Wakefield Calculations for 3D Collimators *

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Abstract

Using a recently developed time domain numerical approach we calculate the short-range geometric wakefields of 3D collimators and compare with analytical models. We find, in the diffractive regime, that the transverse mode kick factor can be approximated from the change in field energy between the beam pipe and the collimator if the collimator is long, or using a "field clipping" estimate if it is short. For collimators of past and present measurements at SLAC, numerical, analytical, and measurement results are compared.

INTRODUCTION

Designing collimators is an important task for accelerator and FEL projects. In this report we study the short-range geometric wake of 3D (rectangular) collimators using a time domain numerical method [1]. This method, together with an indirect 3D integration algorithm, allows one to obtain accurate results for the difficult combination of short bunches and long, gently tapered collimators [2].

In this report, we consider symmetric collimators of the type sketched in Fig. 1. Depending on bunch length, the wake can be thought of as being inductive, diffractive, or of an intermediate type [3]. If the collimator is round and $\rho_1 \equiv \alpha b_2 \sigma^{-1} \ll 1$, it is inductive, with kick factor [4]

$$k_\perp = \frac{Z_0 c\,\alpha}{2\pi^{3/2}\sigma}\left(\frac{1}{b_2} - \frac{1}{b_1}\right) , \tag{1}$$

where σ is the rms bunch length (we consider only Gaussian bunches), $Z0 = 377\,\Omega$, and c is the speed of light. The inductive regime for round collimators was studied numerically in [5], and the results were in agreement with Eq. 1.

If the collimator has a rectangular cross-section, it is inductive when $\rho_2 \equiv \alpha h^2 \sigma^{-1} b_2^{-1} \ll 1$, with h the collimator width; in this case [3]

$$k_\perp = \frac{Z_0 c\,\alpha h}{4\pi^{1/2}\sigma}\left(\frac{1}{b_2^2} - \frac{1}{b_1^2}\right) . \tag{2}$$

If $\rho_1 \ll 1$ but $\rho_2 \gtrsim \pi^2$, then a rectangular collimator is in the intermediate regime and

$$k_\perp = 0.215 A\, Z_0 c \sqrt{\frac{\alpha}{\sigma b_2^3}} , \tag{3}$$

with A a factor to be defined in the next section.

When $\rho_1 \gg 1$ both round and rectangular collimators are diffractive, a regime we study in the next section. Finally, in the last section we demonstrate that, in fact, Eq. 1 agrees well with numerical results for rectangular collimators with arbitrary ρ_2 (only requiring $\rho_1 \ll 1$).

* Work supported by the Department of Energy, contract DE-AC02-76SF00515, the EUROFEL and the EUROTeV projects.

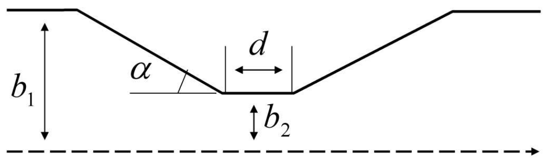

Figure 1: Top half of a symmetric collimator.

DIFFRACTIVE REGIME

When a beam encounters a collimator it takes time for the primary fields of the beam to adjust themselves to a new steady-state configuration. Hence, we expect wakefield effects for short and long collimators to differ.

In the longitudinal case, the impedance of a collimator at high frequencies (the diffractive regime) [6, 7]

$$Z_\parallel \approx 2Z^e , \tag{4}$$

with Z^e the impedance associated with the beam's (static) fields. We suggest that depending on the collimator length d, Z^e should be found in two different ways. For a long collimator, Z^e is equal to the change in the field energy between the beam pipe and the collimator; it has the form

$$Z^e = \frac{1}{Q^2 Z_0}\left(\int_{\Omega_1} (\nabla\varphi_1)^2 ds - \int_{\Omega_2} (\nabla\varphi_2)^2 ds\right) , \tag{5}$$

where φ_i is the solution to the Poisson equation

$$\begin{aligned} \Delta\varphi_i(\vec{x}) &= Z_0 Q \delta(\vec{x} - \vec{x}_0) , \quad \vec{x} \in \Omega_i \\ \varphi_i(\vec{x}) &= 0 , \quad \vec{x} \in \partial\Omega_i , \quad i = 1, 2 ; \end{aligned} \tag{5'}$$

Q is bunch charge, Ω_1, Ω_2, are the beam pipe and collimator cross-sections. For a short collimator, Z^e is obtained from the field energy clipped away by the collimator

$$Z^e = \frac{1}{Q^2 Z_0} \int\limits_{\Omega_1 - \Omega_2} (\nabla\varphi_1)^2 ds . \tag{6}$$

The loss and kick factors of a Gaussian bunch are approximately given by

$$k_\parallel = \frac{c}{\sqrt{\pi}\sigma} Z^e(0) , \tag{7}$$

$$k_\perp = \frac{c}{\Delta^2}[Z^e(\Delta) - Z^e(0)] , \tag{8}$$

where Δ is beam offset from the axis.

For a monopole mode the steady-state field pattern does not depend on pipe radius and both Eqs. 5 and 6 give

$$Z^e = \frac{Z_0}{2\pi} \ln\left(\frac{b_1}{b_2}\right) , \tag{9}$$

and hence

$$k_\parallel = \frac{Z_0 c}{2\pi^{3/2}\sigma} \ln\left(\frac{b_1}{b_2}\right) . \tag{10}$$

The dipole steady-state field pattern, however, depends on the pipe radius and Eqs. 5 and 6 give different results. For a long collimator ($d \to \infty$) we expect

$$k_\perp = \frac{Z_0 c}{2\pi} \left(\frac{1}{b_2^2} - \frac{1}{b_1^2}\right) , \tag{11}$$

for a short collimator ($d \to 0$)

$$k_\perp = \frac{Z_0 c}{4\pi} \left(\frac{1}{b_2^2} - \frac{b_2^2}{b_1^4}\right) . \tag{12}$$

In order to test Eqs. 11, 12, and to obtain a feeling about "long" and "short" we have calculated the loss and kick factors for example collimators as functions of collimator length d. For the first example $\alpha = \pi/2$, $b_1 = 19$ mm, and $b_2 = 1.9$ mm; bunch lengths $\sigma = 0.5$, 0.3, 0.1 mm. We find that the numerically obtained loss factors are not sensitive to collimator length d, and Eq. 9 approximates them well. The calculated kick factors are shown in Fig. 2 (left frame). The straight gray lines give the asymptotic (long, short) approximations, Eqs. 11, 12. The numerical results agree well with the analytical estimates. Also, the direct sums of the kicks for "in"- and "out"-transitions (calculated separately) are shown. We see that they agree with the kick for a sufficiently long collimator, as expected.

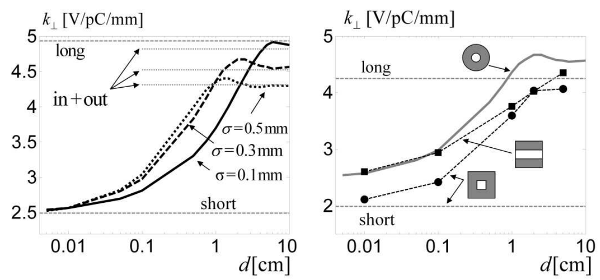

Figure 2: Kick factor *vs.* collimator length. A round collimator (left), a square or rectangular collimator ($\sigma = 0.3$ mm, right).

For 3D collimators, the loss factor for short and long collimators is, in general, no longer nearly the same. In this case the energy impedance Z^e can be found numerically by solving the 2D problem (5') (see, *e.g.* [7]). We have performed both 3D time-dependent and 2D electrostatic calculations for square and rectangular collimators with aperture half-height $b_2 = 1.9$ mm, aperture width $h = 3.8$ mm (square) or 38 mm (rectangular), beam pipe half-height and half-width $b_1 = 19$ mm. The results are presented in Fig. 2 (the right frame) and Table 1. The gray lines in the figure show the approximate kicks for the square collimator. We see from the table that the rectangular collimator gives a kick very similar to the round one. Of the three collimators, the smallest kick is for the square one. We see that Eqs. 11, 12, can be useful for all three types of collimators.

Table 1: Loss and kick factors as estimated by 2D electrostatic calculation. The bunch length $\sigma = 0.3$ mm. "Short" means using Eq. 6, "long" Eq. 5.

Type	$k_\parallel$ [V/pC]		$k_\perp$ [V/pC/mm]	
	short	long	short	long
round	78	78	2.50	5.01
rect.	56	72	2.43	6.11
square	74	78	1.99	4.25

From the above results we see that the kicks for long and short step collimators are related by

$$k_\perp^{short} \approx \frac{1}{2} k_\perp^{long} . \tag{13}$$

Hence, in Eq. 3 we suggest one use $A = 1$ for a long collimator ($d \to \infty$) and $A = \frac{1}{2}$ for a short one ($d \to 0$). For a collimator in the inductive regime, however, the result should be independent of d.

The good agreement we have found between direct time-domain calculation [1] and the approximations (5, 6), suggests that the latter method can be used to approximate short-bunch wakes for a large class of 3D collimators.

RECTANGULAR COLLIMATORS

In our simulations for 3D collimators we used a time-domain numerical scheme [1] combined with an indirect integration algorithm [2]. We consider now two sets of collimators that were measured in experiment at SLAC. The first set includes four collimators measured in 2001 [8]. The parameters are given in Table 2. All rectangular collimators have width $h = 2b_1 = 38$ mm and no flat region ($d = 0$).

Table 2: Geometry of SLAC collimators of 2001.

Coll. #	1	2	3	4
Type	rect.	square	rect.	rect.
b_2 [mm]	1.9	1.9	1.9	3.8
α [mrad]	168	335	335	298

In order to check the accuracy of the 3D discretization we have calculated wakefields for round collimators with $b_1 = 19$ mm, $b_2 = 1.9$ mm, $\alpha = 335$ mrad using the 2.5D and 3D codes. The two results are nearly indistinguishable (see Fig. 3 on the left). The wakes for square collimator #2 and rectangular collimator #3 are also plotted, for two bunch lengths, in Fig. 3.

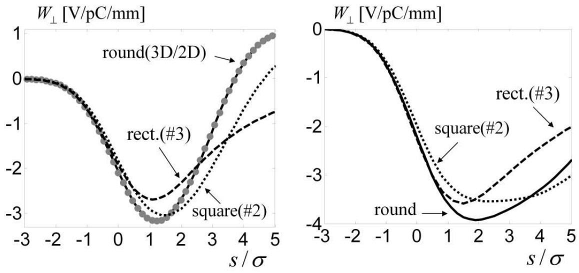

Figure 3: Transverse wake of Gaussian bunch, with $\sigma = 0.65$ mm (left) and $\sigma = 0.3$ mm (right).

In Fig. 4 a comparison of simulation (open symbols) and experiment (closed symbols) for the SLAC collimators is made. We estimate the accuracy in calculated $k_\perp$ to be about 1%. We note good agreement for rectangular collimators #1 and #4. On the other hand the short bunch results for collimators #2 and #3 agree very well with the calculations of the previous section, but they disagree with the experimental data (by 20%). Tables 3, 4, compare simulated results, measurements, and the analytical estimates. Entries in bold type are the appropriate estimates for the simulated results. Note that Eq. 1 results agree surprisingly well with simulation for rectangular collimators with arbitrary ρ_2 (only requiring $\rho_1 \ll 1$).

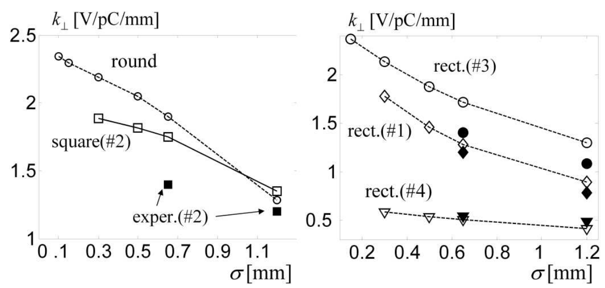

Figure 4: Kick factor *vs.* σ for various collimators.

Finally, we have calculated loss and kick factors for the set of collimators used in the recent, 2006 SLAC experiment. The collimator description and measured data can be found in [9]. (During the process of preparing this paper the data had not yet been analyzed.) Perhaps the most interesting result in Table 5 is a noticeable difference in $k_\perp$ for collimators #2 and #3. The collimators have identical taper angles and apertures. However, for collimator #2 $d = 0$, for collimator #3 $d = 1$ m. In agreement with the analytic models discussed in this paper, a long collimator length results in a kick factor increase by a factor ~ 2. In order to check the results of Table 5 we have done 2.5D simulations for round collimators with the same profiles. The results show the same qualitative behavior as those for the rectangular collimators of Table 5.

Table 3: Kick factor [V/pC/mm]; $\sigma = 0.65$ mm. Measurement errors are given in parentheses.

Coll. #	1	2	3	4
ρ_1/ρ_2	0.5/50.	1.0	1./98.	1.7/43.
simul.	1.28	1.75	1.72	0.50
meas.	1.2(0.1)	1.4(0.1)	1.4(0.1)	0.54(0.05)
Eq. 1	**1.24**	2.5	2.5	1.0
Eq. 3	2.4	3.3	3.3	1.1
Eq. 12	2.5	**2.5**	**2.5**	**0.62**

Table 4: Kick factor [V/pC/mm]; $\sigma = 1.2$ mm. Measurement errors are given in parentheses.

Coll. #	1	2	3	4
ρ_1/ρ_2	0.3/27.	0.5	0.5/53.	0.9/24.
simul.	0.90	1.35	1.30	0.41
meas.	0.78(0.13)	1.2(0.1)	1.08(0.1)	0.49(0.15)
Eq. 1	**0.7**	**1.3**	**1.3**	**0.5**
Eq. 3	1.7	2.4	2.4	0.8
Eq. 12	2.5	2.5	2.5	0.6

Table 5: Loss and kick factors for new set of collimators.

Coll. #	$k_\parallel$ [V/pC]		$k_\perp$ [V/pC/mm]	
	$\sigma = 0.3$ mm	$\sigma = 0.5$ mm	$\sigma = 0.3$ mm	$\sigma = 0.5$ mm
1	50	28	1.9	1.7
2	60	33	3.6	3.1
3	63	33	6.1	5.1
4	40	24	0.7	0.8
5	81	47	7.1	6.8
6	51	24	2.9	2.3
7	60	34	3.1	2.7
8	56	28	3.0	2.4

Acknowledgements

We thank M. Dohlus, G. Stupakov and T. Weiland for helpful discussions on wakes and impedances, and CST GmbH for letting us use CST MICROWAVE STUDIO for the meshing.

REFERENCES

[1] I. Zagorodnov, T. Weiland, Phys Rev STAB **8**, 042001(2005).

[2] I. Zagorodnov, DESY 06-081, May 2006.

[3] G.V. Stupakov, SLAC-PUB-8857, June 2001.

[4] K. Yokoya, CERN-SL/90-88 (AP), July 1990.

[5] I. Zagorodnov et al., PAC'03, p. 3252 (2003).

[6] S. Heifets and S. Kheifets, Rev Mod Phys **63**, 631 (1991).

[7] K.L.F. Bane, I. Zagorodnov, PAC'06, THPCH072.

[8] P. Tenenbaum et al., PAC'01, p. 418 (2001).

[9] N.K. Watson et al., PAC'06, MOPLS066.