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THE SHAPE OF THE ATLAS FORWARD CALORIMETER CURRENT PULSE

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We calculate the shape of the current pulse in an FCal-style tube electrode when the liquid argon in the gap is momentarily, uniformly illuminated with ionizing radiation. To zeroth order the FCal pulse is triangular. The first order correction adds a quadratic term a few percent of the triangle peak.

1. Introduction

When the Large Hadron Collider (LHC) turns on at the CERN laboratory it will become the world's highest energy proton-proton collider. Because the cross sections of interest are small, the LHC must operate at an exceptionally high luminosity. This presents a particular challenge to the two detectors which will focus on "high p_T physics". The plentiful ordinary "low p_T" (aka minimum bias) collisions will illuminate all the detectors with background particles which leads to several problems. For the liquid argon calorimeters in the ATLAS detector, and particularly for the forward calorimeters (FCal) near the beam directions, there will be a nearly constant bombardment of particles creating a low level of ionization in the electrode gaps on every bunch crossing. In order to minimize space-charge effects, the FCal electrodes were designed with smaller than normal gaps. Each of the approximately 61,000 electrodes in the FCal is constructed with a solid rod inside a thin-walled tube. An insulating PEEK fiber is helically wound around this rod to ensure a concentric arrangement. The gap between the rod and tube is filled with liquid argon, the ionizing medium. A positive potential is applied to the rod while the tube (and the surrounding matrix) is held at ground, creating an electric field across the gap. This mechanical design provides a practical method to mass produce narrow gaps with acceptable precision [1].

For planar electrodes the geometry, and hence electric field in the gap, is uniform so that the ionization per energy deposit and drift speed of the ionization electrons are constant across the gap. This leads to a current pulse with a sharp rise and a linearly falling profile, terminating when the last electrons cross the gap. That is, the current pulse versus time has a right triangle shape. In contrast, the cylindrical geometry of the FCal tube electrodes gives an electric field which falls with radius. The non-uniform geometry, the non-uniform drift speed, and the non-uniform ionization density can distort the current pulse. We will show that the first order correction to the triangle pulse, due to the non-uniform drift speed, cancels out but the non-uniform geometry and the non-uniform ionization add to the triangle a small quadratic term which enhances the current between the fixed end points.

We start the discussion with a description of the geometry. Next we lay out the simplifying assumptions employed in the calculation. Parameterizations of the non-uniform drift speed and the non-uniform ionization density are presented. A careful discussion of the general formula for the current pulse is followed by the kinematics of the drifting electrons. At this point all the ingredients are in hand and the result is easily obtained. We conclude with a description of the result.

2. The electrode geometry

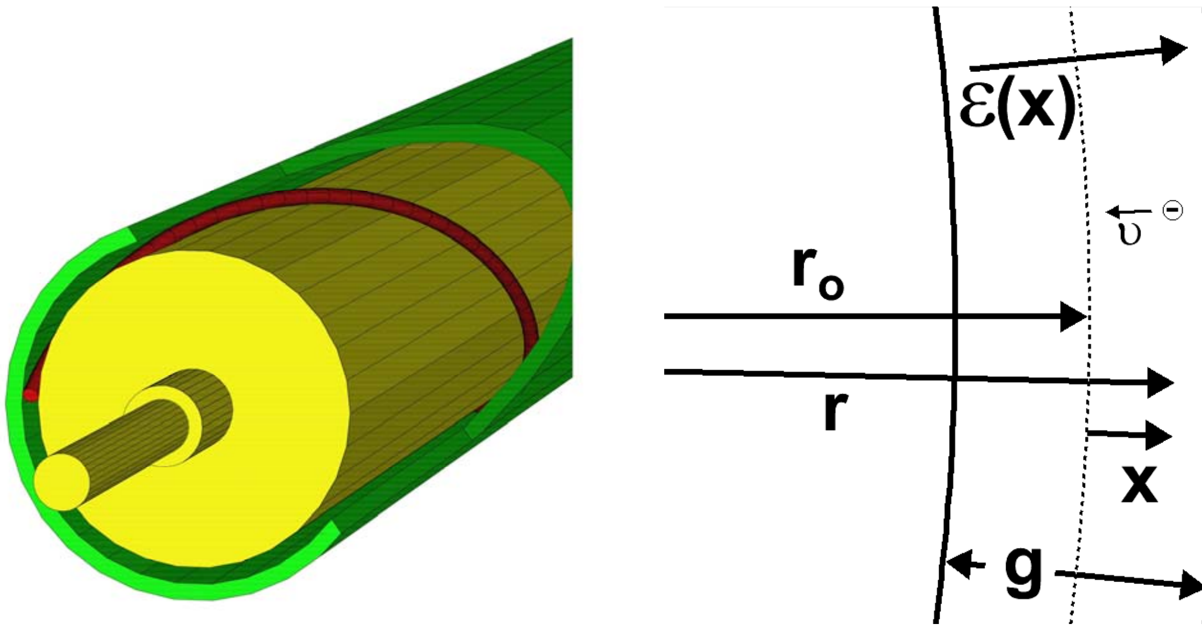

Fig. 1. The ATLAS Forward Calorimeter electrode consists of a solid metal rod inside a thin walled copper tube. A helically wound PEEK fiber holds the rod in place. Liquid argon fills the gap between the outer diameter of the rod and the inner diameter of the tube. A portion of the electrode is shown on the left. On the right, the coordinate system used to specify the geometry of the gap is indicated. The displacement vectors of magnitude r and r_0 originate at the symmetry axis which is outside the frame of the drawing.

Each of the two forward calorimeters (one at each end of the ATLAS detector) is made up of three modules called FCal1, FCal2, and FCal3, one behind the other as viewed from the collision point. The background ionization rate is largest in FCal1 and smallest in FCal3 so the size of the gap must be the smallest in FCal1; it can be larger in FCal2, and even larger in FCal3. Table 1 shows the relevant electrode dimensions for each of the modules.

	FCal1	FCal2	FCal3
g (mm)	0.269	0.376	0.508
r_0 (mm)	2.491	2.653	3.001
h (%)	2.6	3.4	4.1

Table 1. FCal electrode dimensions and triangle pulse correction

We define $r_0 - g/2$ as the radius of the cylindrical surface of the rod and $r_0 + g/2$ as the radius of the inner surface of the tube. Hence g is the width of the liquid argon gap and r_0 marks the radius of an imaginary cylindrical surface mid way across the gap. Sometimes we will denote positions via the coordinate r but we will find it convenient to refer to locations within the gap relative to this mid way cylindrical surface using the coordinate x. Thus $r = r_0 + x$. Because we assume cylindrical symmetry we won't need the θ and z coordinates.

To make the mathematics a bit more transparent we will define the initial time to be $t = -g/2v_0$ where v_0 is the electron drift speed at $x = 0$. The time when the last electrons have drifted out of the gap will be defined to be $t = +g/2v_0$. It will be apparent later that v_0 is defined in a self-consistent manner.

3. Simplifying assumptions

The calculation presented in this paper is carried out in the same spirit as the one for planar electrodes where the current pulse has the shape of a right triangle. We ignore the current from the very slowly moving positive ions. We assume the external circuit has zero impedance, that is, the potential across the electrode is constant, independent of the current. We take the drifting electron charge to be much less than the charge stored on the electrode surfaces so that space-charge distortion of the electric field [2] can be ignored when calculating the drift speed. We neglect the small magnetic fields set up by the currents and any constant external magnetic fields. We ignore signal propagation delays. And we ignore the transition effect [3], that is, we assume the energy deposit in the liquid argon at the initial instant of time is spatially uniform.

4. Drift speed and initial recombination parameterizations

Due to the cylindrical symmetry of the electrodes, the electric field in the gap is not uniform. It falls off with radius r from the center of the rod. Because the electron drift speed depends on electric field, the speed will vary with the location of the electrons. Figure 2 shows the electron drift speed in liquid argon at T = 88 K versus electric field [4]. At the middle of the gap (at $r = r_0$) the electric field is $\mathcal{E}_0 \equiv V_0 / g = 1.0\ \mathrm{kV/mm}$. An arrow is drawn from the origin to this nominal operating point.

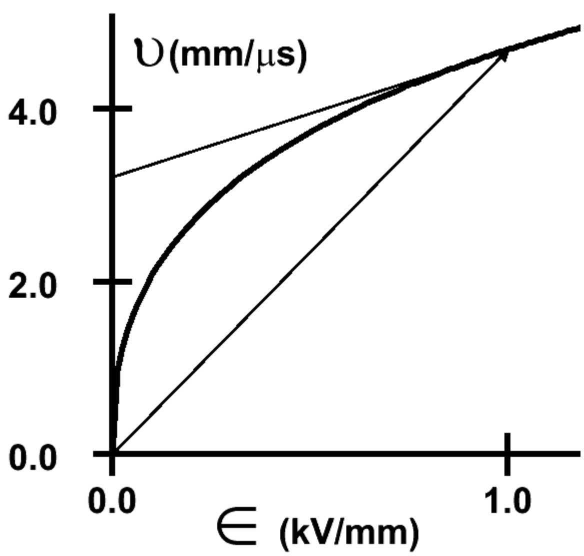

Fig. 2. Electron drift speed v versus relative electric field $\mathcal{E}/\mathcal{E}_0$ in liquid argon at T = 88 K.

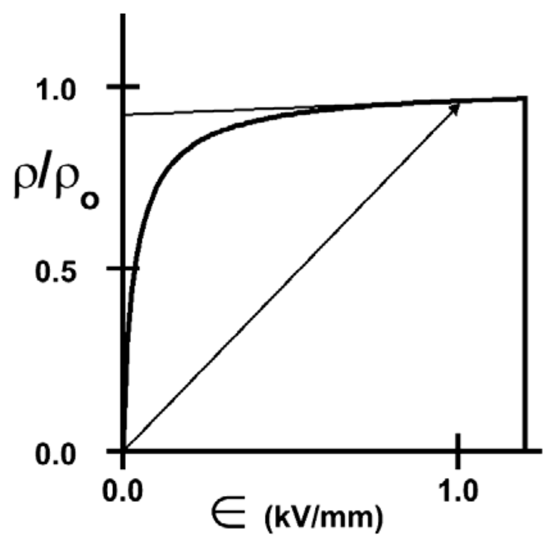

Fig. 3. Relative electron charge density ρ/ρ_0 after initial recombination versus relative electric field $\mathcal{E}/\mathcal{E}_0$.

At infinite electric field the initial ionization density ρ_0 would be uniform throughout the liquid argon in the gap because of our assumption that the argon is uniformly illuminated with radiation at the initial instant of time. But because the electric field is finite, a small fraction of the initial ionization will recombine. Since the electric field varies across the gap, the initial ionization density ρ will vary across the gap. Figure 3 shows one model [5] for the dependence of the ionization on electric field. The vertical axis is the ratio of actual ionization density to the density at infinite electric field. The nominal operating point is again shown via the arrow from the origin.

For small variations of the electric field, the electron drift speed and the initial ionization density will vary to first order via the tangent lines shown in each of figures 2 and 3. We will find it useful to define

$$\alpha \equiv \frac{\mathcal{E}_0}{v_0}\frac{dv}{d\mathcal{E}} = \left(\frac{dv}{v_0}\right)\Big/\left(\frac{d\mathcal{E}}{\mathcal{E}_0}\right) \tag{4.1}$$

and

$$\beta \equiv \frac{V_0}{g v_0}\frac{d\rho}{d\mathcal{E}} = \left(\frac{d\rho}{\rho_0}\right)\Big/\left(\frac{d\mathcal{E}}{\mathcal{E}_0}\right) \tag{4.2}$$

where $dv/d\mathcal{E}$ and $d\rho/d\mathcal{E}$ are evaluated at $\mathcal{E}_0$. Each of these gives the fractional change in v or ρ for a given fractional change in the electric field. Or one can interpret the parameter α or β as the ratio of the slope of the tangent line to the slope of the nominal arrow. For the electron drift speed $\alpha \approx 0.32$ and for the initial ionization density $\beta \approx 0.039$ from reference [5] or $\beta \approx 0.054$ from reference [6].

5. Derivation of the current pulse [7]

The electric field in the liquid argon gap has only a radial component and, in the absence of any free charges, is given by $\mathcal{E}(r) = c/r$ where the constant c is determined by requiring the line integral of the electric field across the gap equal the negative of the electric potential applied across the gap. In our case V_0 is the potential applied to the rod with the tube held at ground.

$$V_0 = \int_{r_0-g/2}^{r_0+g/2} dr\,\mathcal{E}(r) = c\int_{r_0-g/2}^{r_0+g/2}\frac{dr}{r} = c\ln\left(\frac{r_0+g/2}{r_0-g/2}\right)$$

Thus

$$c = \frac{V_0}{\ln\left(\dfrac{r_0+g/2}{r_0-g/2}\right)}$$

To lowest order in g/r_0 and x/r_0,

$$\mathcal{E}(x) = \frac{V_0}{g}\left(1-\frac{x}{r_0}\right) \quad \text{where} \quad -\frac{g}{2} \le x \le +\frac{g}{2} \tag{5.1}$$

Now we imagine a concentric cylindrical surface of radius r_1 $\left(|\, r_1 - r_0 \,| \le g/2\right)$ with uniform surface charge density σ_1. The electric field in the gap is now modified such that

$$\mathcal{E}(r) = \begin{cases} a/r & \text{for } r < r_1 \\ b/r & \text{for } r > r_1 \end{cases}$$

Applying Gauss' Law $\nabla \cdot \mathcal{E} = \dfrac{\partial \mathcal{E}}{\partial r} + \dfrac{\mathcal{E}}{r} = \dfrac{\rho}{\epsilon}$ at that charged surface at r_1 and enforcing the line integral of $\mathcal{E}(r)$ across the gap to equal V_0 we get

$$\mathcal{E}(r) = \frac{c_g}{r} + \theta(r - r_1)\frac{\sigma_1 r_1}{\epsilon r} \quad \text{where } c_g = \frac{V_0 - \dfrac{\sigma_1 r_1}{\epsilon}\ln\left(\dfrac{r_0 + g/2}{r_1}\right)}{\ln\left(\dfrac{r_0 + g/2}{r_0 - g/2}\right)}$$

and where $\theta(r - r_1)$ is the unit step function $\left(\dfrac{d}{dr}\theta(r - r_1) = \delta(r - r_1)\right)$ and $\rho = \sigma_1 \delta(r - r_1)$. At the surface of the inner electrode (at $r = r_0 - g/2$ or equivalently at $x = -g/2$)

$$\mathcal{E}(x = -g/2) = \frac{c_g}{r_0 - g/2}$$

and at the surface of the outer electrode (at $r = r_0 + g/2$, $x = +g/2$)

$$\mathcal{E}(x = +g/2) = \frac{1}{(r_0 + g/2)}\left(c_g + \frac{\sigma_1 r_1}{\epsilon}\right)$$

The surface charge density and charge on the inner electrode (again using Gauss' Law) are

$$\sigma(-g/2) = \epsilon\mathcal{E}(-g/2) = \frac{\epsilon c_g}{r_0 - g/2} \quad \text{and} \quad Q(-g/2) = 2\pi L \epsilon c_g$$

and on the outer electrode

$$\sigma(+g/2) = -\epsilon\mathcal{E}(+g/2) = -\frac{\epsilon c_g + \sigma_0 r_0}{r_0 + g/2} \quad \text{and} \quad Q(+g/2) = -2\pi L\left(\epsilon c_g + \sigma_1 r_1\right)$$

With the charge at r_1 of $Q(r_1) = 2\pi r_1 L \sigma_1$ we see that the total charge $Q = Q(-g/2) + Q(+g/2) + Q(r_1)$ is zero.

Now we allow the charges of surface charge density σ_1 at r_1 (or x_1) to drift in the electric field to $r_1 + dr_1$ (or $x_1 + dx_1$ where $dx_1 = dr_1$). Then the surface charge density σ_1 will decrease as the area of the cylindrical surface increases. If σ_0 is the surface charge density when the radius of the cylindrical surface is at r_0 then $\sigma_1 r_1 = \sigma_0 r_0$ specifies how σ_1 changes as r_1 changes such that the total charge on the surface remains fixed. Our assumption that the external circuit presents zero impedance guarantees that V_0 is fixed as r_1 increases. The change in electric field at the surface of the inner electrode per change in r_1 is

$$\frac{d\mathcal{E}(-g/2)}{dr_1} = \frac{1}{(r_0 - g/2)} \frac{1}{\ln\left(\frac{r_0 + g/2}{r_0 - g/2}\right)} \left(\frac{-\sigma_0 r_0}{\epsilon}\right) \frac{d}{dr_1}\left[\ln(r_0 + g/2) - \ln(r_1)\right] = \frac{1}{(r_0 - g/2)} \frac{1}{\epsilon r_1} \frac{\sigma_0 r_0}{\ln\left(\frac{r_0 + g/2}{r_0 - g/2}\right)}$$

and for the outer electrode

$$\frac{d\mathcal{E}(+g/2)}{dr_1} = \frac{1}{(r_0 + g/2)} \frac{1}{\epsilon r_1} \frac{\sigma_0 r_0}{\ln\left(\frac{r_0 + g/2}{r_0 - g/2}\right)}$$

If the charges making up σ_1 at r_1 drift with speed v_1 then the charges on the electrodes change as

$$\frac{dQ(-g/2)}{dt} = -\frac{dQ(+g/2)}{dt} = 2\pi L \frac{\sigma_1 v_1}{\ln\left(\frac{r_0 + g/2}{r_0 - g/2}\right)} \approx 2\pi r_0 L \frac{\sigma_1 v_1}{g}$$

This produces a current in the external circuit of

$$i(t) = \frac{dQ(-g/2)}{dt} = -\frac{dQ(+g/2)}{dt} \approx 2\pi r_0 L \frac{\sigma_1 v_1}{g} \tag{5.2}$$

Because σ_1 and v_1 change as the charges drift across the gap, $i(t)$ will not be constant (in contrast to the case with planar electrodes). For additional simple cases see Ref. [2], section 6.

Now we assume that at one instant of time the liquid argon in the gap is uniformly illuminated with ionizing radiation. The ionization electrons will not be uniform across the gap because initial recombination depends on the electric field which varies across the gap. These free electrons start to drift across the gap creating an induced current $i(t)$ in the external circuit. The electrons produced nearest the outer electrode (the cathode) are the ones which cross the whole gap and are the last to reach the anode. We define $x_e(t)$ to be the radial coordinate of these last electrons. Then the current pulse (the current as a function of time) is given by

$$i(t) = \frac{2\pi r_0 L}{g} \int_{-g/2}^{x_e(t)} dx \rho(x,t) v(x) \tag{5.3}$$

where $v(x)$ is the electron drift speed in the liquid argon and $\rho(x,t)$ is the density of drifting electrons in the liquid argon. Note that we've simplified the constant outside the integral using the approximation $g/2 \ll r_0$.

In this section we've seen that the electric field is modified by the presence of unbalanced charges in the liquid argon. In many practical applications, these unbalanced charges are so small that, for purposes of calculating the drift speed and initial recombination, we can neglect them and use the electric field of equation (5.1). In order that this be a good approximation we require

$$\left| 2\pi L \int_{-g/2}^{+g/2} dx (r_0 + x) \rho(x,t) \right| \ll \left| Q(-g/2) \right| \approx \left| Q(+g/2) \right|$$

6. Kinematics of the drifting ionization electrons

Here our goal is to find the expressions we will need to put the integrand of equation (5.3) into a form we can easily integrate and interpret. This requires that we find simple expressions for the kinematics of the drifting ionization electrons. We will take all expressions to lowest order in $g/2r_0$, x/r_0, and $v_0 t / r_0$ for $-g/2 < x < +g/2$ and $-g/2v_0 < t < +g/2v_0$ (see the end of section 2 for the definition of time). Starting with equation (5.1) we see that

$$\frac{\mathcal{E}(x) - \mathcal{E}_0}{\mathcal{E}_0} = -\frac{x}{r_0} \tag{6.1}$$

Now applying the result from section 4 we see that

$$\frac{v(x) - v_0}{v_0} = \alpha \left[\frac{\mathcal{E}(x) - \mathcal{E}_0}{\mathcal{E}_0} \right] = -\alpha \frac{x}{r_0} \tag{6.2}$$

which immediately gives

$$v(x) = v_0 \left(1 - \alpha \frac{x}{r_0} \right) \tag{6.3}$$

Now integrating this expression we find

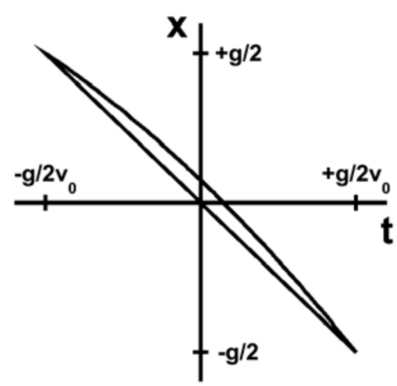

Fig. 4. Path followed by the last ionization electron. Straight line path is to 0th order while the quadratic path is to 1st order.

$$x_e(t) = -v_0 t + \frac{\alpha}{2r_0}\left(\frac{g^2}{4} - v_0^2 t^2\right) \tag{6.4}$$

which describes the motion of the last electrons to leave the gap. For an electron which starts at position x_1 at the initial time $t = -g/2v_0$ its subsequent motion is described by

$$x(t) = (x_1 - v_0 t - g/2) + \frac{\alpha}{2r_0}\left(\frac{g^4}{4} - (x_1 - v_0 t - g/2)^2\right) \tag{6.5}$$

Now we consider an interval Δx_1 at x_1. Electrons at the head of this interval starting at time $t = -g/2v_0$ will separate a bit from those electrons starting at the same time at the tail of this interval such that the interval $\Delta x(t)$ stretches with time as

$$\Delta x(t) = \left[1 + \frac{\alpha}{r_0}(g/2 + v_0 t)\right]\Delta x_1 \tag{6.6}$$

This will cause the density of electrons to tend to decrease with the drift time. Now given $x(t)$ at time t we can let time run backwards to find x_1 at time $t = -g/2v_0$ via

$$x_1 = (x + v_0 t + g/2) + \frac{\alpha}{2r_0}\left[x^2 - (x + v_0 t + g/2)^2\right] \tag{6.7}$$

Figure 4 shows a plot of $x_e(t)$ versus t in zeroth order and in first order. These last electrons to drift out of the gap start at $x = g/2$ next to the cathode at time $t = -g/2v_0$ and hit the anode at $x = -g/2$ at time $t = +g/2v_0$. To zeroth order, the drift speed is constant at v_0. To first order, these electrons start with speed

$$v(x = +g/2) = v_0\left(1 - \frac{\alpha g}{2r_0}\right) \tag{6.8}$$

and end with speed

$$v(x = -g/2) = v_0\left(1 + \frac{\alpha g}{2r_0}\right) \tag{6.9}$$

7. The current pulse

Here we gather all the expressions we need to insert into equation (5.3) and we redefine ρ_0 to be the electron charge density at $x = 0$ at time $t = -g/2v_0$ after initial recombination.

$$i(t) = \frac{2\pi r_0 L}{g}\int_{-g/2}^{x_e(t)} dx\,\rho_0\left(1 + \frac{x_1 - x}{r_0}\right)\left(1 - \frac{\beta x_1}{r_0}\right)\left(1 - \frac{\alpha}{r_0}(g/2 + v_0 t)\right)v_0\left(1 - \frac{\alpha x}{r_0}\right) \tag{7.1}$$

The first bracket corrects the electron charge density for the bunching of the electrons as they drift to smaller r, the "geometry effect". (See equation (6.7) for the definition of x_1.) To first order this correction depends only on time. The second bracket corrects the electron charge density for the dependence of the initial ionization on electric field. (See section 4 for the definition of β.) The third bracket corrects the electron charge density for the stretching of x intervals between drifting electrons as calculated in equation (6.6). The last bracket corrects the electron drift speed for the dependence on electric field as displayed in equation (6.3). The upper limit of integration $x_e(t)$ is given by equation (6.4) and itself includes a first order term depending on α. Keeping only first order terms the expression becomes

$$i(t) = \frac{2\pi r_0 L}{g} \rho_0 v_0 \int_{-g/2}^{x_e(t)} dx \left[1 + \frac{1}{r_0} \left\{ (1-\alpha-\beta) \left(\frac{g}{2} + v_0 t \right) - (\alpha+\beta) x \right\} \right] \tag{7.2}$$

The integral is straightforward and yields

$$i(t) = \frac{2\pi r_0 L \rho_0 v_0}{g} \left(\frac{g}{2} - v_0 t \right) \left[1 + \frac{(1-\beta/2)}{r_0} \left(\frac{g}{2} + v_0 t \right) \right] \tag{7.3}$$

Note that the dependence on α, the variation of speed with electric field, has cancelled out.

8. Interpretation

Equation (7.3), normalized such that the initial current (at time $t = -g/2v_0$) is 1.0, is plotted as the top curve in Figure 5. This is the first order pulse. Progressing down the plot, the next is the zeroth order right triangle current pulse. Shown at the bottom is the first order correction which, when added to the zeroth order pulse gives the first order pulse. Values of $h = g(1-\beta/2)/4r_0$ are shown in Table 1.

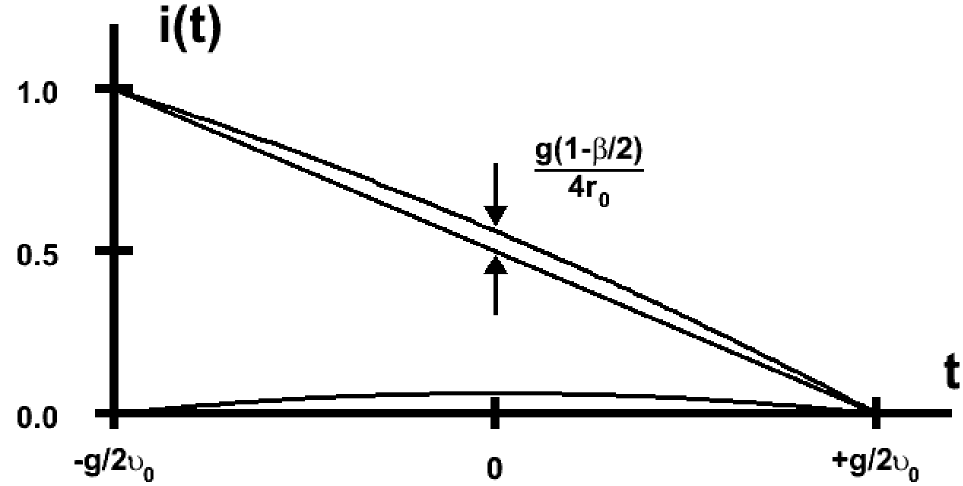

Fig. 5. The normalized zeroth order pulse (right triangle), the first order pulse, and the first order correction are plotted versus time.

The dominant contribution to the first order correction comes from the "geometry effect". Let us start by neglecting the drift speed gradient α (which has canceled out anyway) and the initial recombination variation β for the moment. Then if we divide the initial charge in the gap into bins of equal width dx, the bins at larger x will contain more charge because the volume of these bins is larger. As the electrons in a given bin drift to smaller values of x the electron charge density increases because the volume of the bin shrinks. This increase in density boosts the current pulse relative to the triangle.

Initial recombination is larger at large x where the electric field $\mathcal{E}$ is smaller. This small effect tends to counter the "geometry effect".

The insensitivity of the current pulse shape to the speed gradient α might have been anticipated. As the electrons speed up the electron density drops (relative to the case with $\alpha = 0$). Since the current depends on the product, these effects tend to cancel. Or one could argue that the way we have defined v_0 ensures the α-dependence will cancel. It is straightforward to show that the total charge in the current pulse is independent of α. This total charge constrains the integral under the first order correction. Since the first order correction is quadratic equaling zero at the ends of the time interval, the shape is completely determined.

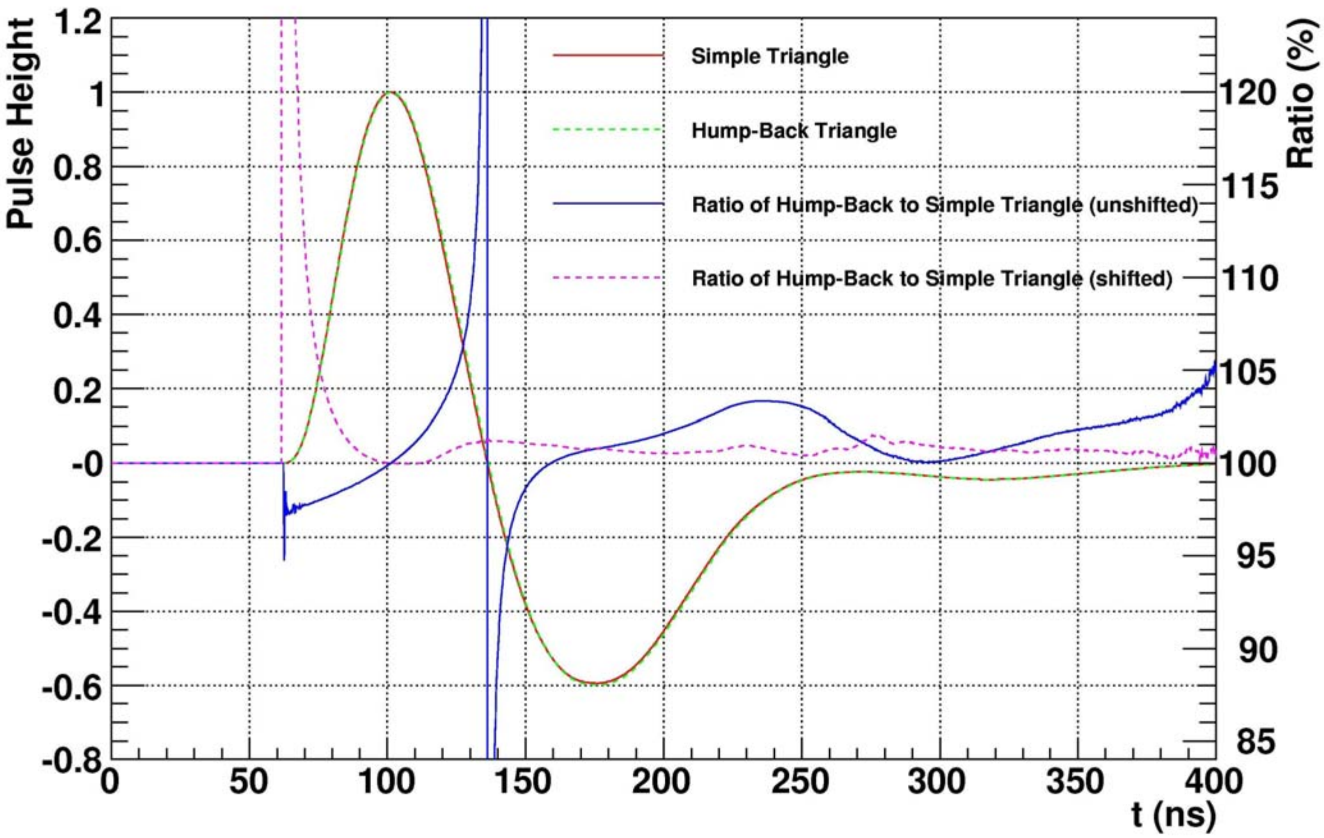

Fig. 6. Normalized pulse shape at the ADC without and with the first order correction. Also shown is the ratio of these pulses before and after time-shifting to line up the peaks and zero crossing points.

We have fed these current pulses through a spice simulation of the FCal electronics chain. At the end of the chain, i.e. at the output to the ADC on the Front End Board (after pedestal subtraction), the shaped pulse for FCal1 is as shown in Figure 6. The solid (red) line shows the 0^{th} order triangle and the dashed (green) line the triangle with the 1^{st} order correction. These two pulses have been normalized so that both peak heights are unity. On this scale it is difficult to see much difference. To highlight the differences we have calculated the ratios of the two pulses. The solid (dark blue) curve shows the ratio of the 1^{st} order pulse to the 0^{th} order pulse. Here one can see that the peaks and the zero crossing points of the two pulses do not line up. The 1^{st} order pulse is pushed to slightly later times relative to the starting point. The spice program reports the shaped pulse relative to the start of the triangle pulse on the electrode. But when we compare spice shapes to measured shapes we will use some other method to line up the pulses. For instance we might line up the peaks.

If we shift the 1^{st} order pulse earlier so that the peaks and zero crossing points line up, then the ratio is as shown by the dashed (purple) curve. The scale for these ratio curves is at the right of the plot. One can see that the shape of the time-shifted 1^{st} order pulse relative to the 0^{th} order pulse is within a few percent except at the beginning where the 1^{st} order pulse is much larger. There are additional, subtle effects which contribute to this excess at the leading edge of the measured pulse shape [8].

9. References

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