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SIMULATION STUDY ON THE ENERGY DEPENDENCE OF THE TMCI THRESHOLD IN THE CERN-SPS

G. Rumolo, E. Métral, E. Shaposhnikova, CERN, Geneva, Switzerland

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This paper concentrates on theoretical studies of Transverse Mode Coupling Instability (TMCI) at the SPS. It shows the expected thresholds based on the HEADTAIL tracking model and on impedance values estimated from previous measurements. First, the effect of space charge is addressed as an important ingredient at the low energies. Subsequently, the change of TMCI threshold possibly induced by a higher injection energy into the SPS (plausible according to the upgrade studies) is investigated and a scaling law with energy is derived.

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This paper concentrates on theoretical studies of Transverse Mode Coupling Instability (TMCI) at the SPS. It shows the expected thresholds based on the HEADTAIL tracking model and on impedance values estimated from previous measurements. First, the effect of space charge is addressed as an important ingredient at the low energies. Subsequently, the change of TMCI threshold possibly induced by a higher injection energy into the SPS (plausible according to the upgrade studies) is investigated and a scaling law with energy is derived.

INTRODUCTION AND MOTIVATIONS

Studies for the LHC performance upgrade include the improvement of the existing LHC injectors and/or the design of possible new rings in the injector chain [1]. Several scenarios, aimed at overcoming the existing bottlenecks, are presently being taken into consideration. The crucial point of the most promising option consists in raising the injection energy into the existing SPS from the present 26 GeV/c to 40 or 60 GeV/c. This would allow first to overcome the existing limitations and secondly, a future upgrade of the SPS to a higher extraction energy ring (1 TeV). This scenario would require the corresponding upgrade of the present SPS injector – the PS ring [2].

One question is how exactly a higher injection energy would affect instability thresholds in the SPS. In particular, TMCI is expected to be a potential danger in the SPS with the enhancement of the broad-band impedance due to the installation of 4 new MKE kickers in the ring [3]. In addition, the vertical electron cloud single bunch instability (also TMC type) has been a limiting factor for a long time for the number of batches that could be injected into the machine (it can be overcome by operating the ring with a rather high vertical chromaticity, which nonetheless is possibly harmful in terms of beam lifetime) [4].
The study of the effect of higher injection energy into the SPS is carried out following the steps outlined below. We will consider an LHC-type beam interacting with a broad-band impedance or an electron cloud. The essential parameters are listed in Table 1.

Table 1: Parameters used in our study

Name	Symbol	Unit	Value
Momentum	p_0	GeV/c	26, 40, 60
Norm. transv. emitt.	$\epsilon_{x,y}$	μm	2.8
Long. emitt. (2σ)	ϵ_z	eVs	0.35
Bunch length	σ_z	m	0.3
Bunch population	N		1.1×10^{11}
Vertical tune	Q_y		26.13
Momentum comp.	α		0.00192
Shunt impedance	R_T	MΩ/m	20
Quality factor	Q		1.
Resonance frequency	$\omega_r/2\pi$	GHz	1.3
E-cloud density	ρ_e	m^{-3}	10^{12}

Taking the reference at 26 GeV/c (which corresponds to $\gamma = 27.7$), the main assumptions of our study are:
1) The longitudinal emittance and the bunch length are kept constant at the values of 0.35 eVs and 0.3 m (having assumed the same beam production scheme in the PS). The momentum spread $\Delta p/p_0$ is scaled by $27.7/\gamma$ for 40 and 60 GeV/c and the matched voltage is re-adjusted according to $|\eta|/\gamma$ (η is the slip factor) for each set of simulations.
2) The normalised transverse emittances are constant, 2.8 μm. Consequently the transverse beam sizes need to be re-scaled by $\sqrt{27.7/\gamma}$ for 40 and 60 GeV/c.

In Section II we review the models used for the TMCI studies. Starting from a brief summary of the analytical formulae used for threshold estimations, we subsequently present the macroparticle model of the HEADTAIL code [5], with which we can simulate the interaction of the SPS bunch at the different energies both with a conventional broad-band impedance and with an electron cloud of known density. Simulation results are given in Section III and conclusions are drawn in Section IV.

MODELS FOR THE TMCI STUDIES

Transverse Mode Coupling Instability can be qualitatively described as a transverse instability that shows when the single bunch intensity is sufficiently high to cause tail disruption due to the strong wake fields left behind by the bunch head. As the single bunch current is increased, azimuthal mode lines shift. When the threshold value is reached, coupling between a pair of adjoining modes occurs and the beam motion becomes unstable. When the instability growth time is very short compared with the synchrotron period, this instability is also referred to as "strong head-tail" or "beam break-up", recalling the single bunch instability in a linac, where the intra-bunch synchrotron motion does not play any role [6]. The TMCI threshold is one of the fundamental limitations in the performances of many machines. TMCI was observed in lepton machines, but observation of TMCI for hadrons (without transition crossing) has first occurred in 2002 in the SPS, when a single bunch with very low longitudinal emittance injected into the machine went unstable, showing very similar features to the typical TMCI [7, 8].

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Analytical estimates

Transverse mode coupling by a resonator impedance can be calculated within the Sacherer model using the approximate "Hermitian" modes for Gaussian bunches to get explicit expressions for the frequency shifts and instability thresholds [9]. The approximate expressions for the stability limit (as maximum number of particles per bunch, N_b) with a broad-band impedance, for an ultrarelativistic bunch ($\beta \approx 1$) matched to its bucket and for zero chromaticity, can be written (symbols are explained in Table 1):

$$N_b = \begin{cases} \dfrac{3.75}{\sqrt{2}} \cdot \dfrac{Q^2 Q_y |\eta| \omega_r}{R_T c e} \cdot \epsilon_z [\mathrm{eVs}] \quad \text{if} \quad \omega_r \sigma_z / c \leq 1 \\ 11.25\pi \cdot \dfrac{Q Q_y |\eta| c^2}{\omega_r^2 \sigma_z^3 R_T e} \cdot \epsilon_z [\mathrm{eVs}] \quad \text{if} \quad \omega_r \sigma_z / c \gg 1 \end{cases} \tag{1}$$

Different expressions in the limits of very short or very long bunch are given. The longitudinal emittance ϵ_z is defined as 4π times the product between the r.m.s. values of bunch length and energy spread. For the SPS the N_b should be then multiplied by the inverse Yokoya factor $12/\pi^2$ (which is the scaling factor of the dipole wake field, the one responsible for the collective instability [10]) due to the flat structure of the SPS beam chamber. The dependence on the energy is in the slip factor $\eta = (\alpha - \gamma^{-2})$. Therefore, from Eq. (1) it is evident that, if the longitudinal emittance is kept constant, the TMCI threshold levels off as $\eta \to \alpha$ for $\gamma \gg 1/\sqrt{\alpha}$. In the general case one can show that for fixed beam parameters and zero chromaticity the scaling of the TMCI threshold with energy is $\propto p_0 Q_s$, where Q_s is the synchrotron tune. This translates into an energy dependence like $\sqrt{|\eta|\gamma}$ for fixed voltage or $\propto |\eta|$ for matched voltage.

These formulae could be in principle also applicable for the electron cloud case, modeling the effect of the electron cloud with a broad-band impedance [11] having ω_r equal to the oscillation frequency of the electrons in the bunch

$$\omega_e = \sqrt{\frac{N_b r_e c^2}{2\sigma_z \sigma_y (\sigma_x + \sigma_y)}} \ ,$$

and R_T/Q depending on the electron density (enhanced by the pinching effect). The complication of this approach is that both the electron frequency and density depend on the bunch intensity N_b, which renders Eq. (1) difficult to interpret. In Ref. [12] Eq. (1) is solved for the electron cloud case in the long bunch regime and having assumed that the electron cloud impedance, R_T/Q has an inversely proportional dependence on $\sigma_y(\sigma_x + \sigma_y)/\sigma_z$. The electron cloud instability threshold turns out to have a stronger dependence on energy because the transverse beam sizes scale like $1/\sqrt{\gamma}$, as the normalized emittances are conserved. Following Ref. [12], the overall energy dependence is like $|\eta|^2/\gamma$, if we do not consider the pinch enhancement factor in the cloud impedance expression, whereas it becomes $|\eta|/\gamma$ taking that also into account. Figure 1 summarizes these dependences of the TMC and TMC-like instability thresholds on energy.

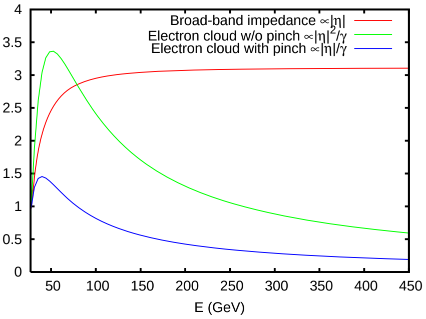

Figure 1: Energy dependence of the TMC and TMC-like instability threholds as theoretically expected. Values are normalized to the 26 GeV/c case.

The decrease of the transverse bunch sizes also plays a role in the classical TMCI threshold through space charge, as long as this effect is significant. Space charge is not included in Eq. (1), and is expected to raise the threshold through head-tail phase mixing at the lower energies.

Simulation: broad-band impedance

We have used the HEADTAIL code to do instability simulations of a single bunch interacting with a broad-band impedance. The model uses a bunch made of macroparticles and then subdivided into N slices, such that each macroparticle in a bunch slice feels the sum of the wakes (those corresponding to a broad-band impedance, with their dipole and quadrupole components appropriately weighed with the Yokoya coefficients) of the preceding slices. Particles also mix longitudinally according to the equations of synchrotron motion in a sinusoidal bucket. Scans for different intensities changed in ranges of plausible values have been done to find the thresholds at different energies. The simulation campaigns were run with and without space charge, so as to assess how the additional tune spread introduced by space charge affects the instability dynamics and threshold.

The main advantages of the macroparticle simulation are:

1) Simulations can be run for particles in a sinusoidal bucket, whereas the analytical formula is only valid in the linear approximation of longitudinal motion. Besides, the stability of an unmatched bunch can also be studied because the longitudinal dynamics is correctly modeled also when the bunch is not matched to the bucket and executes quadrupole oscillations.

2) Both the effect of dipole and quadrupole wake fields for flat pipe can be included.

3) Space charge can be included and its effect disentangled.

Simulation: electron cloud

The dependence of the electron cloud instability on energy has also been studied using the HEADTAIL code. In this model each bunch slice interacts with a two-dimensional electron cloud, which is assumed to be located at one or more points along the ring. At each passage

through the electron cloud, a bunch slice acts on the macroelectrons and its macroparticles feel in turn the effect of the cloud. The next slice will then feel the effect of the cloud as it results from the intereaction with all the preceding slices. After the full passage, the bunch is transported to the next kick point through a linear transport matrix, which can include chromaticity, amplitude detuning, space charge.
The kick given by the electron cloud is similar to the one given by a broad band impedance, since the wake field of an electron cloud has the shape of a damped oscillator, even if this is a simplified picture because, unlike the conventional wakes, the electron cloud wake also depends on the position along the bunch of the slice that creates the wake [13].

SIMULATION RESULTS

A simulation campaign was launched at 26, 40, 60, 270, and 450 GeV/c (using the other parameters from Table 1) in order to study the dependence of the TMCI threshold on energy. Applying Eq. (1) with the long bunch approximation ($\omega_r \sigma_z / c \simeq 8$) to the SPS bunch, we obtain at 26 GeV/c a TMCI threshold of about 1.5×10^{11}. The threshold value scales then like $|\eta|$, i.e. it increases up to about 100 GeV/c and levels off at about 4.5×10^{11} for higher energies. Figure 2 shows the results of the HEADTAIL simulations. Thresholds obtained in simulations have the expected energy dependence ($\propto |\eta|$), but are about a factor 2 below those analytically predicted. This is probably due to the approximations of the analytical model and to the fact that we are at the lower limit of the range of applicability of Eq. (1), close to where it breaks down. Simulations including space charge show that space charge can indeed raise the TMCI threshold up to energies of 60 GeV/c and probably higher, but becomes negligible at very high energies.

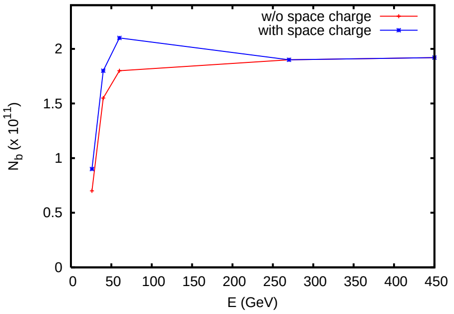

Figure 2: Simulated TMCI threholds at different energies, with and without space charge.

Electron cloud simulations at 26 and 60 GeV/c using an electron cloud density of 10^{12} m^{-3} show that the instability is stronger at higher energy both in field-free and dipole regions (see Fig. 3). Using the energy scaling as $|\eta|^2/\gamma$, the bunch at 26 GeV/c should have been more unstable (the threshold is lower by a factor 3.3, see Fig. 1). Using the energy scaling $|\eta|/\gamma$, we would have expected very similar thresholds for 26 and 60 GeV/c.

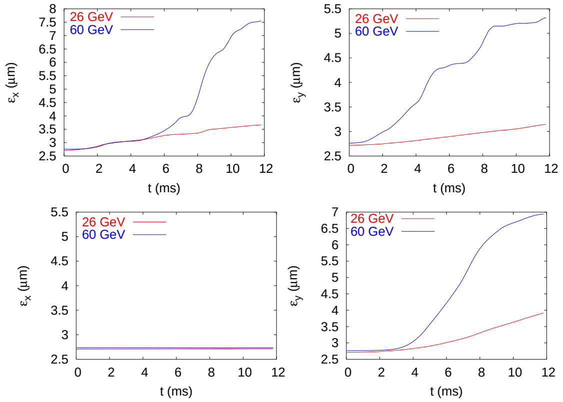

Figure 3: Horizontal (left) and vertical (right) emittance growth due to electron cloud at 26 and 60 GeV/c. Simulation results are shown for a field-free (upper) and a dipole (lower) region.

CONCLUSIONS

The threshold of TMCI increases like $|\eta|$ when raising the injection energy, and at 40 GeV/c it becomes higher than the threshold at 26 GeV/c including space charge. Higher injection energy in the SPS is certainly beneficial against TMCI. On the other hand, due to the lower transverse sizes at higher energy, the TMC-like electron cloud instability seems to become stronger when increasing the energy. More simulation studies with bunch current scans over a broad energy range are needed to define thresholds and scaling laws.

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