

Outer Tracker internal alignment toy Monte Carlo studies

July 11, 2006

Abstract

The results obtained for the Outer Tracker internal alignment with a toy Monte Carlo are presented and fully described.

LHCb Note

Issue	Final
Revision	6

Reference	LHCb-2006-043
Created	April 20, 2006
Last Modified	July 11, 2006

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Contents

1	Introduction	4
2	Introduction to Millepede	5
2.1	The Outer Tracker case	5
2.1.1	Constraining the internal alignment with Millepede . .	6
3	The toy Monte Carlo	7
4	Results	8
4.1	Translations	8
4.2	Rotations	13
4.3	Combined studies	13
5	Conclusion	16
6	Acknowledgments	17
A	APPENDIX A: Constraints in Millepede	18

List of Figures

1	<i>The basic alignment problem. The residual is the distance between the reconstructed clusters (the circles) and the track intercept point.</i>	4
2	<i>Resolution and pull distribution in the misalignment parameter ΔX, obtained with 10 runs with approximately 10000 tracks each, with a X misalignment of 1 mm.</i>	9
3	<i>Resolution in the misalignment parameter ΔX, as a function of the number of tracks used in its determination.</i>	10
4	<i>Resolution in the misalignment parameter ΔY, obtained with 10 runs with approximately 15000 tracks each, with a Y misalignment of 1 mm.</i>	11
5	<i>Resolution in the misalignment parameter ΔZ, obtained with 10 runs with approximately 15000 tracks each, with a Z misalignment of 0.1 mm.</i>	12
6	<i>Resolution in the misalignment parameter Δ_α (left) and Δ_β (right), obtained with 10 runs with approximately 15000 tracks each, with a misalignment of 5 mrad.</i>	14
7	<i>Resolution in the misalignment parameter Δ_γ, obtained with 10 runs with approximately 15000 tracks each, with a misalignment of 1 mrad.</i>	15

List of Tables

1	<i>Resolution in the misalignment parameter ΔZ for different values of the Z misalignment. Every resolution value has been obtained with 10 runs with approximately 15000 tracks each.</i>	12
2	<i>Resolution in the misalignment parameter Δ_α and Δ_β for different values of the X and Y rotations misalignment. The two degrees of freedom are fitted separately. Every resolution value has been obtained with 10 runs with approximately 15000 tracks each.</i>	14
3	<i>Results for a combined fit of the X,Y,Z, translation and X,Y,Z rotations.</i>	16

1 Introduction

The typical alignment problem can be summarized as follows. When a particle passes through a misaligned detector (see Fig. 1 (left)), if the geometry is not corrected, the track will be fitted using the uncorrected geometry (see Fig. 1 (right)). This implies that the wrong hits are assigned to the track and as a consequence the tracking (and consequently the physics) performance deteriorates. This phenomena has been studied in case of misalignments of the VELO [1] and of the Outer Tracker (OT) [2]. As a consequence a clear alignment strategy is needed [3], one of the main steps of the alignment strategy is the software alignment.

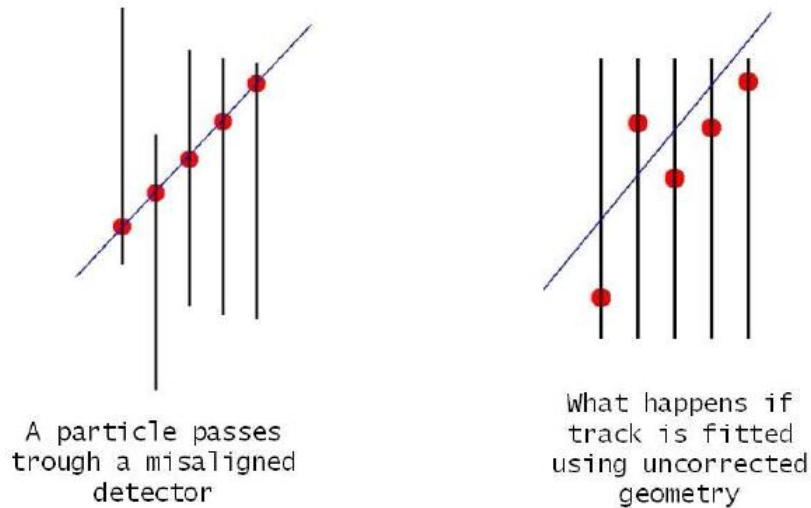


Figure 1: *The basic alignment problem. The residual is the distance between the reconstructed clusters (the circles) and the track intercept point.*

In this note a strategy for the OT [4] internal alignment is described. The internal alignment method taken in exam is a non-iterative least squares fitting method which uses a C++ implementation [5] of the original Millepede algorithm written in Fortran [6] by V. Blöbel for the experiment H1. When using tracks, the standard way to obtain the alignment constants is to minimize the residuals. In Fig. 1, the residual is the distance between the reconstructed clusters (the circles) and the track intercept point.

2 Introduction to Millepede

The main idea behind the Millepede method [6] is that, instead of simply fitting the tracks, the tracks and the residuals are fitted simultaneously. In order to do that a linear relation between the residual and the alignment constants is needed. The measurements along a track can be parameterized as (see Ref. [5]):

$$\mathbf{Y} = X \cdot \alpha + C \cdot \Delta, \quad (1)$$

where \mathbf{Y} is a vector of the measurements of the track, X is a matrix containing the local derivatives of the tracks, α is a vector containing the local parameters of the tracks, Δ contains the global parameters (the alignment constants that need to be determined), and C is a matrix containing the global derivatives. Rewriting (1), in the simple case of a straight track, as

$$y = \sum_j x_j \cdot \alpha_j + \sum_k c_k \cdot \Delta_k, \quad (2)$$

the problem to solve is the minimization of the following χ^2 :

$$\chi^2 = \sum_i w_i \cdot (y^i - \sum_j x_j^i \cdot \alpha_j - \sum_k c_k^i \cdot \Delta_k)^2, \quad (3)$$

where $w_i = 1/\sigma_i^2$. Once again, it is necessary to point out that the χ^2 can be written as in (3) only given a linear relation between the the residual and the alignment constants (as in (1)).

The solution to (3) will contain not only the local track parameters, but also the global alignment parameters. However, since the global alignment parameters are common to all tracks, a proper fit must take into account the correlations between these global alignment parameters and each track's local parameters. Thus a matrix of $n_{tot} \times n_{tot}$ ¹ dimension needs to be inverted. This is done by Millepede.

2.1 The Outer Tracker case

As mentioned in the previous section a linear relation is needed between the residuals and the misalignment constants. For a full derivation of this relation, see Ref. [5] and [7]. The final equation is:

$$\begin{aligned} x_{hit}^{new} &= x_{hit} - \Delta_x + y_{hit} \cdot \Delta_\gamma + t_x \cdot (\Delta_z - x_{hit} \cdot \Delta_\beta + y_{hit} \cdot \Delta_\alpha), \\ y_{hit}^{new} &= y_{hit} - \Delta_y - x_{hit} \cdot \Delta_\gamma + t_y \cdot (\Delta_z + x_{hit} \cdot \Delta_\beta + y_{hit} \cdot \Delta_\alpha), \end{aligned} \quad (4)$$

¹Where $n_{tot} = n_{local} \times n_{tracks} + n_{global}$.

where $x_{hit}^{new}/y_{hit}^{new}$ are the measurements that are observed, x_{hit}/y_{hit} are the values corrected for the misalignment, $\Delta_x, \Delta_y, \Delta_z, \Delta_\alpha, \Delta_\beta$ and Δ_γ are respectively the misalignment for the X, Y, Z translations and X, Y, Z rotations. The parameters t_x and t_y are taken from the parametrization of a track (as a straight line) both in the XZ plane and in the YZ plane. A track can therefore be written as follows:

$$\begin{aligned} x_{measured} &= x_{track} + \epsilon_x = x_0 + t_x \cdot z + \epsilon_x, \\ y_{measured} &= y_{track} + \epsilon_y = y_0 + t_y \cdot z + \epsilon_y, \end{aligned} \quad (5)$$

where $\epsilon = (\epsilon_x, \epsilon_y)$ is the vector of residuals and (t_x, t_y, x_0, y_0) are the track slopes and the intercept points in the XZ, YZ planes. Both Eq. (4) and Eq. (5) need to be adapted for the presence of the stereo-angles. This can be done projecting the $x_{measured}$ and the $y_{measured}$ on the u plane and thus obtaining a value for $u_{measured}$.

For example Eq. (5) can be rewritten as [8]:

$$u_{measured} = (x_0 + t_x \cdot z + \epsilon_x) \cdot \cos \phi + (y_0 + t_y \cdot z + \epsilon_y) \cdot \sin \phi, \quad (6)$$

where ϕ is the stereo-angle defined in [9] and explained in the next section. It appears clear from Eq. (6) that, since $\phi = 0$ for the X-layers, $y_{measured} = 0$ for the X-layers, in other words there's no measurement in y in the X-layers.

2.1.1 Constraining the internal alignment with Millepede

The internal alignment of the OT layers is by definition insensitive to global offsets. It is therefore necessary to introduce a set of constraints into the alignment procedure to prevent correlated movements.

There are nine different deformations that have to be constrained: X, Y, Z translations, X, Y, Z rotations, XZ and YZ shearings and Z scaling. The way this is done in Millepede is explained in [5, 10]. In this section the implementation of the X translation constraint is reported, for more details see Appendix A. Intuitively the idea behind setting the constraint can be explained as follows. For the X translations, for example, before the alignment, one has:

$$\langle \Delta_x \rangle = \Delta_X, \quad (7)$$

where Δ_X is a global offset which cannot be found by the internal alignment and $\langle \Delta_x \rangle$ is the average misalignment in X. In order to avoid a global transformation a new parameter Δ'_x can be introduced:

$$\Delta'_x = \Delta_x - \Delta_X, \quad (8)$$

so that by construction

$$\langle \Delta'_x \rangle = 0. \quad (9)$$

Similarly a constraint equation for any of the nine deformations can be set. The parameters which are given by Millepede are the Δ'_i , the “offset-free” constants.

3 The toy Monte Carlo

The toy Monte Carlo (MC) in use is a useful instrument for having a good understanding of what can “ideally” be achieved by the internal alignment of the OT, using Millepede. However it is necessary to concentrate on the word “ideally”. Since this is a toy MC simulation only, the precision to which the alignment parameters are determined is not indicative of what will eventually be achieved with data. In the toy standalone MC software² for simplicity many things are idealized. The main features of the toy are here described.

- **Geometry**

There are three stations containing four layers each. The z value of the first station is fixed and the $\Delta z_{station}$ and Δz_{layer} are defined. The detector planes are ideal, they contain neither modules nor straws. The layer thickness is set to 0 and the layer’s shape is just rectangular; no special cross-shape hole for the presence of the IT is considered. The hits are also ideal (100% efficiency, no noise, no after-pulses, no cross talk, no left - right ambiguity). The 12 planes mentioned³, which resemble the OT, are the only part of the “LHCb detector” present (i.e. things like the B field, or any other subdetector are missing). In the ideal geometry in use the first OT layer is centered around the point (0,0,7.8 m). The Outer Tracker layers have the angle between the y and the z axis⁴ in the following configuration: $0^\circ, +5^\circ, -5^\circ, 0^\circ$ [9]. The OT resolution is fixed to its nominal value [13], 200 μm .

- **Tracks**

Tracks are simply straight lines between the first and the last layer; no multiple scattering is taken in account. The tracks originate from the primary vertex which is an (x,y,z) point random generated around

²It’s code written in C++, which runs standalone, and not inside the LHCb Software. In these studies the KNOSSOS version 1.1, dated 13th of April, 2006, has been used.

³3 stations \times 4 layers gives 12 planes.

⁴For a clear definition of the reference frame, see [11, 12].

the point (0,0,0). For every primary vertex approximately 15 tracks are generated. The tracks angles are distributed randomly between ± 100 mrad.

- **Event**

For every event one primary vertex is generated.

- **Run**

Every run contains a certain number of events. Typically a run containing 1000 events corresponds to approximately 15000 tracks. In every run each layer is misaligned with a different value, which is random - Gauss generated - with a width which can be set by the user. This is the value to which we will refer when we will say that the misalignment is 1 mm⁵. Millepede is called every run and it executes the matrix inversion and the χ^2 fit every run.

4 Results

Each of the six degrees of freedom ($\Delta_x, \Delta_y, \Delta_z, \Delta_\alpha, \Delta_\beta, \Delta_\gamma$ from Eq. (4)) has first been studied alone (for different misalignment values) and then together with the others.

4.1 Translations

Fig. 2 shows the results obtained for the X translations. On the left side the X resolution (the difference between the misalignment value and the one found by Millepede) is plotted. The width (σ) of the distribution is 2 μ m. On the right the corresponding pull distribution is plotted. The width (σ) of the distribution is 1.021 ± 0.181 , compatible with the expected value of 1. The plot in Fig. 2 has been obtained with 10 runs⁶ with approximately 10000 tracks each for a misalignment value of 1 mm, but similar results have been obtained for other misalignment values. Fig. 3 shows the X resolution as a function of the number of tracks.

The resolution values obtained with Millepede (the triangles in Fig. 3) follow very well the expected behavior (the straight line in Fig. 3) given by the equation:

⁵It means that for every run, the 12 layers have been misaligned by a value which is random, generated with a Gaussian distribution, which has a width of 1 mm.

⁶Observe that the plot in Fig. 2 has 120 entries. 120 is obtained as 10×12 , where 10 is the number of runs and 12 is the number of layers. Also the plots in Fig. 5, 6, 7 have 120 entries each.

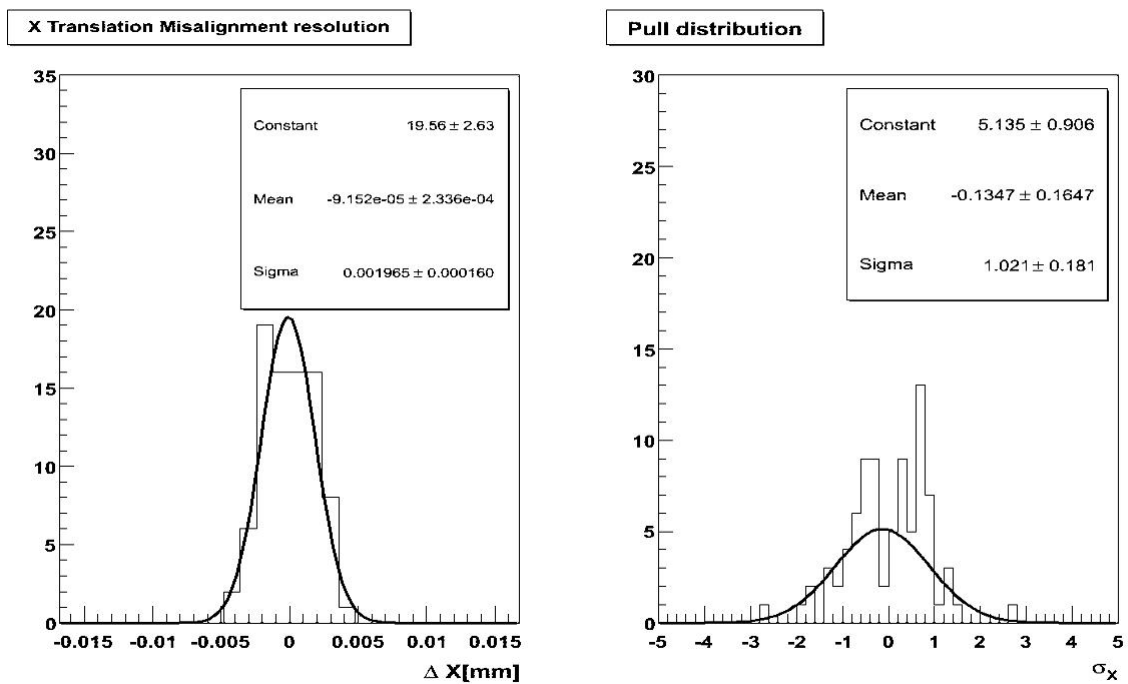


Figure 2: Resolution and pull distribution in the misalignment parameter ΔX , obtained with 10 runs with approximately 10000 tracks each, with a X misalignment of 1 mm.

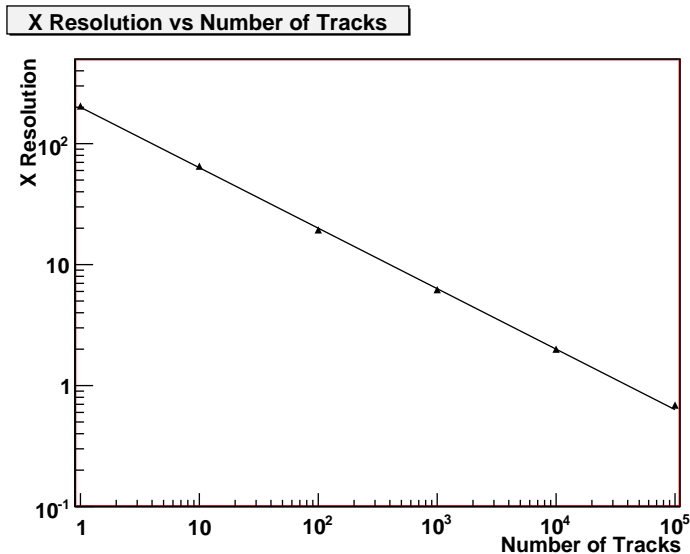


Figure 3: Resolution in the misalignment parameter ΔX , as a function of the number of tracks used in its determination.

$$R(R_0, N_0, N_{tracks}) = R_0 \cdot \sqrt{\frac{N_0}{N_{tracks}}}, \quad (10)$$

where R is the resolution, the R_0 , N_0 coordinate is the $2 \mu\text{m}$ X resolution, 10^4 tracks coordinate point and N_{tracks} is the number of tracks.

Similar studies have been performed for Y translations as is shown in Fig. 4. The OT is sensitive to Y translations only in the stereo-layers while the X-layers are not sensitive to Y misalignments. The width (σ) of the distribution of the Y misalignment resolution as shown in the plot is $17.44 \pm 2.59 \mu\text{m}$. This value has been obtained with a misalignment value of 1 mm, but similar results have been obtained for other misalignment values.

This value can be “qualitatively” understood as follows:

$$Y_{resolution} \approx \frac{X_{resolution}}{\sin \phi}, \quad (11)$$

where ϕ is the stereo angle⁷. The value of Y resolution mentioned has been obtained with 10 runs⁸ with approximately 15000 tracks each. The corre-

⁷Observe that from Eq. (11) follows that the $Y_{resolution}$ is infinite for the X-layers since $\phi = 0$.

⁸Observe that the plot in Fig. 4 has 60 entries. 60 is obtained as 10×6 , where 10 is the number of runs and 6 is the number of stereo-layers, the X-layers are not considered.

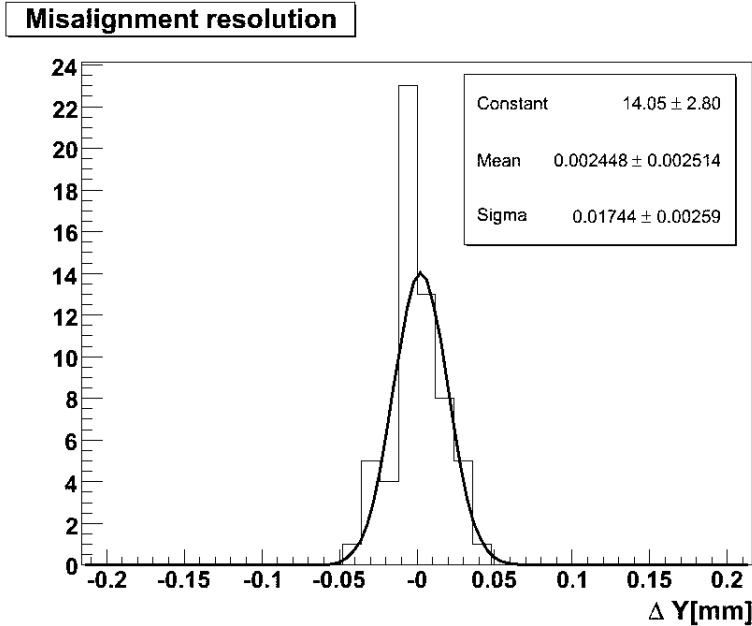


Figure 4: Resolution in the misalignment parameter ΔY , obtained with 10 runs with approximately 15000 tracks each, with a Y misalignment of 1 mm.

sponding X resolution for the same number of tracks (as obtained from Eq. (10)) is $1.63 \mu\text{m}$. Using this value of X resolution, the corresponding value for the Y resolution (as obtained from Eq. (11)) is $18.74 \mu\text{m}$ which is compatible with the value shown in Fig. 4 of $17.44 \pm 2.59 \mu\text{m}$.

Similarly, the Z alignment procedure has been studied. Fig. 5 shows a Z misalignment resolution of $1 \mu\text{m}$ obtained with 10 runs with approximately 15000 tracks each with a misalignment value of 0.1 mm. It is interesting to observe that for Z translations (this was not the case for X and Y translations) the value of resolution increases with the value of the misalignment given as input. This is due to the non-linear behavior of such a degree of freedom (see Ref. [8, 14]).

This non-linear behavior of this degree of freedom has not been taken into consideration in the version of the MC toy in use. This results in a clear dependence of the Z resolution on the Z misalignment. This has been studied and the results are reported in Table 1.

Furthermore the correlation between the initial misalignment and the fitted result has also been studied. This correlation can be quantified in

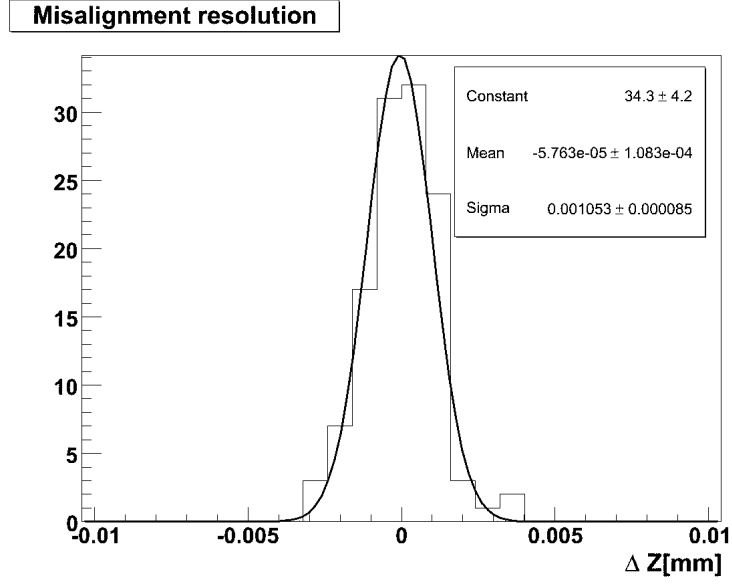


Figure 5: Resolution in the misalignment parameter ΔZ , obtained with 10 runs with approximately 15000 tracks each, with a Z misalignment of 0.1 mm.

Z Misalignment value (mm)	Z Resolution (μm)
0.1	1.1
0.5	3.1
1.0	4.9
5.0	27.8

Table 1: Resolution in the misalignment parameter ΔZ for different values of the Z misalignment. Every resolution value has been obtained with 10 runs with approximately 15000 tracks each.

terms of a correlation factor CF defined as:

$$CF = \frac{\sum_{i=1}^N (\text{Input value})_i \times (\text{Output value})_i}{\sqrt{\sum_{i=1}^N (\text{Input value})_i^2} \times \sqrt{\sum_{i=1}^N (\text{Output value})_i^2}}, \quad (12)$$

where N is the total number of fitted points. The value found is 0.73. This value shows a clear correlation between the initial misalignment and the fitted result.

A simultaneous determination of X, Y and Z translations yields misalignment resolutions that are similar to those obtained when these degrees of freedom are treated in isolation.

4.2 Rotations

Both the X and the Y rotation degree of freedom suffer from the same non-linear effects as the Z translation since their derivatives depend on the slope of the tracks, as indicated in Eq. (4). The result is that the resolution increases with the value of the misalignment given as input.

Fig. 6 (left) shows the X rotation resolution obtained with 10 runs with approximately 15000 tracks each, with a misalignment of 5 mrad. The resolution is $25.7 \mu\text{rad}$. Similarly Fig. 6 (right) shows the Y rotation resolution obtained with 10 runs with approximately 15000 tracks each, with a misalignment of 5 mrad. The resolution is $28.7 \mu\text{rad}$. The dependence of the resolutions on X and Y rotational misalignments has been studied and the results are reported in Table 2. Observe that all the results obtained for the X rotation resolution are always compatible with the ones for Y. Also in these cases the correlation factors have been calculated. The results are $CF_{\Delta\alpha} = 0.78$ and $CF_{\Delta\beta} = 0.67$, so also here there is a clear correlation between the initial misalignment and the fitted result.

Analogously the Z rotation has been studied. Fig. 7 shows the Z rotation resolution obtained with 10 runs with approximately 15000 tracks each, with a misalignment of 1 mrad. The width (σ) of the distribution is $7.5 \mu\text{rad}$; similar results have been obtained for other misalignment values.

A combined fit of the X, Y translations and of the Z rotations⁹ at the same time has been performed. The results of the fit do not degrade if compared to the ones obtained for the single degrees of freedom, if considered alone.

4.3 Combined studies

The combined fit of all the six degrees of freedom in exam has been performed with 10 runs with approximately 15000 tracks each. The misalignment value

⁹These are the three degrees of freedom which do not show any non-linear behavior.

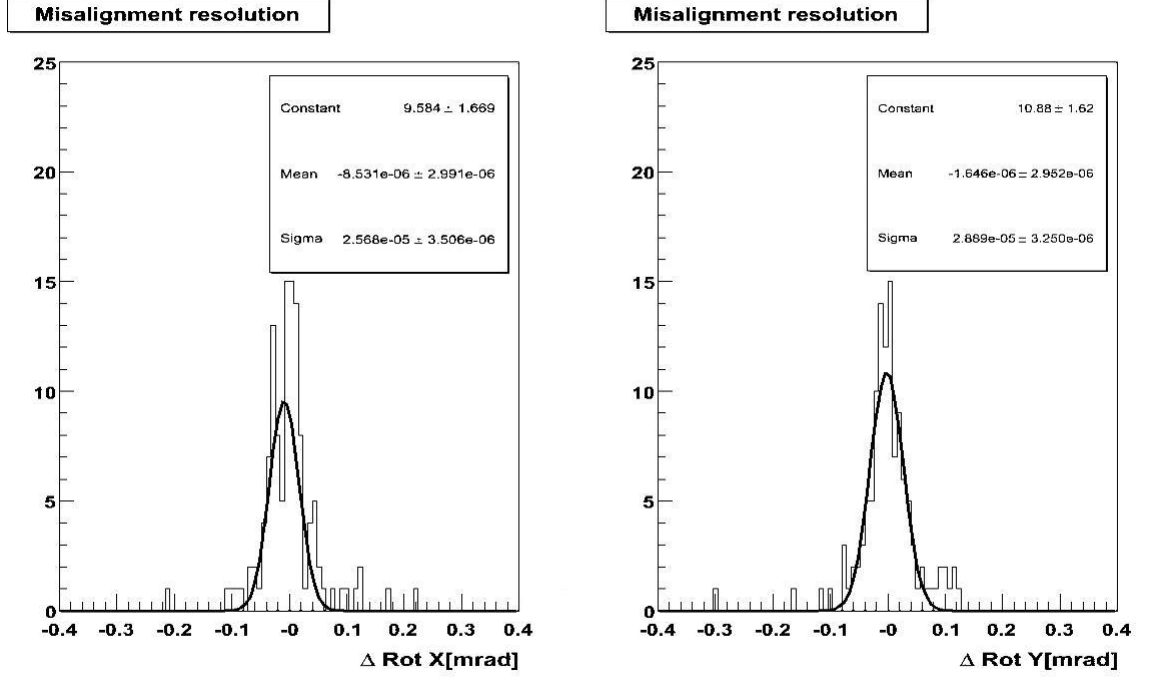


Figure 6: Resolution in the misalignment parameter Δ_α (left) and Δ_β (right), obtained with 10 runs with approximately 15000 tracks each, with a misalignment of 5 mrad.

Misalignment value (mrad)	Y Rotation Resolution: Δ_β (μ rad)	X Rotation Resolution: Δ_α (μ rad)
0.1	1.1	1.0
0.5	3.2	2.9
1	4.7	5.1
5	28.7	25.7
10	98.7	96.4

Table 2: Resolution in the misalignment parameter Δ_α and Δ_β for different values of the X and Y rotations misalignment. The two degrees of freedom are fitted separately. Every resolution value has been obtained with 10 runs with approximately 15000 tracks each.

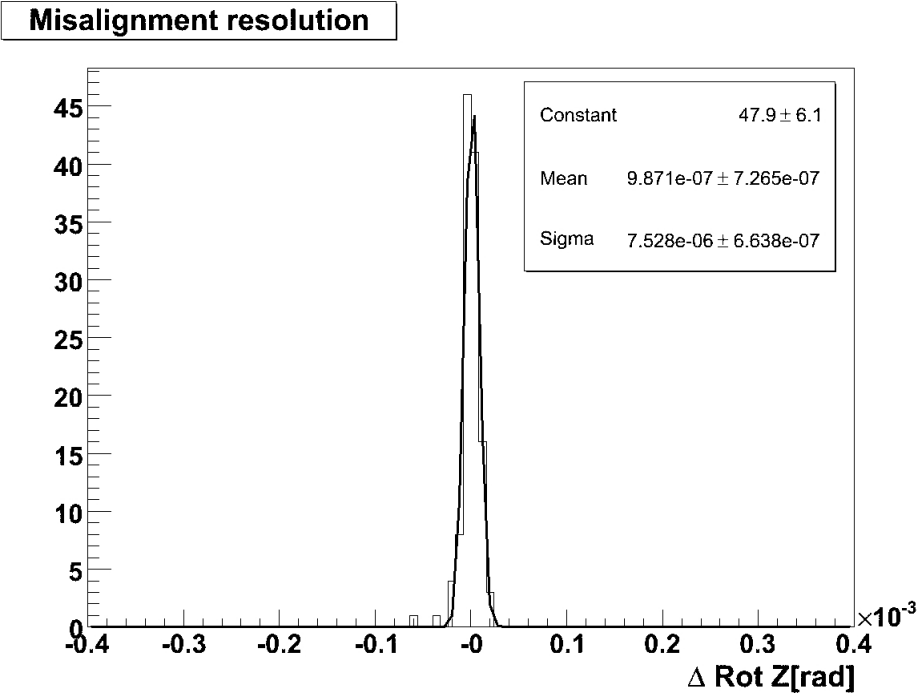


Figure 7: Resolution in the misalignment parameter Δ_γ , obtained with 10 runs with approximately 15000 tracks each, with a misalignment of 1 mrad.

for the translations degrees of freedom has been set to 1.0 mm for X and Y and to 5.0 mm for Z, while for the rotations degree of freedom it has been set to 5 mrad for X and Y rotations and to 1 mrad for Z rotations. The results (together with the results for any of the degrees of freedom if fitted alone, extracted from Section 4.1 and Section 4.2) are shown in Table 3. In the last column the results of a 2^{nd} iteration of the combined fit are shown. In the 2^{nd} iteration, the newly determined misalignment resolutions (the results of the first iteration) become our misalignment constants which need to be determined.

Degree of freedom	Misalignment values	Resolution		
		<i>Considered alone</i>	<i>Combined fit</i>	<i>Combined fit 2nd iteration</i>
X translation	1.0 mm	1.6 μm	20.1 μm	3.2 μm
Y translation	1.0 mm	17.4 μm	196.8 μm	47.9 μm
Z translation	5.0 mm	27.8 μm	96.4 μm	27.4 μm
X rotation	5 mrad	25.7 μrad	99.8 μrad	28.8 μrad
Y rotation	5 mrad	28.7 μrad	102.3 μrad	31.4 μrad
Z rotation	1 mrad	7.5 μrad	26.4 μrad	9.6 μrad

Table 3: *Results for a combined fit of the X,Y,Z, translation and X,Y,Z rotations.*

As shown in Table 3, it is clear that when all misalignments are present, a single iteration is insufficient for the accurate determination of the misalignment parameters. Only when a second iteration is included one does obtain a precision in the misalignment parameters which approaches the one obtained when a single misalignment is simulated. This degraded precision after the first iteration results from the correlations between the Z translations, and X and Y rotations with the tracks' slopes (see Eq. (4)).

5 Conclusion

First results for the internal alignment of the Outer Tracker have been presented. The ideal situation in which the results have been produced does not let us draw any conclusion which can be exported to the real case, but a good level of understanding of the possibility of using Millepede for the internal alignment of the OT has been reached. This understanding can be exported for the implementation of the code to the LHCb software.

6 Acknowledgments

I would like to thank Johan Blouw, Steven Blusk and Marcel Merk for the useful comments and Sebastien Viret for his continuous help.

A APPENDIX A: Constraints in Millepede

There are nine different deformations that have to be constrained: X, Y, Z translations, X, Y, Z rotations, XZ and YZ shearings and Z scaling. For the X, Y, Z translations and the X, Y, Z rotations it is possible to write:

$$\langle \Delta_x \rangle = \Delta_{\underline{X}}, \quad (\text{A-1})$$

$$\langle \Delta_y \rangle = \Delta_{\underline{Y}}, \quad (\text{A-2})$$

$$\langle \Delta_z \rangle = \Delta_{\underline{Z}}, \quad (\text{A-3})$$

$$\langle \Delta_\alpha \rangle = \Delta_{\underline{\alpha}}, \quad (\text{A-4})$$

$$\langle \Delta_\beta \rangle = \Delta_{\underline{\beta}}, \quad (\text{A-5})$$

$$\langle \Delta_\gamma \rangle = \Delta_{\underline{\gamma}}. \quad (\text{A-6})$$

where $\Delta_{\underline{X}}$, $\Delta_{\underline{Y}}$, $\Delta_{\underline{Z}}$, $\Delta_{\underline{\alpha}}$, $\Delta_{\underline{\beta}}$ and $\Delta_{\underline{\gamma}}$ are global offset which cannot be found by the internal alignment and $\langle \Delta_x \rangle$, $\langle \Delta_y \rangle$, $\langle \Delta_z \rangle$, $\langle \Delta_\alpha \rangle$, $\langle \Delta_\beta \rangle$ and $\langle \Delta_\gamma \rangle$ are the average misalignment in X, Y, Z translations and X, Y, Z rotations. In order to avoid a global transformation, new parameters Δ'_x , Δ'_y , Δ'_z , Δ'_α , Δ'_β and Δ'_γ can be introduced:

$$\Delta'_x = \Delta_x - \Delta_{\underline{X}}, \quad (\text{A-7})$$

$$\Delta'_y = \Delta_y - \Delta_{\underline{Y}}, \quad (\text{A-8})$$

$$\Delta'_z = \Delta_z - \Delta_{\underline{Z}}, \quad (\text{A-9})$$

$$\Delta'_\alpha = \Delta_\alpha - \Delta_{\underline{\alpha}}, \quad (\text{A-10})$$

$$\Delta'_\beta = \Delta_\beta - \Delta_{\underline{\beta}}, \quad (\text{A-11})$$

$$\Delta'_\gamma = \Delta_\gamma - \Delta_{\underline{\gamma}}, \quad (\text{A-12})$$

so that by construction

$$\Delta'_x = 0, \quad (\text{A-13})$$

$$\Delta'_y = 0, \quad (\text{A-14})$$

$$\Delta'_z = 0, \quad (\text{A-15})$$

$$\Delta'_\alpha = 0, \quad (\text{A-16})$$

$$\Delta'_\beta = 0, \quad (\text{A-17})$$

$$\Delta'_\gamma = 0. \quad (\text{A-18})$$

For the XZ and YZ shearings and the Z scaling, as for Eq. from (A-1) to (A-6), it is possible to write:

$$\langle \Delta_{XZ\text{shearing}} \rangle = \Delta_{\underline{XZ\text{shearing}}}, \quad (\text{A-19})$$

$$\langle \Delta_{YZ\text{shearing}} \rangle = \Delta_{\underline{YZ\text{shearing}}}, \quad (\text{A-20})$$

$$\langle \Delta_{Z\text{scaling}} \rangle = \Delta_{\underline{Z\text{scaling}}}, \quad (\text{A-21})$$

where $\Delta_{XZshearing}$, $\Delta_{YZshearing}$ and $\Delta_{Zscaling}$ are the global offset which cannot be found by the internal alignment and $\langle \Delta_{XZshearing} \rangle$, $\langle \Delta_{YZshearing} \rangle$ and $\langle \Delta_{Zscaling} \rangle$ are respectively the average XZ shearing, the average YZ shearing and the average Z scaling defined as follows:

$$\langle \Delta_{XZshearing} \rangle = \frac{\sum_{i=1}^{N_{layers}} \frac{\Delta_x \cdot (z_i - \bar{z})}{\sigma_{\bar{z}}}}{N_{layers}}, \quad (\text{A-22})$$

$$\langle \Delta_{YZshearing} \rangle = \frac{\sum_{i=1}^{N_{layers}} \frac{\Delta_y \cdot (z_i - \bar{z})}{\sigma_{\bar{z}}}}{N_{layers}}, \quad (\text{A-23})$$

$$\langle \Delta_{Zscaling} \rangle = \frac{\sum_{i=1}^{N_{layers}} \frac{\Delta_z \cdot (z_i - \bar{z})}{\sigma_{\bar{z}}}}{N_{layers}}, \quad (\text{A-24})$$

where \bar{z} and $\sigma_{\bar{z}}$ are defined as

$$\bar{z} = \frac{\sum_{i=1}^{N_{layers}} z_i}{N_{layers}}, \quad (\text{A-25})$$

$$\sigma_{\bar{z}} = \frac{\sum_{i=1}^{N_{layers}} (z_i - \bar{z})^2}{N_{layers}}, \quad (\text{A-26})$$

where, in the OT case, $N_{layers} = 12$. In other words \bar{z} is the average z value and $\sigma_{\bar{z}}$ is its error.

As for Eq. from (A-7) to (A-12) some new parameters can be introduced:

$$\Delta'_{XZshearing} = \Delta_{XZshearing} - \langle \Delta_{XZshearing} \rangle, \quad (\text{A-27})$$

$$\Delta'_{YZshearing} = \Delta_{YZshearing} - \langle \Delta_{YZshearing} \rangle, \quad (\text{A-28})$$

$$\Delta'_{Zscaling} = \Delta_{Zscaling} - \langle \Delta_{Zscaling} \rangle, \quad (\text{A-29})$$

so that by construction

$$\Delta'_{XZshearing} = 0, \quad (\text{A-30})$$

$$\Delta'_{YZshearing} = 0, \quad (\text{A-31})$$

$$\Delta'_{Zscaling} = 0. \quad (\text{A-32})$$

The parameters which are given by Millepede are the Δ'_i , the ‘‘offset-free’’ constants.

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