the universe is characterised by a hierarchy of perturbations nested within perturbations. In the past the perturbations were small on all scales, so the universe was well described by linear perturbation theory around the FRW universe. (And it has been shown that a solution of the linearised equations indeed corresponds to a nearly FRW solution of the the full equations [122].) In the simplest models of inflation, the primordial spectrum of density perturbations is adiabatic and nearly scale-invariant (at horizon entry), and for typical models of supersymmetric weakly interacting dark matter there is a cut-off in the power spectrum at small scales due to collisional damping and free-streaming [123].

The density perturbations grow logarithmically during the radiation-dominated era and linearly during the matter-dominated era. For typical supersymmetric dark matter, the first generation of perturbations becomes non-linear and forms bound structures around a redshift of 40-80 [123]. For other viable dark matter candidates such as axions [124], light dark matter [125] or right-handed neutrinos [126], structure formation may begin at a different time due to differences in the details of the perturbation spectrum, but the qualitative picture is expected to remain the same. For a comprehensive analysis of dark matter candidates, see [127].

The structures which collapse and stabilise (or form voids) are part of larger scale perturbations which are in turn undergoing the process of slowing down and collapsing (or speeding up and becoming emptier), and so on, with ever larger non-linear overdense structures and underdense voids forming over time. Since total mass is conserved, the formation of high-density regions is accompanied by the formation of larger underdense regions.

One possibility is that the gravitational collapse associated with the formation of the first generation of structures would lead to acceleration. Since acceleration smoothens inhomogeneities and anisotropies and impedes structure formation, backreaction-driven acceleration will eventually end. This is implicit in the limit $Ht < 1$, which rules out eternal acceleration [65]. However, in the case of the real universe the physical reason for the end of the acceleration is not as clear as in the two-region toy model, though according to the perturbation equation (17) the negative spatial curvature will work against structure formation. After acceleration ends, inhomogeneities and anisotropies can become important again, as perturbations are nested inside each other,

with new modes constantly entering the horizon. This could lead to oscillations between deceleration and acceleration. Such oscillations are not ruled out observationally, and might be detectable in the next generation of experiments [128]. Oscillation between acceleration and deceleration is a possible solution to the coincidence problem, as one could observe acceleration in the recent past at all times after the start of structure formation.

The question of whether acceleration does start with the collapse of the first generation of structures and then undergo oscillations should be addressed in a realistic model. At first sight, such early acceleration seems unlikely because backreaction involves dynamical spatial curvature, as we have discussed. It seems likely that significant spatial curvature from redshift $\approx$ 40-80 onwards would have been apparent in observations, particularly in the CMB [129]. However, precisely because the spatial curvature does not behave in the same way as in FRW models, the CMB bounds cannot be straighthforwardly applied; see section 3.3 for discussion of spatial curvature and observations.

Oscillations aside, backreaction offers another way of addressing the coincidence problem. In addition to the time when the first generation of objects forms, structure formation involves a second preferred time, near the era $\sim$ 10 billion years when acceleration has been observed.

The process of hierarchical structure formation is not entirely self-similar even when the primordial spectrum of perturbations is scale-invariant. In addition to the cut-off scale due to collisional damping and free-streaming, there is at least one other scale present, related to the transition from radiation domination to matter domination (the physics of dark matter or inflation can of course involve further scales). The scale k_{eq} corresponding to the wavenumber of the mode which entered the horizon at matter-radiation equality is imprinted on the perturbation spectrum because metric perturbations which entered the horizon during the radiation-dominated era ($k > k_{eq}$) are damped relative to those which enter during matter domination ($k < k_{eq}$). In the real universe, the matter-radiation equality scale is $k_{eq}^{-1} \approx$ 60-160 h^{-1}Mpc (for $0.15 \lesssim \Omega_{m0} \lesssim 0.35$ [26] and the updated value $H_0 = 62.3 \pm 5.2$ km/s/Mpc [28]). Therefore the amplitude of the metric perturbations grows with increasing wavelength, asymptotically approaching the value $A \approx 10^{-5}$ set by inflation (or some other process in the primordial universe).

There is a factor $k^2/(aH)^2$ in going from the metric perturbations to density perturbations, and the suppression for small wavelengths is approximately $1/k^2$ up to a logarithm, so there is only a logarithmic difference in the amplitude of the density perturbations at small wavelengths, as mentioned above. As a result, the size of structures which are just starting to collapse relative to the Hubble radius, $R_{nl}/(aH)^{-1}$, grows rapidly at first and rises monotonically, slowing down as structure formation proceeds and saturating at the value $\approx \sqrt{A}$ once all perturbations which entered the horizon during the radiation-dominated era have collapsed. (Here $R_{nl}(t)$ is the scale at which the mean square of the density contrast at time t is unity, $\langle \delta^2 \rangle (R_{nl}(t), t) = 1$.)

If the universe followed the Einstein–de Sitter behaviour, the evolution would enter the saturation regime around 10-100 billion years.

Once structure formation has started, part of the universe is in a constant state of collapse, and part is always becoming more empty. Based on the Buchert equations (10)–(13) and the analysis of the two-region toy model, we would expect the backreaction in the real universe to be strongest when the collapsing objects contract from having occupied the maximum fraction of volume. Neglecting evolution in number density, this naively occurs after the objects have reached their maximum size. The issue is not entirely clear, however, and it could be that the important factor is the slowing down of the growth of the collapsing regions as their size relative to the horizon becomes practically saturated. At any rate, regarding the coincidence problem, it is encouraging that these simple arguments lead to a time which is in rough agreement with the era when acceleration has been observed.

There is another reason to think that acceleration will not occur with the formation of the first bound objects, but later in the history of the universe. If we set up the two-region toy model discussed in section 3.1 so that the initial volumes and absolute magnitudes of the density perturbations in the overdense and underdense region are equal (which is arguably closer to the real situation at early times), the expansion goes smoothly from the Einstein–de Sitter behaviour ($a \propto t^{2/3}$) to the empty universe behaviour ($a \propto t$). There is no slowdown period and no acceleration, as the variance of the expansion rate is too small. The calculation that was done in the toy model is more representative of structure formation in the late-time universe, when voids already occupy a large fraction of the volume.

So, it is plausible that acceleration only occurs once structure formation takes place in an environment dominated by voids. In view of this it is, again, encouraging that simulations and analytical work show that voids grow to fill practically the whole space, with the void distribution dominated by voids of a characteristic size, which grows as larger structures become non-linear and smaller voids merge [93, 119, 130, 131]. Not all analyses of simulations agree on the distribution of void sizes, evolution of the fraction of space occupied by voids or the fraction of space in voids today. In [94] the growth of the void volume fraction is clear, but the value today is only ≈ 0.6, whereas the authors of [132] find that the fraction reaches ≈ 1 at present day. Some of the simulations use the ΛCDM model, where the present day is singled out by construction, so care must be taken in applying the results to a dust-dominated universe. Nevertheless, given that the contribution of the cosmological constant in small at high redshifts, one would expect the feature that voids do not dominate early on to remain valid, while the behaviour at low redshifts should be checked more carefully.

These issues should be looked at in detail both analytically and with simulations. However, while simulations are a useful complement to observations and analytical work in determining what the structures present in the universe are actually like, finding out about backreaction with simulations is not straightforward. Because simulations use Newtonian gravity and periodic boundary conditions, backreaction vanishes identically,

as discussed in section 2.2. Thus the effect of inhomogeneity and anisotropy on the mean expansion rate would not be visible in simulations even if the background evolution were dynamically adjusted to take into account the structures, instead of being predetermined. For work on the local effects of inhomogeneities and anisotropies on the expansion in a backreaction context, see [42].

The backreaction conjecture. Let us summarise the physical picture of backreaction-driven acceleration. As perturbations become non-linear, overdense regions collapse and form stable structures, and underdense regions form voids which become ever emptier. The geometry of the hypersurfaces of constant proper time is no longer everywhere perturbatively near homogeneous and isotropic, but a fraction of space is taken up by non-linear structures in various stages of expansion or collapse. A given proper time no longer corresponds to a single expansion rate, but to a distribution of expansion rates. Nevertheless, the description of the expansion in terms of an overall scale factor remains valid, because the individidual non-linear regions are small compared to the scales over which we are measuring observables. This distribution evolves until it reaches a self-similar regime where the regions grow at the same rate as the visual horizon. This happens when the universe is around 10-100 billion years old.

As light rays coming to us pass through hypersurfaces of proper time, the fraction of space in each stage of expansion (or collapse) that they encounter changes as new non-linear regions emerge and old regions evolve. The conjecture is that the fraction of space in collapsing objects first grows, diminishing the average expansion rate. As these regions collapse and their contribution to the expansion rate is overcome by that of underdense regions, the average expansion accelerates, as demonstrated with the toy model in section 3.1. (Note that this mechanism relies on the structures we know to exist in the universe, as opposed to speculation of a globally inhomogeneous state [70].)

This backreaction conjecture should be quantitatively checked by studying realistic models which can be compared with observations. However, let us briefly discuss what we can say regarding compatibility with cosmological observations on the basis of the Buchert equations and the qualitative description we have outlined.

3.3. Comparison with observations

Spatial curvature and the density parameters. Acceleration driven by backreaction necessarily involves spatial curvature. It is instructive to look at the behaviour of the universe in terms of the density parameters. Dividing (10) and (11) by $3H^2$, we have [41,55]

$$q \equiv -\frac{1}{H^2}\frac{\ddot{a}}{a} = \frac{1}{2}\Omega_m + 2\Omega_{\mathcal{Q}} \tag{21}$$

$$1 = \Omega_m + \Omega_R + \Omega_{\mathcal{Q}} , \tag{22}$$

where $\Omega_m \equiv 8\pi G_N \langle \rho \rangle /(3H^2)$, $\Omega_R \equiv -\langle^{(3)}R\rangle/(6H^2)$ and $\Omega_{\mathcal{Q}} \equiv -\mathcal{Q}/(6H^2)$ are the density parameters of matter, spatial curvature and the backreaction variable,

respectively. As seen from the definition of $\mathcal{Q}$ in (14), the backreaction density parameter is just minus the relative variance of the expansion rate, plus the contribution of shear: $\Omega_{\mathcal{Q}} = -(\langle\theta^2\rangle - \langle\theta\rangle^2)/\langle\theta\rangle^2 + 3\langle\sigma^2\rangle/\langle\theta\rangle^2$.

According to a variety of observations, today we have $0.15 \lesssim \Omega_{m0} \lesssim 0.35$ [26]. A rough estimate of present-day acceleration from SNIa data is $-1.2 \lesssim q_0 \lesssim -0.3$ [133], though the lower limit could be extended to at least -1.6, and the upper limit to above zero, so q_0 could even be positive today [3, 14]. Strictly speaking, results from even fairly model-independent analyses which do not assume the FRW equations but only the FRW metric [3,4] cannot be directly applied to backreaction, because of the different behaviour of the spatial curvature. In general, the results of SNIa data analysis depend strongly on the parametrisation used [12], and the data seems to contain little model-independent information about the expansion beyond the fact that it has accelerated in the recent past [3, 4]. The quoted value of q_0 should be understood as a rough estimate which assumes that the universe has not decelerated strongly since it started accelerating, sufficient for our purposes.

Given the values of Ω_{m0}, q_0, the relations (21), (22) imply $0.9 \lesssim \Omega_{R0} \lesssim 1.5$, $-0.7 \lesssim \Omega_{\mathcal{Q}0} \lesssim -0.2$. It might seem that such a large negative spatial curvature would be in clear conflict with CMB observations [129]. However, as with SNIae, CMB analysis is very model- and prior-dependent. In particular, the analysis leading to tight bounds on spatial curvature assumes that the equation of state of the negative-pressure medium which is driving the acceleration does not vary rapidly in time. In contrast, at the level of the average equations, backreaction looks like a medium which evolves from the dust equation of state to having negative pressure. Such behaviour allows a significant contribution from negative spatial curvature [13], as the evolution of the equation of state can cancel the geometrical effect of spatial curvature on the angular diameter distance.

As with the SNIa data, the non-FRW behaviour of the spatial curvature means that the CMB analysis in FRW models cannot be applied to backreaction as is. Spatial curvature would be expected to be negligible before backreaction becomes important and then evolve rapidly, rather than going smoothly like a^{-2}, and it could even change sign. This makes analysis of the CMB anisotropies more involved than in the FRW case, as the basis functions for slices of constant curvature cannot be straightforwardly used. However, if backreaction becomes important only at redshifts of order unity and below, one might expect the spatially flat FRW framework to be a good first approximation, as the spatial curvature becomes significant only on the last leg of the journey of the CMB from the last scattering surface to us. At any rate, the issue of light propagation in such an inhomogeneous and anisotropic spacetime should be studied in the context of a realistic quantitative model.

Dynamical spatial curvature is a qualitative feature which is unique to backreaction. It might be useful in distinguishing it observationally from FRW models, which can share many of the other features of backreaction, including slowdown before acceleration, oscillations between acceleration and deceleration and a low Hubble parameter (see

e.g. [134]).

Large-scale variance. It has been argued that backreaction cannot explain the observed acceleration since the geometry of the universe is so smooth, i.e. so near the FRW metric [66,71]. However, after non-linear structure formation has started, the geometries of all regions of the universe are not perturbatively near each other, as measured by invariant quantities such as the scalar curvature. For example, the difference in the expansion rate between expanding and collapsing regions is of order one and their behaviour is qualitatively different; as we have noted, the associated breakdown of perturbation theory is well known in the context of the spherical collapse model. A perhaps even simpler example is that the space inside stabilised structures such as galaxies does not expand (or collapse), so the relative difference between the local expansion rate and the mean expansion rate is exactly unity. This difference between the local static metric and the global expanding metric is also well known, see [135] for discussion and references, and has recently been studied in the context of spacetime-dependent couplings [136].

It is true that despite the presence of non-linear regions, the density field is smooth when averaged over large scales, as positive and negative density perturbations cancel due to conservation of mass. For this reason the energy density of dust necessarily goes like $\langle \rho \rangle \propto a^{-3}$, as shown by (12), and can never lead to acceleration. However, as we have discussed, it is not the variance of the density but the variance of the expansion rate which contributes to the acceleration. As estimated above, a variance of $(\langle \theta^2 \rangle - \langle \theta \rangle^2)/\langle \theta \rangle^2 \approx 0.2 - 0.7$ (plus the contribution of shear) today could explain the observations. The important question is then what is the fraction of space occupied by regions with non-linear perturbations. If it is small, the variance can be negligible, whereas if it is of order one, the expansion will accelerate, unless the effect of the large variance is overcome by the effect of shear.

Analysis of voids in the two-degree Field Galaxy Redshift Survey found that 40% of the survey volume is taken up by voids [92], which were required to be very underdense, $\delta \leq -0.9$, and to have a minimum radius of 10 h^{-1}Mpc. The mean density contrast of voids was found to be $\delta = -0.94 \pm 0.02$ and the mean radius was 14.89 ± 2.65 h^{-1}Mpc. These numbers are in rough agreement with analysis of earlier surveys, some of which found larger mean sizes and a bigger fraction of the volume in voids [90,91]. The estimate of the volume fraction is conservative, both because of the limit on δ and radius, and because the void-finding algorithm looks for spherical voids (some of which are then combined). Simulations have indicated that voids can have complicated shapes, and that large voids in particular are typically non-spherical [93,137] (though see also [119,131]). The minimum size may also be important. Analysis of voids in the Millennium simulation (using the ΛCDM model) using the same void-finding algorithm, but with a minimum radius of 6 h^{-1}Mpc, found the mean radius to be 10.45 h^{-1}Mpc, while the mean density contrast was as low as before, $\delta = -0.92$ [138]. The fact that the mean size is not only smaller than in [91,92], but near the minimum radius of those studies suggests that the contribution of small voids may be important. The prevalence

of small voids is also suggested by some simulations [94], though not by others [93]. In any case, one should be careful when comparing observations of the real universe with simulations which use the ΛCDM model. It also bears emphasising that there is no generally agreed definition of a void [137], so it is not straightforward to compare void properties between different studies.

We can make a rough order of magnitude estimate for the lower limit of the variance implied by the density distribution found in [92] by assuming that the volume taken up by voids is a single homogeneous and isotropic region and the rest of the universe is another homogeneous and isotropic region. This is like the toy model in section 3.1, but with region 1 being a general underdense FRW universe instead of being completely empty. Putting in the numbers $\delta_1 = -0.94, v_1 = 0.4$ and demanding that the mean density equals that of a spatially flat background gives $\langle\theta^2\rangle/\langle\theta\rangle^2 \approx 1.08$, or $\Omega_{\mathcal{Q}0} \approx -0.08$. Taking, more consistently, the background to be the sum of the two regions, we get $\langle\theta^2\rangle/\langle\theta\rangle^2 \approx 1.04$, or $\Omega_{\mathcal{Q}0} \approx -0.04$. (In the latter case, we have to take $\Omega_{m0} = 0.45$, as the toy model does not permit a lower value for the given density distribution.) These are not far from the the value required by observations, $\Omega_{\mathcal{Q}0} \lesssim -0.2$. These are likely to be severe underestimates of the variance, as the essential collapsing regions have been smoothed over. A realistic estimate should also account for the shear, which could counter the effect of the variance.

We can also estimate the evolution of the expansion rate on the basis of observations and ΛCDM simulations of Lyman-α absorbers at redshifts between 2 and 4.5 [140]. It seems that the Lyman-α absorbers follow the overall Hubble flow, and allow a fairly direct observation of the expansion rate. The distribution of expansion rates analysed in [140] displays the qualitative features which we have identified as crucial for acceleration. Most of the volume is underdense and expanding faster than the mean, and there is a small fraction of space which is collapsing. The fraction of volume taken up by both the rapidly expanding and the collapsing regions grows with decreasing redshift, and space becomes increasingly dominated by voids which expand faster than average. According to the backreaction conjecture, the volume fraction occupied by collapsing regions should decrease at low redshifts so that the mean expansion accelerates. As the redshifts analysed in [140] do not go below 2, it is not possible to check the behaviour in the redshift range where the SNIa data indicate acceleration. However, we do see from the simulations that the variance grows with decreasing redshift, implying increasing backreaction. At $z = 3.8$ we have $\langle\theta^2\rangle/\langle\theta\rangle^2 \approx 1.05$, at $z = 3.4$ we have $\langle\theta^2\rangle/\langle\theta\rangle^2 \approx 1.07$, and at $z = 2.0$ we have $\langle\theta^2\rangle/\langle\theta\rangle^2 \approx 1.26$. In other words, $\Omega_{\mathcal{Q}}$ evolves from -0.05 to -0.26, plus the contribution of shear (which it has not yet been possible to observationally determine).

Having voids dominate a large fraction of space not only contributes to the variance of the expansion rate and thus to acceleration, it is also necessary for boosting Ht above the Einstein–de Sitter value of 2/3. Fitting the parameters of the ΛCDM model to the three-year WMAP data gives $H_0 = 73.4^{+2.8}_{-3.8}$ km/s/Mpc and $t_0 = 13.73^{+0.13}_{-0.17}$ billion years [129], resulting in (neglecting correlation in the errors) $H_0 t_0 = 1.03^{+0.05}_{-0.06}$. This value

is very model-dependent, and we can get a more robust estimate using the Hubble Space Telescope measurement of the Hubble parameter, which was recently revised downwards to $H_0 = 62.3 \pm 5.2$ km/s/Mpc [28] from the old value $H_0 = 72 \pm 8$ km/s/Mpc [139]. Taking $t_0 = 13 \pm 1$ billion years, the new value of H_0 gives $0.70 \lesssim H_0 t_0 \lesssim 0.97$.

The updated lower value for H_0 is easier for backreaction to accommodate, since $Ht \approx 1$ requires that almost all of the volume of the universe is taken up by voids which are almost completely empty. The volume domination of voids demonstrates what the decreasing spatial curvature which accompanies backreaction-driven acceleration implied by the Buchert equations means in physical terms. Having $H_0 t_0 \lesssim 1$ is a natural outcome of the backreaction framework, related to the domination of the space by voids, whereas in the ΛCDM model it is a coincidence. Given that backreaction cannot raise Ht above 1 (assuming that matter can be treated as dust and vorticity can be neglected), one can rule out backreaction as an explanation for the acceleration by showing, in a model-independent manner, that $Ht > 1$. (In fact, it is enough to show that the local expansion rate satisfies the inequality $\theta t/3 > 1$ somewhere.)

The above estimates show that the qualitative picture of backreaction outlined on the basis of the Buchert equations and the two-region toy model is not in obvious disgreement with observations and simulations. In fact, naive quantitative estimates of the variance of the expansion rate give the right order of magnitude required for the backreaction acceleration mechanism to work. A serious comparison with observations and simulations will require both a realistic quantitative model for the backreaction as well as more careful interpretation of the observations (since the FRW metric cannot be used) and simulations (since in backreaction in the Newtonian limit vanishes identically as an artifact of periodic boundary conditions, and simulations with a ΛCDM background may be misleading). However, the naive estimates above show that the backreaction explanation for the acceleration is plausible given what we know about inhomogeneity and anisotropy in the universe.

Deviations from the average. In a homogeneous and isotropic spacetime, it is guaranteed that a comoving observer will measure the average values for the expansion rate, energy density and other observables, since there is no difference between the average and local values. One would expect this to also hold for spacetimes which are perturbatively near FRW everywhere. However, in a universe which contains large inhomogeneities and anisotropies, one can ask what is the relevance of the averages for an observer making measurements at one particular location. This is essentially the question of why the approximation of looking only at an overall scale factor is valid, discussed as assumption 1 in section 2.1. There are two answers. The first is that most measurements of cosmological quantities are made indirectly via observations of the CMB, LSS or SNIa, which are mostly sensitive to large-scale properties of the universe, rather than local ones. The second, more pragmatic, argument is the proven success of the scale factor ansatz in fitting observations, and the fact that the size of non-linear structures is small compared to the visual horizon guarantees the consistency of the

ansatz.

However, as the size of structures relative to the visual horizon is not entirely negligible and the variance of the expansion rate is large, one could expect to see directional differences in observables which are sensitive to the large-scale expansion rate and density distribution, such as the low multipoles of the CMB and the optical depth. One can think of this as analysing the statistical scatter around the average behaviour given by the Buchert equations. It is tempting to speculate that backreaction could thus link the observed acceleration to the directional large-angle anomalies seen in the CMB [5–9]. In the first-year WMAP temperature map, the southern hemisphere has an optical depth of $\tau = 0.24^{+0.06}_{-0.07}$, while on the northern hemisphere the optical depth is consistent with zero, with an upper limit of $\tau < 0.08$ (in the frame which maximises the asymmetry) [6]. A better studied feature is the presence of a preferred direction in the low multipoles, which is correlated with the dipole and the ecliptic plane. The former suggests an effect related to the structures which are responsible for our proper motion with respect to the CMB [112] or systematics regarding the calibration with the dipole [8], while the latter points towards a systematical effect associated with the motion of the WMAP satellite.

From cosmology one cannot get a correlation with the ecliptic, and probably not with the dipole either. The ecliptic correlation could be a coincidence, as the low multipoles are still anomalous even if the ecliptic correlation is neglected, and its significance has gone down in the three-year WMAP data [7, 9]. Even neglecting the ecliptic correlation, it is not clear how backreaction would produce such a distinct preferred direction. On the other hand, backreaction could tie the directional anomalies with the observed lack of power at large scales, unlike most proposals, which explain the directional anomalies by adding a new contribution which aggravates the amplitude problem. Since the behaviour of the perturbations is directly related to the spatial curvature, which is in turn tied to the acceleration, a connection with directional variation of the Integrated Sachs–Wolfe effect (and the inferred optical depth) seems plausible. It is interesting that the template of the anisotropic Bianchi VII$_h$ model fits the anomalies quite well [141]. The fitting template corresponds to a homogeneous but anisotropic universe with shear, vorticity and negative spatial curvature. Though the details of the Bianchi VII$_h$ universe are different from what is expected from backreaction, this does show that a globally inhomogeneous and/or anisotropic universe can explain the anomalies.

Whether any of the CMB directional anomalies are related to backreaction should be studied in a quantitative model of light propagation in an inhomogeneous and anisotropic spacetime. One would also expect to obtain a prediction for the directional variation of SNIa luminosity distances, which should be different from that of a linearly perturbed FRW model [23, 24], and which might be testable in the next generation of observations. Such directional variation could be another clear way, in addition to dynamical spatial curvature, of distinguishing between acceleration driven by backreaction and a homogeneous and isotropic medium with negative pressure (or

an equivalent modification of gravity).

4. Conclusion

Acceleration and inhomogeneity/anisotropy. The observational evidence for the acceleration of the universe is usually interpreted in the framework of linearly perturbed Friedmann–Robertson–Walker (FRW) models, which describe a universe that is everywhere almost homogeneous and isotropic. In the context of such models, a medium with negative pressure or modified gravity is needed to explain the observations. This leads to the coincidence problem: why has the exotic matter or strange gravity become important only recently? The most significant qualitative change in the universe around the era where acceleration has been observed is the formation of non-linear structures, so it seems a natural possibility that the observed deviation from the general relativistic prediction of the homogeneous and isotropic cosmological models with normal matter could be related to the breakdown of homogeneity and isotropy.

The issue of cosmological homogeneity and isotropy has been extensively discussed over the years by George Ellis and collaborators, notably in the context of the observational program of cosmology [84–87]. One of the issues they have highlighted is that averaging and applying the field equations do not commute: in other words, the average properties of an inhomogeneous and/or anisotropic spacetime do not satisfy the Einstein equation. The task of finding the model that best describes the average behaviour of the inhomogeneous universe has been termed the fitting problem.

The relativistic equations which describe the behaviour of average quantities in an inhomogeneous and/or anisotropic, but irrotational, ideal fluid universe have been derived by Thomas Buchert [41, 47]. The Buchert equations show that it is possible for inhomogeneities and/or anisotropies to lead to accelerating average expansion in a dust universe, even though the local acceleration decelerates everywhere. They also show that the fraction of space occupied by non-linear regions is the determining quantity, not the size of the individual regions. Even when the average properties of space can be described in terms of an overall scale factor, the evolution of the scale factor does not necessarily follow the FRW equations.

The possibility that inhomogeneities and/or anisotropies could lead to acceleration was studied in the context of linear perturbation theory in [54], and it was suggested that acceleration could be due to perturbations which have entered the non-linear regime but haven't yet stabilised. The possibility of acceleration via backreaction has been numerically verified [76, 79, 80]. However, the physics of how structure formation leads to acceleration and the question of why acceleration begins much later than structure formation have been unclear.

We have now clarified these issues, which turn out to be intimately associated with the process of gravitational collapse. With a simple toy model, we have explicitly shown how overdense regions can first slow down the expansion, which then accelerates as these regions shrink and their contribution to the expansion rate decreases rapidly as they

collapse.

We have also noted that the matter-radiation equality scale imprinted on the dark matter power spectrum leads to a preferred time for structure formation that is near the observed acceleration era. The typical size of collapsing structures relative to the visual horizon grows rapidly at the start of structure formation, but then slows down, saturating around 10-100 billion years. A naive look at observations and simulations of structure in the universe shows that the degree of inhomogeneity required for backreaction to yield acceleration is plausible.

The backreaction conjecture. The backreaction conjecture for the acceleration is simple. According to the Buchert equations, large variance of the expansion rate leads to acceleration. The physical interpretation is simply that the relative volume taken up by the regions of space which are expanding faster will come to dominate over the slower expanding regions, so the average expansion rate will rise. Collapsing regions, i.e. regions with a negative expansion rate, give a large contribution to the variance, since they contribute positively to the mean square but negatively to the square of the mean. Structure formation involves gravitational collapse, and the size of the collapsing regions is largest at late times when acceleration has been observed.

Such an explanation keeps the phenomenological successes of the FRW scale factor in fitting the observations, while avoiding the failure of the FRW equations, which has required the introduction of a medium with negative pressure or modified gravity. This is in contrast to models which propose explaining the observations by the effect of inhomogeneities on the propagation of light without having accelerated expansion [20, 21], where the success of the ansatz that one needs only to look at an overall scale factor is accidental.

In the context of FRW models, there have been attempts to connect the late-time acceleration to inflation (via making the same scalar field responsible for both), the era of matter-radiation equality (via a tracker field which reacts to the change in the background equation of state) and dark matter (via unified dark matter and dark energy). Backreaction involves a subtle link to all these issues. Inflation determines the initial amplitude of the density perturbations, matter-radiation equality starts the clock for structure formation, and the nature of dark matter determines the processed form of the power spectrum and the time of formation of the first generation of structures.

With many previously unclear conceptual and qualitative issues settled, the task is now to build a realistic model and make quantitative estimates that can be compared with observations. The relevant aspects of observations and simulations should also be understood better. On the basis of general considerations we can already state that we should have $Ht < 1$, and that there should be observable amounts of spatial curvature (assuming that vorticity is negligible and that matter can be treated as dust) [65]. There may be a slowdown period preceding the acceleration, and the expansion may oscillate between deceleration and acceleration, but these issues have to be worked out in the context of a detailed model.

Note that there are no new fundamental parameters to adjust, and any unknowns are due to existing uncertainties about the power spectrum, the modelling of structure formation and so on: the backreaction conjecture is eminently falsifiable. Backreaction analysis simply entails doing the usually implicit averaging in cosmology in a way that is both mathematically consistent and takes into account the structures that are known to be present in the universe, as has been advocated over the years in the context of the program of observational cosmology and related work. Backreaction offers an elegant possible explanation for late-time acceleration. Whether or not this possibility turns out to be realised, the effect of structure formation on the expansion rate should be carefully evaluated to solve the fitting problem and complete the program of determining the right equations for describing the overall behaviour of the universe.

Acknowledgments

[1] Eisenstein D J *et al* , *Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies*, 2005 *Astrophys. J.* **633** 560 [astro-ph/0501171]

Cole S *et al* (The 2dFGRS Team), *The 2dF Galaxy Redshift Survey: Power-spectrum analysis of the final dataset and cosmological implications*, 2005 *Mon. Not. Roy. Astron. Soc.* **362** 505 [astro-ph/0501174]

[2] Padmanabhan T and Choudhury T R, *A theoretician's analysis of the supernova data and the limitations in determining the nature of dark energy*, 2003 *Mon. Not. Roy. Astron. Soc.* **344** 823 [astro-ph/0212573]

Choudhury T R and Padmanabhan T, *Cosmological parameters from supernova observations: A critical comparison of three data sets*, 2005 *Astron. & Astrophys.* **429** 807 [astro-ph/0311622]

[3] Shapiro C and Turner M S, *What Do We Really Know About Cosmic Acceleration?* [astro-ph/0512586]

[4] Elgarøy Ø and Multamäki T, *Bayesian analysis of Friedmannless cosmologies* [astro-ph/0603053]

[5] de Oliveira-Costa A, Tegmark M, Zaldarriaga M and Hamilton A, *The significance of the largest scale CMB fluctuations in WMAP*, 2004 *Phys. Rev.* **D69** 063516 [astro-ph/0307282]

Eriksen H K, Hansen F K, Banday A J, Górski K M and Lilje P B, *Asymmetries In The CMB Anisotropy Field*, 2004 *Astrophys. J.* **605** 14 Erratum 2004 *Astrophys. J.* **609** 1198 [astro-ph/0307507]

Copi C J, Huterer D and Starkman G D, *Multipole Vectors–a new representation of the CMB sky and evidence for statistical anisotropy or non-Gaussianity at $2 \le \ell \le 8$*, 2004 *Phys. Rev.* **D70** 043515 [astro-ph/0310511]

Schwarz D J, Starkman G D, Huterer D and Copi C J, *Is the low-l microwave background cosmic?*, 2004 *Phys. Rev. Lett.* **93** 221301 [astro-ph/0403353]

Land K and Magueijo J, *The axis of evil*, 2005 *Phys. Rev. Lett.* **95** 071301 [astro-ph/0502237]

Copi C J, Huterer D, Schwarz D J and Starkman G D, *On the large-angle anomalies of the microwave sky*, 2006 *Mon. Not. Roy. Astron. Soc.* **367** 79 [astro-ph/0508047]

de Oliveira-Costa A and Tegmark M, *CMB multipole measurements in the presence of foregrounds*, 2006 *Phys. Rev.* **D74** 023005 [astro-ph/0603369]

Hinshaw G *et al* , *Three-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Temperature analysis* [astro-ph/0603451]

Abramo L R, Bernui A, Ferreira I S, Villela T and Wuensche C A, *Alignment Tests for low CMB multipoles*, [astro-ph/0604346]

Magueijo J and Sorkin R D, *Occam's razor meets WMAP* [astro-ph/0604410]

[6] Hansen F K, Balbi A, Banday A J and Górski K M, *Cosmological parameters and the WMAP data revisited*, 2004 *Mon. Not. Roy. Astron. Soc.* **354** 905 [astro-ph/0406232]

[7] Bielewicz P, Eriksen H K, Banday A J, Górski K M and Lilje P M, *Multipole vector anomalies in the first-year WMAP data: a cut-sky analysis*, 2005 *Astrophys. J.* **635** 750 [astro-ph/0507186]

[8] Freeman P E, Genovese C R, Miller C J, Nichol R C and Wasserman L, *Examining the Effect of the Map-Making Algorithm on Observed Power Asymmetry in WMAP Data*, 2006 *Astrophys. J.* **638** 1 [astro-ph/0510406]

Helling R C, Schupp P and Tesileanu T, *CMB statistical anisotropy, multipole vectors and the influence of the dipole* [astro-ph/0603594]

[9] Copi C J, Huterer D, Schwarz D J and Starkman G D, *The Uncorrelated Universe: Statistical Anisotropy and the Vanishing Angular Correlation Function in WMAP Years 1-3* [astro-ph/0605135]

[10] Vauclair S C *et al* , *The XMM–NEWTON Omega Project: II. Cosmological implications from the high redshift L-T relation of X-ray clusters*, 2003 *Astron. & Astrophys.* **412** L37 [astro-ph/0311381]

Blanchard A, *Cosmological Interpretation from High Redshift Clusters Observed Within the XMM-Newton Ω-Project* [astro-ph/0502220]

Sadat R *et al* , $XMM-Newton$ Ω project: III. Gas mass fraction shape in high redshift clusters, 2005 *Astron. & Astrophys.* **437** 31 [astro-ph/0503426]

Nuza S E and Blanchard A, *On the baryon mass function for galaxy clusters*, 2006 *Astron. & Astrophys.* **452** 47 [astro-ph/0601402]

[11] Mukherjee P, Parkinson D, Corasaniti P S, Liddle A R and Kunz M, *Model selection as a science driver for dark energy surveys*, 2006 *Mon. Not. Roy. Astron. Soc.* **369** 1725 [astro-ph/0512484]

[12] Bassett B A, Kunz M, Silk J and Ungarelli C, *A late-time transition in the cosmic dark energy?*, 2002 *Mon. Not. Roy. Astron. Soc.* **336** 1217 [astro-ph/0203383]

Alam U, Sahni V and Starobinsky A A, *Is there Supernova Evidence for Dark Energy Metamorphosis?*, 2004 *Mon. Not. Roy. Astron. Soc.* **354** 275 [astro-ph/0311364]

Alam U, Sahni V and Starobinsky A A, *The case for dynamical dark energy revisited*, 2004 JCAP06(2004)008 [astro-ph/0403687]

Jonsson J, Goobar A, Amanullah R and Bergstrom L, *No evidence for Dark Energy Metamorphosis?*, 2004 JCAP09(2004)007 [astro-ph/0404468]

Corasaniti P S, Kunz M, Parkinson D, Copeland E J and Bassett B A, *The foundations of observing dark energy dynamics with the Wilkinson Microwave Anisotropy Probe*, 2004 *Phys. Rev.* **D70** 083006 [astro-ph/0406608]

Alam U, Sahni V and Starobinsky A A, *Rejoinder to No Evidence of Dark Energy Metamorphosis*, [astro-ph/0406672]

Hannestad S and Mörtsell E, *Cosmological constraints on the dark energy equation of state and its evolution*, 2004 JCAP09(2004)001 [astro-ph/0407259]

Bassett B A, Corasaniti P S and Kunz M, *The essence of quintessence and the cost of compression*, 2004 *Astrophys. J.* **617** L1 [astro-ph/0407364]

Stephan-Otto C, *Optimized supernova constraints on dark energy evolution* [astro-ph/0605403]

[13] Ichikawa K, Kawasaki M, Sekiguchi T and Takahashi T, *Implication of dark energy parametrizations on the determination of the curvature of the universe* [astro-ph/0605481]

[14] Gong Y and Wang A, *Observational constraints on the acceleration of the Universe*, 2006 *Phys.*

Rev. **D73** 083506 [astro-ph/0601453]

[15] Hogg D W *et al* , *Cosmic homogeneity demonstrated with luminous red galaxies*, 2005 *Astrophys. J.* **642** 54 [astro-ph/0411197]

[16] Pietronero L and Labini F S, *Statistical physics for complex cosmic structures*, 2006 *AIP Conf. Proc.* **822** 294 [astro-ph/0406202]

Joyce M, Labini F S, Gabrielli A, Montuori M and Pietronero L, *Basic properties of galaxy clustering in the light of recent results from the Sloan Digital Sky Survey*, 2005 *Astron. & Astrophys.* **443** 11 [astro-ph/0501583]

[17] Kerscher M, Schmalzing J, Buchert T and Wagner H, *Fluctuations in the IRAS 1.2 Jy catalogue*, 1998 *Astron. & Astrophys.* **333** 1 [astro-ph/9704028]

Kerscher M, Mecke K, Schmalzing J, Beisbart C, Buchert T and Wagner H, *Morphological fluctuations of large-scale structure: The PSCz survey*, 2001 *Astron. & Astrophys.* **373** 1 [astro-ph/0101238]

Hikage C *et al* (SDSS collaboration), *Minkowski Functionals of SDSS galaxies I : Analysis of Excursion Sets*, 2003 *Publ. Astron. Soc. Jap.* **55** 911 [astro-ph/0304455]

[18] Lemaître A G, *The Expanding Universe*, 1933 *Ann. Soc. Sci Bruxelles* **A53** 51 (in French) Reprinted in 1997 *Gen. Rel. Grav.* **29** 641

Tolman R C, *Effect of Inhomogeneity on Cosmological Models*, 1934 *Proc. Nat. Acad. Sci. USA* **20** 169 Reprinted in 1997 *Gen. Rel. Grav.* **29** 935

Bondi H, *Spherically symmetrical models in general relativity*, 1947 *Mon. Not. Roy. Astron. Soc.* **107** 410

[19] Krasiński A, *Inhomogeneous Cosmological Models*, 1997 Cambridge University Press, Cambridge

[20] Partovi M H and Mashhoon B, *Toward Verification Of Large Scale Homogeneity In Cosmology*, 1984 *Astrophys. J.* **276** 4

Goicoechea L J and Martin-Mirones J M, *Magnitude-redshift test: cosmological inhomogeneity effects*, 1987 *Astron. & Astrophys.* **186** 22

Humphreys N, Maartens R and Matravers D, *Anisotropic Observations in Universes with Nonlinear Inhomogeneity*, 1997 *Astrophys. J.* **477** 47 [astro-ph/9602033]

Célérier M-N, *Do we really see a cosmological constant in the supernovae data?*, 1999 *Astron. & Astrophys.* **353** 63 [astro-ph/9907206]

Iguchi H, Nakamura T and Nakao K-i, *Is Dark Energy the Only Solution to the Apparent Acceleration of the Present Universe?*, 2002 *Prog. Theor. Phys.* **108** 809 [astro-ph/0112419]

Alnes H, Amarzguioui M and Grøn Ø, *An inhomogeneous alternative to dark energy?*, 2006 *Phys. Rev.* **D73** 083519 [astro-ph/0512006]

Bolejko K, *Supernovae Ia observations in the Lemaître-Tolman model* [astro-ph/0512103]

Mansouri R, *Structured FRW universe leads to acceleration: A non-perturbative approach* [astro-ph/0512605]

Vanderveld R A, Flanagan É É and Wasserman I, *Mimicking Dark Energy with Lemaître-Tolman-Bondi Models: Weak Central Singularities and Critical Points*, 2006 *Phys. Rev.* **D74** 023506 [astro-ph/0602476]

Apostolopoulos P S, Brouzakis N, Tetradis N and Tzavara N, *Cosmological Acceleration and Gravitational Collapse*, 2006 JCAP06(2006)009 [astro-ph/0603234]

Garfinkle D, *Inhomogeneous spacetimes as a dark energy model* [gr-qc/0605088]

Alnes H and Amarzguioui M, *CMB anisotropies seen by an off-center observer in a spherically symmetric inhomogeneous universe* [astro-ph/0607334]

Enqvist K and Mattsson T, *The effect of inhomogeneous expansion on the supernova observations* [astro-ph/0609120]

[21] Biswas T, Mansouri R and Notari A, *Nonlinear Structure Formation and "Apparent" Acceleration: an Investigation*, 2006 [astro-ph/0606703]

[22] Bolejko K, *Structure formation in quasispherical Szekeres model*, 2006 *Phys. Rev.* **D73** 123508 [astro-ph/0604490]

Bolejko K, *Cosmological applications of the Szekeres model* [astro-ph/0607130]

[23] Barausse E, Matarrese S and Riotto A, *The Effect of Inhomogeneities on the Luminosity Distance-Redshift Relation: is Dark Energy Necessary in a Perturbed Universe?*, 2005 *Phys. Rev.* **D71** 063537 [astro-ph/0501152]

[24] Bonvin C, Durrer R and Gasparini M A, *Fluctuations of the luminosity distance*, 2006 *Phys. Rev.* **D73** 023523 [astro-ph/0511183]

Bonvin C, Durrer R and Kunz M, *The dipole of the luminosity distance: a direct measure of H(z)*, 2006 *Phys. Rev. Lett.* **96** 191302 [astro-ph/0603240]

[25] Moffat J W and Tatarski D C, *Redshift And Structure Formation In A Spatially Flat Inhomogeneous Universe*, 1992 *Phys. Rev.* **D45** 3512

Moffat J W and Tatarski D C, *Cosmological observations in a local void*, 1995 *Astrophys. J.* **453** 17 [astro-ph/9407036]

Wu X-P, Deng Z, Zou Z, Fang L-Z and Qin B, *On the measurement of the Hubble constant in a local low density universe*, 1995 *Astrophys. J.* **448** L65 [astro-ph/9412082]

Suto Y, Suginohara T and Inagaki Y, *Limits on the Global Hubble Constant and the Age of the Universe from the Local Hubble Constant Measurement*, 1995 *Prog. Theor. Phys.* **93** 839 [astro-ph/9412090]

Nakao K-i, Gouda N, Chiba T, Ikeuchi S, Nakamura T and Shibata M, *The Hubble parameter in void universe: Effect of the peculiar velocity*, 1995 *Astrophys. J.* **453** 541 [astro-ph/9502054]

Shi X, Widrow L M and Dursi L J, *Measuring Hubble's constant in our inhomogeneous Universe*, 1996 *Mon. Not. Roy. Astron. Soc.* **281** 565 [astro-ph/9506120]

Zehavi I, Riess A G, Kirshner R P and Dekel A, *A Local Hubble Bubble from Type Ia Supernovae?*, 1998 *Astrophys. J.* **503** 483 [astro-ph/9802252]

Tomita K, *Bulk flows and cosmic microwave background dipole anisotropy in cosmological void models*, 2000 *Astrophys. J.* **529** 26 [astro-ph/9905278]

Tomita K, *Distances and lensing in cosmological void models*, 2000 *Astrophys. J.* **529** 38 [astro-ph/9906027]

Goodwin S P, Thomas P A, Barber A J, Gribbin J and Onuora L I, *The local to global H_0 ratio and the SNe Ia results* [astro-ph/9906187]

Tomita K, *Anisotropy of the Hubble Constant in a Cosmological Model with a Local Void on Scales of $\sim$ 200 Mpc*, *Prog. Theor. Phys.* **105** 419 [astro-ph/0005031]

Tomita K, *A Local Void and the Accelerating Universe*, 2001 *Mon. Not. Roy. Astron. Soc.* **326** 287 [astro-ph/0011484]

Tomita K, *Analyses of Type Ia Supernova Data in Cosmological Models with a Local Void*, 2001 *Prog. Theor. Phys.* **106** 929 [astro-ph/0104141]

Tomita K, *Luminous Matter Distribution, Bulk Flows and Baryon Contents in Cosmological Models with a Local Void*, 2002 *Prog. Theor. Phys.* **108** 103 [astro-ph/0203125]

Tomita K, *Dipole anisotropies of* IRAS *galaxies and the contribution of a large-scale local void*, 2003 *Astrophys. J.* **584** 580 [astro-ph/0211137]

[26] Peebles P J E, *Probing General Relativity on the Scales of Cosmology* [astro-ph/0410284]

[27] Blanchard A, Douspis M, Rowan-Robinson M and Sarkar S, *An alternative to the cosmological 'concordance model'*, 2003 *Astron. & Astrophys.* **412** 35 [astro-ph/0304237]

Sarkar S, *Measuring the cosmological density perturbation*, 2005 *Nucl. Phys.* **B** *Proc. Suppl.* 148 1 [hep-ph/0503271]

[28] Sandage A, Tammann G A, Saha A, Reindl B, Macchetto F D and Panagia N, *The Hubble Constant: A Summary of the HST Program for the Luminosity Calibration of Type Ia Supernovae by Means of Cepheids* [astro-ph/0603647]

[29] Blanchard A, Douspis M, Rowan-Robinson M and Sarkar S, *Large-scale galaxy correlations as a test for dark energy*, 2006 *Astron. & Astrophys.* **449** 925 [astro-ph/0512085]

[30] Kristian J and Sachs R H, *Observations In Cosmology*, 1966 *Astrophys. J.* **143** 379

[31] Csáki C, Kaloper N and Terning J, *Dimming supernovae without cosmic acceleration*, 2002 *Phys.*

Rev. Lett. **88** 161302 [hep-ph/0111311]

[32] Evslin J and Fairbairn M, *Photon Mixing in Domain Walls and the Cosmic Coincidence Problem*, 2006 JCAP02(2006)011 [hep-ph/0507020]

[33] Shirokov M F and Fisher I Z, *Isotropic Space with Discrete Gravitational-Field Sources. On the Theory of a Nonhomogeneous Universe*, 1963 *Sov. Astron. J.* **6** 699 Reprinted in 1998 *Gen. Rel. Grav.* **30** 1411

[34] Ellis G F R, *Relativistic cosmology: its nature, aims and problems*, 1983 The invited papers of the 10th international conference on general relativity and gravitation p 215
Ellis G F R and Stoeger W, *The 'fitting problem' in cosmology*, 1987 *Class. Quant. Grav.* **4** 1697

[35] Kasai M, *Construction of Inhomogeneous Universes Which Are Friedmann-Lemaître-Robertson-Walker On Average*, 1992 *Phys. Rev. Lett.* **69** 2330
Kasai M, *Inhomogeneous cosmological models which are homogeneous and isotropic on average*, 1993 *Phys. Rev.* **D47** 3214

[36] Buchert T and Ehlers J, *Averaging inhomogeneous Newtonian cosmologies*, 1997 *Astron. & Astrophys.* **320** 1 [astro-ph/9510056]

[37] Tsamis N C and Woodard R P, *Quantum Gravity Slows Inflation*, 1996 *Nucl. Phys.* **B474** 235 [hep-ph/9602315]
Tsamis N C and Woodard R P, *The Quantum Gravitational Back-Reaction on Inflation*, 1997 *Annals Phys.* **253** 1 [hep-ph/9602316]
Abramo L R, Tsamis N C and Woodard R P, *Cosmological Density Perturbations From A Quantum Gravitational Model Of Inflation*, 1999 *Fortsch. Phys.* **47** 389 [astro-ph/9803172]
Woodard R P, *Effective Field Equations of the Quantum Gravitational Back-Reaction on Inflation* [astro-ph/0111462]
Woodard R P, *A leading logarithm approximation for inflationary quantum field theory*, 2005 *Nucl. Phys.* **B** *Proc. Suppl.* 148 108 [astro-ph/0502556]
Miao S P and Woodard R P, *Leading log solution for inflationary Yukawa* [gr-qc/0602110]

[38] Ehlers J and Buchert T, *Newtonian cosmology in Lagrangian formulation: Foundations and perturbation theory*, 1997 *Gen. Rel. Grav.* **29** 733 [astro-ph/9609036]

[39] Unruh W, *Cosmological long wavelength perturbations* [astro-ph/9802323]

[40] Takada M and Futamase T, *How is the local-scale gravitational instability influenced by the surrounding large-scale structure formation?*, 1999 *Gen. Rel. Grav.* **31** 461 [astro-ph/9901079]

[41] Buchert T, *On average properties of inhomogeneous fluids in general relativity I: dust cosmologies*, 2000 *Gen. Rel. Grav.* **32** 105 [gr-qc/9906015]

[42] Sicka C, Buchert T and Kerscher M, *Backreaction in cosmological models* [astro-ph/9907137]
Buchert T, Kerscher M and Sicka C, *Backreaction of inhomogeneities on the expansion: the evolution of cosmological parameters*, 2000 *Phys. Rev.* **D62** 043525 [astro-ph/9912347]

[43] Taruya A and Soda J, *Spherical infall model in a cosmological background density field*, 2000 *Mon. Not. Roy. Astron. Soc.* **317** 873 [astro-ph/9911116]

[44] Buchert T, *On average properties of inhomogeneous cosmologies*, 2000 *Proc. 9th JGRG conference* 306 Hiroshima 1999, ed Eriguchi Y *et al* [gr-qc/0001056]

[45] Kerscher M, Buchert T and Futamase T, *On the abundance of collapsed objects*, 2001 *Astrophys. J.* **558** L79 [astro-ph/0007284]

[46] Buchert T and Carfora M, *Matter seen at many scales and the geometry of averaging in relativistic cosmology*, 2002 General Relativity, Cosmology, and Gravitational Lensing, Napoli Series on Physics and Astrophysics, ed Marmo G *et al* Naples, Bibliopolis p 29 [gr-qc/0101070]
Buchert T and Carfora M, *Regional averaging and scaling in relativistic cosmology*, 2002 *Class. Quant. Grav.* **19** 6109 [gr-qc/0210037]

[47] Buchert T, *On average properties of inhomogeneous fluids in general relativity II: perfect fluid cosmologies*, 2001 *Gen. Rel. Grav.* **33** 1381 [gr-qc/0102049]

[48] Tatekawa T, Suda M, Maeda K-i and Kubotani H, *Inhomogeneities in Newtonian Cosmology and its Backreaction to the Evolution of the Universe* [astro-ph/0109501]

[49] Geshnizjani G and Brandenberger R, *Back Reaction And Local Cosmological Expansion Rate*, 2002 *Phys. Rev.* **D66** 123507 [gr-qc/0204074]

[50] Schwarz D J, *Accelerated expansion without dark energy* [astro-ph/0209584]
Balakin A B, Pavon D, Schwarz D J and Zimdahl W, *Curvature force and dark energy*, 2003 *New J. Phys.* **5** 085 [astro-ph/0302150]

[51] Buchert T and Carfora M, *Cosmological parameters are dressed*, 2003 *Phys. Rev. Lett.* **90** 031101 [gr-qc/0210045]

[52] Brandenberger R H, *Back Reaction of Cosmological Perturbations and the Cosmological Constant Problem* [hep-th/0210165]

[53] Geshnizjani G and Brandenberger R, *Back Reaction Of Perturbations In Two Scalar Field Inflationary Models*, 2005 JCAP04(2005)006 [hep-th/0310265]

[54] Räsänen S, *Dark energy from backreaction*, 2004 JCAP02(2004)003 [astro-ph/0311257]
Räsänen S, *Backreaction of linear perturbations and dark energy* [astro-ph/0407317]

[55] Buchert T and Carfora M, *The Cosmic Quartet - Cosmological Parameters of a Smoothed Inhomogeneous Spacetime*, 2002 *Proc. 12th JGRG conference* 157 Tokyo, ed Shibata M *et al* [astro-ph/0312621]

[56] Hosoya A, Buchert T and Morita M, *Information Entropy in Cosmology*, 2004 *Phys. Rev. Lett.* **92** 141302 [gr-qc/0402076]

[57] Räsänen S, *Backreaction in the Lemaître-Tolman-Bondi model*, 2004 JCAP11(2004)010 [gr-qc/0408097]

[58] Kolb E W, Matarrese S, Notari A and Riotto A, *The effect of inhomogeneities on the expansion rate of the universe*, 2005 *Phys. Rev.* **D71** 023524 [hep-ph/0409038]

[59] Kolb E W, Matarrese S, Notari A and Riotto A, *Cosmological influence of super-Hubble perturbations*, 2005 *Mod. Phys. Lett.* **A20** 2705 [astro-ph/0410541]

[60] Kolb E W, Matarrese S, Notari A and Riotto A, *Primordial inflation explains why the universe is accelerating today* [hep-th/0503117]

[61] Flanagan É É, *Can superhorizon perturbations drive the acceleration of the Universe?*, 2005 *Phys. Rev.* **D71** 103521 [hep-th/0503202]

[62] Geshnizjani G, Chung D J H and Afshordi N, *Do Large-Scale Inhomogeneities Explain Away Dark Energy?*, 2005 *Phys. Rev.* **D72** 023517 [astro-ph/0503553]

[63] Hirata C M and Seljak U, *Can superhorizon cosmological perturbations explain the acceleration of the universe?*, 2005 *Phys. Rev.* **D72** 083501 [astro-ph/0503582]

[64] Notari A, *Late time failure of Friedmann equation* [astro-ph/0503715]

[65] Räsänen S, *Backreaction and spatial curvature in dust universes*, 2006 *Class. Quant. Grav.* **23** 1823 [astro-ph/0504005]

[66] Siegel E R and Fry J N, *Effects of inhomogeneities on cosmic expansion*, 2005 *Astrophys. J.* **628** L1 [astro-ph/0504421]

[67] Martineau P and Brandenberger R H, *The effects of gravitational back-reaction on cosmological perturbations*, 2005 *Phys. Rev.* **D72** 023507 [astro-ph/0505236]

[68] Tsamis N C and Woodard R P, *A measure of cosmological acceleration*, 2005 *Class. Quant. Grav.* **22** 4171 [gr-qc/0506089]

[69] Kolb E W, Matarrese S and Riotto A, *On cosmic acceleration without dark energy* [astro-ph/0506534]

[70] Buchert T, *A cosmic equation of state for the inhomogeneous universe: Can a global far-from-equilibrium state explain dark energy?*, 2005 *Class. Quant. Grav.* **22** L113 [gr-qc/0507028]
Buchert T, *On globally static and stationary cosmologies with or without a cosmological constant and the dark energy problem*, 2006 *Class. Quant. Grav.* **23** 817 [gr-qc/0509124]

[71] Ishibashi A and Wald R M, *Can the acceleration of our universe be explained by the effects of inhomogeneities?*, 2006 *Class. Quant. Grav.* **23** 235 [gr-qc/0509108]

[72] Losic B and Unruh W G, *Long-wavelength metric backreactions in slow-roll inflation*, 2005 *Phys. Rev.* **D72** 123510 [gr-qc/0510078]

Losic B and Unruh W G, *On leading order gravitational backreactions in de Sitter spacetime*, 2006 *Phys. Rev.* **D74** 023511 [gr-qc/0604122]

[73] Martineau and Brandenberger R, *Back-Reaction: A Cosmological Panacea* [astro-ph/0510523]

[74] Kolb E W, Matarrese S and Riotto A, *Comments on Backreaction and Cosmic Acceleration* [astro-ph/0511073]

[75] Kolb E W, Matarrese S and Riotto A, *Comments on long-wavelength backreaction and dark energy* [astro-ph/0511124]

[76] Chuang C H, Gu J A and Hwang W Y, *Inhomogeneity-Induced Cosmic Acceleration in a Dust Universe* [astro-ph/0512651]

[77] Kasai M, Asada H and Futamase T *Toward a No-go Theorem for Accelerating Universe by Nonlinear Backreaction*, 2006 *Prog. Theor. Phys.* **115** 827 [astro-ph/0602506]

[78] Parry M, *A rule of thumb for cosmological backreaction*, 2006 JCAP06(2006)016 [astro-ph/0605159]

[79] Kai T, Kozaki H, Nakao K-i, Nambu Y and Yoo C M, *Can inhomogeneities accelerate the cosmic volume expansion?* [gr-qc/0605120]

[80] Paranjape A and Singh T P, *The Possibility of Cosmic Acceleration via Spatial Averaging in Lemaître-Tolman-Bondi Models* [astro-ph/0605195]

[81] Räsänen S, *Cosmological acceleration from structure formation* [astro-ph/0605632]

[82] Buchert T, Larena J and Alimi J M, *Correspondence between kinematical backreaction and scalar field cosmologies - the 'morphon field'* newblock [gr-qc/0606020]

[83] Ellis G F R and Buchert T, *The universe seen at different scales*, 2005 *Phys. Lett.* **A347** 38 [gr-qc/0506106]

[84] Ellis G F R, *Cosmology and Verifiability*, 1975 *Q.Jl R. astr. Soc.* **16** 245

[85] Ellis G F R, Nel S D, Maartens R, Stoeger W R and Whitman A P, *Ideal observational cosmology*, 1985 *Phys. Rept.* **124** 315

   Stoeger W R, Ellis G F R and Nel S D, *Observational cosmology. III. Exact spherically symmetric dust solutions*, 1992 *Class. Quant. Grav.* **9** 509

   Stoeger W R, Stanley S J, Nel D and Ellis G F R, *Observational cosmology. IV. Perturbed spherically symmetric dust solutions*, 1992 *Class. Quant. Grav.* **9** 1711

   Stoeger W R, Stanley S J, Nel D and Ellis G F R, *Observational cosmology. V. Solution of the first-order general perturbation equations*, 1992 *Class. Quant. Grav.* **9** 1725

   Stoeger W R, Ellis G F R and Xu C, *Observational cosmology. VI. The microwave background and the Sachs-Wolfe effect*, 1994 *Phys. Rev.* **D49** 1845

   Maartens R and Matravers D R, *Isotropic and semi-isotropic observations in cosmology*, 1994 *Class. Quant. Grav.* **11** 2693

   Maartens R, Humphreys N P, Matravers D R and Stoeger W R, *Inhomogeneous universes in observational coordinates*, 1996 *Class. Quant. Grav.* **13** 253 Erratum 1996 *Class. Quant. Grav.* **13** 1689 [gr-qc/9511045]

[86] Bonnor W B and Ellis G F R, *Observational homogeneity of the universe*, 1986 *Mon. Not. Roy. Astron. Soc.* **218** 605

   Stoeger W R, Ellis G F R and Hellaby C, *The relationship between continuum homogeneity and statistical homogeneity in cosmology*, 1987 *Mon. Not. Roy. Astron. Soc.* **226** 373

   Mustapha N, Hellaby C and Ellis G F R, *Large scale inhomogeneity versus source evolution: Can we distinguish them observationally?*, 1997 *Mon. Not. Roy. Astron. Soc.* **292** 817 [gr-qc/9808079]

[87] Matravers D R, Ellis G F R and Stoeger W R, *Complementary Approaches to Cosmology: Relating Theory and Observations*, 1995 *Q. J. R. astr. Soc.* **36** 29

[88] Ellis G F R, *83 years of general relativity and cosmology: progress and problems*, 1999 *Class. Quant. Grav.* **16** A37

[89] Ellis G F R, *Relativistic Cosmology 1999: Issues and Problems*, 2000 *Gen. Rel. Grav.* **32** 1135

[90] El-Ad H, Piran T and da Costa L N, *Automated Detection of Voids in Redshift Surveys*, 1996

Astrophys. J. **462** L13 [astro-ph/9512070]

El-Ad H, Piran T and da Costa L N, *A catalogue of the voids in the IRAS 1.2-Jy survey*, 1997 *Mon. Not. Roy. Astron. Soc.* **287** 790 [astro-ph/9608022]

El-Ad H and Piran T, *Voids in the Large-Scale Structure*, 1997 *Astrophys. J.* **491** 421 [astro-ph/9702135]

El-Ad H and Piran T, *A case devoid of bias: Optical Redshift Survey voids vs. IRAS voids*, 2000 *Mon. Not. Roy. Astron. Soc.* **313** 553 [astro-ph/9908004]

[91] Hoyle F and Vogeley M S, *Voids in the Point Source Catalogue Survey and the Updated Zwicky Catalog*, 2002 *Astrophys. J.* **566** 641 [astro-ph/0109357]

[92] Hoyle F and Vogeley M S, *Voids in the Two-Degree Field Galaxy Redshift Survey*, 2004 *Astrophys. J.* **607** 751 [astro-ph/0312533]

[93] Shandarin S F, Sheth J V and Sahni V, *Morphology of the supercluster-void network in ΛCDM cosmology*, 2004 *Mon. Not. Roy. Astron. Soc.* **353** 162 [astro-ph/0312110]

Shandarin S F, *Morphological Statistics of the Cosmic Web* [astro-ph/0405303]

[94] Colberg J M, Sheth R K, Diaferio A, Gao L and Yoshida N, *Voids in a ΛCDM universe*, 2005 *Mon. Not. Roy. Astron. Soc.* **360** 216 [astro-ph/0409162]

[95] Maartens R, Ellis G F R and Stoeger W R, *Limits on anisotropy and inhomogeneity from the cosmic background radiation*, 1995 *Phys. Rev.* **D51** 1525 [astro-ph/9501016]

Maartens R, Ellis G F R and Stoeger W R, *Improved limits on anisotropy and inhomogeneity from the cosmic background radiation*, 1995 *Phys. Rev.* **D51** 5942

Maartens R, Ellis G F R and Stoeger W R, *Anisotropy and inhomogeneity of the universe from $\Delta T/T$*, 1996 *Astron. & Astrophys.* **309** L7 [astro-ph/9510126]

Stoeger W R, Araujo M and Gebbie T, *The Limits on Cosmological Anisotropies and Inhomogeneities from COBE Data*, 1997 *Astrophys. J.* **476** 435 [astro-ph/9904346]

[96] Stoeger W R, Maartens R and Ellis G F R, *Proving almost homogeneity of the universe: An Almost Ehlers-Geren-Sachs theorem*, 1995 *Astrophys. J.* **443** 1

[97] Wainwright J, Hancock M J and Uggla C, *Asymptotic self-similarity breaking at late times in cosmology*, 1999 *Class. Quant. Grav.* **16** 2577 [gr-qc/9812010]

Nilsson U S, Uggla C, Wainwright J and Lim W C, *An almost isotropic cosmic microwave temperature does not imply an almost isotropic universe*, 1999 *Astrophys. J.* **522** L1 [astro-ph/9904252]

Lim W C, Nilsson U S and Wainwright J, *Anisotropic universes with isotropic cosmic microwave background radiation*, 2001 *Class. Quant. Grav.* **18** 5583 [gr-qc/9912001]

[98] Clarkson C A and Barrett R K, *Does the Isotropy of the CMB Imply a Homogeneous Universe? Some Generalised EGS Theorems*, 1999 *Class. Quant. Grav.* **16** 3781 [gr-qc/9906097]

Barrett R K and Clarkson C A, *Undermining the Cosmological Principle: Almost Isotropic Observations in Inhomogeneous Cosmologies*, 2000 *Class. Quant. Grav.* **17** 5047 [astro-ph/9911235]

Clarkson C A, Coley A A, O'Neill E S D, Sussman R A and Barrett R K, *Inhomogeneous cosmologies, the Copernican principle and the cosmic microwave background: More on the EGS theorem*, 2003 *Gen. Rel. Grav.* **35** 969 [gr-qc/0302068]

[99] Ellis G F R and Solomons D M, *Caustics of compensated spherical lens models*, 1998 *Class. Quant. Grav.* **15** 2381 [gr-qc/9802005]

[100] Lieu R and Mittaz J P D, *Are the WMAP angular magnification measurements consistent with an inhomogeneous critical density Universe?*, 2005 *Astrophys. J.* **623** L1 [astro-ph/0409048]

[101] Kibble T W B and Lieu R, *Average magnification effect of clumping of matter*, 2005 *Astrophys. J.* **632** 718 [astro-ph/0412275]

[102] Lieu R and Mittaz J P D, *On the absence of gravitational lensing of the cosmic microwave background*, 2005 *Astrophys. J.* **628** 583 [astro-ph/0412276]

[103] Ehlers J, *Contributions to the relativistic mechanics of continuous media*, 1961 *Abh. Akad. Wiss. Lit. Mainz. Nat. Kl.* **11** 792 (in German) Reprinted in 1993 *Gen. Rel. Grav.* **25** 1225

[104] Ellis G F R, *Relativistic Cosmology*, 1971 General Relativity and Cosmology, ed Sachs R K Academic Press Inc., London p 104

[105] Ellis G F R and Matravers D R, *General Covariance in General Relativity?*, 1995 *Gen. Rel. Grav.* **27** 777

[106] Del Popolo A, Ercan E N and Xia Z, *Ellipsoidal collapse and previrialization*, 2001 *Astron. J.* **122** 487 [astro-ph/0108080]

[107] Buchert T and Domínguez A, *Adhesive Gravitational Clustering*, 2005 *Astron. & Astrophys.* **438** 443 [astro-ph/0502318]

[108] Trodden M and Vachaspati T, *Causality and cosmic inflation*, 2000 *Phys. Rev.* **D61** 023502 [gr-qc/9811037]
Trodden M and Vachaspati T, *What is the homogeneity of our universe telling us?*, 1999 *Mod. Phys. Lett.* **A14** 1661 [gr-qc/9905091]

[109] Davis T M and Lineweaver C H, *Expanding Confusion: common misconceptions of cosmological horizons and the superluminal expansion of the universe*, 2004 *Publ. Astr. Soc. Australia* **21** 97 [astro-ph/0310808]

[110] Ellis G F R, *On horizons and the cosmic landscape*, 2006 *Gen. Rel. Grav.* **38** 1209 [astro-ph/0603266]

[111] Moffat J W, *Cosmic microwave background, accelerating universe and inhomogeneous cosmology*, 2005 JCAP10(2005)012 [astro-ph/0502110]
Tomita K, *Second-order gravitational effects of local inhomogeneities on CMB anisotropies in nonzero-Λ flat cosmological models*, 2005 *Phys. Rev.* **D72** 043526 [astro-ph/0505157]
Tomita K, *Second-order gravitational effects of local inhomogeneities on CMB anisotropies and non-Gaussian signatures*, 2005 *Phys. Rev.* **D72** 103506 erratum 2006 *Phys. Rev.* **D73** 029901 [astro-ph/0509518]
Inoue K T and Silk J, *Local Voids as the Origin of Large-angle Cosmic Microwave Background Anomalies* [astro-ph/0602478]
Abramo L R, Sodré Jr L and Wuensche C A, *Anomalies in the low CMB multipoles and extended foregrounds* [astro-ph/0605269]

[112] Vale C, *Local Pancake Defeats Axis of Evil* [astro-ph/0509039]
Cooray A and Seto N, *Did WMAP see Moving Local Structures?*, 2005 JCAP12(2005)0512 [astro-ph/0510137]
Rakić A, Räsänen S and Schwarz D J, *Microwave sky and the local Rees-Sciama effect*, 2006 *Mon. Not. Roy. Astron. Soc.* **369** L27 [astro-ph/0601445]

[113] Zalaletdinov R M, *Averaging out the Einstein equations*, 1992 *Gen. Rel. Grav.* **24** 1015
Mars M and Zalaletdinov R M, *Space-time averages in macroscopic gravity and volume-preserving coordinates*, 1997 *J. Math. Phys.* **38** 4741 [dg-ga/9703002]
Zalaletdinov R M, *Averaging problem in general relativity, macroscopic gravity and using Einstein's equations in cosmology*, 1997 *Bull. Astron. Soc. India* **25** 401 [gr-qc/9703016]
Zalaletdinov R, *Space-time averages of classical physical fields* [gr-qc/0411004]
Coley A A, Pelavas N and Zalaletdinov R M, *Cosmological solutions in macroscopic gravity*, 2005 *Phys. Rev. Lett.* **95** 151102 [gr-qc/0504115]

[114] Padmanabhan T, *A possible approach to understand nonlinear gravitational clustering in expanding background* [astro-ph/0511536]

[115] Raychaudhuri A, *Relativistic Cosmology. I*, 1955 *Phys. Rev.* **98** 1123

[116] van Elst H and Ellis G F R, *Quasi-Newtonian dust cosmologies*, 1998 *Class. Quant. Grav.* **15** 3545 [gr-qc/9805087]

[117] Padmanabhan T, *Structure formation in the universe*, 1993 Cambridge University Press, Cambridge p 273

[118] Liddle A R and Lyth D H, *Cosmological Inflation and Large-Scale Structure*, 2000 Cambridge University Press, Cambridge p 277

[119] Sheth R K and van de Weygaert R, *A hierarchy of voids: Much ado about nothing*, 2004 *Mon.*

Not. Roy. Astron. Soc. **350** 517 [astro-ph/0311260]

[120] Engineer S, Kanekar N and Padmanabhan T, *Nonlinear density evolution from an improved spherical collapse model*, 2000 *Mon. Not. Roy. Astron. Soc.* **314** 279 [astro-ph/9812452]

[121] Sato H and Maeda K-i, *The Expansion Law of the Void in the Expanding Universe*, 1983 *Prog. Theor. Phys.* **70** 119

Maeda K-i and Sato H, *Expansion of a Thin Shell around a Void in Expanding Universe*, 1983 *Prog. Theor. Phys.* **70** 772

Maeda K-i and Sato H, *Expansion of a Thin Shell around a Void in Expanding Universe. II*, 1983 *Prog. Theor. Phys.* **70** 1276

Suto Y, Sato K and Sato H, *Expansion of Voids in a Matter-Dominated Universe*, 1984 *Prog. Theor. Phys.* **71** 938

Bertschinger E, *The Self-Similar Evolution Of Holes In An Einstein-De Sitter Universe*, 1985 *Astrophys. J. Suppl.* **58** 1

[122] D'Eath P D, *On the Existence of Perturbed Robertson-Walker Universes*, 1976 *Ann. Phys.* **98** 237

[123] Hofmann S, Schwarz D J and Stoecker H, *Formation of small-scale structure in SUSY CDM* [astro-ph/0211325]

Green A M, Hofmann S and Schwarz D J, *The power spectrum of SUSY-CDM on sub-galactic scales*, 2004 *Mon. Not. Roy. Astron. Soc.* **353** L23 [astro-ph/0309621]

Green A M, Hofmann S and Schwarz D J, *The first WIMPy halos*, 2005 JCAP08(2005)003 [astro-ph/0503387]

Green A M, Hofmann S and Schwarz D J, *Small scale WIMP physics* [astro-ph/0508553]

[124] Kolb E W and Tkachev I I, *Femtolensing and Picolensing by Axion Miniclusters*, 1996 *Astrophys. J.* **460** L25 [astro-ph/9510043]

Sikivie P, *The Search for Dark Matter Axions* [hep-ph/0606014]

[125] Boehm C and Fayet P, *Scalar dark matter candidates*, 2004 *Nucl. Phys.* **B683** 219 [hep-ph/0305261]

Boehm C, Mathis H, Devriendt J and Silk J, *WIMP matter power spectra and small scale power generation*, 2003 [astro-ph/0309652]

Boehm C, *Structure formation for light annihilating dark matter*, 2004 *Int. J. Mod. Phys.* **A19** 4355

[126] Asaka T, Blanchet S and Shaposhnikov M, *The νMSM, Dark Matter and Neutrino Masses*, 2005 *Phys. Lett.* **B631** 151 [hep-ph/0503065]

Abazajian K, *Production and evolution of perturbations of sterile neutrino dark matter*, 2006 *Phys. Rev.* **D73** 063506 [astro-ph/0511630]

Asaka T, Kusenko A and Shaposhnikov M, *Opening a new window for warm dark matter*, 2006 *Phys. Lett.* **B638** 401 [hep-ph/0602150]

[127] Boehm C, Fayet P and Schaeffer R, *Constraining dark matter candidates from structure formation*, 2001 *Phys. Lett.* **B518** 8 [astro-ph/0012504]

Boehm C, Fayet P and Schaeffer R, *Constraining the strength of dark matter interactions from structure formation* [astro-ph/0205406]

Boehm C and Schaeffer R, *Constraints on dark matter interactions from structure formation: Damping lengths*, 2005 *Astron. & Astrophys.* **438** 419 [astro-ph/0410591]

[128] Barenboim G, Requejo O M and Quigg C, *Observational constraints on undulant cosmologies*, 2006 JCAP04(2006)008 [astro-ph/0510178]

Linder E V, *On oscillating dark energy*, 2006 *Astropart. Phys.* **25** 167 [astro-ph/0511415]

[129] Spergel D N *et al* , *Wilkinson Microwave Anisotropy Probe (WMAP) three year results: Implications for cosmology* [astro-ph/0603449]

[130] Dubinski J, da Costa L N, Goldwirth D S, Lecar M and Piran T, *Void evolution and the large scale structure*, 1993 *Astrophys. J.* **410** 458

[131] van de Weygaert R, Sheth R and Platen E, *A Hierarchy of Voids* [astro-ph/0404397]

[132] Furlanetto S and Piran T, *The evidence of absence: galaxy voids in the excursion set formalism*, 2006 *Mon. Not. Roy. Astron. Soc.* **366** 467 brtastro-ph/0509148

[133] Riess A G *et al* (Supernova Search Team Collaboration), *Type Ia Supernova Discoveries at z > 1 From the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution*, 2004 *Astrophys. J.* **607** 665 [astro-ph/0402512]

[134] Ferrer F and Räsänen S, *Dark energy and decompactification in string gas cosmology*, 2006 JHEP02(2006)016 [hep-th/0509225]

[135] Carrera M and Giulini D, *On the influence of the global cosmological expansion on the local dynamics in the Solar System* [gr-qc/0602098]

[136] Mota D F and Barrow J D, *Varying Alpha in a More Realistic Universe*, 2004 *Phys. Lett.* **B581** 141 [astro-ph/0306047]

Mota D F and Barrow J D, *Local and Global Variations of The Fine Structure Constant*, 2004 *Mon. Not. Roy. Astron. Soc.* **349** 291 [astro-ph/0309273]

Clifton T, Mota D F and Barrow J D, *Inhomogeneous gravity*, 2005 *Mon. Not. Roy. Astron. Soc.* **358** 601 [gr-qc/0406001]

Shaw D J and Barrow J D, *The local effects of cosmological variations in physical 'constants' and scalar fields. I: Spherically symmetric spacetimes*, 2006 *Phys. Rev.* **D73** 123505 [gr-qc/0512022]

Shaw D J and Barrow J D, *Local experiments see cosmologically varying constants* [gr-qc/0512117]

Shaw D J and Barrow J D, *The local effects of cosmological variations in physical 'constants' and scalar fields. II: Quasi-spherical spacetimes*, 2006 *Phys. Rev.* **D73** 123506 [gr-qc/0601056]

[137] Shandarin, Feldman H A, Heitmann K and Habib S, *Shapes and Sizes of Voids in the ΛCDM Universe: Excursion Set Approach*, 2006 *Mon. Not. Roy. Astron. Soc.* **367** 1629 [astro-ph/0509858]

[138] Lee J and Park D, *Rotation of Cosmic Voids and Void-Spin Statistics* [astro-ph/0606477]

[139] Freedman W L *et al* , *Final Results from the Hubble Space Telescope Key Project to Measure the Hubble Constant*, 2001 *Astrophys. J.* **553** 47 [astro-ph/0012376]

[140] Rauch M *et al* , *Expansion and Collapse in the Cosmic Web*, 2005 *Astrophys. J.* **632** 58 [astro-ph/0509262]

[141] Jaffe T R, Banday A J, Eriksen H K, Górski K M and Hansen F K, *Evidence of vorticity and shear at large angular scales in the WMAP data: A violation of cosmological isotropy?*, 2005 *Astrophys. J.* **629** L1 [astro-ph/0503213]

Land K and Magueijo J, *Template fitting and the large-angle CMB anomalies*, 2006 *Mon. Not. Roy. Astron. Soc.* **367** 1714 [astro-ph/0509752]

Ghosh T, Hajian A and Souradeep T, *Unveiling Hidden Patterns in CMB Anisotropy Maps* [astro-ph/0604279]

Bridges M, McEwen J D, Lasenby A N and Hobson M P, *Markov chain Monte Carlo analysis of Bianchi VII_h models* [astro-ph/0605325]

Jaffe T R, Banday A J, Eriksen H K, Górski K M and Hansen F K, *Bianchi Type VII_h Models and the WMAP 3-year Data* [astro-ph/0606046]