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ON THE IMPLEMENTATION OF EXPERIMENTAL SOLENOIDS IN MAD-X AND THEIR EFFECT ON COUPLING IN THE LHC

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INTRODUCTION

The transverse coupling introduced by an experimental solenoid in the LHC can be quantified [1] by the complex coupling coefficient c,

$$c^{\mp}_{\text{sol}} = -\frac{i}{4\pi}\,\frac{B_s l}{B\rho}\left(\sqrt{\frac{\beta^*_y}{\beta^*_x}} \pm \sqrt{\frac{\beta^*_x}{\beta^*_y}}\right), \qquad (1)$$

where $B_s l$ is the integrated solenoid field strength and $B\rho$ the beam rigidity. $\beta^*_{z=x|y}$ denotes the beta function value at the center of the solenoid, which is the IP (interaction point). The LHC is designed to operate with round beams, hence $\beta^*_x = \beta^*_y$ and the sum coupling resonance ($c^+_{\text{sol}} = 0$) is not exited. At injection energy (450 GeV), where the effect is most pronounced, this coefficient amounts to

$$c^{-}_{\text{CMS, 450 GeV}} = -\frac{i}{2\pi}\,\frac{B_s l}{B\rho} = -0.0053\, i, \qquad (2)$$

for the strongest experimental solenoid (CMS-magnet). This is small and other sources of coupling, in particular the a_2 errors in the main dipoles are expected to give a coupling coefficient which is more than one order of magnitude larger. A global coupling correction for the whole machine is planned [2]. The solenoidal effects are too small to justify dedicated magnets for local solenoid compensation. Still, it may be desirable to allow to adjust coupling separately for each solenoid which may be turned on or off.

An optics design code such as MADX [3] can be used to study the coupling effects. Several recent developments in MADX concerning solenoids have been done, in particular tracking with solenoids can be performed and the use of PTC allows to compute the coupled lattice functions defined by Ripken [4]. Implementation details regarding solenoids will be discussed in the last section.

Relevant LHC (beam) parameters are summarized in Tab. 1, values referring to the experimental solenoids are given in Tab. 2. A schematic layout of the IP5 region (CMS) and its optics is depicted in Fig. 1.

Table 1: LHC general parameters

Parameter			Inj.	Coll.
Momentum	p	[GeV/c]	450	7000
Trans. norm. emittance	ε_{N}	[μm rad]	3.5	3.75
Horizontal Tune	Q_x		64.28	64.31
Vertical Tune	Q_y		59.31	59.32
Max. β H/V (cell)	$\beta_{\text{max.}}$	[m]	177/180	
Min. β H/V (cell)	$\beta_{\text{min.}}$	[m]	30/30	
Average β ($= R/Q$)	$\langle\beta\rangle$	[m]	66/72	
Max. Dispersion H/V (cell)	$D_{\text{max.}}$	[m]	2.018/0.0	
RMS beam size IP5	σ_{rms}	[μm]	375.2	16.7
Half crossing angle IP1/IP5		[μrad]	$\pm$ 160	$\pm$ 142.5
Half parallel separation IP1/IP5		[mm]	$\pm$ 2.5	0.0
Plane of crossing IP1			vertical	
Plane of crossing IP5			horizontal	
β at IP1/IP5	β^*	[m]	17	0.55

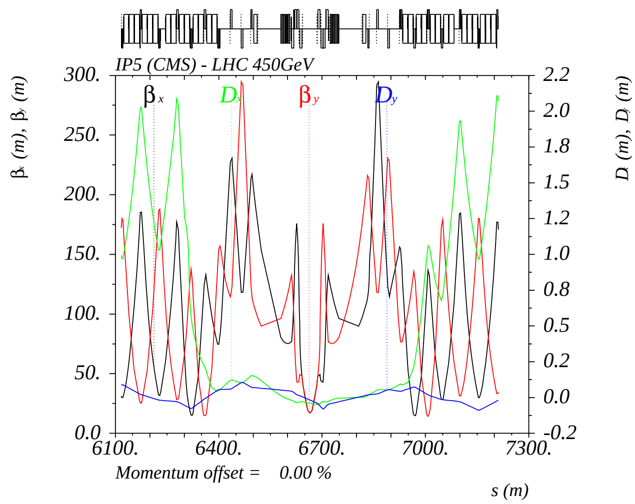

Figure 1: LHC IP5 (CMS) layout and optics.

COUPLING CORRECTION

There are different strategies to compensate coupling in an accelerator. A solenoid modifies the transverse oscillation modes and rotates the beams by an angle $\theta = \frac{B_s l}{2B\rho}$. A compensation should eliminate or minimize these effects. An obvious solution is the use of anti-solenoids left and right of the main solenoids. This is not practical for high energy machines with very large solenoids like the LHC.

Another standard technique is based on the use of skew quadrupoles. This *resonance method* [1] uses Hamiltonian formalism and treats the coupling fields as a perturbation of the uncoupled optics. The skew quadrupoles are placed and adjusted such that the main resonance coupling terms disappear.

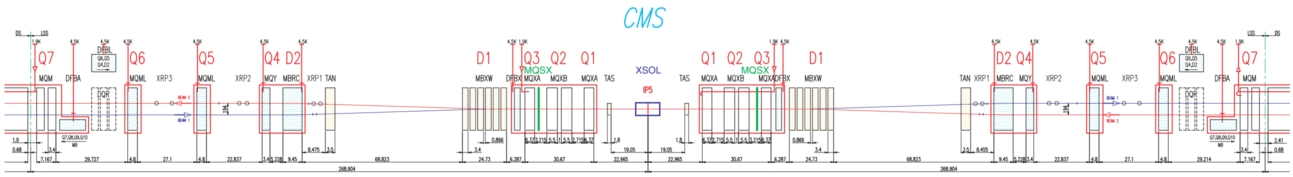

Figure 2: LHC IP5 layout. CMS solenoid (XSOL) and nearby skew quadrupoles (MQSX)

LHC AND SOLENOIDS

In the LHC a global coupling correction scheme will be used to minimize the impact of field imperfections and tilt errors on the beam quality [2]. We now look at the effect of the strongest LHC experimental solenoid (CMS, IP5) on beam optics at 450 GeV.

At injection energy (450 GeV) beams will generally be separated by means of separation bumps. Additionally, the beams cross from inside to outside and vice versa at each IP. The solenoid field slightly tilts the crossing plane. The half separation of the two beams is decreased by 15 μm, which is less than 1 % of the nominal separation. If no separation bumps are used, e.g. in an early collision run at 450 GeV, the solenoid will introduce a separation of this order. This can easily be corrected by adjusting the separation bumps.

Table 2: Parameters of experimental solenoids in LHC

Property/Experiment		ATLAS	CMS
Magnetic induction at IP	[T]	2.0	4.0
Coil length	[m]	5.3	12.5

The β-beating which is induced by the CMS solenoid is shown for the LHC machine in Fig. 3. The peak β-beating $(\Delta\beta/\beta)_{peak} = 0.1\,\%$ is well below and within the accepted margin [5] of 21 %.

Equally the dispersion beating is well within the budget of 30 %. To analyze we use the normalized dispersion function $D_{x|y,N} = D_{x|y}/\sqrt{\beta_{x|y}}$,

$$\frac{\Delta D_x(s)}{\sqrt{\beta_x(s)}} \Bigg/ \frac{D_{x,\mathrm{qf}}}{\sqrt{\beta_{x,\mathrm{qf}}}} \text{ and } \frac{\Delta D_y(s)}{\sqrt{\beta_y(s)}} \Bigg/ \frac{D_{y,\mathrm{qd}}}{\sqrt{\beta_{y,\mathrm{qd}}}}\,, \tag{3}$$

where $D_{x,\mathrm{qf}} = 2.1$ m, $D_{y,\mathrm{qd}} = 16$ cm and $\beta_{x,\mathrm{qf}} = \beta_{y,\mathrm{qd}} = 180$ m. The peak dispersion beating expressed in these quantities amounts to $D_{x,N}/D_{x,\mathrm{qf},N} = 0.2\,\%$ and $D_{y,N}/D_{y,\mathrm{qd},N} = 2.5\,\%$.

To summarize, we can say that the impact on beam optics is small and will hardly be visible in standard operation.

We note, that the local solenoid compensation used in LEP with two skew quadrupole pairs relied on the symmetry of the optical functions around the IP [6]. This scheme cannot simply be employed for the antisymmetric LHC optics.

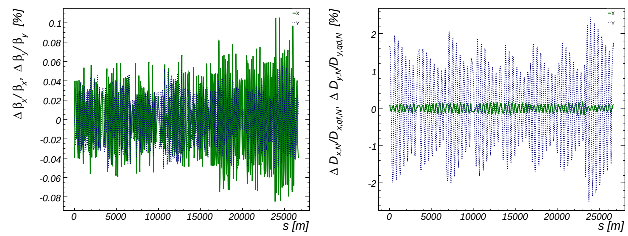

Figure 3: Induced β- and dispersion-beating by the CMS solenoid in the LHC at 450 GeV.

SOLENOIDS AND MAD-X

Following a request on the last MADX day [7], we implemented a *thin* solenoid in the tracking module as well as in the twiss module. A *thick* version was available already, however since the tracking module exclusively operates with thin elements, this became necessary.

The implementation follows closely the formulae given in [8], where the canonical equations of motion are derived directly from the Hamiltonian. We only report the relevant formulae here and note some important facts:

- The hard-edge model is used. Fringe fields of the solenoid are taken into account, however they are finite 'hard-edge' steps of the B-field;
- the solution of the equations of motion are obtained from the *expanded* Hamiltonian, and it that sense the solutions or not 'exact';
- and although this is a *thin* element, both the normalized magnetic strength k_s *and* its product $k_s \cdot L$ with the length L have to be known and used in the equations. This makes the solenoid different from e.g. the multipoles.

If z^i, z^f denote initial and final (canonical) coordinates, where $z \in [x, p_x, y, p_y, \sigma, p_\sigma]$, we can write the solution of the equations of motion in a solenoid of length $L = \Delta s$ in the *thin-lens approximation* as

$$\begin{aligned}
x^f &= \quad x^i \cdot \cos\Delta\Theta + y^i \cdot \sin\Delta\Theta, \\
p_x^f &= \quad \hat{p}_x^f \cdot \cos\Delta\Theta + \hat{p}_y^f \cdot \sin\Delta\Theta, \\
y^f &= -x^i \cdot \sin\Delta\Theta + y^i \cdot \cos\Delta\Theta, \\
p_y^f &= -\hat{p}_x^f \cdot \sin\Delta\Theta + \hat{p}_y^f \cdot \cos\Delta\Theta, \\
\sigma^f &= \quad \hat{\sigma}^f + \left\{x^i \cdot \hat{p}_y^f + y^i \cdot \hat{p}_x^f\right\} \cdot \frac{H(s_0)\cdot\Delta s}{\left[1+f(p_\sigma^i)\right]^2} \cdot f'(p_\sigma^i), \\
p_\sigma^f &= \quad p_\sigma^i,
\end{aligned} \tag{4}$$

where the intermediate variables $\hat{z}$ are given by

$$\begin{aligned}\hat{p}_x^f &= p_x^i - \frac{x^i}{[1+f(p_\sigma^i)]} \cdot H(s_0)^2 \cdot \Delta s, \\ \hat{p}_y^f &= p_y^i - \frac{y^i}{[1+f(p_\sigma^i)]} \cdot H(s_0)^2 \cdot \Delta s, \\ \hat{\sigma}^f &= \sigma^i - \frac{f'(p_\sigma^i)}{[1+f(p_\sigma^i)]^2} \cdot H(s_0)^2 \cdot \Delta s \cdot \frac{1}{2}\left\{(x^i)^2 + (y^i)^2\right\}.\end{aligned} \tag{5}$$

The normalized solenoid strength $k_s = \frac{e \cdot B}{p_0 \cdot c}$ is related to $H(s_0)$ by

$$H(s_0) = \frac{1}{2}\frac{e}{p_0 \cdot c} \cdot B_s(0,0,s_0) = \frac{1}{2} \cdot k_s. \tag{6}$$

$H(s_0)$ is *not* the Hamiltonian, but an abbreviation for the expression defined above, see also [8, (2.23e, p.9)]. Accordingly the quantity $\Delta\Theta$ is then related to $k_{si} = k_s \cdot L$ ($= k_s \cdot \Delta s$ to conform to the notation in [8]) and given by

$$\begin{aligned}H(s_0) \cdot \Delta s &= \frac{1}{2} k_{si}, \\ \Delta\Theta &= \frac{H(s_0) \cdot \Delta s}{[1+f(p_\sigma^i)]}.\end{aligned} \tag{7}$$

RIPKEN [8] uses the function $f(p_\sigma)$ to stress the fact that the relative momentum deviation $\Delta p/p_0$ depends on the longitudinal canonical variable p_σ. It is given by

$$\begin{aligned}f(p_\sigma) &= \sqrt{1 + 2p_\sigma + \beta^2 p_\sigma^2} - 1, \\ f'(p_\sigma) &= \frac{1 + \beta^2 p_\sigma}{\sqrt{1 + 2p_\sigma + \beta^2 p_\sigma^2}}.\end{aligned} \tag{8}$$

There is an important difference in the set of canonical variables used in MADX and by RIPKEN [8], but noting that

$$\begin{aligned}p_\sigma &= \frac{p_T}{\beta}, \\ \sigma &= \beta \cdot T,\end{aligned} \tag{9}$$

where (T, p_T) are the longitudinal variables used in MADX and β is the relativistic quantity of the beam, we can completely integrate the formulae from [8] into MADX.

The transverse transport map can also be written in matrix form, where we use $K = k_s/2$.

$$M_{\text{thin, sol}} = \underbrace{\begin{bmatrix}\cos\Delta\Theta & 0 & \sin\Delta\Theta & 0 \\ 0 & \cos\Delta\Theta & 0 & \sin\Delta\Theta \\ -\sin\Delta\Theta & 0 & \cos\Delta\Theta & 0 \\ 0 & -\sin\Delta\Theta & 0 & \cos\Delta\Theta\end{bmatrix}}_{\text{pure rotation}} \cdot \underbrace{\begin{bmatrix}1 & 0 & 0 & 0 \\ -\frac{K^2 L}{1+f(p_\sigma^i)} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{K^2 L}{1+f(p_\sigma^i)} & 0\end{bmatrix}}_{\text{focusing in both planes}} \tag{10}$$

The last decomposition of the thin-lens solenoid transfer matrix shows, that it can be separated into a pure rotational part, which refers to the solenoid body without any edge effects and a focusing part, which give the effect of the fringe fields. It should be also apparent, that slicing a thick solenoid into several thin ones and replacing the effect of the thick solenoid by

$$M_{\text{thick, sol}} \approx M_{\text{Drift}} \cdot M_{\text{thin, sol}} \cdot M_{\text{Drift}} \ldots M_{\text{Drift}} \cdot M_{\text{thin, sol}} \cdot M_{\text{Drift}}$$

does in general not give the correct edge focusing effect. However, it can be shown numerically that such a slicing converges to the thick lens solution.

It should be noted that the Hamiltonian for the solenoid can also be solved exactly, see e.g. [9], however, the solution is not symplectic. Therefore the solution from the expanded Hamiltonian is used.

CONCLUSION AND OUTLOOK

Solenoid coupling effects in the LHC are small and global compensation at injection should be sufficient. We discussed the size of the effects and propose a global compensation which allows to correct each solenoid individually. We also reported on the status and implementation of solenoids in MADX.

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